Pairs Trading - Cointegration

Motivation:

- Security A - (Something Correlated to it) is more mean-reverting. Similar securties should trade similarily. Hence, any discrepancies between the correlated pairs should converge
- When there is a discrepancy, we form a long-short spread trade and bet on the discrepancy converging
- Pairs trading is one of the most popular statistical arbitrage strategies in traditional markets. We want to test the performance of this strategy in the cryptocurrency market, as it is still relatively new and should be fertile grounds for finding market inefficiencies
- We will use cointegration to find similar pairs as it is a robust statistical approach for identifying long-term equilibrium relationships between assets

In [1]:
```
from datetime import datetime
import pandas as pd
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
from matplotlib.pyplot import figure

import statsmodels.api as sm
from statsmodels.tsa.stattools import adfuller

import warnings
warnings.filterwarnings("ignore", category=FutureWarning)

from joblib import Parallel, delayed
from itertools import combinations
```

Step 0: Data Cleaning

We selected the top 1,000 currencies sorted by volume from CoinGecko. Then, we filtered out coins with fewer than 90% of daily price data points between 2018 and 2024.

In [2]:
```
from_pickle = pd.read_pickle('Data/CoinGecko_px_vol_1D.pkl')
```

In [3]:
```
price_columns = [col for col in from_pickle.columns if 'price' in col]
crypto_px = from_pickle[price_columns]
crypto_px.columns = crypto_px.columns.droplevel(1)
```

In [4]:
```
crypto_px = crypto_px.loc['2018-01-01':]
crypto_px = crypto_px.drop(columns=['TUSD', 'DAI', 'WBTC', 'WETH', 'USDC', 'BSV']
```

In [5]:
```python
# Calculate the total number of data points
total_data_points = len(crypto_px)

# Calculate the number of non-NA/null entries for each coin
non_null_counts = crypto_px.notnull().sum()

# Calculate the threshold for 90% of the data points
threshold = 0.90 * total_data_points

# Filter out columns with fewer than 90% of data points
crypto_px = crypto_px.loc[:, non_null_counts >= threshold]

crypto_px
```

Out[5]:

Date	BTC	XRP	BNB	DOGE	ADA	TRX	LTC	LINK	BCH
2018-01-01	14093.606831	2.310120	8.828099	0.009091	0.747140	0.051654	230.462120	0.751033	2426.970077
2018-01-02	15321.932852	2.455290	9.090393	0.009335	0.807430	0.080893	255.048185	0.689388	2627.026940
2018-01-03	15583.885538	3.125710	9.886323	0.009592	1.075401	0.098107	248.042194	0.704623	2630.511811
2018-01-04	15976.365194	3.220050	9.675758	0.010098	1.179347	0.218139	244.834372	1.036826	2458.894372
2018-01-05	18336.922980	2.931380	16.488523	0.013841	1.077821	0.231673	254.138525	0.996575	2551.321685
...	...	...	...	...	...	...	...	...	...
2024-08-12	58804.234500	0.552884	503.472306	0.100570	0.328140	0.127716	59.691360	10.012537	330.440521
2024-08-13	59350.074333	0.568612	518.752217	0.107647	0.338874	0.126661	61.414267	10.561648	354.155288
2024-08-14	60601.223178	0.576440	523.553455	0.106469	0.340317	0.128873	63.411783	10.577495	352.234629
2024-08-15	58739.193822	0.568488	523.842262	0.102529	0.335251	0.130504	63.906408	10.396984	338.005714
2024-08-16	57624.116929	0.561137	519.989791	0.100275	0.325393	0.130136	65.236601	10.171003	334.436579

2568 rows × 86 columns

We have 86 coins and will select pairs from among them.

In [6]:
```python
# Daily returns
coins_ret = crypto_px / crypto_px.shift() - 1
```

In [7]:
```python
end_of_insample = pd.Timestamp('2021-12-31')

# in-sample price
in_sample_px = crypto_px.loc[:end_of_insample]
```

In [8]:
```python
def compute_turnover(port):
    to = (port.fillna(0)-port.shift().fillna(0)).abs().sum(1)
    return to
```

In [9]:
```
def compute_sharpe_ratio(rets):
    mean_rets = rets.mean()*252
    vol = rets.std()*np.sqrt(252)
    sharpe_ratio = mean_rets / vol
    return sharpe_ratio
```

In [10]:
```
def compute_stats(rets):
    stats={}
    stats['avg'] = rets.mean()*252
    stats['vol'] = rets.std()*np.sqrt(252)
    stats['sharpe'] = stats['avg']/stats['vol']
    stats['hit_rate'] = rets[rets>0].count() / rets.count()
    stats = pd.DataFrame(stats)
    return stats
```

In [11]:
```
def drawdown(px):
    return (px / (px.expanding(min_periods=1).max()) - 1)
```

In [12]:
```
def duration(px):
    peak = px.expanding(min_periods=1).max()
    res = pd.DataFrame(index=px.index,columns=px.columns)
    for col in px.columns:
        for dt in px.index:
            if px.loc[dt,col] >= peak.loc[dt,col]:
                 res.loc[dt,col] = 0
            else:
                res.loc[dt,col] = res.loc[:dt,col].iloc[-2] + 1
    return res
```

In [13]:
```
def plot_with_signals(log_px_i, log_px_j, z_score, exit_threshold=0.5):
    # Define entry signals (when z_score crosses ±1)
    buy_signals_i = (z_score < -1)
    sell_signals_i = (z_score > 1)
    buy_signals_j = (z_score > 1)
    sell_signals_j = (z_score < -1)

    # Define exit signals (when z_score moves back within exit_threshold)
    exit_signals_i = (z_score > -exit_threshold) & (z_score < exit_threshold)
    exit_signals_j = (z_score > -exit_threshold) & (z_score < exit_threshold)

    plt.figure(figsize=(14, 8))

    # Plot log prices
    plt.plot(log_px_i.index, log_px_i, label='Log Price of Asset i', color='blue'
    plt.plot(log_px_j.index, log_px_j, label='Log Price of Asset j', color='orang

    # Plot z-score
    plt.plot(z_score.index, z_score, label='Z-score', color='grey', linestyle='--

    # Plot buy and sell signals for asset i
    plt.plot(log_px_i.index[buy_signals_i], log_px_i[buy_signals_i], '^', color='
    plt.plot(log_px_i.index[sell_signals_i], log_px_i[sell_signals_i], 'v', color

    # Plot buy and sell signals for asset j
    plt.plot(log_px_j.index[buy_signals_j], log_px_j[buy_signals_j], '^', color='
    plt.plot(log_px_j.index[sell_signals_j], log_px_j[sell_signals_j], 'v', color

    # Add labels and legend
    plt.title('Log Prices and Z-score with Entry and Exit Signals')
    plt.xlabel('Date')
    plt.ylabel('Value')
    plt.legend(loc='best')
    plt.grid(True)
    plt.tight_layout()
    plt.show()
```

Step 1: Pairs Selection

The formation period is from January 1, 2018, to December 31, 2021. We use daily in-sample frequency to select pairs through the following steps:

1. **OLS Regression**: Perform Ordinary Least Squares (OLS) regression on the log prices of two securities to obtain the residuals. $\log(p_{i,t}) = \alpha + \beta \log(p_{j,t}) + \epsilon_t$
2. **Stationarity Test**: Apply the Augmented Dickey-Fuller (ADF) test to the residuals to check for stationarity. A p-value < 0.05 suggests that the residuals are stationary.

