SMOOTHED AGGREGATION AMG USING METIS

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1. Problem Statement. In smoothed aggregation-based AMG, the aggregation procedure often require a second pass to aggregate the leftover nodes. These leftover nodes are aggregated using different strategies like aggregating to the largest or smallest neighbouring cluster. The scope of this project is an exploratory investigation on whether METIS can be used to partition the graph of strongly connected nodes.

METIS is a set of programs for partitioning graphs, whose algorithms are based on multilevel k-way partitioning schemes and multilevel recursive bisection. They are faster and produce high quality partitions.[1] Using METIS as an aggregation technique could avoid nodes being left over when a conventional aggregation strategy is used and in turn avoid the second pass needed for such conventional strategies.

2. Approach. The implementation is similar to that of the Smoothed Aggregation based AMG (Algebraic Multi-Grid). The only change is that the graph of the strength of connection matrix obtained via symmetric strength measure with threshold $\theta = 0.15$ was partitioned using METIS to obtain the sparsity pattern. The strength of connection matrix has 1's for strongly connected nodes and 0's for the rest. In this case, the $||.||_\infty$ of strength matrix gives the size of the largest aggregate. In general, the sum of rows of strength matrix could give an estimate of size of aggregates. This could be useful since METIS requires the number of partitions to be specified prior to partitioning. "METIS for python" (python wrapper for METIS) was used for calling METIS library in python. For the purpose of comparison of the results between METIS and a conventional aggregation strategy, following approaches were adopted:

- The number of partitions required for METIS was taken from the number of aggregates produced by the conventional aggregation strategy.
- Only one level of AMG was used for solving the problems.

Weighted Jacobi was used as smoother with weight, $\omega = 2/3$. The iterations were run until residual fell below a tolerance of $1e-6$. The conventional aggregation strategy used for left over nodes was to aggregate it to the smallest neighbouring cluster.

3. Numerical Results. METIS uses two algorithms to partition a graph: multilevel recursive-bisection and multilevel k-way partitioning. K-way partitioning can be can done either such that it reduces the edge-cuts or minimizes the communication volume arising out of partitioning during parallel computing. These three schemes were tested on a 2D Poisson problem with Dirichlet boundary conditions. Figure 3.1 shows the partitioning of the initial strength matrix using these algorithms. From these figures it was observed that multilevel recursive bisection provided better aggregates compared to other METIS algorithms.

METIS's recursive bisection was used to solve a 2D Poisson problem on unstructured grid with Dirichlet boundary conditions and a 2D rotated anisotropic diffusion

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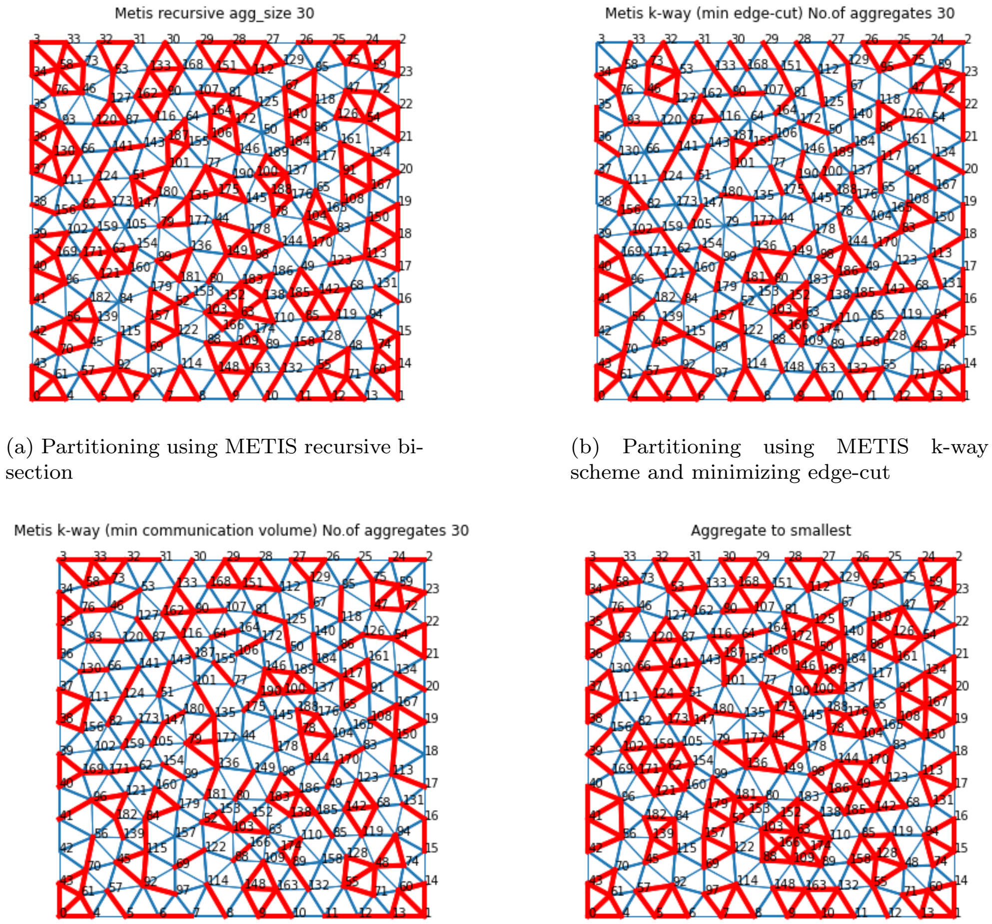


