

## Annual Worth

# Goals

- ▶ Present AW as a frequently preferred measure of costs and benefits.
- ▶ Show details of AW calculation.
- ▶ Give warnings about interpretation
- ▶ Show a computational advantage in problems of repeated purchases.

$$AW(Asset) = PW(Asset)(A|P, i, \text{Life of Asset})$$

- ▶ AW is a transformation of present worth.
- ▶ AW is, except in one edge case, smaller than PW.
  - ▶ The edge case is an asset that is installed at time zero and only lasts through time 1.
  - ▶  $(A|P, i, 1) = \frac{i(1+i)}{(1+i)-1} = (1+i)$
- ▶ Many synonyms
  - ▶ Levelized cost
  - ▶ Capitalized cost (Danger PW is sometimes called this too)
  - ▶ Equivalent annual cost

## Simple Example

The car costs 10K and will last five years. What is the per-period cost of the car when the MARR is 10%?

$$10K(A|P, i = 10\%, 5) = 2.64K$$

- ▶ The purchase price – now – is a present worth.
- ▶ Notice that it is not  $\frac{10K}{5}$ . It does take into account that you paid for the car in time zero.

# The AW Criteria

- ▶ Annual Worth
  - ▶ Per-period costs and benefits (\$)
  - ▶ Unconstrained: If  $AW \geq 0$  get it.
  - ▶ Exclusive: If assets have the same life, choose asset with largest AW.

# Unconstrained Choice

If  $AW \geq 0$  get it.

$$AW(Asset) = PW(Asset)(A|P, i, Life\ of\ Asset)$$

- ▶ Note that  $(A|P, i, Life\ of\ Asset)$  is always positive.
- ▶ That means  $PW \geq 0 \Rightarrow AW \geq 0$

## Exclusive Choice

If assets have the same life, choose asset with largest AW.

- ▶ The “If” is critical
- ▶ Example why

Year	A	B
0	0	0
1	10	9
2		9
3		9
4		9
5		9
AW	10	9

- ▶ With moderate MARR, B is better but has lower AW.

# The If

- ▶ It is less restrictive than you think and often useful.
- ▶ You can construct assets from other assets.
- ▶ Example  $\{\text{Car, Kayak Rack, Kayaks}\}$ 
  - ▶ Doesn't make sense to get a rack without a car or kayaks.
  - ▶ Re-frame as exclusive choice:
    - ▶  $A = \{\text{Car}\}$
    - ▶  $B = \{\text{Car, Kayaks}\}$
    - ▶  $C = \{\text{Car, Rack, Kayaks}\}$



## How this helps with repeated purchases

- ▶ Remember the Roof Example?
  - ▶ Metal Roof (Like the old one but 'done correctly') would last 40 years and cost \$1.5M.
  - ▶ PVC Roof would last 20 years and cost 800K to install. It also requires 15K in additional inspection and maintenance in years 16-20.
- ▶ These have unequal lives but you can use AW by constructing a new asset
  - ▶  $A = \{\text{Metal now}\}$
  - ▶  $B = \{\text{PVC now, PVC in 20 years}\}$

# The Metal Roof

What is the levelized cost of the \$1.5M metal roof when the MARR is 10%?

## Answer

$$1500K(A|P, i = .1, 40) = 153.39K$$

Interpret this as the annual cost of roofing services.

## The PVC Roof

What is the levelized cost of the the two PVC roofs when the MARR is 10%?

PW of the first roof is:

$$PW(PVC) = 800K + \frac{15K(P|A, i = .1, 5)}{(1 + .1)^{15}} = 813.61K$$

## Trick Question

The answer is the same if you make the calculation with two roofs or one.

- ▶ One Roof: \$ 813.61K ( $A|P, i = 10\%, 20$ ) = 95.57K\$
- ▶ Two Roofs:
  - ▶  $PW(PVC|40\text{ Years}) = 813.61K + \frac{813.61K}{1.1^{20}} = 934.55K$
  - ▶  $AW(PVC|40\text{ Years}) = 934.55K(A|P, i = 10\%, 40) = 95.57K$

This is the computational advantage of  $AW$  in the case of repeated purchases.