Selection Criteria

Select the top pair with the most negative test statistic from the ADF test for each coin, ensuring all selected pairs have p-values less than 0.05.

In [14]:
```python
def adf_for_pair(symbol_i, symbol_j, in_sample_px):
    """
    Perform the ADF test on the residuals of the OLS regression between two asset

    Parameters:
    symbol_i (str): The first asset's symbol.
    symbol_j (str): The second asset's symbol.
    in_sample_px (DataFrame): The DataFrame containing in-sample price data for t

    Returns:
    tuple: A tuple containing the pair (symbol_i, symbol_j) and a tuple of (p_val
    """
    # Convert raw price to log price
    in_sample_log_px = np.log(in_sample_px)

    # Handle missing data
    X = in_sample_log_px[symbol_i].fillna(0).values
    Y = in_sample_log_px[symbol_j].fillna(0).values

    # OLS regression
    model = sm.OLS(Y, sm.add_constant(X)).fit()
    alpha = model.params[0]
    beta = model.params[1]
    residuals = Y - beta * X - alpha

    # ADF test on residuals
    adf_result = adfuller(residuals)
    p_value = adf_result[1]
    test_statistic = adf_result[0]

    return (symbol_i, symbol_j), (p_value, test_statistic)
```

In [15]:
```python
def cointegration(in_sample_px, n_jobs=-1):
    """
    Compute ADF test results for all possible pairs of assets.

    Parameters:
    in_sample_px (DataFrame): DataFrame containing in-sample prices of assets.
    n_jobs (int): Number of parallel jobs to run.

    Returns:
    DataFrame: DataFrame containing ADF test results for all pairs.
    """
    symbols = in_sample_px.columns.tolist()
    pairs = list(combinations(symbols, 2))

    # Parallel computation of ADF tests for all pairs
    adf_results = Parallel(n_jobs=n_jobs)(
        delayed(adf_for_pair)(symbol_i, symbol_j, in_sample_px) for symbol_i, sym
    )

    # Create a DataFrame with ADF results
    pairs, results = zip(*adf_results)
    adf_df = pd.DataFrame(results, columns=['p_value', 'test_statistic'], index=p
    adf_df = adf_df.sort_values(by='test_statistic')

    return adf_df
```

In [16]:
```
def pair_selection(adf_df, significance_level=0.05, top_n=1):
    """
    Select the top `n` cointegrated pairs for each individual coin.

    Parameters:
    adf_df (DataFrame): DataFrame containing ADF test results for all pairs.
    significance_level (float): The significance level threshold for p-values to
    top_n (int): The number of top pairs to select for each coin.

    Returns:
    dict: A dictionary where each key is a coin and each value is a DataFrame
          containing the top `n` pairs with that coin.
    """
    # Filter pairs with p-values less than the significance level
    filtered_df = adf_df[adf_df['p_value'] < significance_level]

    # Initialize dictionary to store top pairs for each coin
    top_pairs_per_coin = {}

    # Iterate over all unique coins in the filtered pairs
    all_coins = set(sum([list(pair) for pair in filtered_df.index], []))
    for coin in all_coins:
        # Filter pairs where the coin is involved
        coin_pairs = filtered_df.loc[[(coin in pair) for pair in filtered_df.inde

        # Sort the pairs by the ADF test statistic (smaller test statistic is bet
        sorted_pairs = coin_pairs.sort_values(by='test_statistic')

        # Select the top `n` pairs for this coin
        top_pairs = sorted_pairs.head(top_n)

        # Store the result
        top_pairs_per_coin[coin] = top_pairs

    return top_pairs_per_coin
```

In [17]:
```
def print_top_cointegrated_pairs(selected_cointegrated_pairs_per_coin):
   # Sort the coins alphabetically
    sorted_coins = sorted(selected_cointegrated_pairs_per_coin.keys())

    # Display the top cointegrated pairs for each coin
    for coin in sorted_coins:
        df = selected_cointegrated_pairs_per_coin[coin]
        top_coins = [pair[1] if pair[0] == coin else pair[0] for pair in df.index
        print(f"Top cointegrated coins with {coin}: {top_coins}")
```

