EE447 Mobile Internet Homework1

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1 Strategy

From the description, from A to B there exists n edges, and $n-1$ points, with no other branches
The strategy is to sort $(1-p)/c$ in descending order, such that the order in the checking list satisfies:

$$\frac{1-p_1}{c_1} \geq \frac{1-p_2}{c_2} \cdots \geq \frac{1-p_{n-1}}{c_{n-1}} \geq \frac{1-p_n}{c_n}$$

This strategy can meet the requirement that it will obtain the least expected checking cost.

2 Proof

Use Proof of contradiction, that no other strategies can be more inexpensive.

Suppose there exists one solution which obtain less expected costs, then there must exists at least one neighboring pair: $(p_j, c_j), (p_k, c_k)$: *s.t.* $j \leq k$, $\frac{1-p_j}{c_j} \geq \frac{1-p_k}{c_k}$. However in the new solution, their position has been switched, *i.e.* edge k is arranged before edge$j$. Now we compare the expeced costs between the two solutions.

The original expected mean value must conclude this term, *i.e* for edge j,

- if after testing there isn't one, then we should abort the testing since there cannot be a path from A to B;
- if after testing there is one, then we should go on testing edges in the sorted list.

So when it comes to edge j and edge$k$, there must exists such term:

$$\ldots \left[(1-p_j)c_j + p_j \Big[c_j + c_k + \ldots \Big] \right]$$

And the new solution must include for switching pairs:

$$\ldots \left[(1-p_k)c_k + p_k \Big[c_k + c_j + \ldots \Big] \right]$$

Do substraction between new solution and the proposed solution:

$$\Delta = c_k + p_k c_j - (c_j + p_j c_k) = c_j c_k \left(\frac{1-p_j}{c_j} - \frac{1-p_k}{c_k} \right) \geq 0$$

We can see that new solution can not generate a less expensive cost.

Now, we consider that is the swtiching apppears betwwen any pair instead of neighboring two edges. The situation is like bubble sort. We can start from the proposed solution to the new situation by switching neighboring pairs several times. So, there can't be better solutions, either.