(a) Partitioning using METIS recursive bisection

(b) Partitioning using METIS k-way scheme and minimizing edge-cut

(c) Partitioning using METIS k-way scheme and minimizing communication volume.

(d) Partitioning by aggregating the left-over nodes to the smallest aggregate.

FIG. 3.1. *Aggregates formed by different partitioning algorithms.*

problem on structured grids. The later was obtained from pyamg.

3.1. 2D Poisson. A problem of 191 unknowns on an unstructured grid was solved using METIS's recursive bisection for a single level using 1 pre-smoothing and 0 post-smoothing of Jacobi. The same was done by using the conventional aggregation strategy. Figure 3.2a and Figure 3.2b shows the residual plot of these two cases. The iterations required for the residual to fall below a tolerance of 1e-6 was comparable for both aggregation strategies. Keeping everything else same, a test for 2 pre-smoothing and 0 post-smoothing was also done. Corresponding residual plots are shown in Figure 3.2c and Figure 3.2d. Both aggregation strategies showed improvement with increasing pre-smooth runs of weighted Jacobi, with conventional aggregation performing slightly better in terms of convergence factor.

Table 3.1 shows the convergence factors obtained for the above cases.

TABLE 3.1
Convergence factors for 2D Poisson problem

Pre-smooth runs	**Post-smooth runs**	**Convergence Factor (METIS)**	**Convergence Factor (Smallest Aggregation)**
1	0	0.828	0.809
2	0	0.705	0.679

3.2. 2D rotated anisotropic diffusion. An anisotropic diffusion problem ($\epsilon = 0.01$) with rotation angle 45° was solved on structured grids of size 32x32 and 16x16. For this problem two pre-smoothing and zero post-smoothing of weighted jacobi was applied. Figure 3.3a and Figure 3.3c shows the aggregates obtained for the conventional aggregation strategy and METIS respectively. Figure 3.3b and Figure 3.3d shows the corresponding residual plots. METIS was found to have a better convergence factor for 16x16 grid size. However, no such superiority in performance was seen for the case of 32x32 grid. Figure 3.4 shows the corresponding results for 32x32 grid.

Table 3.2 shows the convergence factors obtained for the above cases.

TABLE 3.2
Convergence factors for 2D rotated anisotropic problem

Grid Size	**Convergence Factor (METIS)**	**Convergence Factor (Smallest Aggregation)**
16x16	0.803	0.836
32x32	0.886	0.870

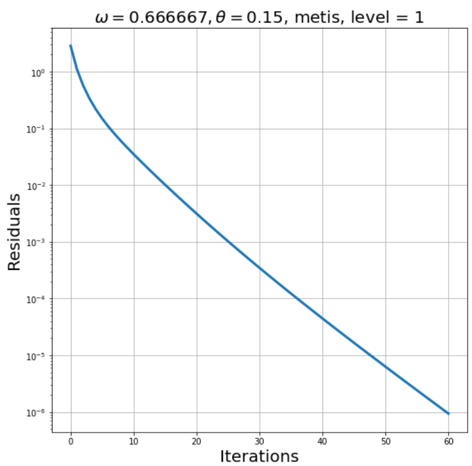
4. Conclusion. The results obtained indicate that the multilevel recursive bisection algorithm of METIS could potentially be used in Smoothed Aggregation based AMG. However, the quality of aggregates produced by recursive bisection could be poor in some cases. On close inspection of Figure 3.1a and Figure 3.1d it can be observed that some aggregates produced by METIS are lines rather than clusters. Since the user does not have control over how METIS partitions a graph, it is possible that for some problems such poor aggregates could dominate and cause poor convergence or maybe even divergence.

Though the current investigation only used one level of heirarchy, METIS can be made to work for multi-level AMG using a coarsening factor. An initial estimate of number of partitions required by METIS can be obtained as described in section 2. Using the initial estimate and coarsening factor, number of partitions required for the next level can be obtained and so on for further levels.

