INDIAN INSTITUTE OF SCIENCE
भारतीय विज्ञान संस्थान

AEROSERVOELASTICITY LABORATORY

PANEL METHODS AND STATIC AERO ELASTIC BEHAVIOUR OF A 2D RIGID AIRFOIL

DIVERGENCE

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1. PANEL METHODS

2. DIVERGENCE

3. UNSTEADY INCOMPRESSIBLE POTENTIAL FLOWS

PANEL METHODS

Panel methods are ideal for calculating the flow field over an airfoil executing unsteady time-dependent motion in an inviscid incompressible medium.

- Hess and Smith(distinct constant source strength for each panel and one vortex of constant strength on each panel).
- Linear vortex (vortex strength on each panel varies linearly from one corner to the other and is continuous across the corner)

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PANEL GEOMETRY

- $\frac{x}{c} = \frac{1}{2}(cos\theta + 1), \theta = 0 - 2\pi$
- x/c begins at 1, passes through 0 and then goes back to 1.
- The numbering scheme is from lower trailing edge to lower leading edge, upper leading edge to upper trailing edge.

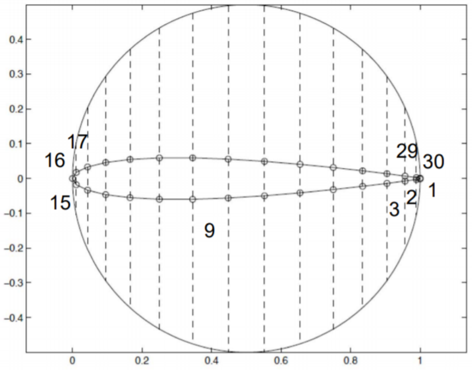

Figure 1.1: Airfoil section divided into 30 panels.

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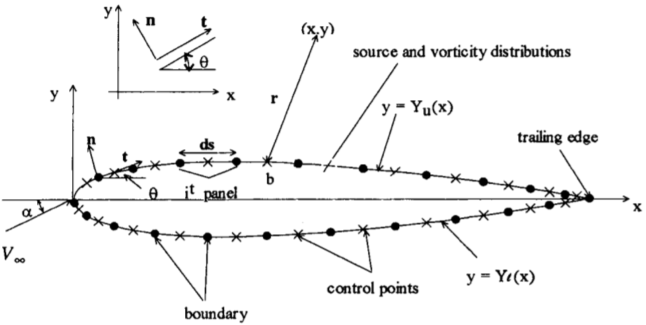
PANEL REPRESENTATION



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POTENTIAL EQUATION

- There are N source panels and N vortex panels.
- The vorticity on each panel is equal. So there are $N+1$ unknowns. (N source strengths and 1 vortex strength)
- The potential equation is:
  $\phi_{total} = \phi_{freestream} + \phi_{source} + \phi_{vortex}$, ie.
  $\phi(x,y) = U(xcos\alpha + ysin\alpha) + \sum_{j=1}^{m} \frac{\sigma}{2\pi} \int_0^{S_j} ln(r)ds_j - \sum_{j=1}^{m} \frac{\gamma}{2\pi} \int_0^{S_j} \theta ds_j$

BOUNDARY CONDITIONS

- The Neumann impermeability boundary condition applied at N panels yields N equations.

$$\frac{\partial \phi}{\partial n} = 0$$

BOUNDARY CONDITIONS

- The Kutta condition can be applied to this flow by enforcing that the pressures just above and just below the trailing edge must be equal.

