New Environments

Goals

Our Environments

- Unconstrained Choice: Pick the *acceptable* assets.
- Exclusive Choice: Pick the *best* asset.
- *Increasing Cost of Funds*: Pick acceptable assets when you face many interest rates.
- *Capital Budgeting*: Pick the best group of assets when limited on how much you can spend.

There is a general statement of the problem

- Our environments can be interpreted as a mixed integer programming problem.
- You get the good stuff in ETM 535/635, Advanced Engineering Economics
- The solutions are NP-Hard
  - Translate from technical, adding more options requires more time to find a solution than adding the previous option.
  - Except for trivial problems, you need a heuristic.
- Heuristics are specialized rules and techniques that find a solution.
  - Simulated annealing
  - Ant Colony optimization
  - Genetic Algorithms

The optimization algorithms are fun to use

- Simulated Annealing
  - Random bounces in parameter space.
  - More likely to keep those that have smaller evaluations of the objective function than large.
  - Sizes of bounces get smaller over time.
  - Wikipedia has a great graphic `https://en.wikipedia.org/wiki/Simulated_annealing#/media/File:Hill_Climbing_with_Simulated_Annealing.gif`
- Genetic Algorithms
  - Like simulated annealing but
  - Many solutions tried at once.
  - They compete and merge with each other
  - I used these for stochastic dynamic programming problems.

Not Doing Any of that

- We will be using some specialized heuristics to find the best sets of assets in the new environments.
- The two/three heuristics have some fragile points.
  - Increasing cost of funds has trouble when an indivisible asset (defined later), that could be funded by a blend of funds from two sources is rejected and the next asset can be funded by a single source of funds.
  - The capital budgeting problem is exhaustive search and the solution requires $2^N - 1$ comparisons.

Increasing Cost of Funds

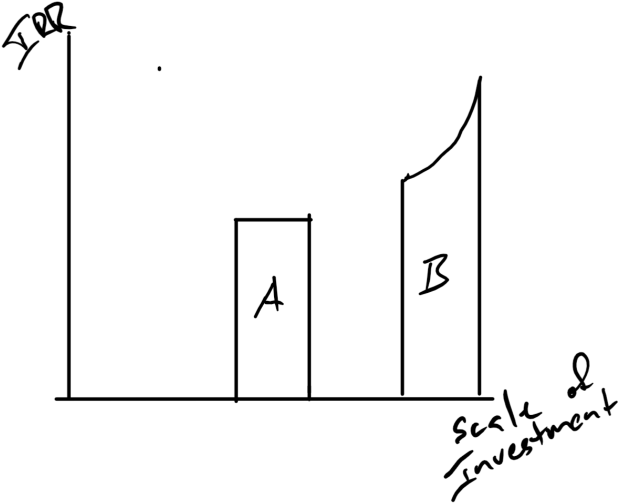
- This is an environment similar to unconstrained choice, where you choose as many assets as you wish but no more than one of any given asset.
- But, you face multiple MARRs from many sources.
  - A MARR for retained earnings
  - Another MARR for bank loans
  - Another MARR for new stock and bond issuance.

Distinguishing Divisible and Indivisible Assets

- Divisible assets
  - You can take some or all.
  - Does not mean any fraction but close.
  - Think of an apartment complex where you get to chose the number of units.
- Indivisible Assets
  - You can take all or none.
  - Think of a car.

Additional Assumption about divisible assets.

No change in IRR dependent on scale, but this often happens in real life.

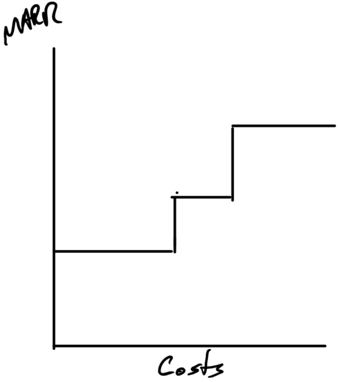


Logic of the heuristic

- Not a continuous problem that can be solved by calculus but you can use calc style logic.
- Your objective is to maximize *net* benefits, i.e., benefits less costs.
- In calc land you would set up a net benefit function, differentiate, set it equal to zero and solve.
- But, that net benefit function has to have the right shape, e.g., $NB'' < 0$, to be sure that your local extrema is a max and the local max.
- For this problem move around the funds and the assets to make sure that it has the right shape for a maximum global net benefit.

