From Bohren and Hufmanf, p. 121,

$$S_2(180°) = -S_1(180°) = \frac{1}{2}\sum_n (-1)^n(2n+1)(a_{n1} - a_{n2})$$

$$Q_b = \frac{\sigma_b}{\pi a^2} = \frac{1}{x^2}\left|\sum_n (-1)^n(2n+1)(a_{n1} - a_{n2})\right|^2$$

Exercises

(1) For a sphere of radius 5 μm and real refractive index 1.59, use your code to provide graphs of Q_e, Q_s, Q_a, and Q_b as a function of size parameter for $\Im\,\mathsf{m} = 10^{-6}$, .001 and .1 over ka=[.01, 100].

(2) Produce graphs of Q_e, Q_s, Q_a, and Q_b for a 5 μm silicon sphere over λ =[280, 2500] μm.

(3) Compare Q_b with normal incidence reflectance from the bulk material.

(4) The absorption cross section of a sphere may be written as (ReadMie.pdf)

$$\sigma_a = \frac{2\pi}{|\mathsf{m}|^2 k^2}\sum_{n=1}^{\infty}(2n+1)\,\Re\; i\psi'_n(\eta)\,\psi_n^*(\eta)\left(\mathsf{m}\,|c_{n1}|^2 + \mathsf{m}^*\,|c_{n2}|^2\right),$$

where $\eta = \mathsf{m}kr\big|_{r=a}$. Rather than just looking at η, graph Q'_a as a function of kr for $kr = [0, ka+1]$. Use the 5 μm Si sphere of Ex. (2) and $\lambda = 350, 400$ and 600 nm. In other words plot the function

$$\sigma'_a = \frac{2\pi}{|\mathsf{m}|^2 k^2}\sum_{n=1}^{\infty}(2n+1)\,\Re\; i\psi'_n(\mathsf{m}kr)\,\psi_n^*(\mathsf{m}kr)\left(\mathsf{m}\,|c_{n1}|^2 + \mathsf{m}^*\,|c_{n2}|^2\right).$$

Discuss your results.

(5) Use the following normalization scheme to calculate the normalized Mueller matrix for a 1 μm Si sphere at $\lambda = 350, 400$ and 600 nm. This normalization is used to produce values, such as the ones shown below, where the S_{ij} elements are bounded between minus one and one. Use an angular resolution of at least 1°.

$$S_{11}(\theta) = S_{11}(\theta)/S_{11}(0°)$$

$$S_{ij}(\theta) = S_{ij}(\theta)/S_{11}(0)/S_{ij}(\theta)$$

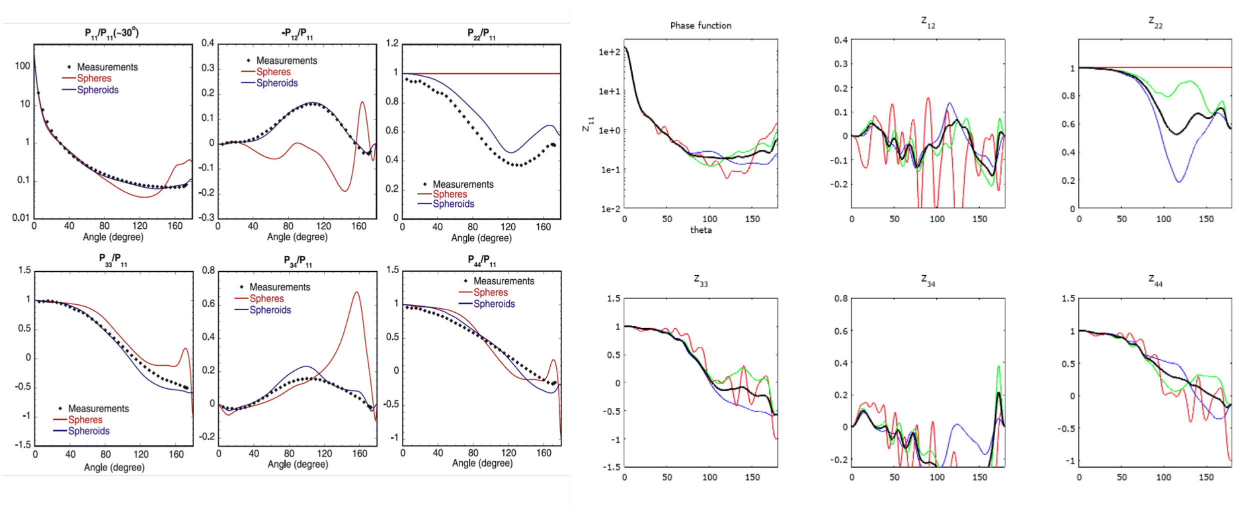

Figure 1: Representive Mueller matrices for populations of randomly oriented dust grains. On the left, a representative comparison between measurement and theory by Dubovic, et al. On the right, representative calculations made for the present study. The green and blue curves are, respectively, for oblate and prolate spheroids, the black curves are the average of the two, and the red curves are for equivalent volume distributions of spheres.