Uncertainty in Cash Flows

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Why Uncertainty and Sensitivity?

- We really don't know
- Some of our guesses may be critical and sensitivity analysis may guide us to spend more time supporting those assumptions.
- Clients may ask for risk assessments
- Clients may have other assumptions about critical parameters

What do you mean “I don’t know”?

- We are way more certain than we should be.
- Example:
  - Make your best guess at the population of Kenya
  - Give yourself a reasonable upper and lower bound so you are 80% sure the true value is there.
  - Not zero to a billion

Answer

48.46M. (2016)

- One in five of you should be out of bounds.
- Usually, if honest, more than one in five will be out.
- If less, Hawthorn Effect

Some parameters are more critical than others

- Cost effectiveness depends on estimating costs and benefits.
- Example: Cost effectiveness of an Energy Efficiency improvement
  - Costs, commonly install costs are easy to get.
  - Benefits, due to hours used, are critical.
- Low use, not cost effective.
- High use, cost effective.

Client Requests

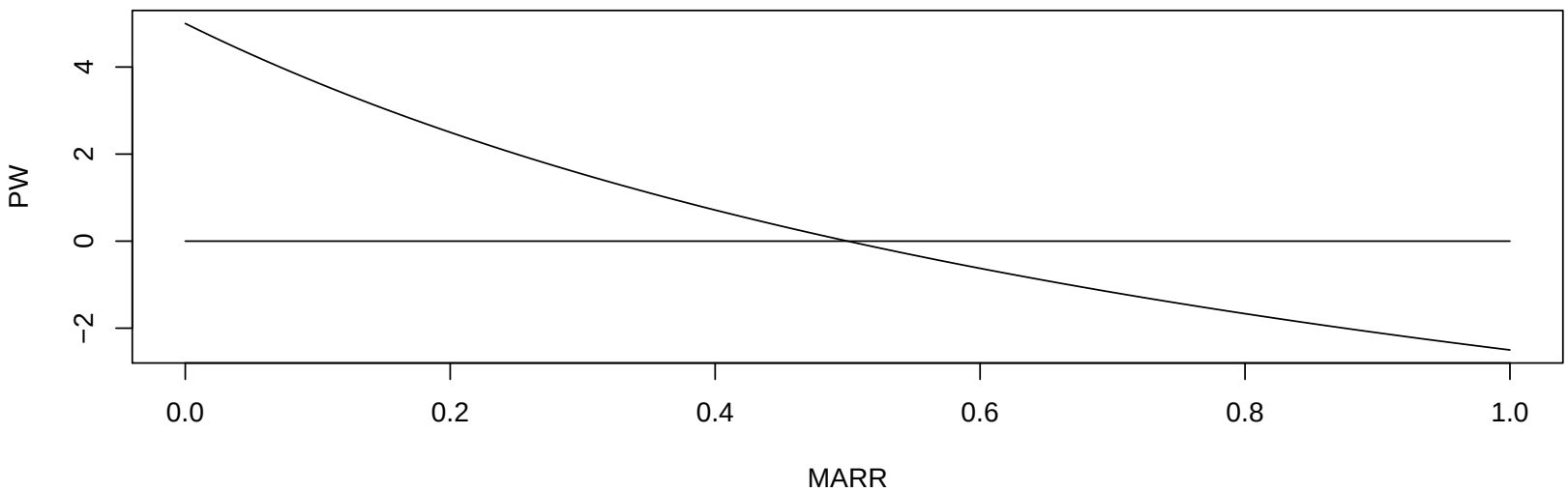
- It is not that they don't trust you.
- Risk plays into setting a MARR.
  - High risk, high MARR
  - Low Risk, low MARR
- They want to know the probability of success
- It is part of a portfolio of risks:
  - They want to know how risky
  - They want to know how correlated it is with other risks.
- They like independent risks
- They like risks that are negatively correlated with existing risk

Clients Have Different Assumptions

- They may disagree with you but you still want to show them that the results are valid
- There may be actual diversity.
  - Different costs at different locations.
  - Different experiences in different years.
- You need to do an analysis that covers all that.

Warnings: You Have Seen Sensitivity Analysis

- Recall the PW diagrams we used in learning IRR?
- Shows how PW changes with changes in MARR.
  $PW = -10 + \frac{15}{(1+r)}$



Other Parameters

There are some parameters that are usually of concern:

- Initial costs
- Salvage value
- Scale

Some are harder than others:

- Planning horizon
- Uncertainty about Risks

Why planning horizon is harder

- Mostly you assume time is discrete – not continuous.
- You need to either change to continuous discounting $\frac{1}{e^{N(1+r)}}$ rather than $\frac{1}{(1+r)^N}$
- Or create an equivalent MARR with smaller compounding periods but equivalent with their nominal statement.