In [18]:

```
# Compute ADF test results for all pairs
adf_df = cointegration(in_sample_px)
print(adf_df)

# Select the top n cointegrated pairs for each coin
selected_cointegrated_pairs_per_coin = pair_selection(adf_df, significance_level=
print_top_cointegrated_pairs(selected_cointegrated_pairs_per_coin)
```

```
                  p_value  test_statistic
STMX  ARDR  6.265094e-09       -6.614248
      IOST  1.745421e-07       -5.991039
TRX   ARDR  3.525891e-07       -5.854503
STEEM CTXC  3.811222e-07       -5.839269
ONT   NULS  1.392020e-06       -5.581681
...                  ...             ...
AMB   REQ   9.515168e-01       -0.078725
NEO   GNO   9.560108e-01       -0.029644
BCH   GNO   9.844215e-01        0.485368
AMB   SYS   9.860765e-01        0.541327
DASH  GNO   9.908005e-01        0.750882

[3655 rows x 2 columns]
Top cointegrated coins with ADA: ['SBD']
Top cointegrated coins with ADX: ['XLM']
Top cointegrated coins with AMB: ['PIVX']
Top cointegrated coins with ANT: ['UQC']
Top cointegrated coins with ARDR: ['STMX']
Top cointegrated coins with ARK: ['STRAX']
Top cointegrated coins with BAT: ['DOGE']
Top cointegrated coins with BCH: ['SWFTC']
Top cointegrated coins with BLZ: ['STRAX']
Top cointegrated coins with BNB: ['NEO']
Top cointegrated coins with BORG: ['REN']
Top cointegrated coins with BTC: ['SWFTC']
Top cointegrated coins with BTG: ['BNB']
Top cointegrated coins with CTXC: ['STEEM']
Top cointegrated coins with CVC: ['XRP']
Top cointegrated coins with DASH: ['SWFTC']
Top cointegrated coins with DCR: ['SWFTC']
Top cointegrated coins with DENT: ['STRAX']
Top cointegrated coins with DGB: ['ZIL']
Top cointegrated coins with DOGE: ['BAT']
Top cointegrated coins with ELF: ['STEEM']
Top cointegrated coins with ENJ: ['VIC']
Top cointegrated coins with EOS: ['TRX']
Top cointegrated coins with ETC: ['BTG']
Top cointegrated coins with FUN: ['XVG']
Top cointegrated coins with GAS: ['VGX']
Top cointegrated coins with GFT: ['CTXC']
Top cointegrated coins with GLM: ['ICX']
Top cointegrated coins with GNO: ['BNB']
Top cointegrated coins with ICX: ['GLM']
Top cointegrated coins with IDEX: ['REQ']
Top cointegrated coins with IOST: ['STMX']
Top cointegrated coins with IOTA: ['ZRX']
Top cointegrated coins with LINK: ['NMR']
Top cointegrated coins with LRC: ['TRAC']
Top cointegrated coins with LSK: ['ONT']
Top cointegrated coins with LTC: ['SWFTC']
Top cointegrated coins with MANA: ['STORJ']
Top cointegrated coins with MDT: ['TRX']
Top cointegrated coins with MED: ['XDC']
Top cointegrated coins with MKR: ['SWFTC']
Top cointegrated coins with MLN: ['BNB']
Top cointegrated coins with MTL: ['PRO']
Top cointegrated coins with NEO: ['BNB']
Top cointegrated coins with NMR: ['LINK']
Top cointegrated coins with NULS: ['ONT']
Top cointegrated coins with OMG: ['MANA']
Top cointegrated coins with ONT: ['NULS']
Top cointegrated coins with PIVX: ['ONT']
Top cointegrated coins with POWR: ['STMX']
Top cointegrated coins with PRO: ['GLM']
Top cointegrated coins with QTUM: ['MANA']
Top cointegrated coins with REN: ['BTG']
Top cointegrated coins with REQ: ['POWR']
Top cointegrated coins with RLC: ['VIC']
```

```
Top cointegrated coins with RVN: ['TRX']
Top cointegrated coins with SBD: ['VIC']
Top cointegrated coins with SNT: ['DGB']
Top cointegrated coins with SNX: ['QTUM']
Top cointegrated coins with STEEM: ['CTXC']
Top cointegrated coins with STMX: ['ARDR']
Top cointegrated coins with STORJ: ['MANA']
Top cointegrated coins with STRAX: ['CTXC']
Top cointegrated coins with SWFTC: ['BTC']
Top cointegrated coins with SYS: ['REQ']
Top cointegrated coins with THETA: ['SBD']
Top cointegrated coins with TRAC: ['LRC']
Top cointegrated coins with TRX: ['ARDR']
Top cointegrated coins with UQC: ['ANT']
Top cointegrated coins with UTK: ['ADX']
Top cointegrated coins with VGX: ['GAS']
Top cointegrated coins with VIB: ['XVG']
Top cointegrated coins with VIC: ['SBD']
Top cointegrated coins with WAVES: ['BTG']
Top cointegrated coins with WAXP: ['TRX']
Top cointegrated coins with XDC: ['MED']
Top cointegrated coins with XEM: ['DGB']
Top cointegrated coins with XLM: ['ZIL']
Top cointegrated coins with XMR: ['SWFTC']
Top cointegrated coins with XNO: ['VGX']
Top cointegrated coins with XRP: ['CVC']
Top cointegrated coins with XVG: ['IOST']
Top cointegrated coins with ZEC: ['SWFTC']
Top cointegrated coins with ZEN: ['BTG']
Top cointegrated coins with ZIL: ['XLM']
Top cointegrated coins with ZRX: ['IOTA']
```

In [19]:
```python
final_pairs = []

for i, (coin, df) in enumerate(selected_cointegrated_pairs_per_coin.items()):
    # Extract the index (pairs) from the DataFrame and add to final_pairs
    final_pairs.extend(df.index.tolist())

# Remove duplicates (since a pair might appear in more than one coin's top pairs)
final_pairs = list(set(final_pairs))

# Sort the final list of pairs alphabetically
final_pairs.sort()

print(f"Selected pairs: {final_pairs}")
print(f"Total number of pairs: {len(final_pairs)}")
```

```
Selected pairs: [('ADA', 'SBD'), ('ADX', 'UTK'), ('AMB', 'PIVX'), ('ANT', 'UQ
C'), ('ARK', 'STRAX'), ('BCH', 'SWFTC'), ('BLZ', 'STRAX'), ('BNB', 'BTG'), ('BN
B', 'GNO'), ('BNB', 'MLN'), ('BNB', 'NEO'), ('BTC', 'SWFTC'), ('CTXC', 'STRAX'),
('DASH', 'SWFTC'), ('DCR', 'SWFTC'), ('DENT', 'STRAX'), ('DGB', 'SNT'), ('DGB',
'XEM'), ('DGB', 'ZIL'), ('DOGE', 'BAT'), ('ELF', 'STEEM'), ('ENJ', 'VIC'), ('ET
C', 'BTG'), ('GFT', 'CTXC'), ('GLM', 'ICX'), ('GLM', 'PRO'), ('IOST', 'XVG'),
('LINK', 'NMR'), ('LRC', 'TRAC'), ('LTC', 'SWFTC'), ('MANA', 'OMG'), ('MANA', 'Q
TUM'), ('MANA', 'STORJ'), ('MED', 'XDC'), ('MKR', 'SWFTC'), ('ONT', 'LSK'), ('ON
T', 'NULS'), ('ONT', 'PIVX'), ('POWR', 'REQ'), ('POWR', 'STMX'), ('PRO', 'MTL'),
('REN', 'BORG'), ('REN', 'BTG'), ('REQ', 'IDEX'), ('RLC', 'VIC'), ('SNX', 'QTU
M'), ('STEEM', 'CTXC'), ('STMX', 'ARDR'), ('STMX', 'IOST'), ('SYS', 'REQ'), ('TH
ETA', 'SBD'), ('TRX', 'ARDR'), ('TRX', 'EOS'), ('TRX', 'MDT'), ('TRX', 'RVN'),
('TRX', 'WAXP'), ('VGX', 'GAS'), ('VGX', 'XNO'), ('VIC', 'SBD'), ('WAVES', 'BT
G'), ('XLM', 'ADX'), ('XLM', 'ZIL'), ('XMR', 'SWFTC'), ('XRP', 'CVC'), ('XVG',
'FUN'), ('XVG', 'VIB'), ('ZEC', 'SWFTC'), ('ZEN', 'BTG'), ('ZRX', 'IOTA')]
Total number of pairs: 69
```

