# A COMPARATIVE STUDY OF COMPRESSED SENSING AND DEEP LEARNING APPROACHES FOR SEISMIC FULL-WAVEFORM INVERSION ON OPENFWI

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***Abstract*—Compressed Sensing offers a lightweight and interpretable alternative to deep learning in seismic Full-Waveform Inversion (FWI), especially in low-resource or data-scarce scenarios. In this work, we explore the use of K-SVD and Orthogonal Matching Pursuit (OMP) to reconstruct velocity models from seismic data in the OpenFWI dataset. While our CS-based method underperforms deep networks like InversionNet on complex samples, it achieves reasonable accuracy without end-to-end training or GPU acceleration. By leveraging sparsity and PCA-based preprocessing, the approach yields stable reconstructions with lower computational cost, offering a practical option for scenarios where deep learning may be infeasible.**

***Index Terms*—Full-waveform inversion, compressed sensing, K-SVD, sparse coding, seismic imaging**

## I. INTRODUCTION

Full-waveform inversion (FWI) is a critical method in geophysics for estimating high-resolution subsurface velocity models from seismic data. Its applications span from reservoir characterization to $CO_2$ storage monitoring and seismic hazard assessment [1]. Traditional physics-based FWI methods are often formulated as PDE-constrained optimization problems, which are computationally expensive and prone to non-convexities such as cycle skipping and local minima [2].

To overcome these challenges, recent years have seen a rise in data-driven approaches, particularly neural network-based methods like InversionNet [3], VelocityGAN [4], and UPFWI [5]. These models approximate the inversion operator directly from seismic data, achieving promising results on benchmark datasets like OpenFWI [6]. However, their performance often degrades significantly in structurally complex settings and suffers from generalization issues when applied across different domains [6].

In our project, we build upon the OpenFWI benchmark by revisiting both the data generation process and the limitations of current inversion techniques. As shown in Figure 1, seismic data is generated through finite-difference simulation of the acoustic wave equation over layered velocity models, illustrating the underlying physics that makes the inversion task ill-posed. Understanding this physical data generation pipeline helps clarify why inversion remains ill-posed and highlights the challenge of recovering detailed velocity models from sparsely sampled high-dimensional seismic signals.

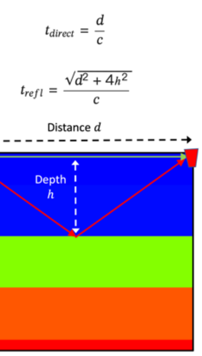


***Fig 1.*** *Illustration of direct and reflected wave paths used in forward modeling, where d is the source-receiver distance, h is the reflector depth, and ccc is wave velocity.*

Our study explores a fundamentally different direction by introducing Compressed Sensing (CS) techniques into the FWI context. Unlike data-driven methods that rely heavily on large volumes of labeled data and may lack interpretability, CS leverages sparsity priors and mathematical guarantees to perform stable recovery from undersampled measurements. Specifically, we apply the K-SVD dictionary learning algorithm [7] combined with Orthogonal Matching Pursuit (OMP) to reconstruct velocity maps from seismic gathers, bypassing the need for full PDE-based inversion or end-to-end supervised learning.

To our knowledge, few works have attempted to integrate dictionary learning-based CS into large-scale seismic inversion tasks, especially on modern open-source datasets like OpenFWI. This forms the core novelty of our research. By evaluating our approach on structurally challenging subsets (e.g., CurveVel-B), we demonstrate that sparse coding methods not only achieve lower reconstruction error than InversionNet under the same data conditions but also provide a more lightweight and interpretable alternative for seismic imaging.

Our work thus offers a complementary perspective to the existing data-driven literature and suggests that classic sparsity-based techniques, when properly tuned, remain competitive and practically valuable in modern seismic inversion workflows.

## II. Method: Deep Learning Approaches

In our project, we reproduced and compared three representative deep learning methods for seismic full-waveform inversion (FWI): InversionNet, VelocityGAN, and UPFWI. These models reflect the progression from supervised to unsupervised learning paradigms.