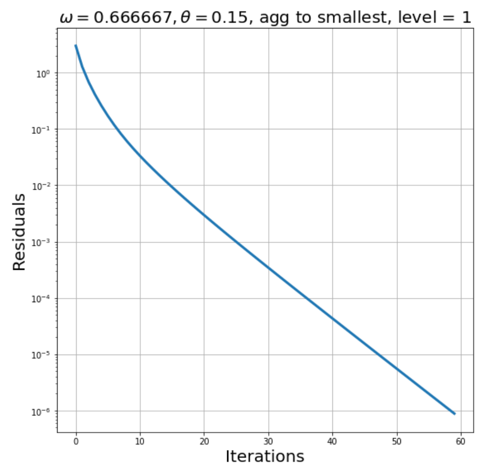
Current investigation is also limited to two kinds of test problems. Further exploration on METIS's viability as an aggregation technique could be carried out for different strength measures and different kinds for test problems.

REFERENCES

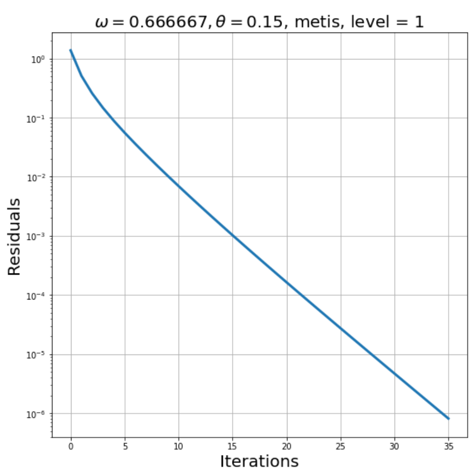
[1] G. KARYPIS AND V. KUMAR, *A fast and high quality multilevel scheme for partitioning irregular graphs*, SIAM Journal on Scientific Computing, 20 (1998), pp. 359–392, https://doi.org/10.1137/S1064827595287997.



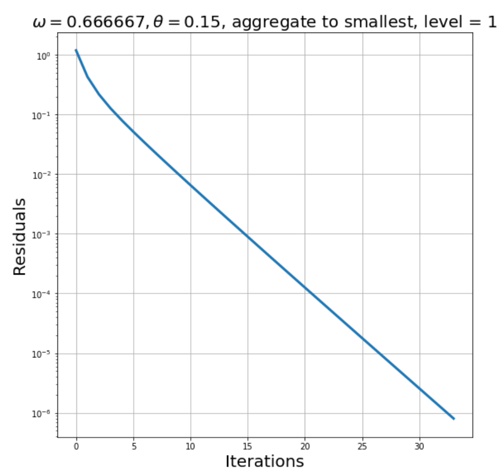
(a) Residual plot using METIS's recursive bisection. 1 pre-smoothing, 0 post-smoothing



(b) Residual plot using conventional aggregation strategy, 1 pre-smoothing, 0 post-smoothing.

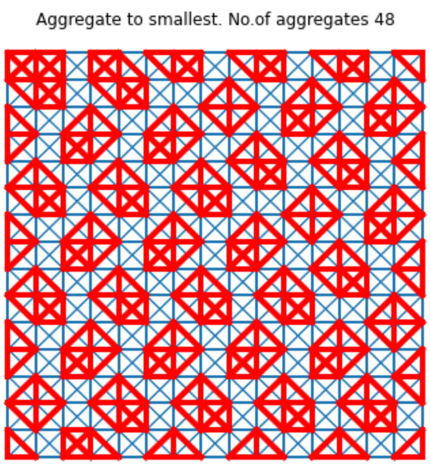


(c) Residual plot using METIS's recursive bisection, 2 pre-smoothing, 0 post-smoothing.

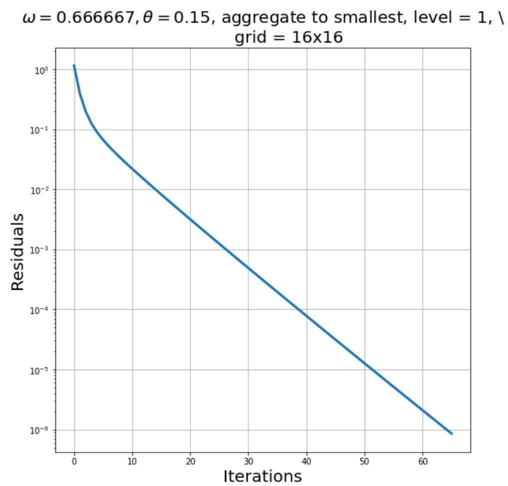


(d) Residual plot using conventional aggregation strategy, 2 pre-smoothing, 0 post-smoothing.