Step 2: Trading Strategy

For the backtesting period from January 1, 2022, to the present, we implement the following strategy to each pairs:

1. Signal Generation

- **Residual Calculation:**

$$\epsilon_t = \log(p_{i,t}) - (\beta_t \log(p_{j,t}) + \alpha_t)$$

  where:
  - $\log(p_{i,t})$ and $\log(p_{j,t})$ are the log prices of coins i and j at time t.
  - β_t is calculated as:

$$\beta_t = \text{Corr}_t \times \frac{\text{Vol}_{j,t}}{\text{Vol}_{i,t}}$$

  with:
  - Corr_t is the 90-day rolling correlation between $\log(p_{i,t})$ and $\log(p_{j,t})$.
  - $\text{Vol}_{i,t}$ and $\text{Vol}_{j,t}$ are the 90-day rolling volatilities of $\log(p_{i,t})$ and $\log(p_{j,t})$, respectively.
  - α_t is calculated as:

$$\alpha_t = \mu_{\log(p_{j,t})} - \beta_t \cdot \mu_{\log(p_{i,t})}$$

  where:
  - $\mu_{\log(p_{j,t},90)}$ is the 90-day rolling mean of $\log(p_{j,t})$.
  - $\mu_{\log(p_{i,t},90)}$ is the 90-day rolling mean of $\log(p_{i,t})$.
- **Z-Score Calculation:**

$$z_t = \frac{\epsilon_t - \mu_t}{\sigma_t}$$

  where:
  - μ_t is the 90-day rolling mean of the spread.
  - σ_t is the 90-day rolling standard deviation of the spread.

2. Portfolio Construction

- **Entry Signals:**
  - **Short** coin i and **long** β_t units of coin j if $z_t > 1$.
  - **Long** coin i and **short** β_t units of coin j if $z_t < -1$.
- **Exit Signals:**
  - Close the position when z_t moves to any of the following thresholds:

$$z_t \geq -\text{threshold} \quad \text{or} \quad z_t \leq \text{threshold}$$

    where threshold is one of the values: 0.1, 0.2, 0.5, or 0.7.

In [20]:
```
def gen_signals(px, pairs, window=90):
    signal_df = {}

    for pair in pairs:
        asset_i, asset_j = pair

        # Forward-fill missing values and replace zeros with NaNs
        px_i = px[asset_i].replace(0, np.nan).ffill()
        px_j = px[asset_j].replace(0, np.nan).ffill()

        # Apply log transformation
        log_px_i = np.log(px_i)
        log_px_j = np.log(px_j)

        # Calculate rolling covariance and variance
        rolling_cov = log_px_i.rolling(window=window, min_periods=1).cov(log_px_j
        rolling_var = log_px_i.rolling(window=window, min_periods=1).var()

        # Calculate beta and alpha
        beta = rolling_cov / rolling_var
        alpha = log_px_j.rolling(window=window).mean() - beta * log_px_i.rolling(

        # Calculate spread for time t using beta and alpha
        spread = log_px_i - (beta * log_px_j + alpha)

        # Calculate rolling mean and standard deviation of the spread using data
        spread_mean = spread.rolling(window=window, min_periods=1).mean()
        spread_std = spread.rolling(window=window, min_periods=1).std()

        # Calculate the z-score for time t using spread, mean, and std
        z_score = (spread - spread_mean) / spread_std

        # Store beta, alpha, spread, and z-score in a multi-level column DataFram
        signal_df[(pair, 'beta')] = beta
        signal_df[(pair, 'alpha')] = alpha
        signal_df[(pair, 'spread')] = spread
        signal_df[(pair, 'z_score')] = z_score

    # Convert the dictionary to a DataFrame
    signal_df = pd.DataFrame(signal_df)

    return signal_df
```

```
In [21]: def gen_port(signal_df, pairs, threshold=0.5):
    # Initialize a DataFrame with the same index and columns as crypto_px, filled
    pos = pd.DataFrame(index=signal_df.index, columns=in_sample_px.columns)
    for pair in pairs:
        asset_i, asset_j = pair
        # Access z-scores and betas for this pair
        z_scores = signal_df[(pair, 'z_score')]
        betas = signal_df[(pair, 'beta')]

        # Set positions based on z-scores
        pos.loc[z_scores > 1, asset_i] = -1  # Short one unit of asset_i
        pos.loc[z_scores < -1, asset_i] = 1  # Long one unit of asset_i
        pos.loc[(z_scores.abs() <= threshold), asset_i] = 0  # Exit signal

        pos.loc[z_scores > 1, asset_j] = betas  # Long beta units of asset_j
        pos.loc[z_scores < -1, asset_j] = -betas  # Short beta units of asset_j
        pos.loc[(z_scores.abs() <= threshold), asset_j] = 0  # Exit signal

    # Forward-fill missing values
    pos = pos.ffill()
    # Normalize to ensure a fully-invested portfolio
    pos = pos.divide(pos.abs().sum(axis=1), axis=0).fillna(0)
    return pos
```

Step 3: Performance Evaluation

We evaluate the strategy's performance across four exit thresholds: 0.1, 0.2, 0.5, and 0.7. For each threshold, the following key metrics are calculated:

- **Sharpe Ratio**: Risk-adjusted return of the strategy.
- **Transaction Costs**: Average costs incurred due to trading.
- **Holding Period**: Average number of days a position is held.
- **Turnover**: Average daily proportion of the portfolio that is traded.
- **Annualized Return**: Average yearly return of the strategy.
- **Annualized Volatility**: Standard deviation of returns on an annual basis, indicating risk.

Cryptocurrencies can have commissions of ~7bps. While total slippage is unknown and will depend on the trader’s volume as well, let’s assume another 13 bps. So total all-in execution costs will be 20 bps for market-orders.