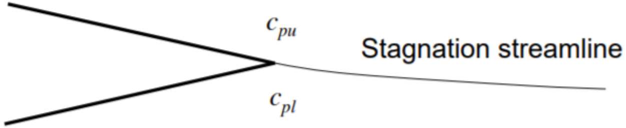
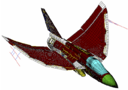


Figure 1.2: If the two pressures are not equal, then the stagnation streamline will wrap itself around the trailing edge.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BOUNDARY CONDITIONS

So,

$$C_{pu} = C_{pl}$$

Since the normal velocity on the surface is zero, vector sum of the two tangential velocities at the trailing edge must be equal to zero, ie,

$$V_N^t + V_1^t = 0$$

Applying these boundary conditions yields $N + 1$ equations which can be solved to find the $N + 1$ unknowns.

PANEL METHODS
PANEL GEOMETRY
HESS AND SMITH
POTENTIAL EQUATION
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LINEAR EQUATIONS
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FIXED PARAMETERS
EQUATIONS
ITERATIVE ANALYSIS
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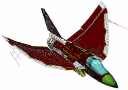
RESULTS
UNSTEADY INCOMPRESSIBLE POTENTIAL FLOWS
BOUNDARY CONDITIONS AND EQUATIONS
RESULTS



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LINEAR EQUATIONS

The boundary conditions can be converted into algebraic equations which can be written in terms of the unknowns, $\sigma_j (j = 1, 2, ..., N)$ (source strength at each panel) and Γ (constant vortex strength),

$$AX = B$$

A is a square matrix of order $(N + 1)$
X = $(\sigma_1, ..., \sigma_i, ..., \sigma_N, \Gamma)^T$
B = $(b_1, ..., b_i, ..., b_N, b_{N+1})^T$

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RESULTS

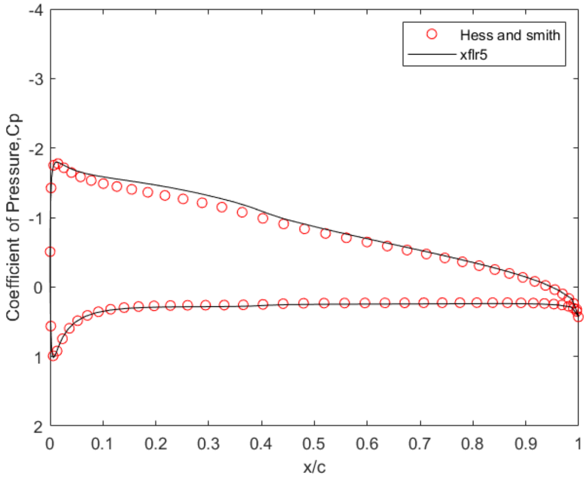


Figure 1.3: Comparison of the Cp plots for 80 panels and AOA = 5 deg for NACA 4412

RESULTS

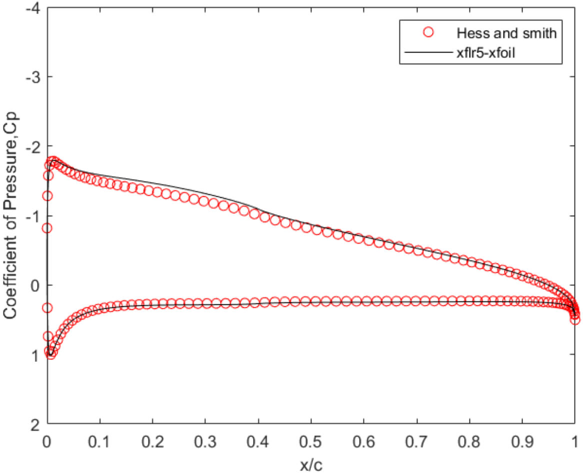
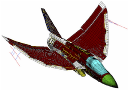


Figure 1.4: Comparison of the Cp plots for 160 panels and AOA = 5 deg for NACA 4412