# Repeated Purchases

- ▶ AW provides a computational shortcut when there are repeated purchases.
  - ▶ If you buy one asset and then replace it with an identical one.
  - ▶ Like replacing lamps in fixtures.
- ▶ Only works if the asset lives are factors of the planning horizon.
- ▶ I usually call these conforming assets.
- ▶ Example:
  - ▶ Planning horizon of 12 years.
  - ▶ Options have lives of 3, 4, 2, and 6 years.

## Repeated Purchases (Con't)

As long as assets are factors of the planning horizon:

$$AW(Asset|Single) = AW(Asset|Repeated)$$

# The planning horizon

Often times you are not given an explicit planning horizon.

- ▶ I've done this to you with abstract assets. The planning horizon is implicitly the life of the longest lived asset.
- ▶ Planning horizon is 2 years.

Year	A	B
0	0	0
1	10	0
2	0	10



# Common Practice

- ▶ Common practice is to set the planning horizon equal to the least common multiple of the asset lives.
  - ▶ Asset lives of 2, 3 and 4 years
  - ▶ Planning horizon of 12.
- ▶ Same as result as infinite horizon without the limits.

## Comments on Common Practice

- ▶ The planning horizon should be reasonable.
- ▶ Backyard Aquaculture Story:
  - ▶ Two potential pumps: 3 and 4 year pumps
  - ▶ Planning horizon 12 years.
  - ▶ If 8 and 9 years? I'm not living another 72 years.
- ▶ Rounding a few years on asset lives often does not matter much for long-lived investments.
  - ▶  $(P|A, i = 1., 100) = 9.9992743$
  - ▶  $(P|A, i = 1., 101) = 9.9993403$
- ▶ Do the math anyway, you are billing, but it probably won't change the decision.

# Common Practice for Non-conforming Assets

- ▶ Two general options:
  - ▶ Repeat purchases followed by an equipment lease for the remaining time.
  - ▶ Repeat purchases followed by salvage/sale of asset before end of life.
- ▶ Calculate PW based on these patterns and convert to AW.
- ▶ Example: 5 Year asset and 12 year planning horizon (Cost Convention, not Net Benefit)
  - ▶ Lease:  $PW = PW(\text{Asset}) + \frac{PW(\text{Asset})}{(1+i)^5} + \frac{Lease(P|A,i,2)}{(1+i)^{10}}$
  - ▶ Salvage:  $PW = PW(\text{Asset}) + \frac{PW(\text{Asset})}{(1+i)^5} + \frac{PW(\text{Asset})}{(1+i)^{10}} - \frac{Salvage}{(1+i)^{12}}$

# Backyard Aquaculture

The idea was to add Tilapia to the composting and garden cycle.  
MARR of 10%

- ▶ Choice of two pumps to keep water moving.
  - ▶ Pump 1: 7 year life, \$100.
  - ▶ Pump 2: 5 year life, \$75.
- ▶ Calculate the AW
- ▶ Choose the best asset to provide this service.

# Answer

- ▶ Note implied planning horizon is 35 years.
- ▶ AW Calculations
  - ▶  $\$AW(\text{Pump 1}) = 100 (A|P, i = .1, 7) = 20.54$
  - ▶  $\$AW(\text{Pump 2}) = 75 (A|P, i = .1, 5) = 19.78$
- ▶ Interpretation
  - ▶ Pump 1 provides pumping services at a cost of \$20.54 per year.
  - ▶ Pump 2 provides pumping services at a cost of \$19.78 per year.

Notice that Pump 2 provides pumping services at the lowest cost.

## You Can Do PW too

PW Calculations for 35 Year planning horizon.

- ▶  $PW(Pump\ 1|35\ Years) = 100 + \frac{100}{1.1^7} + \frac{100}{1.1^{14}} + \frac{100}{1.1^{21}} + \frac{100}{1.1^{28}} = 198.10$
- ▶ Will use the time compression trick.
  - ▶ Effective 5-year rate:  $(1 + \frac{.1}{1})^5 - 1 = 0.6105$
  - ▶  $PW(Pump\ 2|35\ Years) = 75 + \frac{75(P|A, i=0.6105, 6)}{(1+0.6105)} = 146.91$

Notice that Pump 2 provides pumping services at the lowest cost.

# Summary

- ▶ AW and PW give consistent choice but by different means.
- ▶ AW provides a computational advantage when asset lives are factors of planning horizon.
- ▶ Clients often find AW, per-period costs and benefits, easier to interpret than PW, life-cycle costs and benefits.