In [22]:

```python
# Define the thresholds and initialize the arrays for storing metrics
thresholds = [0.1, 0.2, 0.5, 0.7]
metrics = {
    'Sharpe Ratio': np.zeros(len(thresholds)),
    'Return': np.zeros(len(thresholds)),
    'Volatility': np.zeros(len(thresholds)),
    'Holding Period': np.zeros(len(thresholds)),
    'Turnover': np.zeros(len(thresholds)),
    'Transaction Costs': np.zeros(len(thresholds)),
}

# Set the start of the out-of-sample period
start_of_out_sample = end_of_insample + pd.DateOffset(days=1)

# Loop over each threshold to calculate metrics
for i, threshold in enumerate(thresholds):
    # Generate signals
    signal_df = gen_signals(crypto_px, final_pairs, window=90)
    signal_df = signal_df.loc[start_of_out_sample:]

    # Generate portfolio
    port = gen_port(signal_df, final_pairs, threshold)

    # Calculate out-of-sample daily returns
    out_sample_ret = coins_ret.loc[start_of_out_sample:][port.columns]
    strat_gross_ret = (port.shift() * out_sample_ret).sum(1)

    # Calculate net returns
    to = compute_turnover(port)
    tcost_bps = 20  # (commissions + slippage)
    strat_net_ret = strat_gross_ret.subtract(to * tcost_bps * 1e-4, fill_value=0)

    # Compute Sharpe ratio
    sharpe_ratio = compute_sharpe_ratio(strat_net_ret)
    metrics['Sharpe Ratio'][i] = sharpe_ratio

    # Compute return
    returns = strat_net_ret.mean()
    metrics['Return'][i] = returns

    # Compute volatility
    volatility = strat_net_ret.std()
    metrics['Volatility'][i] = volatility

    # Compute holding period (average number of days a position is held)
    metrics['Holding Period'][i] = 2/to.mean()

    # Store turnover
    metrics['Turnover'][i] = to.mean()

    # Compute transaction costs
    total_tcost = to * tcost_bps * 1e-4
    metrics['Transaction Costs'][i] = total_tcost.mean()

# Convert metrics to a DataFrame for easier visualization
metrics_df = pd.DataFrame(metrics, index=thresholds)

# Display the results
print(metrics_df)
```

```
     Sharpe Ratio    Return  Volatility  Holding Period  Turnover  \
0.1      1.303307  0.000713    0.008688       19.918467  0.100409   
0.2      1.503272  0.000824    0.008706       17.729815  0.112804   
0.5      1.482900  0.000902    0.009656       14.840894  0.134763   
0.7      1.299942  0.000845    0.010322       13.296672  0.150414   

     Transaction Costs  
0.1           0.000201  
0.2           0.000226  
0.5           0.000270  
0.7           0.000301  
```

The performance across different thresholds is relatively robust, with only minor variations. Notably, the highest Sharpe Ratio is observed with a threshold of 0.2. Therefore, we will use the 0.2 threshold for further exploration of the strategy.

In [23]:
```
# signal construction
signal_df = gen_signals(crypto_px, final_pairs, window=90)
signal_df = signal_df.loc[start_of_out_sample:]
signal_df.head()
```

Out[23]:

	(ADA, SBD)				(ADX, UTK)				(AMB, PIVX)	
	beta	alpha	spread	z_score	beta	alpha	spread	z_score	beta	alpha
Date										
2022-01-01	0.564480	1.592742	-2.267152	-1.961986	0.301923	-0.852495	0.610650	-0.735134	0.907025	2.621252
2022-01-02	0.574016	1.587555	-2.245130	-1.813594	0.305609	-0.851739	0.638341	-0.541876	0.910049	2.629780
2022-01-03	0.580821	1.584301	-2.272021	-1.871088	0.305537	-0.851759	0.622449	-0.630763	0.915705	2.647043
2022-01-04	0.586767	1.581526	-2.312552	-1.978738	0.306342	-0.851069	0.596247	-0.789343	0.927266	2.682824
2022-01-05	0.592077	1.578483	-2.309751	-1.907372	0.300725	-0.852211	0.547709	-1.099652	0.939755	2.721377