Getting the right shape of net benefit function.

- Order your assets from highest IRR to lowest IRR.
- Order your funds from lowest MARR to highest MARR.



Use analogy from maximization to pick assets

- In calc land you would set up a maximization problem

$$\max_q B(q) - C(q)$$

- Get the first derivative and set it equal to zero

$$B'(q) = C'(q)$$

- You just find where marginal benefit is equal to marginal cost.
- You can also think in terms of incremental costs and incremental benefits.
- *If incremental benefits is greater than incrmental costs, do it.*

Incremental Benefits and Costs

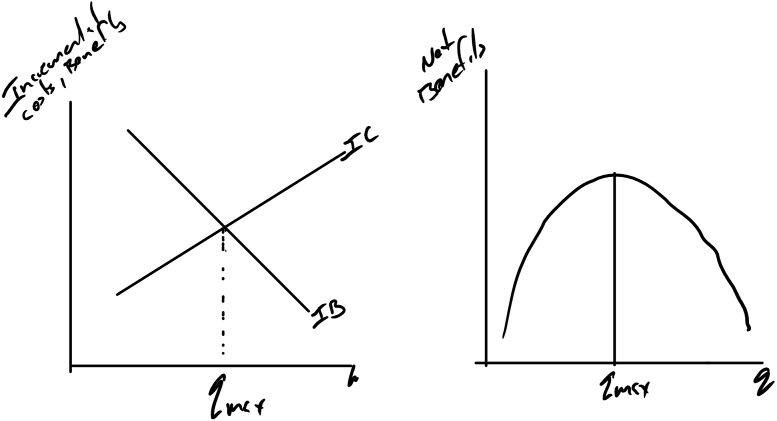


Figure 2

Why the "IB>IC Do it" rule

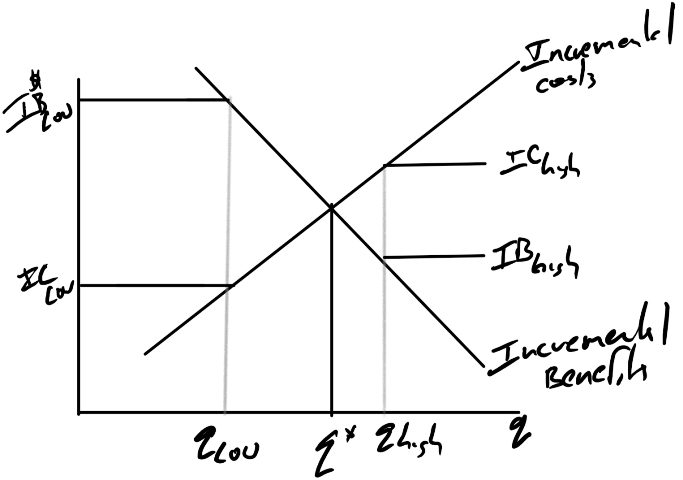


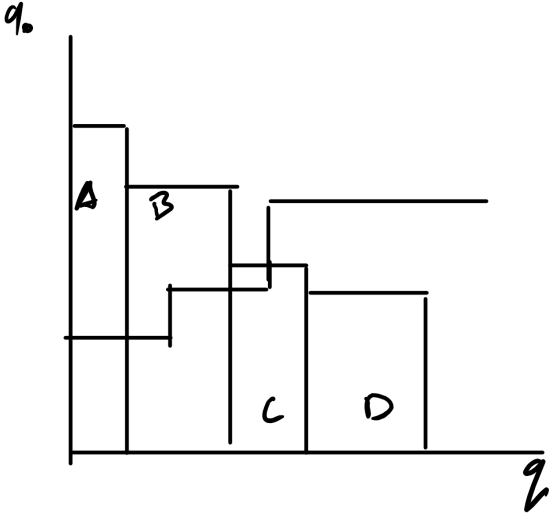
Figure 3

Application to Divisible Assets

- This is all dependent on getting the curvature of the net benefit function right.
- Right means that the local extrema is a global max.
- Incremental costs should be non-decreasing.
- Incremental benefits should be non-increasing.
- Nothing odd happens with divisible assets.