Example: MARR 10% per year compounded daily

Start with 10% per year MARR and find equivalent annual MARR compounded daily.

$$\left(1 + \frac{r}{365}\right)^{365} = 1.1$$

$$365\left[1.1^{\frac{1}{365}} - 1\right] = 0.0953226$$

Allows you to vary time on a smaller time frame.

Why uncertainty about risks is harder

- Risk is when you know the odds but don't know the outcome.
  - You know that each side of a true die has a 1/6 chance of showing.
- Uncertainty is when you don't know the odds.
  - Uncertain if it is your trick die that rolls a 6 half the time or the true die.
- You end up describing your beliefs about the probability as a Beta distribution.
  - Hint, you just walked into the land of Bayesian statistics.
  - We will do some Monte Carlo simulations later so you can see what to do.
- Talk about your sensitivity analysis as about the mean of the present worth rather than the present worth.

Simple How To: PW Example

Consider a simple constant series that we can describe the present worth with factor notation:

$$P = -C + A(P|A, i = r, N = n)$$

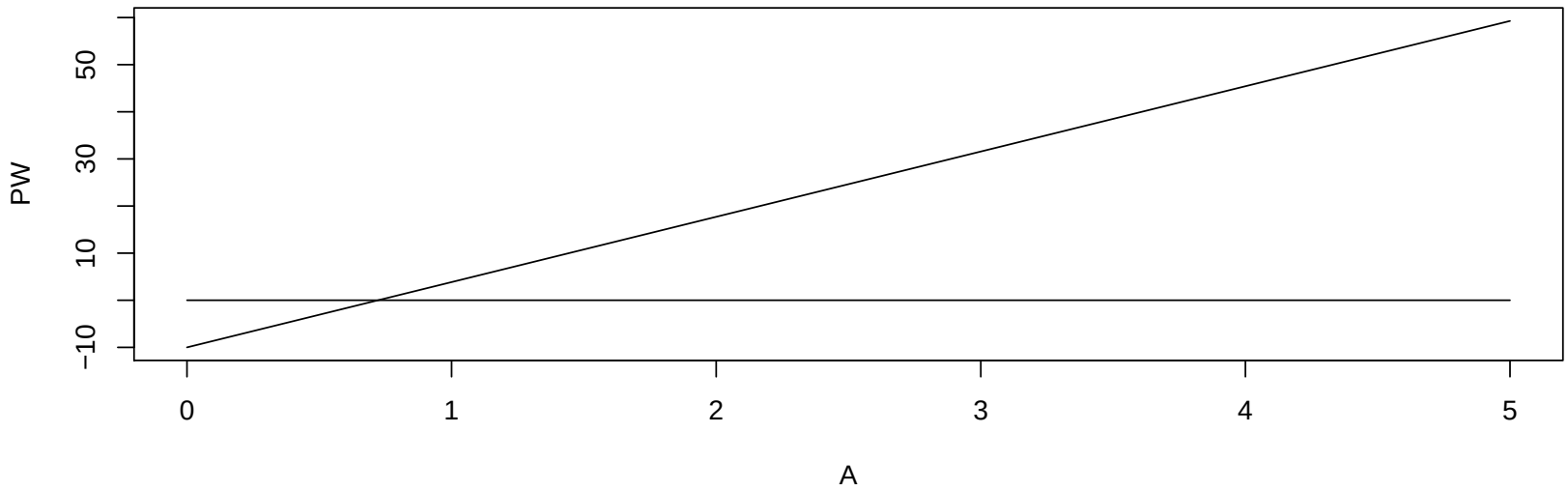
Lots of parameters to vary:

- A, the size of the constant benefits.
- r, the discount rate
- n, how long you get payments
- C, the initial cost

We warned you about r and n so lets vary just A

PW Figure

r = .1, n = 10, C = -10, $A \in [0, 5]$



BTW PW is zero when A =0.7217385

What Do You Learn from this?

- If you are pretty sure that A is not near 0.7217385, you are done spending time getting a better estimate of A.
  - Refining you estimate of the parameter with more data, more detailed calculations, more bids, more literature review will not change the decision.
  - Looking for more precision or less uncertainty is a waste of resource.
- If not, then you need to decide if the effort is worth the refinement. That depends on your other options.