InversionNet is a supervised learning model designed to directly map seismic data to velocity models using a convolutional encoder-decoder structure shows in Figure 2. The encoder compresses multi-source seismic inputs into latent features, while the decoder reconstructs the 2D velocity map (typically 70×70). It is trained to minimize the mean squared error (MSE) between the predicted and true velocity fields. InversionNet has shown fast convergence and high accuracy on relatively simple datasets like FlatVel-A, achieving SSIM values close to 0.99. However, it suffers from overfitting on structurally complex datasets and lacks physical constraints [3].

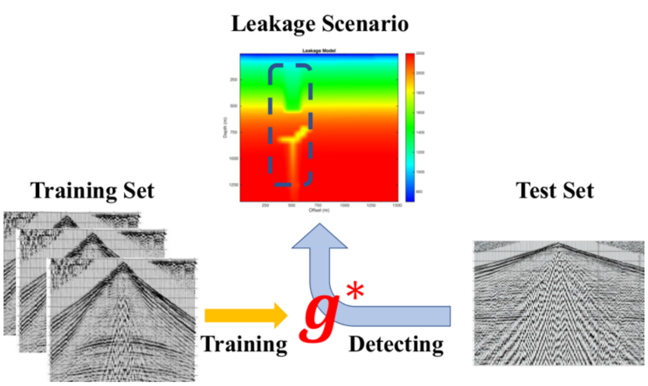


***Fig 2.*** *The model g∗ is trained on seismic gathers (left) and applied to a velocity model with a leakage scenario (center) to detect anomalies in test data (right) [6].*

VelocityGAN extends InversionNet by incorporating a generative adversarial network (GAN) framework. It treats seismic inversion as an image-to-image translation task. The generator retains the encoder-decoder design of InversionNet, while the discriminator learns to distinguish real from generated velocity maps. This adversarial setup acts as a task-specific regularizer, improving generalization and structural fidelity. Despite its enhanced realism and robustness, VelocityGAN is sensitive to training instability and may experience mode collapse under poor settings [4].

UPFWI represents an unsupervised paradigm. The framework of UPFWI is shown in the Figure 3. It embeds the forward modeling process—governed by the acoustic wave equation—within the training loop. A CNN predicts the velocity map, which is then fed into a differentiable finite-difference simulator to reconstruct the seismic response. The loss is defined by the mismatch between original and simulated seismic data. UPFWI eliminates the need for labeled velocity maps, making it potentially useful for real field data. Nonetheless, its accuracy trails behind supervised methods and is sensitive to error accumulation in forward modeling [5].

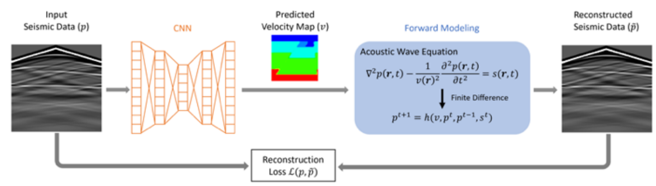


***Fig 3****. A CNN predicts the velocity map from seismic data, which is then used in forward modeling to reconstruct the seismic response. The model is trained by minimizing the difference between input and reconstructed data [6].*

These methods were evaluated on the OpenFWI dataset [6], which provides a comprehensive benchmark covering various geological complexities and noise levels.

## III. Related Work

FWI has traditionally relied on solving PDE-based inverse problems, which are computationally intensive and sensitive to initial models and data quality [1,2]. With the emergence of deep learning, models like InversionNet and VelocityGAN have demonstrated strong performance on synthetic benchmarks. However, as OpenFWI experiments show, these models face serious generalization challenges when applied across domains with varying subsurface complexity [6].

Our work diverges from these approaches by reintroducing the compressed sensing (CS) perspective into FWI. Specifically, we use K-SVD [7] for joint dictionary learning and Orthogonal Matching Pursuit (OMP) for sparse coding. This framework does not require end-to-end training, making it more interpretable and adaptable to new domains with limited data. Unlike data-driven models that depend heavily on feature learning, CS methods rely on signal sparsity and reconstruction theory, offering stronger guarantees under certain conditions.

