1) Gamma

$$\theta \sim \text{Gamma}(\alpha,\beta),\ \ \alpha > 0,\ \beta > 0$$

$$p(\theta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\theta\beta},\ \theta > 0$$

$$E(\theta) = \frac{\alpha}{\beta}$$

$$V(\theta) = \frac{\alpha}{\beta^2}$$

$$\text{mode}(\theta) = \frac{\alpha-1}{\beta},\ \ \alpha \geq 1$$

2) Inverse Gamma

$$\theta \sim \text{Inv}-\text{Gamma}(\alpha,\beta),\ \ \alpha > 0,\ \beta > 0\ \ \Rightarrow\ \ 1/\theta \sim \text{Gamma}(\alpha,\beta)$$

$$p(\theta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)}\theta^{-(\alpha+1)}e^{-\beta/\theta},\ \theta > 0$$

$$E(\theta) = \frac{\beta}{\alpha-1},\ \ \alpha > 1$$

$$V(\theta) = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)},\ \ \alpha > 2$$

$$\text{mode}(\theta) = \frac{\beta}{\alpha+1}$$

3) Chi-Square

$$\theta \sim X^2_{\nu},\ \nu > 0\ \ \Rightarrow\ \ \theta \sim \text{Gamma}(\alpha = \frac{\nu}{2}, \beta = \frac{1}{2})$$

$$p(\theta) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)}\theta^{\nu/2-1}e^{-\theta/2},\ \theta > 0$$

$$E(\theta) = \nu$$

$$V(\theta) = 2\nu$$

$$\text{mode}(\theta) = \nu - 2,\ \ \nu \geq 2$$

4) **Inverse Chi-Square**

$$\theta \sim \text{Inv}-X_\nu^2,\ \nu > 0 \quad \Rightarrow \quad \theta \sim \text{Inv}-\text{Gamma}(\alpha = \frac{\nu}{2}, \beta = \frac{1}{2})$$

$$p(\theta) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)} \theta^{-(\nu/2+1)} e^{-1/2\theta},\ \theta > 0$$

$$E(\theta) = \frac{1}{\nu - 2},\ \nu > 2$$

$$V(\theta) = \frac{2}{(\nu-2)^2(\nu-4)},\ \nu > 4$$

$$\text{mode}(\theta) = \frac{1}{\nu+2}$$

5) **Scaled - Inverse Chi-Square**

$$\theta \sim \text{Inv}-X^2(\nu, s),\ \nu > 0 \text{ (degrees of freedom)},\ \ s > 0 \text{ (scale)}$$

$$\Rightarrow \quad \theta \sim \text{Inv}-\text{Gamma}(\alpha = \frac{\nu}{2}, \beta = \frac{\nu s^2}{2})$$

$$\text{If } X \sim X_\nu^2 \Rightarrow \theta = \frac{\nu s^2}{X} \sim \text{Inv}-X^2(\nu, s)$$

$$p(\theta) = \frac{(\nu/2)^{\nu/2}}{\Gamma(\nu/2)} s^\nu \theta^{-(\nu/2+1)} e^{-\nu s/2\theta},\ \theta > 0$$

$$E(\theta) = \frac{\nu s^2}{\nu - 2},\ \nu > 2$$

$$V(\theta) = \frac{2\nu^2 s^4}{(\nu-2)^2(\nu-4)},\ \nu > 4$$

$$\text{mode}(\theta) = \frac{\nu s^2}{\nu+2}$$

5) **Scaled - t - distribution**

$\theta \sim t_\nu(\mu, \sigma^2)$, $\nu > 0$ (degrees of freedom), $\mu \in \mathbb{R}$ (location), $\sigma > 0$ (scale)

If $\mu = 0$ and $\sigma = 1$ we have the usual Student distribution (t_ν) with ν degrees of freedom.

If $X \sim t_\nu \Rightarrow \theta = \mu + \sigma X \sim t_\nu(\mu, \sigma^2)$.

$$p(\theta) = \frac{\Gamma\left[\frac{\nu+1}{2}\right]}{\sigma\sqrt{\nu\pi}\,\Gamma(\nu/2)}\left[1+\frac{1}{\nu}\left(\frac{\theta-\mu}{\sigma}\right)^2\right], \theta \in \mathbb{R}$$

$$E(\theta) = \mu,\ \nu > 1$$

$$V(\theta) = \frac{\sigma^2\nu}{(\nu-2)},\ \nu > 2$$

$$\text{mode}(\theta) = \mu$$