Exercise 3.2 Given that $I_{X_i \leq t}$ is a Bernoulli random variable equal to 1 with probability $\Phi(t)$, show that the variance of the normalized estimator $I_{X_i \leq t}/\Phi(t)$ goes to infinity when t decreases to $-\infty$. Deduce the number of simulations (as a function of t) that are necessary to achieve a variance less than 10^{-8}.

$$\hat{\Phi}(t) \sim N\left(\Phi(t), \frac{\Phi(t)(1-\Phi(t))}{n}\right)$$

$$\hat{\Phi}(t)/\Phi(t) \sim N\left(1, \frac{1-\Phi(t)}{\Phi(t)n}\right)$$

since $\Phi(t) \to 0$ as $t \to -\infty$,

$$lim_{t \to -\infty} \frac{1-\Phi(t)}{\Phi(t)n} = \infty$$

Solving for n with the normalized estimate,

$$n = \frac{1-\Phi(t)}{\Phi(t)10^{-8}}$$

Solving for n when we are interested in estimating Φ,

$$n = \frac{\Phi(t)(1-\Phi(t))}{10^{-8}}$$