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RESULTS

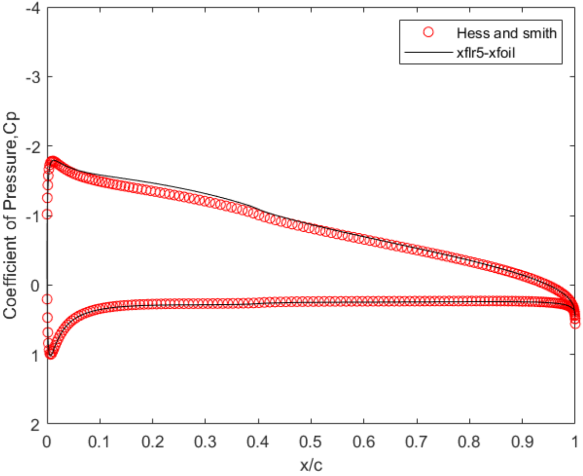


Figure 1.5: Comparison of the Cp plots for 300 panels and AOA = 5 deg for NACA 4412

RESULTS

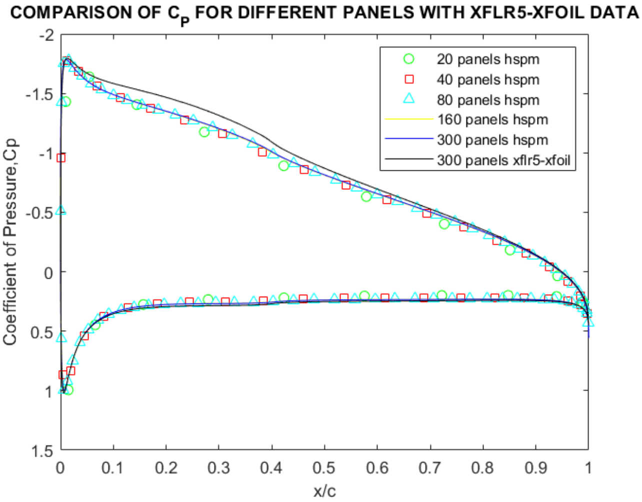
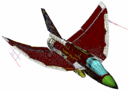


Figure 1.6: Comparison of the Cp plots for different panels and AOA = 5 deg for NACA 4412



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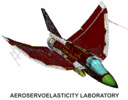
DIVERGENCE

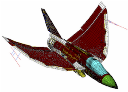
Divergence is the phenomenon that occurs when the moments due to aerodynamic forces overcome the restoring moments due to structural stiffness, so resulting in structural failure.
At a critical airspeed known as the torsional divergence speed, the incremental aerodynamic pitching moment is equal to the incremental elastic torsional restoring moment due to an elastic twist of the lifting surface.Above the divergence speed a static instability is created leading to divergence.
Infinite deflections are not possible, and in practice the structure will fail.

DIVERGENCE



PARAMETERS FIXED FOR THE STUDY OF DIVERGENCE

1. Torsional Stiffness = 3000 $Nrad^{-1}$
2. Density of air = 1 kgm^{-3}
3. Shear center was fixed at mid chord.
4. Rigid angle of attack = 5 degrees.
5. The airfoil is assumed to be symmetric.NACA 0012
6. Chord length = 1 m.
7. Hess and Smith panel method was used for computing the aerodynamic moment about shear centre.



EQUATIONS

- Restoring moment equation $M_e = K_\theta \theta$
  - M_e is the elastic restoring moment per unit span due to the elastic twist of the wing section.
  - K_θ is the torsional spring stiffness per unit span of the wing.

ITERATIVE ANALYSIS

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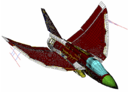
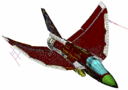
- For the first iteration the moment due to aerodynamic forces for the rigid angle of attack is calculated about the shear center.The moment for 2D airfoil is obtained from Hess and Smith panel method.
- This moment is then equated to the moment due to structural stiffness to obtain the twist.
- The new angle of attack is taken as sum of the initial rigid angle and the previously obtained twist.
- New moment is calculated from this angle of attack and the iteration continues until it converges.
- This convergence happens only below the divergence speed.
- The iterative analysis is carried out for different free stream velocities.

