Annual Worth

Goals

- Present AW as a frequently preferred measure of costs and benefits.
- Show details of AW calculation.
- Give warnings about interpretation
- Show a computational advantage in problems of repeated purchases.

AW

$$AW(Asset) = PW(Asset)(A|P, i, Life\ of\ \ Asset)$$

- AW is a transformation of present worth.
- AW is, except in one edge case, smaller than PW.
  - The edge case is an asset that is installed at time zero and only lasts through time 1.
  - $(A|P, i, 1) = \frac{i(1+i)}{(1+i)-1} = (1 + i)$
- Many synonyms
  - Levelized cost
  - Capitalized cost (Danger PW is sometimes called this too)
  - Equivalent annual cost

Simple Example

The car costs 10K and will last five years. What is the per-period cost of the car when the MARR is 10%?

$$10K(A|P, i = 10\%, 5) = 2.64K$$

- The purchase price – now – is a present worth.
- Notice that it is not $\frac{10K}{5}$. It does take into account that you paid for the car in time zero.

The AW Criteria

- Annual Worth
  - Per-period costs and benefits ($)
  - Unconstrained: If $AW \geq 0$ get it.
  - Exclusive: If assets have the same life, choose asset with largest AW.

Unconstrained Choice

If $AW \geq 0$ get it.

$$AW(Asset) = PW(Asset)(A|P, i, Life\ of\ Asset)$$

- Note that $(A|P, i, Life\ of\ Asset)$ is always positive.
- That means $PW \geq 0 \Rightarrow AW \geq 0$

Exclusive Choice

If assets have the same life, choose asset with largest AW.

- The "If" is critical
- Example why

Year	A	B
0	0	0
1	10	9
2		9
3		9
4		9
5		9
AW	10	9

- With moderate MARR, B is better but has lower AW.

The If

- It is less restrictive than you think and often useful.
- You can construct assets from other assets.
- Example {Car, Kayak Rack, Kayaks}
  - Doesn't make sense to get a rack without a car or kayaks.
  - Re-frame as exclusive choice:
    - A = {Car}
    - B = {Car, Kayaks}
    - C = {Car, Rack, Kayaks}

How this helps with repeated purchases

- Remember the Roof Example?
  - Metal Roof (Like the old one but ‘done correctly’) would last 40 years and cost \$1.5M.
  - PVC Roof would last 20 years and cost 800K to install. It also requires 15K in additional inspection and maintenance in years 16-20.
- These have unequal lives but you can use AW by constructing a new asset
  - A = {Metal now}
  - B = {PVC now, PVC in 20 years}

The Metal Roof

What is the levelized cost of the \$1.5M metal roof when the MARR is 10%?

Answer

$$1500K(A|P, i = .1, 40) = 153.39K$$

Interpret this as the annual cost of roofing services.

The PVC Roof

What is the levelized cost of the the two PVC roofs when the MARR is 10%?

PW of the first roof is:

$$PW(PVC) = 800K + \frac{15K(P|A, i = .1, 5)}{(1 + .1)^{15}} = 813.61K$$

Trick Question

The answer is the same if you make the calculation with two roofs or one.

- One Roof: \$ 813.61K (A|P, i = 10%, 20) = 95.57K\$
- Two Roofs:
  - $PW(PVC|40Years) = 813.61K + \frac{813.61K}{1.1^{20}} = 934.55K$
  - $AW(PVC|40Years) = 934.55K(A|P, i = 10\%, 40) = 95.57K$

This is the computational advantage of AW in the case of repeated purchases.

Repeated Purchases

- AW provides a computational shortcut when there are repeated purchases.
  - If you buy one asset and then replace it with an identical one.
  - Like replacing lamps in fixtures.
- Only works if the asset lives are factors of the planning horizon.
- I usually call these conforming assets.
- Example:
  - Planning horizon of 12 years.
  - Options have lives of 3, 4, 2, and 6 years.

Repeated Purchases (Con't)

As long as assets are factors of the planning horizon:

$$AW(Asset|Single) = AW(Asset|Repeated)$$

The planning horizon

Often times you are not given an explicit planning horizon.

- I've done this to you with abstract assets. The planning horizon is implicitly the life of the longest lived asset.
- Planning horizon is 2 years.

Year	A	B
0	0	0
1	10	0
2	0	10

Common Practice

- Common practice is to set the planning horizon equal to the least common multiple of the asset lives.
  - Asset lives of 2, 3 and 4 years
  - Planning horizon of 12.
- Same as result as infinite horizon without the limits.

Comments on Common Practice

- The planning horizon should be reasonable.
- Backyard Aquaculture Story:
  - Two potential pumps: 3 and 4 year pumps
  - Planning horizon 12 years.
  - If 8 and 9 years? I'm not living another 72 years.
- Rounding a few years on asset lives often does not matter much for long-lived investments.
  - $(P|A, i = 1., 100) = 9.9992743$
  - $(P|A, i = 1., 101) = 9.9993403$
- Do the math anyway, you are billing, but it probably won't change the decision.

Common Practice for Non-conforming Assets

- Two general options:
  - Repeat purchases followed by an equipment lease for the remaining time.
  - Repeat purchases followed by salvage/sale of asset before end of life.
- Calculate PW based on these patterns and convert to AW.
- Example: 5 Year asset and 12 year planning horizon
  - Lease: $PW = PW(Asset) + \frac{PW(Asset)}{(1+i)^5} + \frac{Lease(P|A,i,2)}{(1+i)^{10}}$
  - Salvage: $PW = PW(Asset) + \frac{PW(Asset)}{(1+i)^5} + \frac{PW(Asset)}{(1+i)^{10}} + \frac{Salvage}{(1+i)^{12}}$

Backyard Aquaculture

The idea was to add Tilapia to the composting and garden cycle.
MARR of 10%

- Choice of two pumps to keep water moving.
  - Pump 1: 7 year life, $100.
  - Pump 2: 5 year life, $75.
- Calculate the AW
- Choose the best asset to provide this service.

Answer

- Note implied planning horizon is 35 years.
- AW Calculations
  - \$AW(Pump 1) $= 100\ (A|P, i = .1, 7) = 20.54$
  - \$AW(Pump 2) $= 75\ (A|P, i = .1, 5) = 19.78$
- Interpretation
  - Pump 1 provides pumping services at a cost of \$20.54 per year.
  - Pump 2 provides pumping services at a cost of \$19.78 per year.

Notice that Pump 2 provides pumping services at the lowest cost.

You Can Do PW too

PW Calculations for 35 Year planning horizon.

- $PW(Pump\ 1|35\,Years) = 100 + \frac{100}{1.1^7} + \frac{100}{1.1^{14}} + \frac{100}{1.1^{21}} + \frac{100}{1.1^{28}} = 198.10$
- Will use the time compression trick.
  - Effective 5-year rate: $(1 + \frac{.1}{1})^5 - 1 = 0.6105$
  - $PW(Pump\ 2|35\,Years) = 75 + \frac{75(P|A, i=0.6105, 6)}{(1+0.6105)} = 190.81$

Notice that Pump 2 provides pumping services at the lowest cost.

Summary

- AW and PW give consistent choice but by different means.
- AW provides a computational advantage when asset lives are factors of planning horizon.
- Clients often find AW, per-period costs and benefits, easier to interpret than PW, life-cycle costs and benefits.