Cost benefit analysis of doing cost benefit analysis

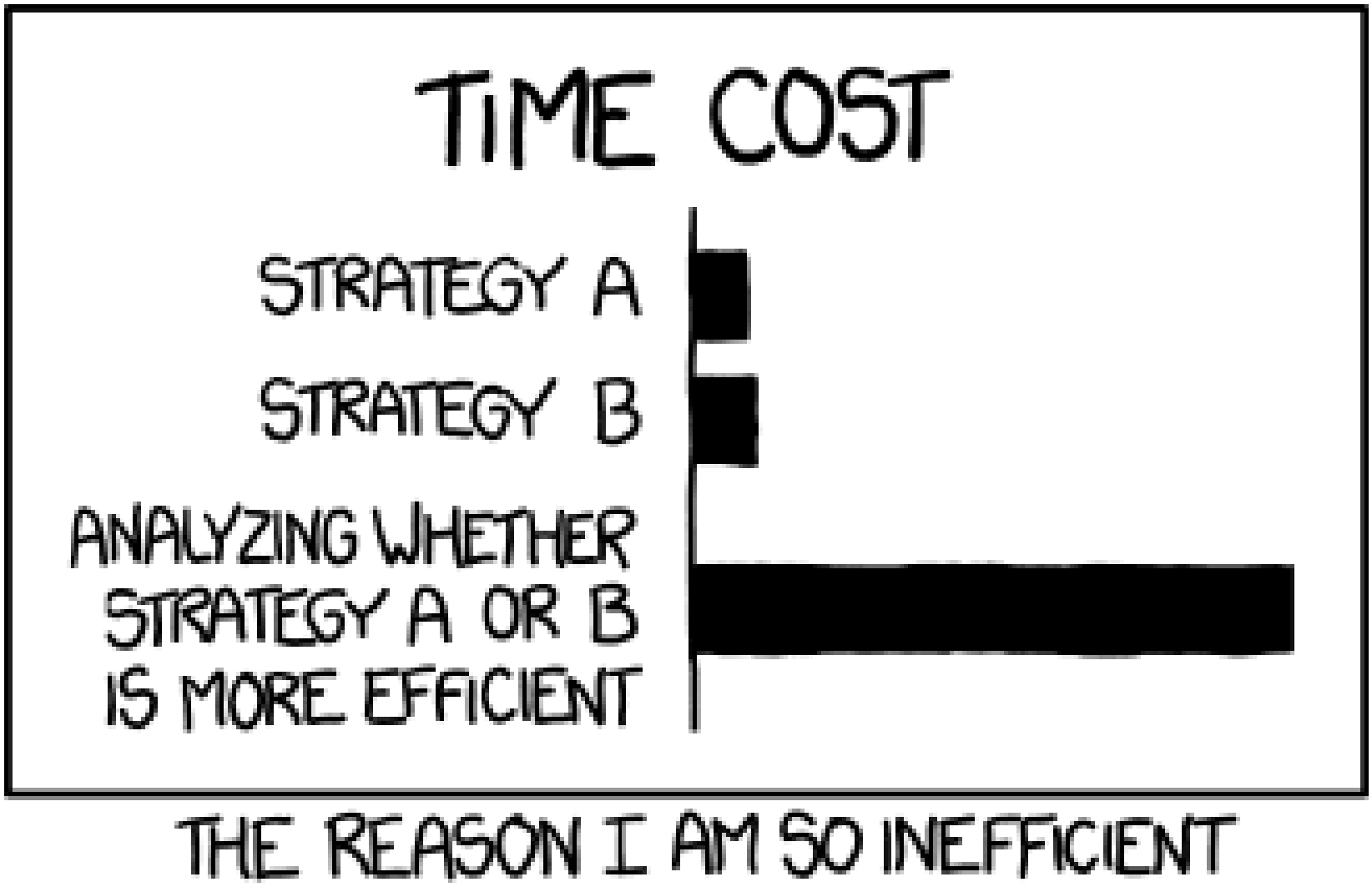


Figure 1

What Do You Learn from this? (Con't)

- The client can check on the consequences of their beliefs about A.
- Note we still don't know risk, for that we need to know the distribution of our beliefs about A.
  - Is it N(2, .5)?
  - Is it U(1,4)?
  - We will do this later.
- Sensitivity with present worth is easiest of the three.