Divisible asset case

Get all assets and parts of assets where the IRR is greater than MARR.



Summary of Diagram

- Vertical is IRR and MARR
- Horizontal is the scale of investment
- What gets purchased?
  - A is purchased because $IRR > MARR_{Retained\ Earnings}$
  - B is purchased because $IRR > MARR_{Retained\ Earnings} > MARR_{Loan\ 1}$
  - C is partially purchased.
    - Only the part that can be funded by Loan 1 since $IRR > MARR_{Loan\ 1}$
    - Not the part funded by Loan 2. $IRR < MARR_{Loan\ 2}$

Try Tabular

Asset	A	B	C	D
IRR	20%	10%	12%	8%
A_0	6	5	4	7

MARR $= 9\%$

Easy Answer

Asset	A	B	C	D
IRR	20%	10%	12%	8%
A_0	6	5	4	7

- MARR = 9%
- Note that this is just the IRR criteria. Buy if the IRR is greater than MARR, i.e., A, B and C.

Limited Retained Earnings

Asset	A	B	C	D
IRR	20%	10%	12%	8%
A_0	6	5	4	7

- \$12 of retained earnings at 9%
- Line of credit at 12% available.

Get the Order Right

Asset	A	C	B	D
IRR	20%	12%	10%	8%
A_0	6	4	5	7

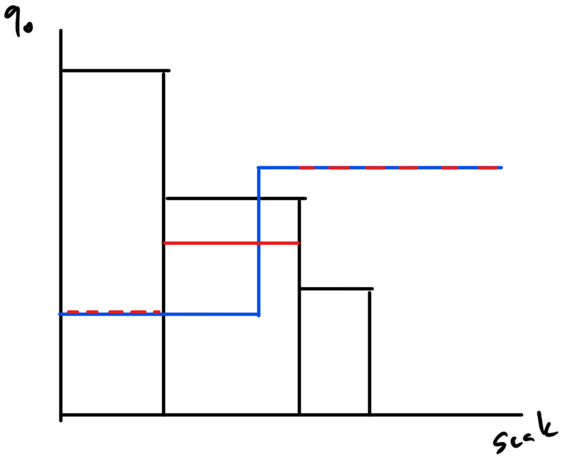
- Use \$10 of the retained earnings to buy all of A and C.
- Use the remaining \$2 to buy 2/5 of B, the part where the $IRR > MARR$.
- Don't use the remainder of the loan to buy remainder of B or D.

Indivisible Assets

- If follows the same rules but cost of funds can get complicated.
- The cost of funds is the weighted average of the funds used.
  - Example: \$7 at 3% and \$3 at 10%.
  - $\frac{7}{10}3\% + \frac{3}{10}10\% = 5.1\%$
- You need to convert the *marginal* cost of funds to *Weighted average* cost of funds.
  - The curvature of the net benefit function is *usually* OK.
  - Problems happen when the "IB>IC, Buy It" rule skips a high IRR asset but says to buy less expensive asset with lower IRR. (Picture Later).

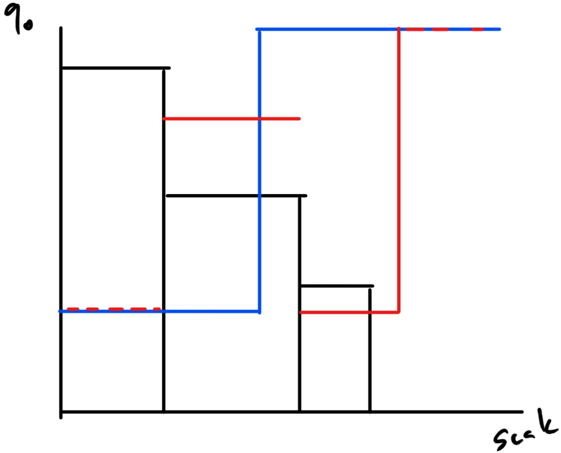
Incremental Converted to Weighted Average

Note that the weighted average is still monotonically increasing.



It can go bad

When middle asset is not purchased, IC is not monotonic.



Indivisible Assets

Asset	A	B	C	D
IRR	20%	10%	12%	8%
A_0	6	5	4	7

- *$14* of retained earnings at 9%
- Line of credit at 12% available.