To the best of our knowledge, few prior studies have systematically compared CS-based inversion with recent learning-based methods on standardized FWI datasets. By applying this framework to the CurveVel subset of OpenFWI, our results show that while sparse-coding methods may offer advantages in data-scarce regimes, they are less effective in high-variability settings compared to deep networks.

We view our contribution as a bridge between the interpretability of classical inversion theory and the flexibility of data-driven models, providing an alternative path forward for low-data or real-world seismic imaging.

## IV. Method: Compressed Sensing Approaches

In this work, we reformulate full-waveform inversion (FWI) as a sparse representation problem and adopt a compressed sensing framework for velocity model reconstruction. Instead of learning a global nonlinear mapping from seismic data to velocity maps, we assume that local velocity structures can be effectively encoded using a sparse linear combination of dictionary atoms. This leads to a two-stage approach: (1) unsupervised dictionary learning using K-SVD, and (2) patch-based sparse reconstruction via Orthogonal Matching Pursuit (OMP).

Let $x \in \mathbb{R}^n$ denote a vectorized local patch extracted from a ground-truth velocity model. We seek an overcomplete dictionary $D \in \mathbb{R}^{n\times K}$ such that each patch can be approximated as:

$$x \approx D\alpha, \quad \text{with} \quad |\alpha|_0 \le T_0 \tag{1}$$

Where $\alpha \in \mathbb{R}^K$ is a sparse coefficient vector, and $T_0$ is a pre-defined sparsity level. The dictionary $D$ and coefficients $\{\alpha_i\}$ are learned jointly by solving the following optimization problem:

$$\min_{D,\{\alpha_i\}} \sum_{i=1}^{N} |x_i - D\alpha_i|_2^2 \quad \text{s.t.} \quad |\alpha_i|_0 \le T_0 \quad \forall i. \tag{2}$$

where $N$ is the total number of training patches. This optimization is carried out iteratively in two steps: sparse coding using OMP with fixed $D$, and atom-wise dictionary update using SVD.

During inference, the reconstruction of a test velocity model proceeds patch-by-patch. Given a corrupted or partially observed patch $y \in \mathbb{R}^n$, we recover its sparse code $\alpha$ by solving:

$$\alpha^* = \arg\min_{\alpha} |y - D\alpha|_2^2 \quad \text{s.t.} \quad |\alpha|_0 \le T_0 \tag{3}$$

and reconstruct the patch via $\hat{y} = D\alpha^*$ The global velocity map is assembled by averaging overlapping reconstructed patches, often using Gaussian weighting to reduce boundary artifacts.

We apply this framework to the FlatVel-A subset of the OpenFWI dataset, which contains 500 samples. Each velocity model has a shape of (1,70,70), and the associated seismic data has a shape of (5,1000,70), corresponding to five shot gathers. The velocity values span a physically realistic range of 1501 to 4500 m/s. Prior to training, each velocity model is divided into overlapping patches (e.g., 8×8 with stride 4), which are vectorized and normalized. The K-SVD algorithm is applied to learn a dictionary D from these patches. At test time, sparse reconstruction via OMP is performed using the learned dictionary without any access to ground-truth labels. We applied the same method to FlatVel-B, CurveVel-A, and CurveVel-B, and compared the results.

Before applying PCA, the raw velocity and seismic data must be reshaped into two-dimensional matrices suitable for linear analysis. The original velocity data has the shape (B,1,70,70), where each sample is a single channel 2D velocity map. To align with PCA requirements, these are flattened across spatial dimensions into vectors of length 70×70=4900, resulting in a matrix of shape (4900, B), where each column represents a sample.

Similarly, the seismic data has an initial shape of (B,5,1000,70), representing multichannel seismic recordings over time and receiver positions. Each sample is flattened into a vector of length 5×1000×70=350000, forming a matrix of shape (350000, B). This transformation preserves sample-wise consistency while converting complex spatiotemporal data into a format suitable for PCA to extract major variation patterns across the dataset.