AW Example

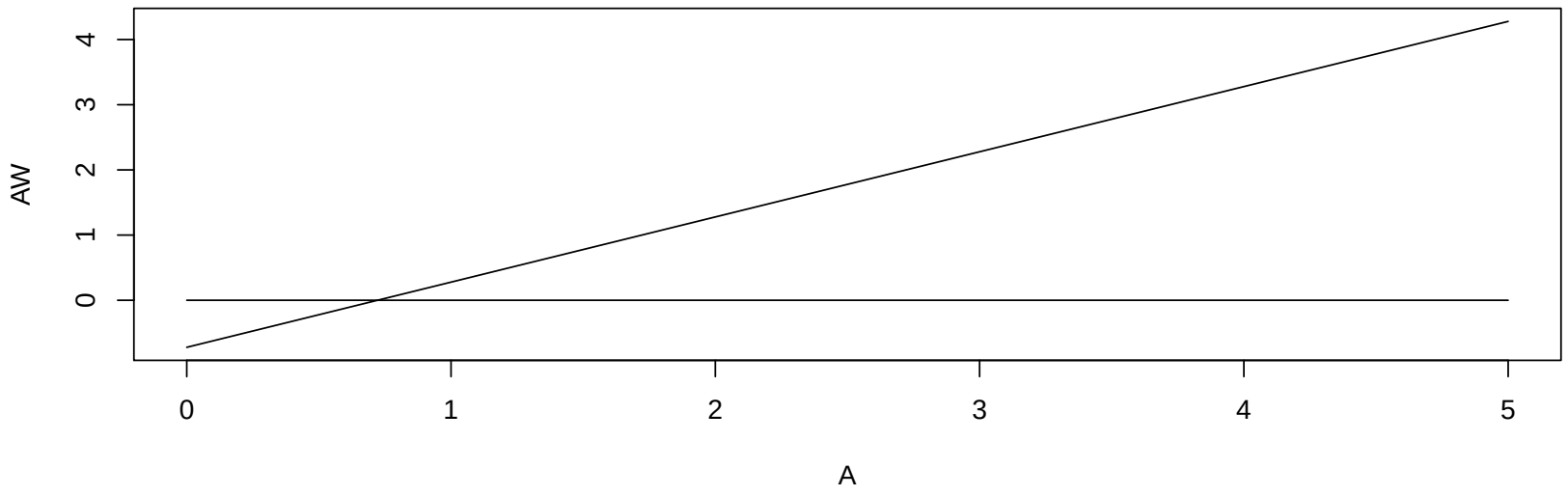
- Stick with the same cash flow as before with present worth factor notation:

$$P = -C + A(P|A, i = r, N = n)$$

- No more difficult than PW sensitivity analysis unless you want to vary the N.
- Annual worth is just

$$AW = A - C(A|P, i = r, N = n)$$

AW Figure



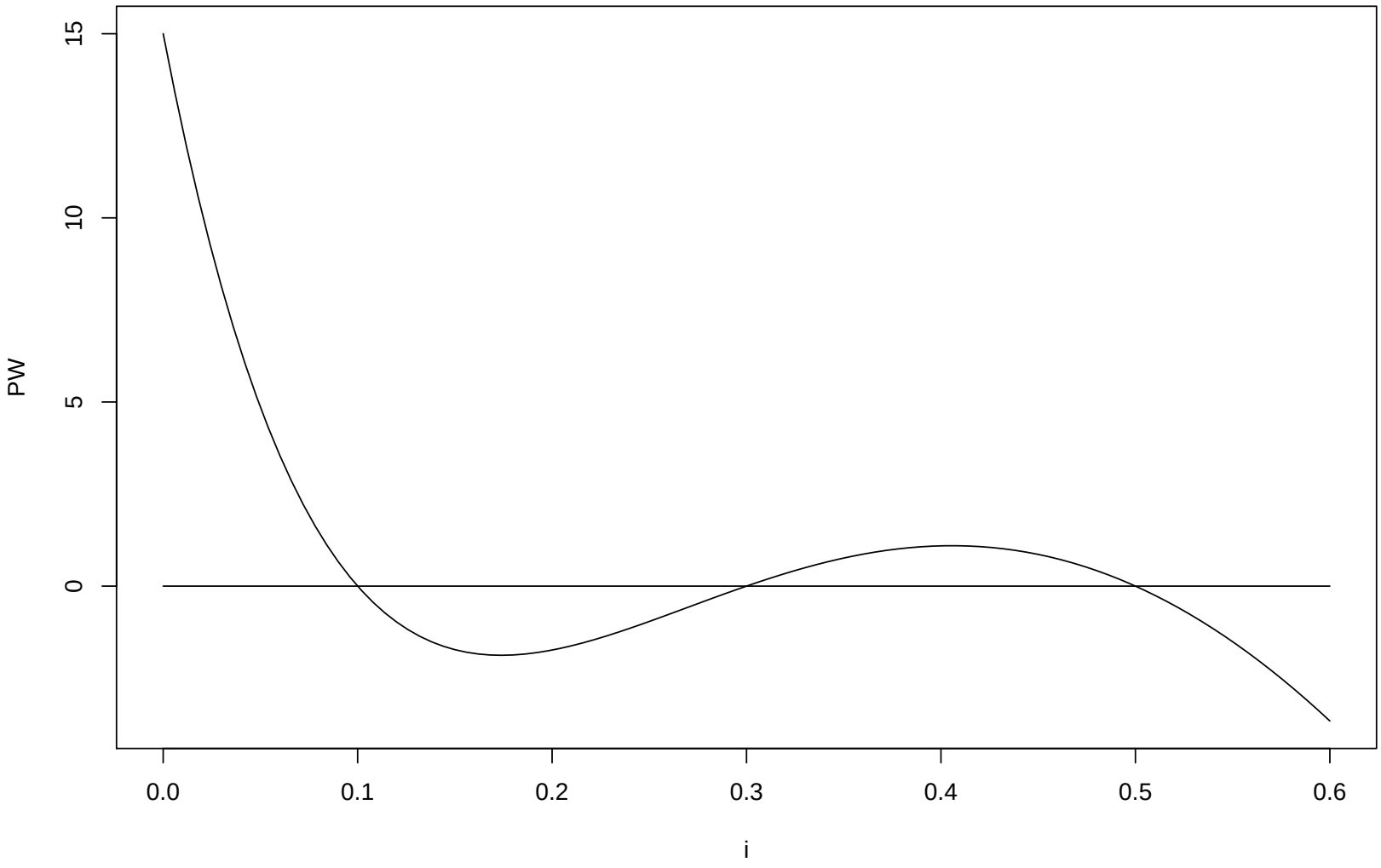
- This is the same information as the PW graph translated into AW.
- Some clients like to see it this way.

