Intro

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Introduction to Monte Carlo

Monte Carlo methods are methods for generating random variables directly or indirectly from target distributions. We generate random variables to estimate p-values or parameters.

Applications of monte carlo methods are in financial engineering and Bayesian computation.

An example of simulation: Gambler's ruin

Consider two gamblers, persons A and B, who start to gamble in a zero-sum game with stakes \$x and \$b-x, respectively. At each round, each gambler puts up a stake of \$h. The probability that A wins a round is p, while the probability that B wins a round is $q = 1 - p$. We wish to compute the probability that A ultimately wins the game. Let us define $v(x, t)$ to the probability that A ultimately wins the game starting with capital \$x on or before the tth round. Similarly, $u(x, t)$ is the probability that B wins the game with their stake of $b - x$ on or before the tth round.

Each of three variables v,u, and w is bounded below by zero and above by 1. Moreover, u and v are nondecreasing in t. w is nonincreasing in t. Thus we can take limits of each of these as t goes to infinity. We shall call these limits v(x), u(x), and w(x), respectively.

Gambler's ruin (fallacy) is the belief that a certain event is *more* likely to occur given the past history. In an experiment where there is a coin toss with probability of seeing heads as 0.5. Each flip of a coin has the same probability of landing on heads regardless of what the previous lands were.

Imagine a gambler on a roulete table. Say the gambler starts with \$10. In this game, the gambler "wins" when they earn a total of \$20 (that is they must play the game until they've earned \$10 on top of their starting \$10). For each game, there is a probability of winning, p = 0.473. Then, can we see how many turns until he/she wins (or loses)?

```
N = 200
income = 10
games = 2*(runif(N)<0.473) - 1 ## generate 1s and -1s
out = cumsum(games) + income

plot(1:N, out, type = "l", xlab = "games", ylab = "income")
abline(h = 0, col = "red")
```
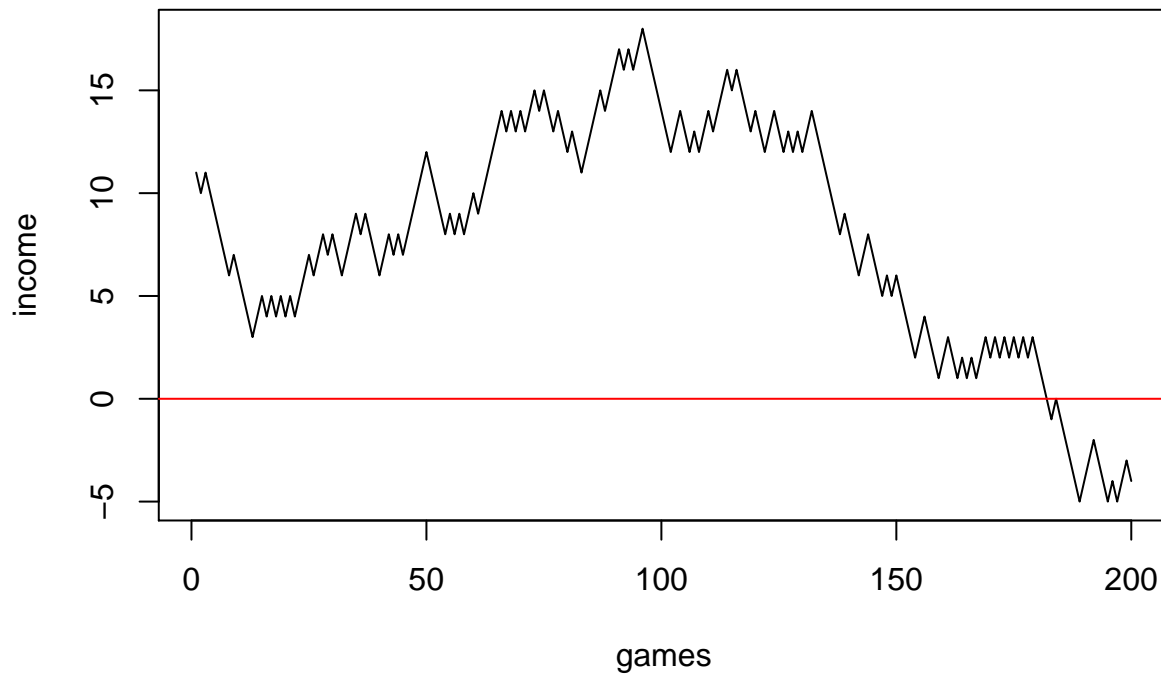
```r
GamblersRuin = function(i){
  income = 10
  n = 0
  while(!(income %in% c(0,20))){
    n = n + 1 ## number of runs till ruin or success
    x = runif(1)
    if(x <= 0.473){
      income = income + 1
    } else{
      income = income - 1
    }
  }
  return(c(n,income))
}

GamblersRuin()
```

```
## [1] 162    0
```

```r
out = lapply(X = 1:100, FUN = GamblersRuin)
out = do.call(rbind, out)

## percentage of success
sum(out[,2] == 20 )
```

```
## [1] 29
```