ANALYTIC CALCULATION OF THE DIVERGENCE SPEED



$$M = qec^2a_1(\theta_0 + \theta)$$

- M = Pitching moment on the airfoil about stiffness centre at initial angle of attack.
- ec = distance between the centre of pressure and stiffness centre.
- q = free stream dynamic pressure.
- a_1 = lift curve slope = 2π (symmetrical airfoil)
- θ_0 = initial angle of incidence.
- θ = unknown aero elastic twist.



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ANALYTIC CALCULATION OF THE DIVERGENCE SPEED

- Equating aerodynamic moment with restoring moment $M_e = K_\theta \theta$.
- Solving this equation for θ gives $\theta = \frac{qec^2a_1}{K_\theta - qec^2a_1}\theta_0$.
- Or $\theta = \frac{qR}{1-qR}\theta_0$.
- The elastic twist becomes infinite as q approaches $\frac{1}{R}$ and this defines the divergence speed as $q_{diV} = \frac{1}{R} = \frac{K_\theta}{ec^2a_1}$.
- So the elastic twist varies with dynamic pressure as $\theta = \frac{q/q_{diV}}{1-q/q_{diV}}\theta_0$.
- For the fixed parameters the divergence speed was calculated to be $61.8ms^{-1}$.

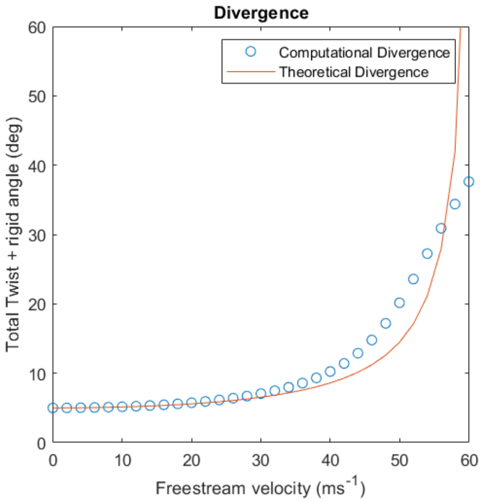
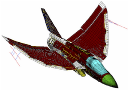


Figure 2.1: Comparison of theoretical and computational values for 8 panels

RESULTS



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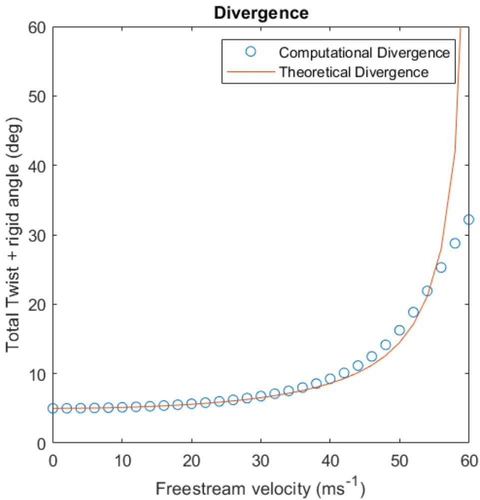

Figure 2.2: Comparison of theoretical and computational values for 16 panels

RESULTS

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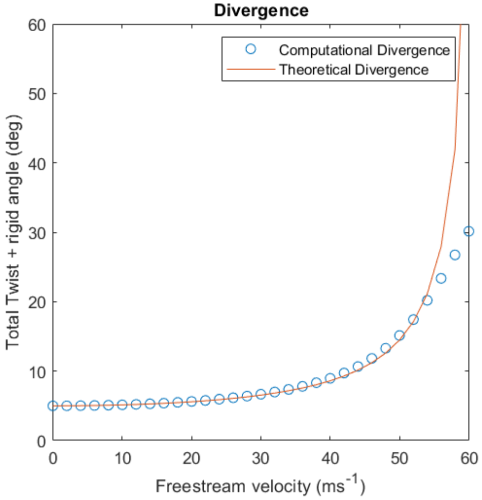