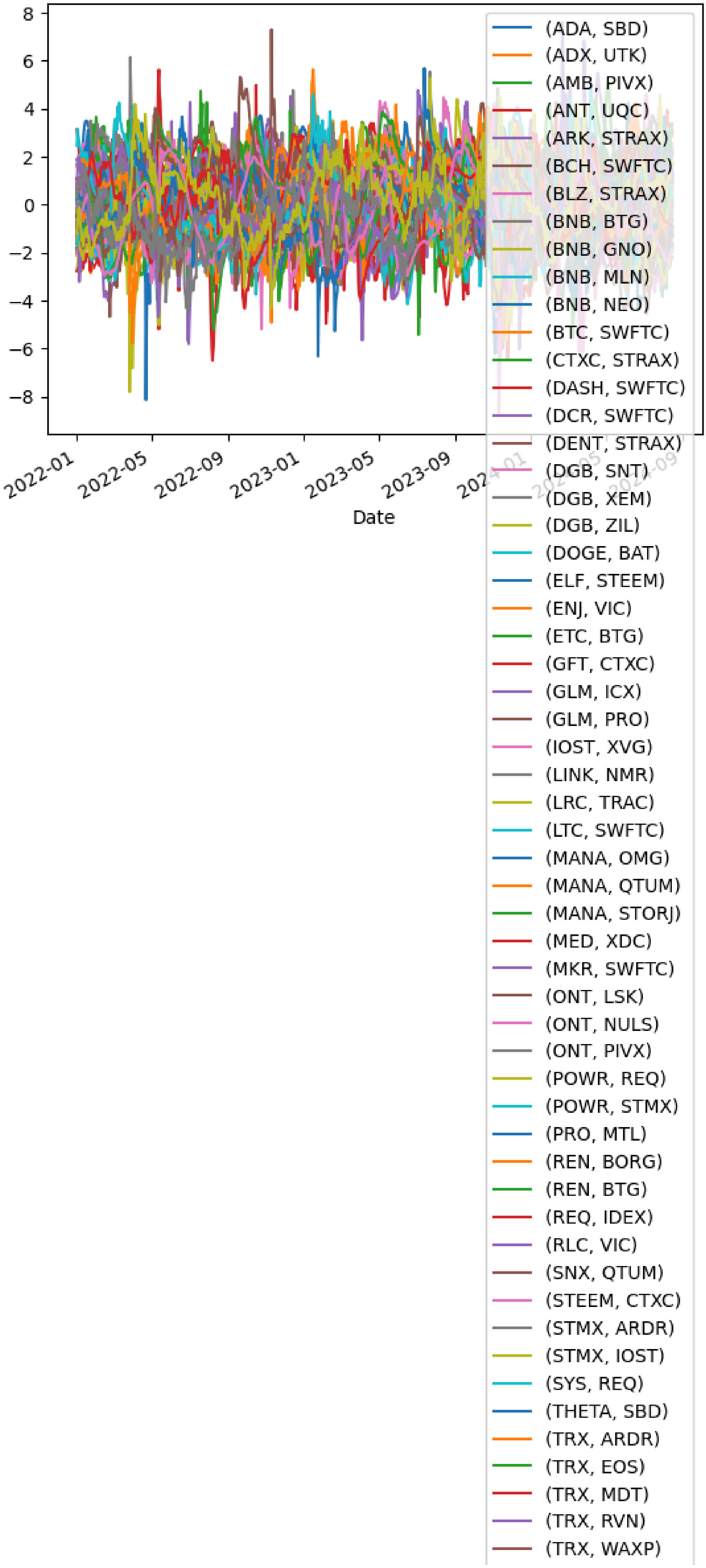
5 rows × 276 columns

In [24]:

```python
# plot the z-score
z_score_df = signal_df.xs('z_score', axis=1, level=1)
z_score_df.plot()
```

Out[24]:

```
<Axes: xlabel='Date'>
```


8
6
4
2
0
−2
−4
−6
−8
2022-01
2022-05
2022-09
2023-01
2023-05
2023-09
Date
(ADA, SBD)
(ADX, UTK)
(AMB, PIVX)
(ANT, UQC)
(ARK, STRAX)
(BCH, SWFTC)
(BLZ, STRAX)
(BNB, BTG)
(BNB, GNO)
(BNB, MLN)
(BNB, NEO)
(BTC, SWFTC)
(CTXC, STRAX)
(DASH, SWFTC)
(DCR, SWFTC)
(DENT, STRAX)
(DGB, SNT)
(DGB, XEM)
(DGB, ZIL)
(DOGE, BAT)
(ELF, STEEM)
(ENJ, VIC)
(ETC, BTG)
(GFT, CTXC)
(GLM, ICX)
(GLM, PRO)
(IOST, XVG)
(LINK, NMR)
(LRC, TRAC)
(LTC, SWFTC)
(MANA, OMG)
(MANA, QTUM)
(MANA, STORJ)
(MED, XDC)
(MKR, SWFTC)
(ONT, LSK)
(ONT, NULS)
(ONT, PIVX)
(POWR, REQ)
(POWR, STMX)
(PRO, MTL)
(REN, BORG)
(REN, BTG)
(REQ, IDEX)
(RLC, VIC)
(SNX, QTUM)
(STEEM, CTXC)
(STMX, ARDR)
(STMX, IOST)
(SYS, REQ)
(THETA, SBD)
(TRX, ARDR)
(TRX, EOS)
(TRX, MDT)
(TRX, RVN)
(TRX, WAXP)

In [25]:
```
# portfolio construction
port = gen_port(signal_df, final_pairs, 0.2)
port.head()
```

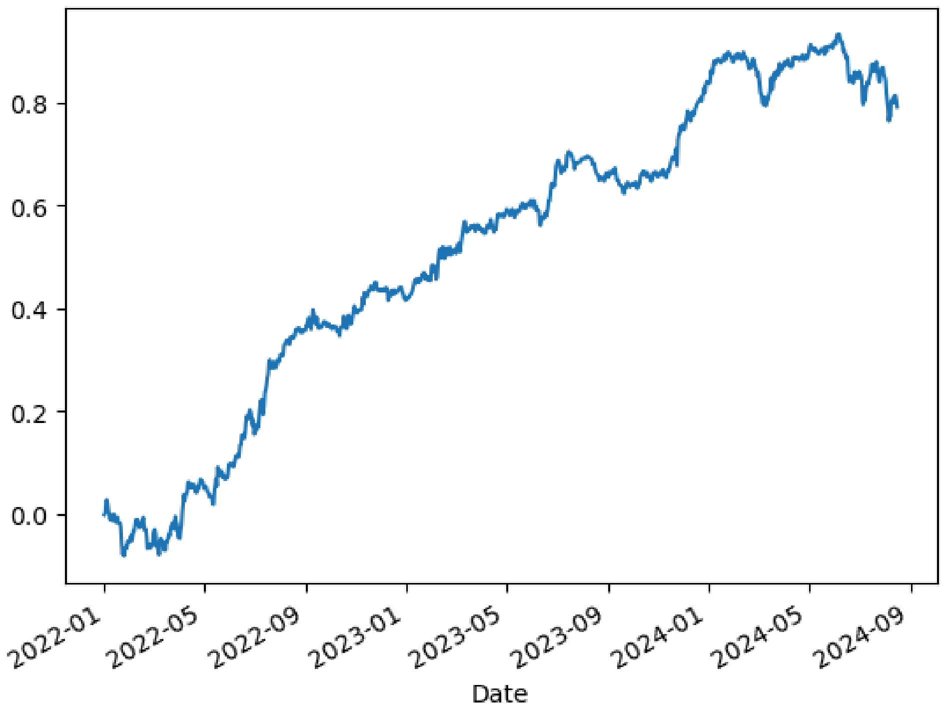
Out[25]:

Date	BTC	XRP	BNB	DOGE	ADA	TRX	LTC	LINK	BCH	EOS	...	UTI
2022-01-01	-0.022301	-0.022301	0.022301	-0.022301	0.022301	-0.022301	0.0	0.0	0.022301	-0.029280	...	0.0
2022-01-02	-0.021542	-0.021542	0.021542	-0.021542	0.021542	-0.021542	0.0	0.0	0.021542	-0.028204	...	0.0
2022-01-03	-0.022096	-0.022096	0.022096	-0.022096	0.022096	-0.022096	0.0	0.0	0.022096	-0.028783	...	0.0
2022-01-04	-0.022303	-0.022303	0.022303	-0.022303	0.022303	-0.022303	0.0	0.0	0.022303	-0.028844	...	0.0
2022-01-05	-0.019778	-0.019778	0.019778	-0.019778	0.019778	-0.019778	0.0	0.0	0.019778	-0.025378	...	-0.0

