STAT 701 Midterm Practice Problems

1. Assume f is a binomial distribution with parameters n and p and g is the poisson distribution with parameter $\lambda = np$.

   - Write the pseudocode for the accept/reject routine to sample from the distribution f using the density g as a candidate. Be sure to discuss in detail how you would come up with the mulitplier M. You do not need to write out the binomial and poisson densities. You can refer to them in terms of f and g.
   - Show that the probability of acceptance of your algorithm is $1/M$.

2. Write the pseudocode for an inverse transform routine for sampling from an ex ponential distribution with pdf $g(x) = \beta e^{-\beta x}, x > 0, \beta > 0$.

3. If you have two densities g_1 and g_2 that you are considering using as importance function to use importance sampling to find the mean of the target density f, discuss at least one means by which you could compare g_1 and g_2 to determine which will lead to more accurate estimation for a fixed sample size.

4. You plan to use importance sampling to find the tail probability of a normal distribution $P(Z > 6)$ using importance sampling via a truncated exponential. Identify the following

   - The function $h(x)$ whose expectation is the goal of the analysis
   - The target density, f(x)
   - The importance function, g(x)
   - The weights, w

5. The code below gives a Monte Carlo estimate of the mean of a beta distribution. Two options are provided for estimating the error. Which do you prefer and why?

```
> Nsim=10^3
> x=rbeta(Nsim, 4, 6)
> estint=mean(x)
> ### Variance estimate A
> estvar1=var(x)/Nsim
> ### Variance estimate B
> xm=matrix(nrow=Nsim, ncol=100, data=sample(x, Nsim*100, replace=T))
> estvar2=mean(apply(xm,2,var)/Nsim)
```