Figure 2.3: Comparison of theoretical and computational values for 100 panels

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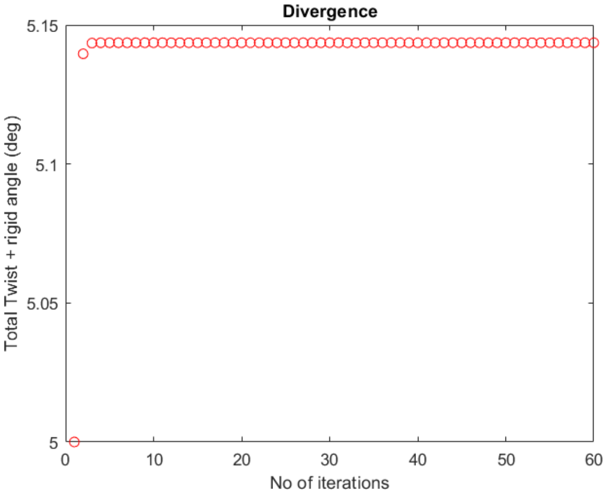
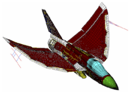


Figure 2.4: Converges after 4 iterations at $10ms^{-1}$ for 100 panels

RESULTS



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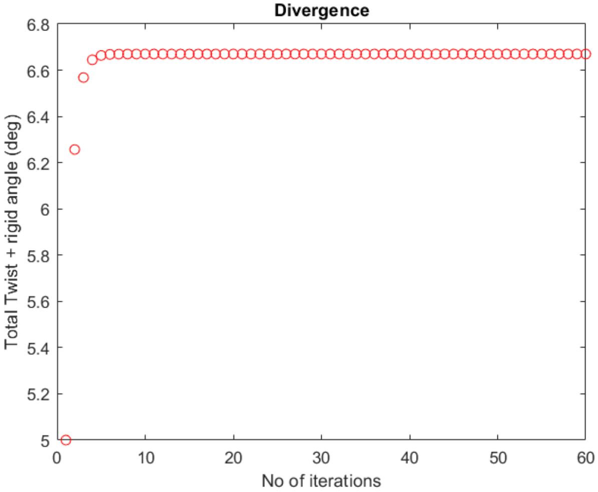

Figure 2.5: Converges after 9 iterations at $30ms^{-1}$ for 100 panels

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RESULTS

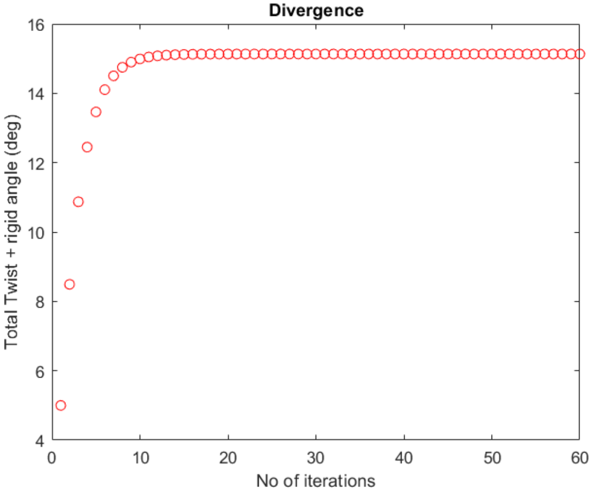

Figure 2.6: Converges after 26 iterations at $50ms^{-1}$ for 100 panels

SUDDEN ACCELERATION OF FLAT PLATE

- The flat plate consists of a single lumped-vortex element which is placed at the quarter chord and Kutta condition is assumed to be satisfied.
- A discrete vortex is selected to model the wake.
- The angle of attack is considered to be small. Here, $\alpha = 5deg$.
- The zero normal flow boundary condition is satisfied at the collocation point at the plate's three-quarter chord point.
- The Kelvin circulation theorem is applied to the vortices.
- The flat plate is suddenly accelerated to a constant velocity $U_\infty = 10ms^{-1}$.

