Reflection Report

Nash equilibria, existence & complexity, and a global frontier reflection

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Abstract

This report consolidates three tasks: (1) a short NashPy exercise on the Prisoner's Dilemma with a public Colab notebook for replication; (2) formal statements for the Nash equilibrium definition and existence theorem, plus a computational-complexity note and a flowchart (Mermaid placeholder); (3) a comparative reflection on cloud platforms, regulatory approaches, and the role of liberal-arts training in responsible computational economics. All attributions use author–date citations handled by `biblatex` and compiled via `biber`.

1 Section 1: NashPy task — Prisoner's Dilemma

Game specification

Two players P_1 and P_2 with strategies $S_1 = S_2 = \{C, D\}$. We adopt the canonical ordering $T > R > P > S$ and set

$$T = 5, \quad R = 3, \quad P = 1, \quad S = 0.$$

Payoff matrices (row = P_1, column = P_2):

$$A = \begin{pmatrix} 3 & 0 \\ 5 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 5 \\ 0 & 1 \end{pmatrix}.$$

NashPy outcome

Using support enumeration (NashPy), the one-shot simultaneous game has a unique Nash equilibrium at (D, D) with payoff profile $(1, 1)$. Defection strictly dominates cooperation for both players, so each best-responds by choosing D with probability 1 and the unique pure equilibrium is (D, D). This presentation follows standard pedagogy (Osborne 2004) and encyclopedic exposition (Stanford Encyclopedia of Philosophy 1997).

Note (150 words). Because D strictly dominates C for each player, the unique Nash equilibrium is the pure profile (D, D). That profile is Pareto-inefficient relative to (C, C) since $R > P$; the example highlights the tension between individual incentives and collective welfare (Osborne 2004; Stanford Encyclopedia of Philosophy 1997). Replication is available via the public Colab notebook (Zhang 2025).

Replicability

Public Colab notebook: `https://colab.research.google.com/drive/1AQzxblcOnihCCeELw2Sltb0sTeDbFOp1?usp=sharing` (Zhang 2025).

2 Task 2: Existence & Complexity of Equilibria

2.1 Definition (quoted)

Nash's formal definition: an n-tuple s is an equilibrium point iff for every player i,

$$p_i(s) \;=\; \max_{r_i} p_i(r_i, s_{-i}),$$

i.e. no player can unilaterally increase her payoff by deviating from s (Nash 1951, p. 287).

Paraphrase (what / why)

- A profile is an equilibrium when each player is playing a best response to the other players' strategies.
- It is a unilateral-deviation stability condition: no single agent can gain by changing strategy alone.
- Mixed strategies (probability simplices) permit convexity required for fixed-point proofs.
- The definition specifies the precise mathematical object targeted by theoretical and computational analysis.

2.2 Existence theorem

Nash's Theorem: "Every finite game has an equilibrium point" (Theorem 1) (Nash 1951, p. 288). The standard proof reduces the problem to a Brouwer fixed-point argument, showing at least one (possibly mixed) equilibrium exists.

Paraphrase (what / why)

- Existence is guaranteed by a fixed-point argument but the proof is non-constructive.
- The theorem elevates equilibrium search to a total search problem—solutions are guaranteed to exist in finite games.
- The result underpins comparative-statics and welfare analysis across a broad class of models.
- Practically, the theorem motivates computational investigation into how to find or approximate equilibria.

2.3 Computational complexity statement

Exact Nash equilibrium computation is PPAD-complete (Daskalakis et al. 2009), meaning the general problem is unlikely to admit a polynomial-time algorithm. Hardness results apply even to two-player (bimatrix) games and inform choices of tractable substitutes (e.g., correlated equilibria).

Paraphrase (what / why)

- PPAD-completeness indicates worst-case algorithmic difficulty for computing exact Nash equilibria.
- The contrast between guaranteed existence and computational hardness motivates approximations, problem-structure exploitation, and alternative equilibrium notions.

- In applied settings, modelers must choose equilibrium concepts and algorithms with tractability in mind.

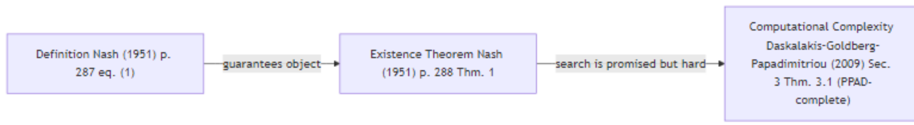

Figure 1: Flowchart

Concise reflection (<200 words)

Existence ensures that finite games admit at least one Nash equilibrium, providing a conceptual baseline for economic modelling and comparative statics (Nash 1951). Complexity results, however, show that finding such equilibria is PPAD-complete in the worst case (Daskalakis et al. 2009). This tension has two concrete consequences: (1) researchers exploit structural restrictions (potential games, convexity, monotone operators) to recover efficient algorithms; (2) practitioners rely on approximations, correlated equilibria, or domain-specific solvers when exact computation is infeasible. Consequently, computational economics must treat equilibrium computation as an algorithmic design problem—selecting the equilibrium notion, data representation, and solver are modelling choices with substantive implications.

3 Task 3: Global frontier reflection

Part A — Comparative reflection (collage placeholder)

I did not attend the field trip; the joint collage (placeholder below) pairs prominent cloud platforms—Amazon Web Services (AWS) and Tencent Cloud—which both provide scalable infrastructure for agent-based simulation, large-scale econometric pipelines, and managed ML services (Amazon Web Services (AWS) n.d. Tencent Cloud n.d.).

Figure 2: Joint AWS & Tencent Cloud collage

Promise and peril of computational game theory. Algorithmic game theory enables large-scale simulation and policy testing (e.g., market design, climate interventions) while also raising risks such as tacit algorithmic collusion and the amplification of biases. Responsible deployment requires governance frameworks and technical safeguards (UNESCO 2021; National Institute of Standards and Technology (NIST) 2023).

U.S. vs. Chinese approaches (short). U.S. frameworks emphasize voluntary, sector-engaged guidance (e.g., NIST's AI RMF) that organizations can adopt to manage AI risks (National Institute of Standards and Technology (NIST) 2023). China has pursued a combination of strategic guidance and prescriptive measures for algorithms and recommendation services (China 2022). Both approaches converge on trustworthy AI goals while differing in instruments and institutional design.

Ethical & SDG connections. Ethical AI governance bears on Sustainable Development Goals such as SDG 10 (reduced inequalities) and SDG 16 (peace, justice and strong institutions) through fairness, transparency, and accountable decision systems (United Nations 2015; UNESCO 2021).

Part B — Liberal arts & global leadership

How can we together do better? Cross-disciplinary collaboration and liberal-arts training cultivate critical reasoning and ethical literacy. These capacities help technologists surface normative assumptions in models and design decisions that promote equitable outcomes.

How liberal arts guides responsible innovation. The liberal-arts emphasis on context, history, and ethics supports dataset stewardship, interpretability, and stakeholder engagement, producing computational work that is both technically rigorous and socially aware.

DKU's joint model and global leadership. Duke Kunshan's integrated curriculum fosters cross-cultural and interdisciplinary problem solving, preparing graduates to deploy computational economics tools responsibly across varied regulatory and social contexts.

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