IRR Example

- Will use a more complex model for this because IRR can get odd.
- Changing the parameters can change not just the location of the roots, i.e., the IRRS but also the number of roots.
- If you want to learn more, read up on bifurcations.
- Reuse the example for multiple IRRs we had in class:
  - -1000, 3900, -5030, 2145

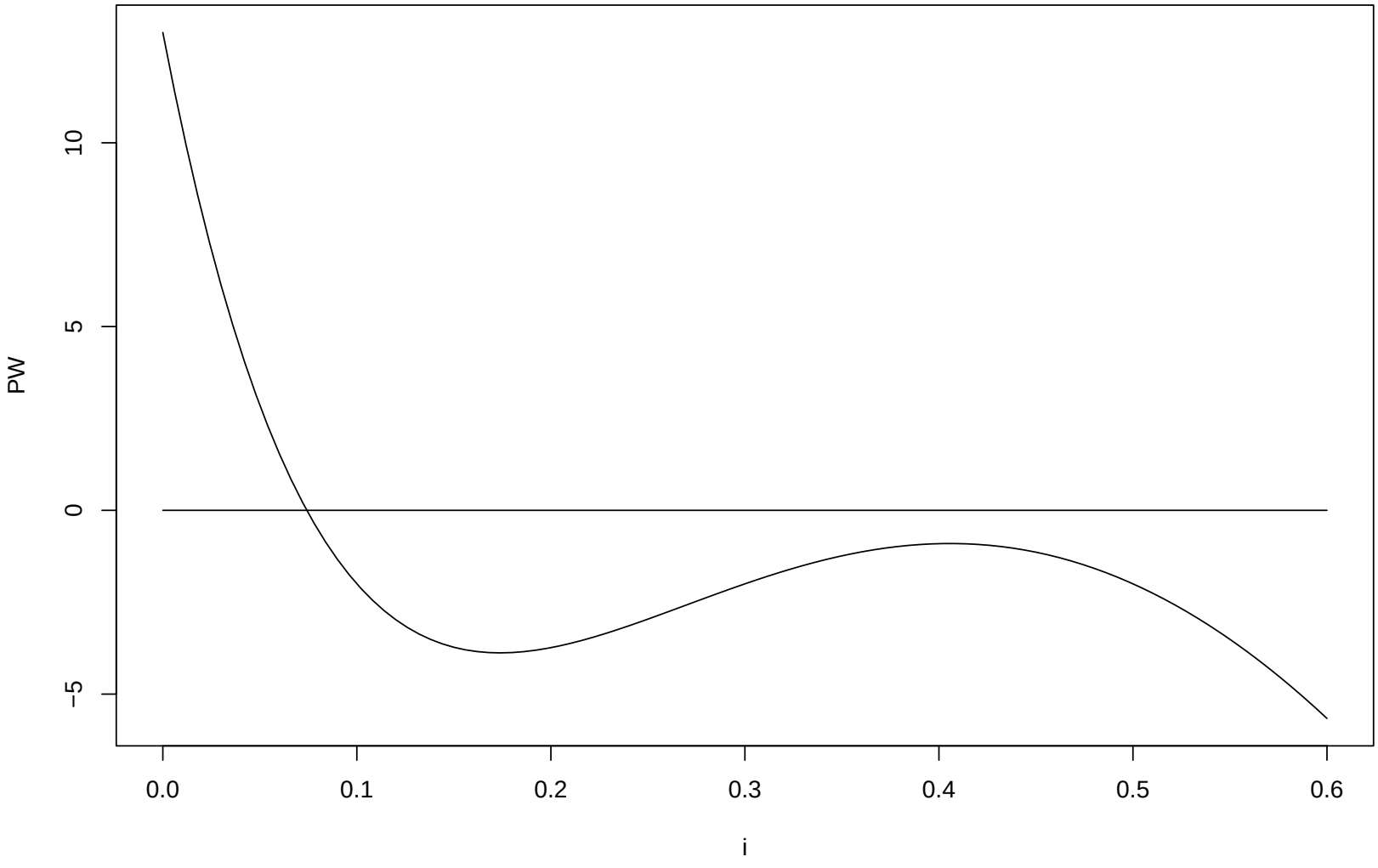
Recall the PW function

This has IRRs at 10%, 30% and 50%



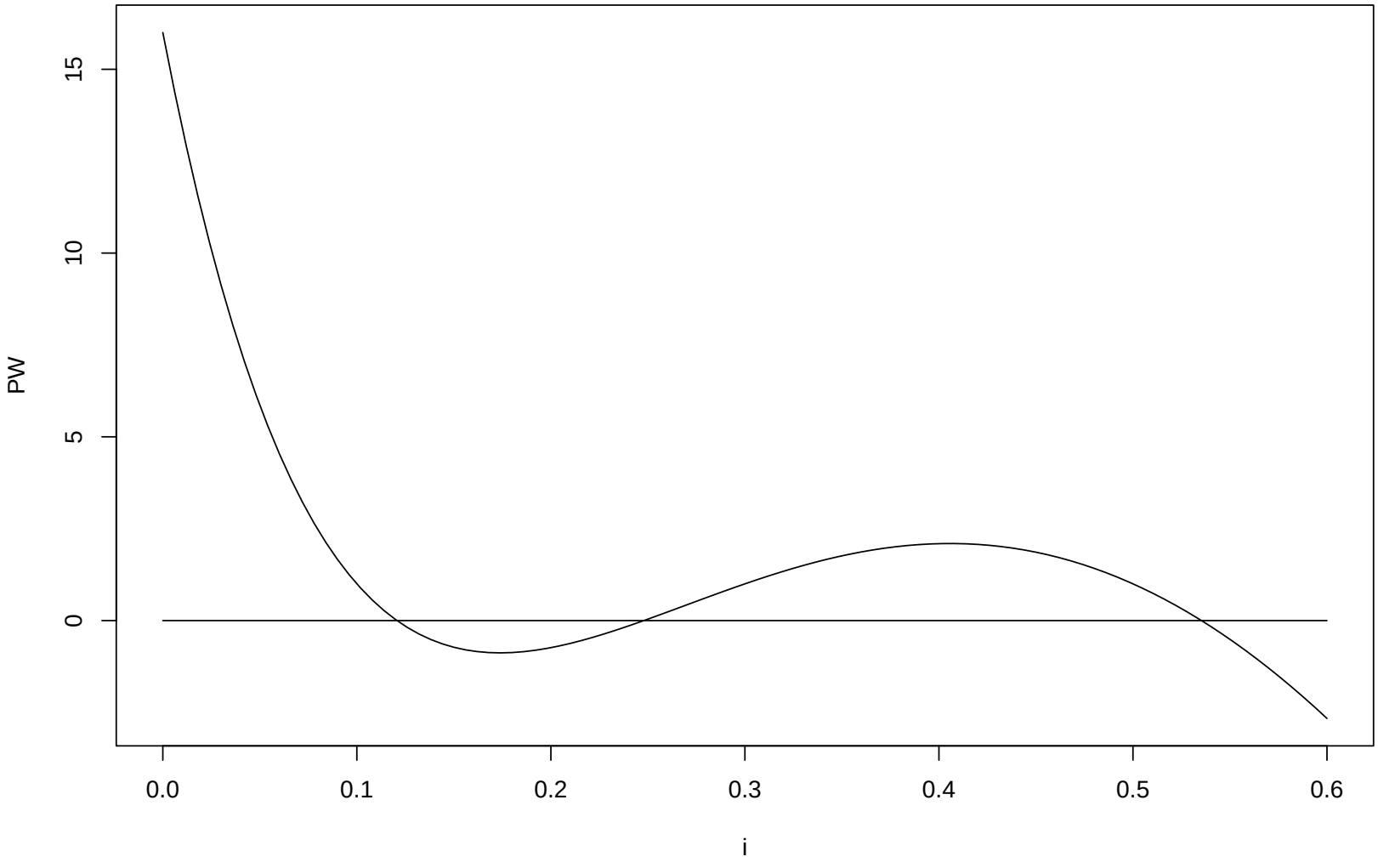
Watch the roots change

$A_0 = -1002$ One root



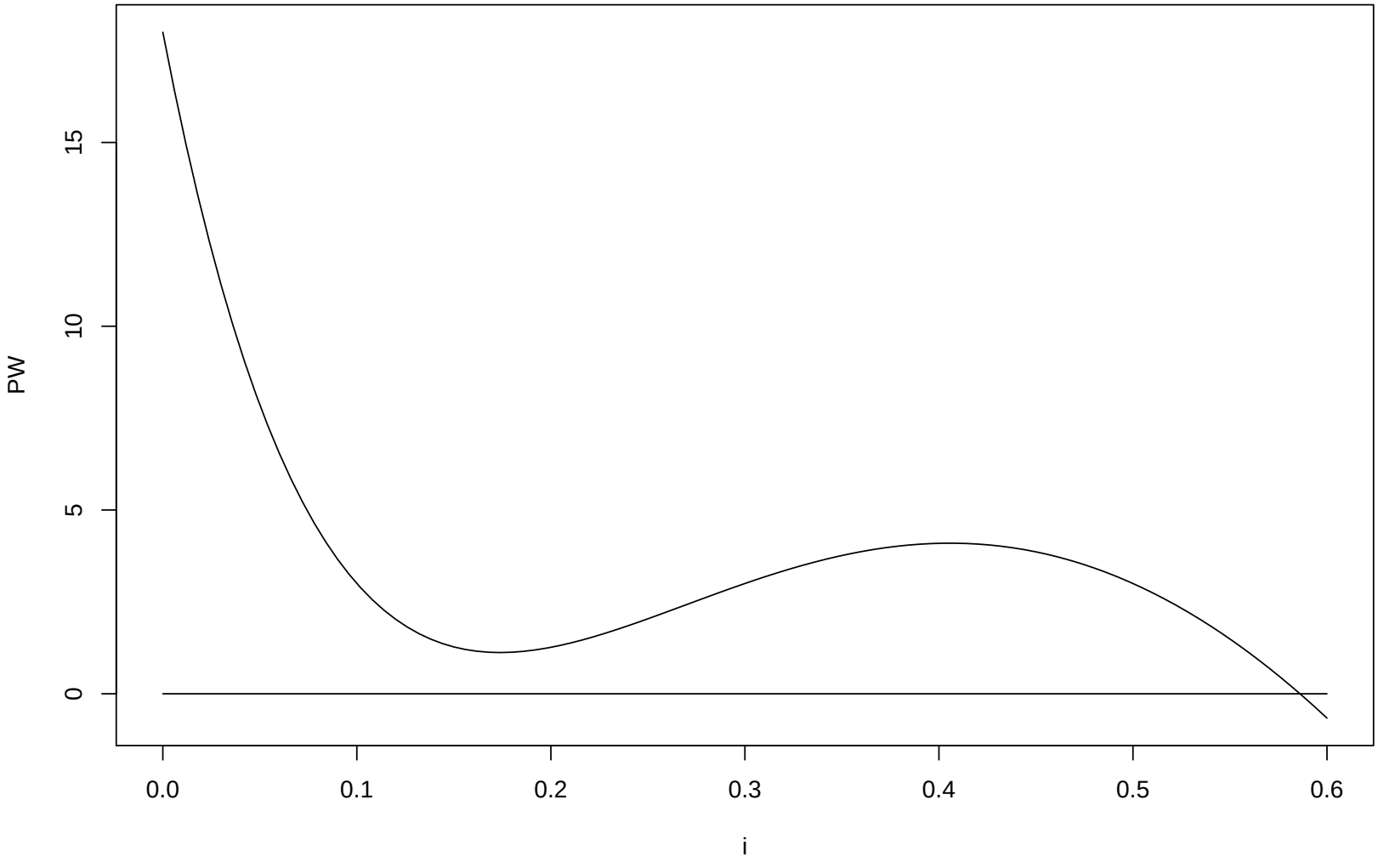
Watch the roots change

$A_0 = -999$ Three roots



Watch the roots change

$A_0 = -997$ One root again



IRR Problems

- Your IRR sensitivity functions are not always continuous.
- It requires a lot of time with clients to explain this.
- Recommend talking your clients out of it.

