Data Representation

Learning Objectives

- Understand how different number systems can represent the same information

- Translate binary numbers to decimal, and vice versa

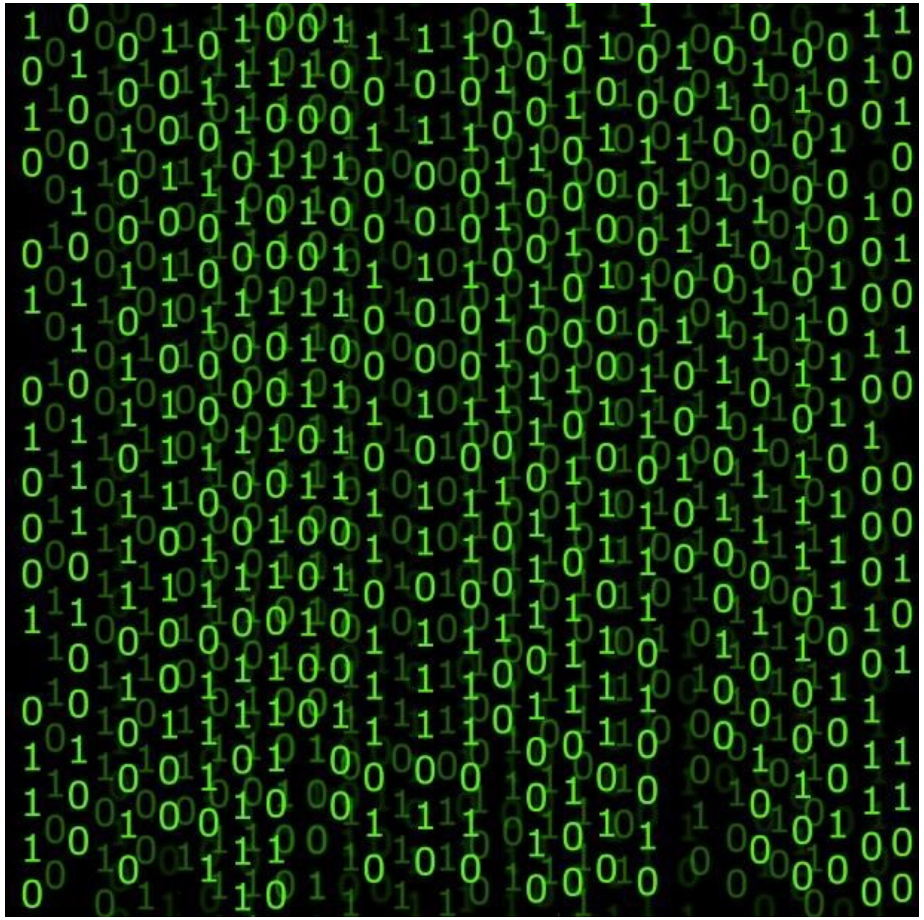
- Interpret binary numbers as abstracted types, including colors and text

Number Systems

Computers Run on 0s and 1s

Computers represent everything by using 0 and 1. You've likely seen references to this before.

How can we represent text, or images, or sound with 0 and 1?



Number Systems – Coins

A number system is a way of representing numbers using symbols.

One example of a number system is US currency. How much is each of the following symbols worth?

Penny 1 cent

Nickel 5 cents

Dime
10 cents

Quarter
25 cents

Number Systems – Dollars

Alternatively, we can represent money using dollars and cents, in decimal form.

For example, a medium coffee at Starbucks is $3.65.

Converting Coins to Dollars

We can convert between number systems by translating a value from one system to the other.

For example, the coins on the left represent the same value as $0.87

Using pictures is clunky. Let's make a new representation system for coins.

Coin Number Representation

To represent coins, we'll make a number with four digits.

The first represents quarters, the second dimes, the third nickels, and the fourth pennies.

c.3.1.0.2 =

3*$0.25 + 1*$0.10 + 0*$0.05 + 2*$0.01
=$0.87

	Q	D	N	P
c	3	1	0	2

Converting Dollars to Coins

Suppose you need to create an algorithm to convert money from dollars to coins, minimizing the number of coins used.

How did your algorithm work?

Conversion Example

What is $0.59 in coin representation?

$0.59 = 2*$0.25 + 0*$0.10 + 1*$0.05 + 4*$0.01 = c.2.0.1.4

Activity: Coin Conversion

Now try the following calculations with your discussion group:

What is c.1.1.1.2 in dollars?

What is $0.61 in coin representation?

Number Systems – Binary

Now let's go back to computers. We can represent numbers using only 0s and 1s with the binary number system.

Instead of counting the number of 1s, 5s, 10s, and 25s in coins you need, count the number of 1s, 2s, 4s, and 8s.

Why these numbers? They're powers of 2. This is a number in base 2, which only needs the digits 0 and 1. In contrast, our usual decimal system uses base 10 (with digits 0-9).

2^3 8	2^2 4	2^1 2	2^0 1
1	1	0	1

Bits and Bytes

When working with binary and computers, we often refer to a set of binary values used together to represent a number.

A single binary value is called a bit.

A set of 8 bits is called a byte.

We commonly use some number of bytes to represent data values.

Counting in Binary

0 =

128	64	32	16	8	4	2	1
0	0	0	0	0	0	0	0

1 =

128	64	32	16	8	4	2	1
0	0	0	0	0	0	0	1

2 =

128	64	32	16	8	4	2	1
0	0	0	0	0	0	1	0

3 =

128	64	32	16	8	4	2	1
0	0	0	0	0	0	1	1

4 =

128	64	32	16	8	4	2	1
0	0	0	0	0	1	0	0

5 =

128	64	32	16	8	4	2	1
0	0	0	0	0	1	0	1

6 =

128	64	32	16	8	4	2	1
0	0	0	0	0	1	1	0

7 =

128	64	32	16	8	4	2	1
0	0	0	0	0	1	1	1

Converting Binary to Decimal

To convert a binary number to decimal, just add each power of 2 that is represented by a 1.

For example, 00011000 = 16 + 8 = 24

128	64	32	16	8	4	2	1
0	0	0	1	1	0	0	0

Another example:

10010001 = 128 + 16 + 1 = 145

128	64	32	16	8	4	2	1
1	0	0	1	0	0	0	1

Converting Decimal to Binary

Converting decimal to binary uses the same process as converting dollars to coins.

Look for the largest power of 2 that can fit in the number and subtract it from the number. Repeat with the next power of 2, etc., until you reach 0.

For example, 36 = 32 + 4 = 00100100

128	64	32	16	8	4	2	1
0	0	1	0	0	1	0	0

Another example:

103 = 64 + 32 + 4 + 2 + 1

128	64	32	16	8	4	2	1
0	1	1	0	0	1	1	1

Activity: Converting Binary

Now try converting numbers on your own.

First: what is 01011011 in decimal?

Second: what is 75 in binary?