上　海　交　通　大　学　试　卷

（ 2019　至 2020　学年 第_2_学期 ）

班级号________________　学号__________　姓名 ________

课程名称 　计算机科学中的数学基础（CS499）　 成绩 ________

我承诺，我将严格遵守考试纪律。

承诺人：________

注意：一定要写清楚 姓名、学号、题号、诚信承诺。

答案发送到邮箱： sjtu_mfcs@163.com

(一) （10 分）

Let $le(X,\preccurlyeq)$ denote the number of linear extensions (线性扩充) of a partially ordered set （偏序集）$(X,\preccurlyeq)$. Prove:

(1) $le(X,\preccurlyeq)=1$ if and only if $\preccurlyeq$ is a linear ordering;

(2) Can $le(X,\preccurlyeq)>n!$ （where $n=|x|$）? Why?

(二) （10 分）

(1) How many ways are there to arrange 4 Americans, 3 Russians, and 5 Chinese into a queue, in such a way that no nationality（国籍）forms a single consecutive block?

(2) Express the nth term (第n项) of the sequences given by the following recurrence relation: $a_0=0,\ a_1=1,\ a_{n+2}=4a_{n+1}-4a_n\ \ (n=0,1,2,\ldots)$.

(三) （10 分）

Find two non-isomorphic(非同构) trees with the same score. (Note: Please give their score and briefly explain why they are not isomorphic.)

(四) （10 分）

A (m,n)-*barbell graph* is obtained by taking a complete graph (既：完全图/团/clique) on m labelled nodes and a complete graph on n labelled nodes, and connecting them by a single edge $(m,n\geq 1)$. （本题假定每个点的 label 都不一样）

(1) For a complete graph with $m+n$ labelled nodes K_{m+n}, how many different (m,n)-*barbell graphs* can you find which are subgraphs of K_{m+n}?

(2) For a given (m,n)-*barbell graph* , what is the number of spanning trees (生成树) of it?

(五) （20 分）

A fixed point（不动点） of a permutation（置换） $\pi:\{1,\ldots,n\}\to\{1,\ldots,n\}$ is a value for which $\pi(x)=x$. Find the (1) expectation, and (2) variance（方差）in the number of fixed points of a permutation chosen uniformly at random from all permutations.

(六) （20 分）

In the random graph model $G(n,p)$, suppose that $p = f(n)$. Show that

(1) If $f(n) = o(n^{-2/3})$, then for any $\epsilon > 0$ and for sufficiently large n, the probability that a random graph chosen from $G(n,p)$ has a clique of 4 or more vertices is less than ϵ.

(2) If $f(n) = \omega(n^{-2/3})$, then for sufficiently large n, the probability that a random graph chosen from $G(n,p)$ does not have a clique with 4 or more vertices is less than ϵ.

(七) （20 分）

In $G(n,p)$ model, we have introduced the so-called *increasing property,* which is: 'The probability of a graph having the property increases as edges are added to the graph. Such a property is called an increasing property.' We can similarly define *decreasing property* for random graph as: 'The probability of a graph having the property decreases as edges are added to the graph. Such a property is called an increasing property.'

(1) Give two non-trivial (非平凡) examples which have the decreasing property and explain why they are 'decreasing'. (Note by 'trivial（平凡）property' we mean a property which every graph has, or a property which no graph has.)

(2) Prove: Every non-trivial increasing/decreasing graph property has a threshold. （请注意，此题回答时应给出所有方向的完整证明。）