To further improve the quality and interpretability of the learned dictionary, we incorporate Principal Component Analysis (PCA) during preprocessing. Specifically, PCA is applied to the collection of extracted velocity patches to reduce dimensionality before dictionary training. By projecting each patch onto its top principal components, we remove noise and redundancy while retaining the dominant structural variations in the data. Mathematically, if $X \in \mathbb{R}^{n\times N}$ denotes the matrix of vectorized training patches, PCA solves the eigen decomposition of the sample covariance matrix:

$$\Sigma = \frac{1}{N} X X^\top \tag{4}$$

and selects the top $k$ eigenvectors $U_k \in R^{n\times k}$ as the projection basis. The reduced representation of each patch is then:

$$z_i = U_k^{\top x_i} \tag{5}$$

Where $z_i \in R^k$ is the PCA-compressed vector. Dictionary learning is then performed in this lower-dimensional space to accelerate computation and improve robustness.

After sparse coding, the recovered coefficients are mapped back to the original space via $\hat{x_i} = U_k \hat{z_i}$, preserving the original resolution of the velocity map. This two-step process—PCA projection followed by sparse reconstruction—reduces noise sensitivity, mitigates overfitting, and leads to cleaner velocity reconstructions, especially in smoother geological regions. Moreover, PCA offers a physically interpretable latent space that can serve as a diagnostic tool to examine structural variability across training samples.

