The exam problems will test your ability in two main areas:

1. Constructing simple code or describing what a bit of code is doing
   - manipulation of data frames
   - control structures: for, while, if-else
   - using vectorization instead of loops
2. Being able to understand and apply the algorithms taught in class
   - Inversion method for random number generation
   - Rejection sampling for random number generation
   - Importance sampling
   - Newton-Raphson

Example 1: Suppose `X` contains a 10-by-10 data frame. Describe what each of the following lines of code does:

```
X[c(1,4,7),c(2:5)]
```

```
X[,-c(6:8)]
```

```
X[abs(X[,7]) < 1, ]
```

Example 2: Write a short program to calculate the smallest K such that

$$\sum_{n=1}^{K} n^{3/2} \leq 50$$

Example 3: Choose which of the responses is approximately what this program print at the end and explain why:

```
X <- matrix(rexp(20000, rate=1/3), 1000, 20)
M <- apply(X, 1, mean)
print( mean(M), var(M) )
```

The options are

```
[1] 3 9
```

```
[1] 3 0.45
```

```
[1] .333 .111
```

```
[1] 0.15 0.45
```

Example 4: Describe in terms of a conditional expectation what the following program estimates:

```
X <- rnorm(10000)
X <- X[which( (X > 0) & (X < 2) )]
mean(X)
```

Example 5: Write a program which does the same as the following without using a loop

```
X <- rnorm(10000)
for(j in 1:10000) if(X[j] < 0) X[j] <- 0 else X[j] <- sqrt( X[j] )
```

Example 6: Consider rejection sampling from the standard normal density,

$$p(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

using the Cauchy distribution

$$g(x) = \frac{1}{\pi(1+x^2)}$$

as the trial distribution. If the first uniform generated was $U = .632$ and the first candidate draw generated was $X = 1.36$, would you accept or reject this candidate? Justify your answer.

Note: You may take it for granted that $M = \sup_x p(x)/g(x) \approx 1.52$.

Example 7: Suppose you have a distribution with density

$$p(x) = \lambda x^{-\lambda-1}$$

where $x \geq 1$ and $\lambda \geq 2$. Write a one line **function** based on the inversion method to generate n samples from the distribution where λ is an input the the function.

What line could you type to estimate $E(X^{2.736})$ if X has the distribution described above with $\lambda = 3$?

Example 8: Suppose you wanted to estimate the integral

$$\int_{-3}^{3} e^{-|x|} dx$$

by monte carlo. Write two short programs to do this by

1. viewing this as an expectation against the Uniform$(-3, 3)$ density
2. importance sampling with $N(0, 1)$ as your trial density

Which of these do you expect to provide a smaller standard error for the integral approximation?

Example 9: Consider optimization of the function $f(x) = \exp(-x^2 + 3x + 4)$. Suppose your start value is $x_0 = 1$. Where would you be after a single Newton-Raphson update?