Get the Order Right and then

Asset	A	C	B	D
IRR	20%	12%	10%	8%
A_0	6	4	5	7
RE (9%)	6	4	4	0
Loan (12%)	0	0	1	7
IC Funds	9%	9%	9.6%	12%

- Buy A since $IRR > MARR_{RE}$
- Buy C since $IRR > MARR_{RE}$
- Buy B since $IRR > MARR_{\frac{4}{5}RE+\frac{1}{5}Loan}$
- Do not buy D since $IRR < MARR_{Loan}$

Try again

Asset	A	B	C	D
IRR	20%	10%	12%	8%
A_0	6	5	4	7

- *\$12* of retained earnings at 9%
- Line of credit at 12% available.

Solution

Asset	A	C	B	D
IRR	20%	12%	10%	8%
A_0	6	4	5	7
RE (9%)	6	4	2	*2*
Loan (12%)	0	0	3	*5*
IC Funds	9%	9%	*10.8%*	*11.14%*

You can't calculate the IC funds until you know if the previous asset was purchased.

- Buy A since $IRR > MARR_{RE}$
- Buy C since $IRR > MARR_{RE}$
- Do not Buy B since $IRR > MARR_{\frac{2}{5}RE+\frac{3}{5}Loan}$
- Do not buy D since $IRR < MARR_{\frac{2}{7}RE+\frac{5}{7}Loan}$

Increasing Costs of Funds Summary

- Common environment when funding comes from many sources.
- In the divisible asset case, take the fractions of assets with $IRR > MARR$.
- The described heuristic is generally useful but can break in the divisible asset case.
- The problem is that the weighted average cost of funds is often dependent on which assets were accepted before.

Introduction Capital Budgeting

"Capital Budgeting" refers to a problem where you are strictly limited to the amount that can be spent in period zero, but have a defined MARR.

- Commonly arises in small businesses when you don't have access to lending.
  - The MARR is frequently based on the return on common assets, or
  - the rate on your savings.
- Or, in large organizations when you have separate operating and capital budgets with no ability to transfer.
  - Does not apply in a use it or lose it environment (UIOLI).
  - Very common but assumes that higher ups understand long term needs more than lower downs.
  - In UIOLI you buy dumb things

The Heuristic

It is the stupidest one you can use – "exhaustive search"

- Form all combinations of your assets.
- Eliminate all assets that are not feasible, i.e., cost more than you can spend in period zero.
- Choose the combination with the highest PW.

Why is it stupid?

Because computation cost grows so quickly. There are $2^N - 1$ proper subsets of a set.

- Three elements (A, B, C)
  - Pairs ((A,B),(A,C), (B,C))
  - One Trio (A, B, C)
- 10 Asset Choices, 1023, combinations.
- Typical maintenance list for Parkrose had 150-200 items.
  - $1.42x10^{45}$ to $1.6x10^{60}$ combinations.
  - Only $9.6x10^{56}$ atoms in the solar system.

Example

Asset	A	B	C
A_0	2	4	3
PW(10%)	2	3	1

With a \$7 capital budget. Note ABC is not feasible.

Combo	PW	Combo	PW
A	2	AB	5
B	3	AC	3
C	1	BC	4

AB with \$1 left over is the best combination.

Observations

- The MARR does not account for the full opportunity cost of an asset
  - It accounts for the cost of funds.
  - Does not capture the net benefits of assets that are forced out.
    - \$10 available and two assets, one that costs \$8 and another that costs \$3
    - Buying the \$3 assets makes it so you can't get the \$8 asset.
    - The opportunity cost is not just \$3 but the net benefits of the \$8 asset.
    - If you had Calc IV, Lagrange multiplier type effect.
- Often have money left over because of the Lagrange type effect.

A Frequent Problem

You often have capital budgeting/ combinatorial optimization problems in government budgets.

- Your high IRR project is not funded but other larger lower IRR projects are funded.
- Your project would have ‘bumped’ one of the larger projects.
- Don’t argue against the other projects/programs, argue for a larger overall budget.

You should notice this in many budget processes. There is a strong tendency to commit one-time money, or reduced contingency, to fund a few high IRR programs.