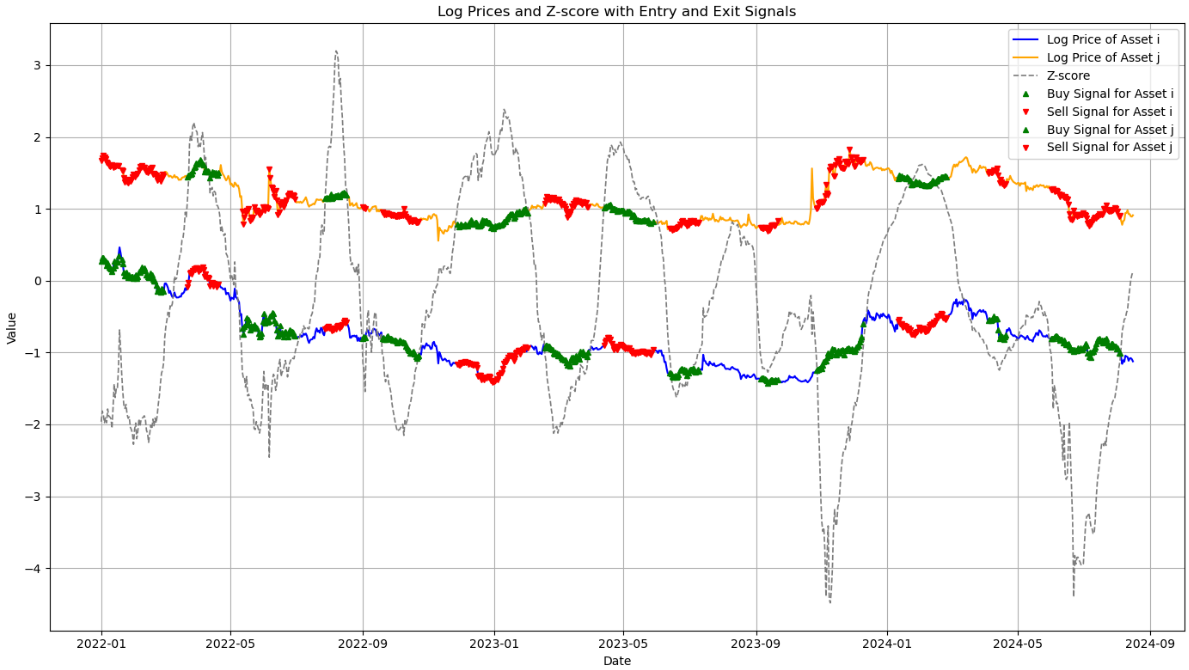
5 rows × 86 columns

In [26]:
```
# Calculate out-of-sample daily returns
out_sample_ret = coins_ret.loc[start_of_out_sample:][port.columns]
strat_gross_ret = (port.shift() * out_sample_ret).sum(1)

# Calculate net returns
to = compute_turnover(port)
tcost_bps = 20  # (commissions + slippage)
strat_net_ret = strat_gross_ret.subtract(to * tcost_bps * 1e-4, fill_value=0)

strat_net_ret.cumsum().plot()
```

Out[26]: <Axes: xlabel='Date'>



In [27]:
```
# select a random pair and visualize their log-prices and z-scores
asset_i, asset_j = final_pairs[0]
log_px_i = np.log(crypto_px.loc[start_of_out_sample:][asset_i])
log_px_j = np.log(crypto_px.loc[start_of_out_sample:][asset_j])
z_score = signal_df[(final_pairs[0], 'z_score')]

plot_with_signals(log_px_i, log_px_j, z_score)
```



We also compare the performance of our strategy against our benchmark, specifically a buy-and-hold strategy for Bitcoin. We will evaluate key metrics including alpha and beta, maximum drawdowns and maximum drawdown duration.

In [28]:
```
buy_and_hold_btc = coins_ret['BTC'][start_of_out_sample:]
buy_and_hold_btc
```

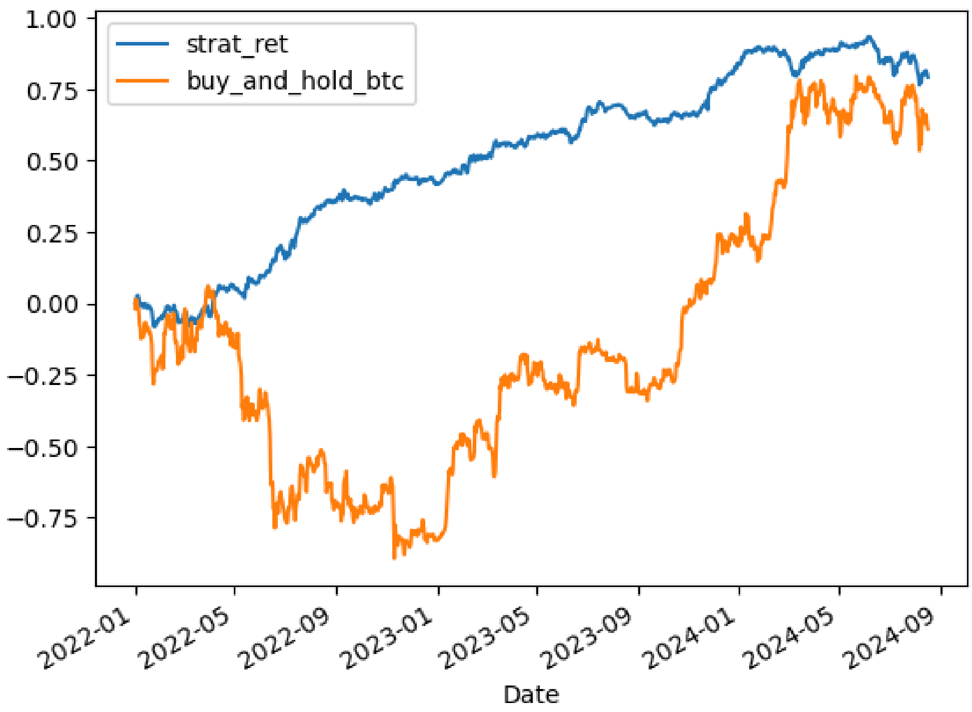
Out[28]:
```
Date
2022-01-01   -0.018482
2022-01-02    0.032307
2022-01-03   -0.008969
2022-01-04   -0.018065
2022-01-05   -0.012747
                ...
2024-08-12   -0.034218
2024-08-13    0.009282
2024-08-14    0.021081
2024-08-15   -0.030726
2024-08-16   -0.018984
Name: BTC, Length: 960, dtype: float64
```

In [29]:
```
full_sample_ret = pd.DataFrame({
    'strat_ret': strat_net_ret,
    'buy_and_hold_btc': buy_and_hold_btc
})

full_sample_stats = compute_stats(full_sample_ret)
print(full_sample_stats)
```

```
                       avg       vol    sharpe  hit_rate
strat_ret         0.207747  0.138197  1.503272  0.531250
buy_and_hold_btc  0.160358  0.457895  0.350208  0.493737
```