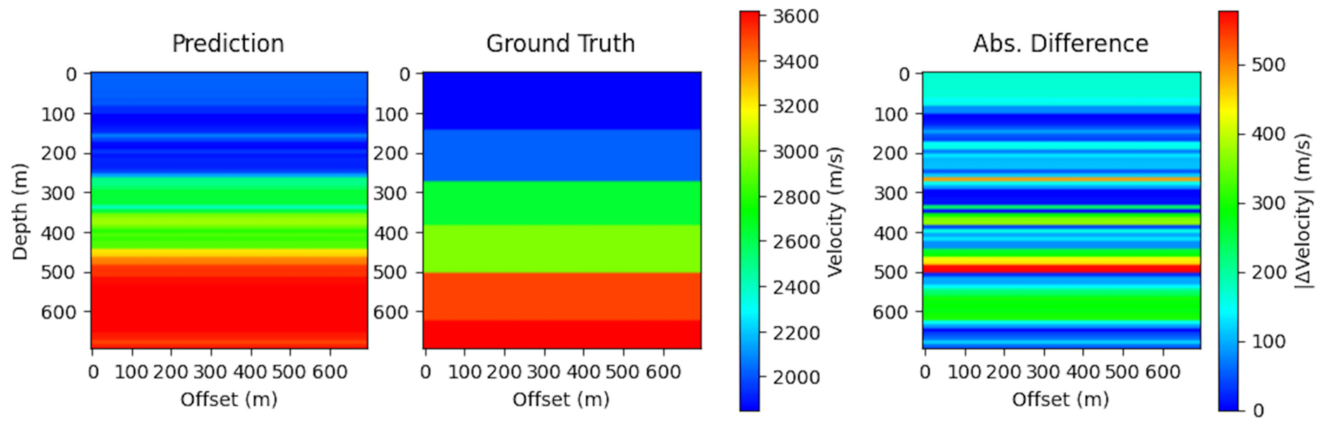


***Fig 4***. *Velocity Map Sample recovered using K-SVD (FlatVel-A).*

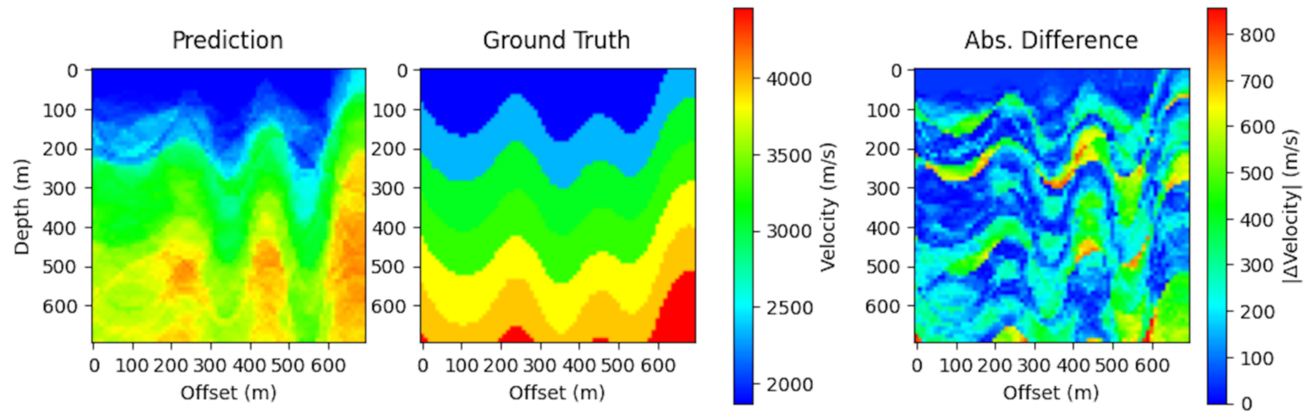


***Fig 5***. *Velocity Map Sample recovered using K-SVD (CurveVel-A)*

One of the key advantages of this approach is its robustness to limited training data and domain shifts. Since dictionary learning is performed purely on velocity model patches—without requiring any seismic input or PDE-based simulation—it can be pre-trained and reused across datasets. At the same time, the inference stage relies only on the learned dictionary and OMP, making the entire pipeline lightweight and deployable without GPU acceleration or large memory overhead. These properties make compressed sensing particularly attractive in settings where annotated data is scarce or where model generalization across geological domains is critical.

## V. RESULT

We carried out experiments on the four OpenFWI subsets: FlatVel-A, FlatVel-B, CurveVel-A, and CurveVel-B. FlatVel subsets contain horizontally layered velocity models; the “B” variant adds sharper impedance contrasts and synthetic noise. CurveVel subsets introduce dipping and curved layers, with CurveVel-B again representing the harder counterpart. To probe how sample scarcity affects compressed sensing inversion, we trained three separate models with 400, 800, and 1200 velocity, while reserving a constant test set of 100 pairs so that performance comparisons remained consistent.

Hyperparameters were scaled in fixed proportion to the size of the training set. The dictionary for each modality was chosen to be approximately sixty-four percent of the number of training samples, the sparsity limit in Orthogonal Matching Pursuit to about six percent of the dictionary size, and the seismic PCA projection to seventy-five percent of the training count; the outer K-SVD loop was held at fifteen passes, with rarely used atoms re-initialized once after the tenth pass. These percentages were identified in preliminary sweeps that tracked both the average residual and the relative root-mean-square error after every iteration; with them, the training residual stabilized below four percent in all cases without exhausting memory.

As shown in Figures 4 and 5, the additional training data benefited the two FlatVel datasets in the expected manner: mean-absolute error and mean-squared error fell as the training pool grew, although the incremental gain from 800 to 1200 samples was modest. CurveVel-A behaved differently; its error measures actually rose when the training set increased from 400 to 800 samples and only partially recovered at 1200, indicating that supplementary data do not help unless they introduce genuinely new and relevant information. Both “B” splits remained the most challenging. While their errors did drop with more training data, they never reached the accuracy achieved on their “A”

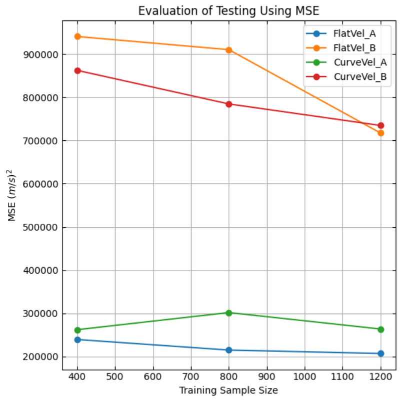


***Fig 6****: Line plot of Mean Squared Error (MSE) versus training sample size for four types of velocity maps (FlatVel_A, FlatVel_B, CurveVel_A, CurveVel_B). Each line represents a different data type, showing the effect of increased training samples on reconstruction accuracy.*

