

# Exercises for the 3rd Morning

## 1 IQ test

Suppose that a  $n$  people have taken an IQ-test. The score  $x_i$  obtained by the  $i$ th person is assume to be normal distributed with known precision  $\tau$  and a mean  $\mu_i$  which corresponds to the true IQ for that person, ie.  $x_i \sim N(\mu_i, \tau)$ . Assume that the people taking the test come from a population where the true IQ can be assume to be normal distributed, ie.  $\mu_i \sim N(\mu_G, \tau_G)$ . Regarding priors we assume a priori that  $\mu_G$  and  $\tau_G$  are independent, and  $\mu_G \sim N(\mu_0, \tau_0)$  and  $\tau_G \sim \text{Gamma}(\alpha, \beta)$ .

1. Determine the joint distribution  $\pi(x_1, \dots, x_n, \mu_1, \dots, \mu_n, \mu_G, \tau_G)$ .
2. Determine the full conditionals,  $\pi(\mu_1 | \mu_2, \dots, \mu_n, \mu_G, \tau_G, x_1, \dots, x_n)$  etc.  
*Hint:* Have a look at the results for the case of  $n$  independent samples  $x_1, \dots, x_n$  from the same normal distributon  $N(\mu, \tau)$ .
3. Specify a Gibbs sampler for sampling  $\pi(\mu_1, \dots, \mu_n, \mu_G, \tau_G | x_1, \dots, x_n)$ .

## 2 Radiocarbon dating

Following Lee (2003, p. 263) consider the following example for archeology: Assume that for each of three samples we have measured the date as  $x_1, x_2$  and  $x_3$  and a reasonable approximation is  $x_i \sim N(\mu_i, \tau_i)$ , where  $\mu_i$  is the true age and  $\tau_i$  is known. It is further known that the age of the samples are positive and below some upper limit  $k$ . In addition the time order of the three samples is known, that is  $\mu_1 < \mu_2 < \mu_3$ . Thus, as a joint prior on  $(\mu_1, \mu_2, \mu_3)$  we propose

$$\pi(\mu_1, \mu_2, \mu_3) \propto \begin{cases} c & \text{if } 0 < \mu_1 < \mu_2 < \mu_3 < k \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

where  $c$  is a positive constant.

1. Determine the joint posterior pdf for the three mean values, ie.  $\pi(\mu_1, \mu_2, \mu_3 | x_1, x_2, x_3)$ .
2. Determine the full conditionals, ie.  $\pi(\mu_1 | \mu_2, \mu_3, x_1, x_2, x_3)$  etc.  
*Notice:* These distributions are non-standard.
3. Specify a Gibbs sampler for sampling of the posterior. How would you generate samples from the non-standard distributions above?