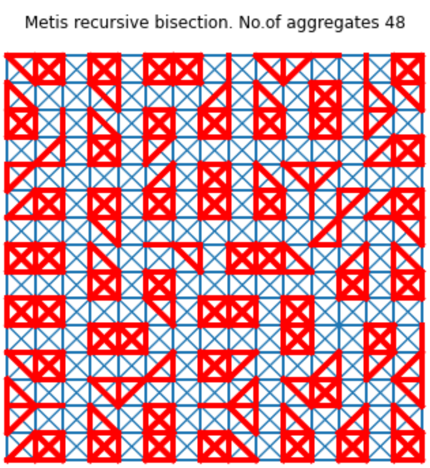
FIG. 3.2. *Residual plot for 2D poisson problem on unstructured grid.*



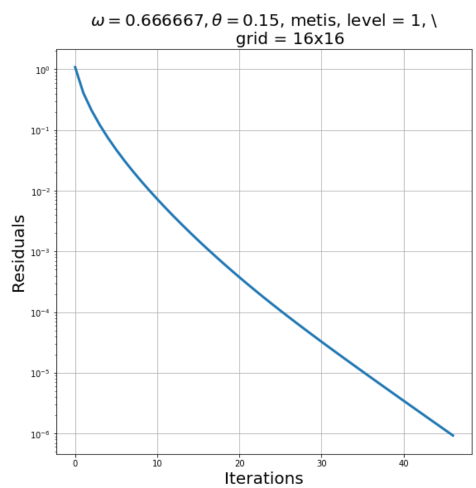
(a) Partitioning using conventional aggregation strategy.



(b) Residual plot for the conventional aggregation strategy.

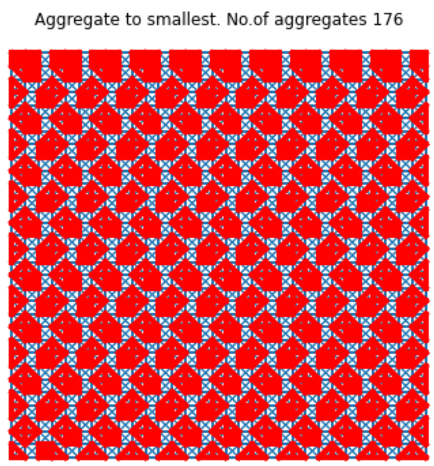


(c) Partitioning using METIS's recursive bisection.

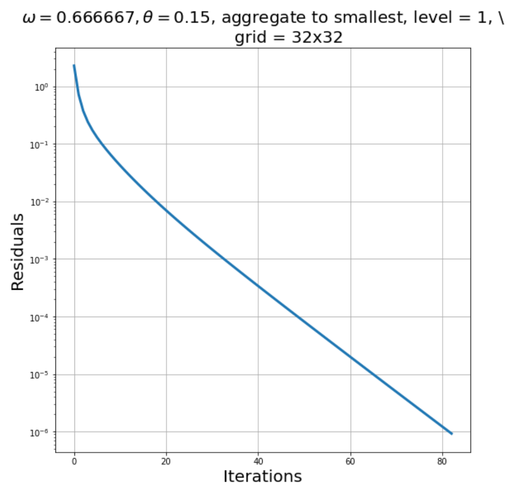


(d) Residual plot using METIS.

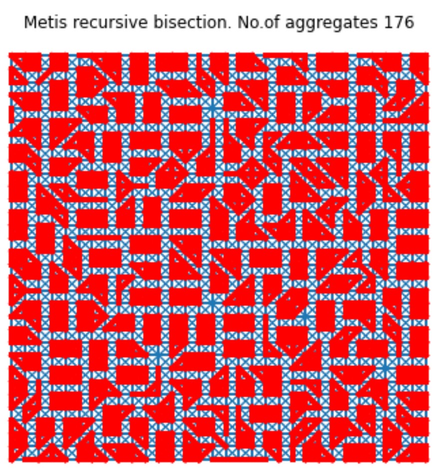
FIG. 3.3. *2D anisotropic diffusion problem on 16X16 structured grid.*



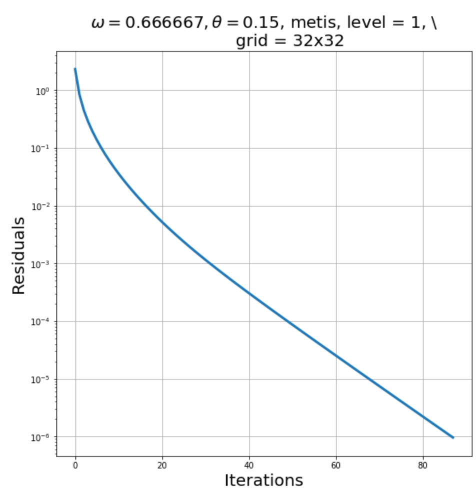
(a) Partitioning using conventional aggregation strategy.



(b) Residual plot for the conventional aggregation strategy.



(c) Partitioning using METIS's recursive bisection.



(d) Residual plot using METIS.

Fig. 3.4. *2D anisotropic diffusion problem on 32X32 structured grid.*