SUDDEN ACCELERATION OF FLAT PLATE

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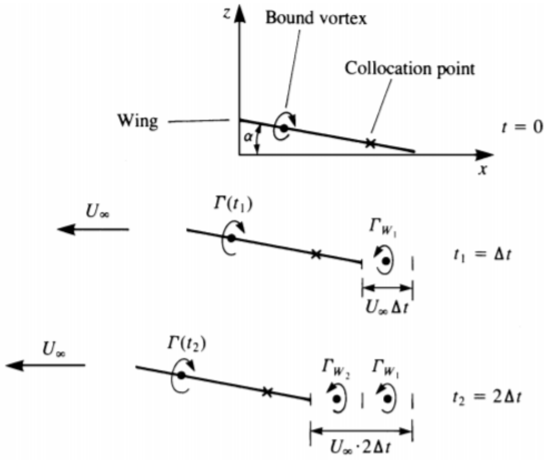
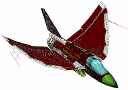


Figure 3.1: Development of wake after the motion of the flat plate for two time steps. $\Delta t = 0.025s$

BOUNDARY CONDITIONS AND EQUATIONS



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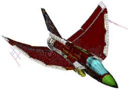
ZERO NORMAL FLOW

- For t_1:

$$\frac{-\Gamma(t_1)}{2\pi(c/2)} + \frac{\Gamma_{W1}}{2\pi[(c/4)+(U_\infty \Delta t/2)]} = -U_\infty \alpha$$

- For $t_2 = t_1 + \Delta t$:

$$\frac{-\Gamma(t_2)}{2\pi(c/2)} + \frac{\Gamma_{W2}}{2\pi[(c/4)+(U_\infty \Delta t/2)]} + \frac{\Gamma_{W1}}{2\pi[(c/4)+(U_\infty 3\Delta t/2)]} = -U_\infty \alpha$$



BOUNDARY CONDITIONS AND EQUATIONS

KELVIN CONDITION

- For t_1:

$$\frac{d\Gamma}{dt} = \Gamma(t_1) + \Gamma_{W1} = 0$$

- For $t_2 = t_1 + \Delta t$:

$$\Gamma(t_2) + \Gamma_{W2} + \Gamma_{W1} = 0$$

- Lift per unit span:

$$L' = \int_0^c \Delta p dx = \rho[U_\infty \Gamma(t) + \frac{\partial}{\partial t}\Gamma(t)c]$$

RESULTS

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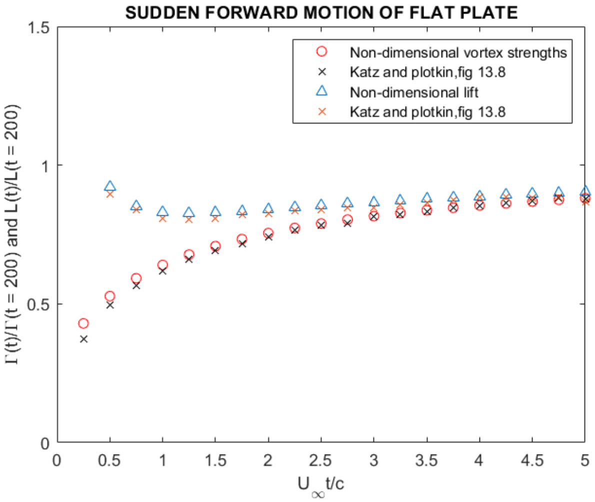

Figure 3.2: Variation of lift and circulation after the initiation of a sudden forward motion of a 2D flat plate

References I

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Katzplotkin-Low speed aerodynamics.
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Mark Drela-Flight Vehicle Aerodynamics.
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