In [30]:
```
full_sample_ret.cumsum().plot()
```

Out[30]:
```
<Axes: xlabel='Date'>
```

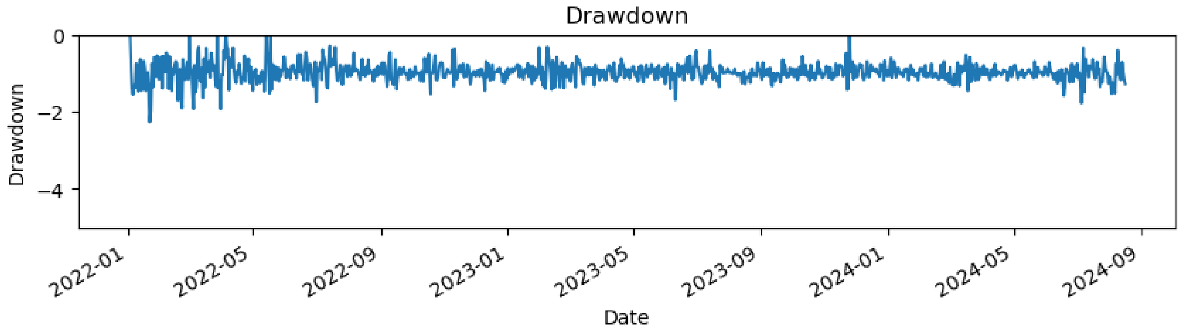


In [31]:
```
corr = full_sample_ret.rolling(252).corr(full_sample_ret['buy_and_hold_btc'])
vol = full_sample_ret.rolling(252).std()

beta = (corr*vol).divide(vol['buy_and_hold_btc'], 0)

# Computing Point-in-Time Residual Returns
resid = full_sample_ret - beta.multiply(full_sample_ret['buy_and_hold_btc'], axis
print(resid)
print(resid.corr())

# The information ratio
IR = resid.mean()/resid.std()*np.sqrt(252)
print(f"Information ratio is {IR['strat_ret']}")
```

```
            strat_ret  buy_and_hold_btc
Date
2022-01-01        NaN               NaN
2022-01-02        NaN               NaN
2022-01-03        NaN               NaN
2022-01-04        NaN               NaN
2022-01-05        NaN               NaN
...               ...               ...
2024-08-12  -0.007699      0.000000e+00
2024-08-13   0.013555      0.000000e+00
2024-08-14  -0.000260      3.469447e-18
2024-08-15  -0.006771      0.000000e+00
2024-08-16  -0.013471      0.000000e+00

[960 rows x 2 columns]
                  strat_ret  buy_and_hold_btc
strat_ret          1.000000          0.069796
buy_and_hold_btc   0.069796          1.000000
Information ratio is 0.6345426373268149
```

The correlation of the residual returns with Bitcoin is 0.07, indicating a low correlation and suggesting that the strategy captures true alpha

In [32]:
```
# max drawdown
dd = drawdown(full_sample_ret['strat_ret'])
plt.figure(figsize=(10, 2))
dd.plot()
plt.ylim(-5, 0)
plt.title('Drawdown')
plt.xlabel('Date')
plt.ylabel('Drawdown')
plt.show()
print(f"The max drawdown is {dd.min()}")
```




```
The max drawdown is -2.2745835868194746
```

In [35]:
```
ddd = duration(full_sample_ret.cumsum())
ddd
```

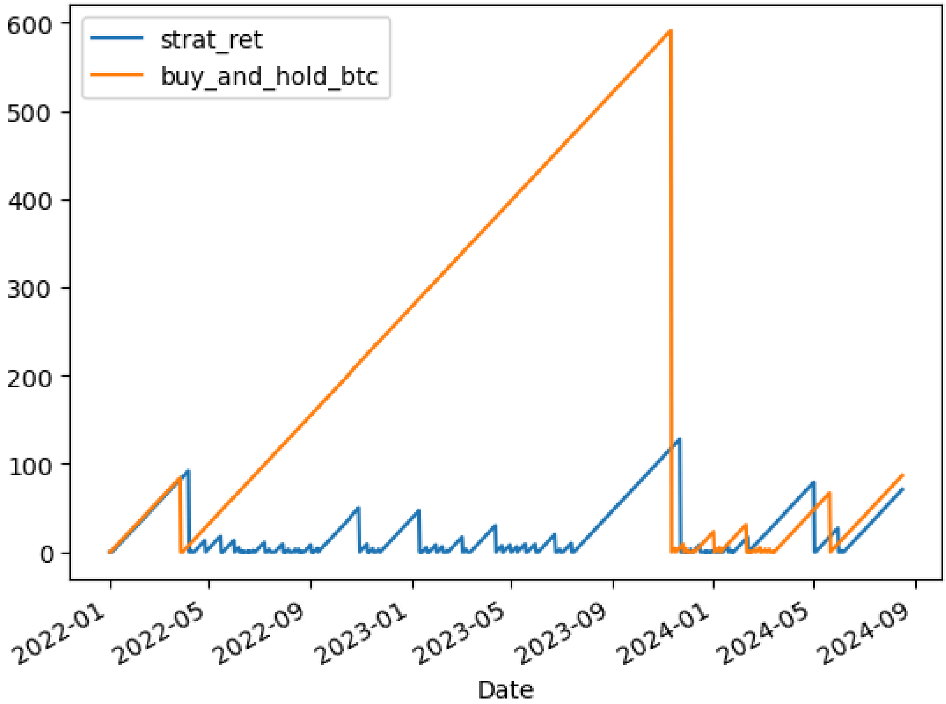
Out[35]:

	strat_ret	buy_and_hold_btc
Date		
2022-01-01	0	0
2022-01-02	0	0
2022-01-03	0	1
2022-01-04	0	2
2022-01-05	1	3
...	...	...
2024-08-12	67	83
2024-08-13	68	84
2024-08-14	69	85
2024-08-15	70	86
2024-08-16	71	87

960 rows × 2 columns

In [36]:
```
ddd.plot()
```

Out[36]: <Axes: xlabel='Date'>



In [37]:
```
# maximum drawdown duration
ddd.max()
```

Out[37]:
```
strat_ret           128
buy_and_hold_btc    591
dtype: object
```

Overall, the cointegration strategy performs well in the crypto market. As a next step, we could explore combining this price/volume-based strategy with fundamental strategies. These strategies are likely to be synergistic and uncorrelated, potentially enhancing the overall performance of the portfolio.

In [ ]: