Homework 9

Problem 1. *Show that, for constant $p \in (0, 1)$, almost no graph in $\mathcal{G}(n, p)$ has a separating complete subgraph.*

Solution. This is a simple application of the 'almost always true of property $P_{i,j}$'. $\square$

Problem 2. *What is the expected number of trees with k vertices in $G \in \mathcal{G}(n, p)$?*

Solution. By *Cayley's formula* and the linearity of expectation, it is $\binom{n}{k}k^{k-2}p^{k-1}$ $\square$

Problem 3. *Show that if almost all $G \in \mathcal{G}(n, p)$ have a graph property $\mathcal{P}_1$ and almost all $G \in \mathcal{G}(n, p)$ have a graph property $\mathcal{P}_2$, then almost all $G \in \mathcal{G}(n, p)$ have both properties.*

Solution. The portion of the graphs have both properties equals 1 minus the portion of the graphs which does not have property $\mathcal{P}_1$ or $\mathcal{P}_2$. However the portion of the graph does not have property $\mathcal{P}_1$ or $\mathcal{P}_2$ is bounded by the sum of the portion of the graphs does not have property $\mathcal{P}_1$ and the the portion of the graphs does not have property $\mathcal{P}_2$, which both tend to 0 as n approaches ∞. The claim in the question then follows. $\square$

Problem 4. *(Optional)*

1. *Prove that the threshold for the existence of cycles in $\mathcal{G}(n, p)$ is $p = \frac{1}{n}$.*
2. *Search the* World Wide Web *to find some real world graphs in machine readable form or data bases that could automatically be converted to graphs.*
   - *(a) Plot the degree distribution of each graph.*
   - *(b) Compute the average degree of each graph.*
   - *(c) Count the number of connected components of each size in each graph.*
   - *(d) Describe what you find.*
3. *Create a simulation (an animation) to show the evolution of the $\mathcal{G}(n, p)$ (Erdös-Rényi) random graph as its density p is gradually increased. Observe the phase transitions for trees of increasing orders, followed by the emergence of the giant component, etc.*