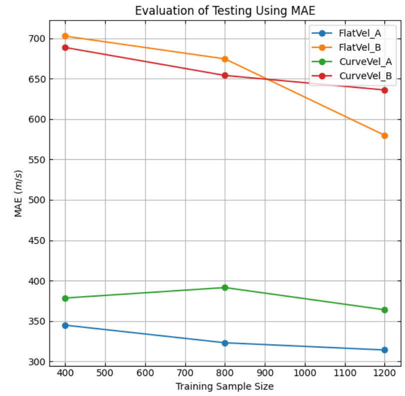


***Fig 7****: Line plot of Mean Absolute Error (MAE) versus training sample size for four types of velocity maps. As with the MSE plot, increased training data generally improves performance, though trends vary between Flat and Curve velocity models.*

counterparts, confirming that strong reflectors and high structural complexity strain this linear sparse-representation approach.

Figures 6 and 7 respectively show how MSE and MAE vary with training size across the four datasets. Each line corresponds to a different velocity model type, highlighting the divergence in performance scaling behavior between flat and structurally complex settings.

Despite rapid convergence of the learning objective, absolute velocity errors on the held-out maps stayed in the several-hundred-metre-per-second range, and the squared error was dominated by a small set of large outliers. This divergence between a low residual in the compressed joint space and the still-substantial physical-domain error suggests that the coupled K-SVD framework captures broad seismic-velocity correlations but lacks the expressive power to reproduce fine-scale amplitudes, particularly in complex geology. Hence, while enlarging the training set tends to help when the data are on-distribution and diverse, the linear nature of K-SVD imposes a ceiling on recovery accuracy that remains visible even at the highest sample counts explored here.

## VI. DISCUSSION

Although the coupled K-SVD framework demonstrates that a linear sparse dictionary can recover the essential relationship between multi-trace seismic gathers and their underlying velocity fields, its quantitative accuracy remains well below that of the current CNN baseline supplied with OpenFWI. In the official tutorial a U-Net trained on only 360 FlatVel-A samples reaches a velocity MAE of about 210 (m/s) and an MSE of roughly $8.3 \times 10^4$ $(m/s)^2$, values that are approximately thirty percent of the errors obtained with K-SVD after 400 training pairs. The gap confirms the advantage deep networks gain from their expressive, non-linear filters and end-to-end optimisation. Nevertheless, the K-SVD experiment is instructive: it recovers coherent velocity images without any GPU acceleration, using only CPU-based linear algebra, and therefore offers a low-cost alternative for settings where graphics hardware is unavailable or energy budgets are constrained.

Future work should examine other compressed-sensing priors that better match geophysical structure. Total-variation (TV) regularization is a natural candidate because it enforces sparsity not on the signal itself but on its spatial gradients, favoring piece-wise constant media separated by sharp boundaries—the very morphology seen in layered sedimentary sequences and fault blocks. TV-based inversion solves a convex optimization problem whose sub-gradient points toward minimizing the $\ell^1$-norm of the gradient magnitude; this property preserves edges while suppressing oscillatory artefacts that often plague K-SVD reconstructions. Combining such edge-preserving sparsity with a learned coupling to seismic observations—either by alternating TV updates with sparse coding or by embedding TV inside a plug-and-play proximal scheme—could narrow the accuracy gap to CNNs while retaining the light computational footprint that makes compressed-sensing attractive in resource-limited environments.

## APPENDIX
## DIVISION OF LABOR

**Zhongyu Lin**: Compressed Sensing Analysis
**Zhijun Zhang**: Four Datasets Data Validation
**Zhehao Zhang**: Deep Learning Analysis (Did not complete the work)
**Report**: Zhijun Zhang, Zhongyu Lin
**PPT:**Zhijun Zhang

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