Bayesian Example

Here is an example taken from *Bayesian Ideas and Data Analysis* by Christensen et al.

$$y|\theta \sim Bin(2430, \theta) \ and \ \theta \sim Beta(12.05, 116.06)$$

This is a beta-binomial problem. There is a beta prior distribution on θ. Beta is conjugate to the binomial distribution (see: https://en.wikipedia.org/wiki/Conjugate_prior#Discrete_distributions). Bayesian anaysis uses prior information combined with observed data to update a probability distribution, posterior distribution, from which we can obtain a probability value. The new probability distribution, posterior, describes knowledge about the unkown parameter θ from historical beliefs (e.g. previous experiments, reports, etc.) and current observed data.

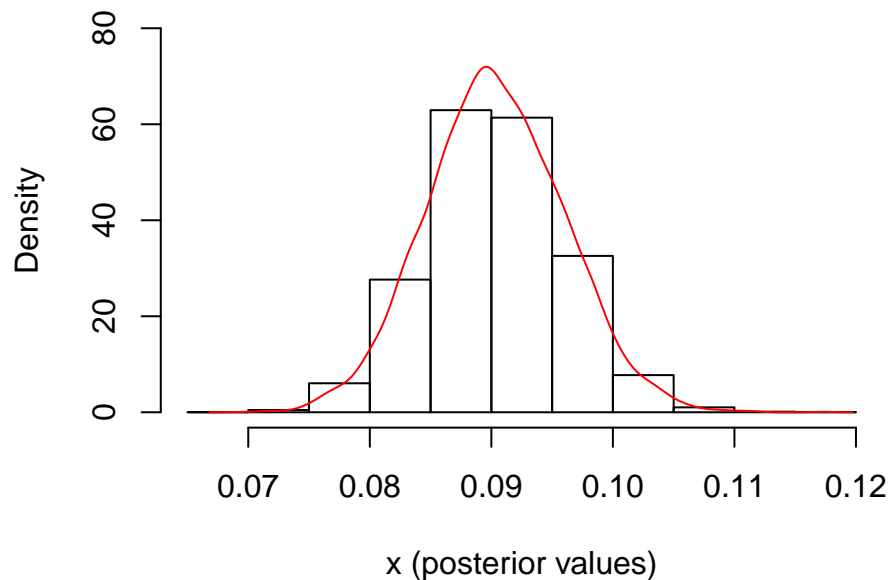$$y|\theta \sim Bin(n, \theta) \ and \ \theta \sim Beta(a, b)$$

The resulting posterior distribution is then

$$\theta|y \sim Beta(y + a, n - y + b)$$

We can now simulate the posterior distribution

```
N = 10^4
set.seed(123)
x = rbeta(n = N, shape1 = 219 + 12.05, shape2 = 2430 - 219 + 116.06)
d = density(x)
hist(x = x, probability = TRUE,
     main = "Beta Posterior Distribution",
     xlab = "x (posterior values)", ylab = "Density",
     ylim = c(0,80))
lines(x = d$x , y = d$y, type = "l", col = 2)
```

Beta Posterior Distribution


```
print("Median: ")
```

```
## [1] "Median: "
```

```
print(quantile(x = x, probs = c(0.025, 0.5, 0.975)))
```

```
##       2.5%        50%      97.5%
## 0.07942339 0.09018049 0.10164574
```

Conclusion

We can tell the VP that the true probability lies between 7.9% and 10.2%, with median probability of 9%.