Priors for Sensitivity Analysis: Finding a Good Range

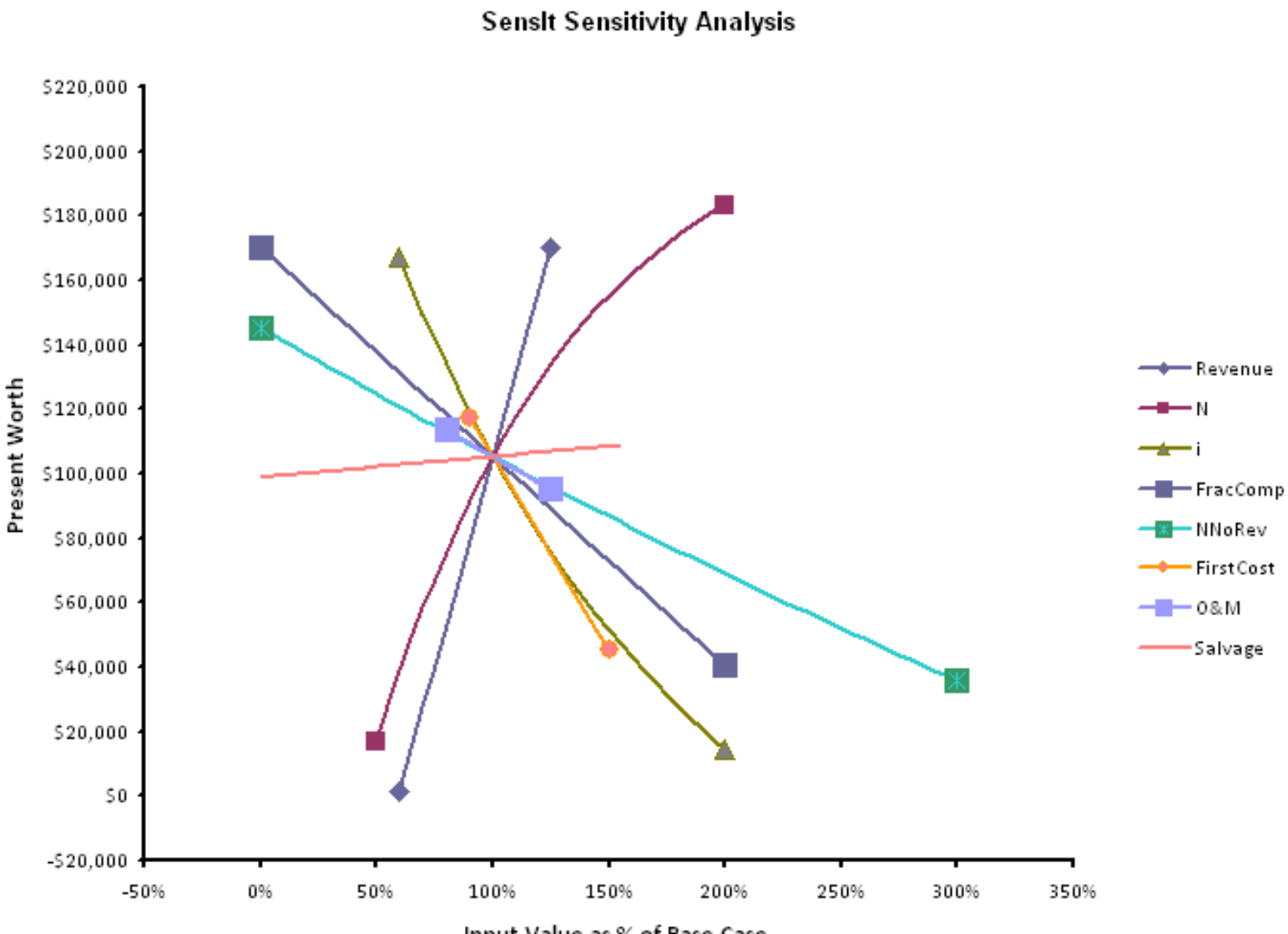
- In the examples above we just picked bounds.
- Book gives some bad advice on how to pick the design points, bounds, in a sensitivity analysis, $\pm$ 20%.
  - Sometimes 20% is way too big, remember the initial cost in the IRR example? \$4 on a scale of \$1000
  - Sometimes 20% is way too small. What is the smallest safety factor you have ever used? 1.25?

Single Person

- A few warnings.
  - This does not work with everyone. They have to be reasonably numeric.
  - They have to be a real expert, with real opinions, real experience with data.
  - It is ok, if they need time, have to make some calculations or want to look a few things up.
- Three questions for each parameter
  - What is your best guess?
  - Largest value that you are 90% sure the value is below?
  - Smallest value that you are 90% sure the value is above?
  - Basically, what are your 80% bounds.

Allows a Spider graph.

- Horizontal axis shows percent difference from best guess.
- Vertical shows the investment criteria of interest.



What to watch for?

- Parameter that cause a decision, sign, change.
  - Can turn a good idea into a bad one without more research
- High sensitivity parameters, high slope.
  - Your expert may be more certain than they should be and should be consulted more about that parameter.
- Low sensitivity parameters, low slope.
  - No more time should be spent on these.
  - Unless, your job is devil's advocate.

Multi-Person: Feasible

You have many choices:

- Ask everyone for their best guess for each parameter:
  - Leads to Monte Carlo Simulations.
  - Captures correlations between beliefs in parameters
- Delphi Survey:
  - Asks not just for numbers, but why you choose the number.
  - Monte Carlo Simulations

Possible but difficult

- Ask everyone for the best guess and 80% bounds:
  - Beyond scope of course.
  - Use with small numbers of experts
  - Welcome again to Bayesian land.
- Ask everyone for joint distributions:
  - Hard to to do.
  - Goes Bayesian again.
  - Never had a client that could do it.
  - I have trouble doing it.

Example Joint Distributions

Ask Everyone for best guess at each parameter.

- You need at least as many experts as parameters.
- If you have a lot of experts, 30+, you can simulate the objective function directly on the data or bootstrap for the uncertainty.
- If you have few, you can approximate with a multivariate normal distribution.

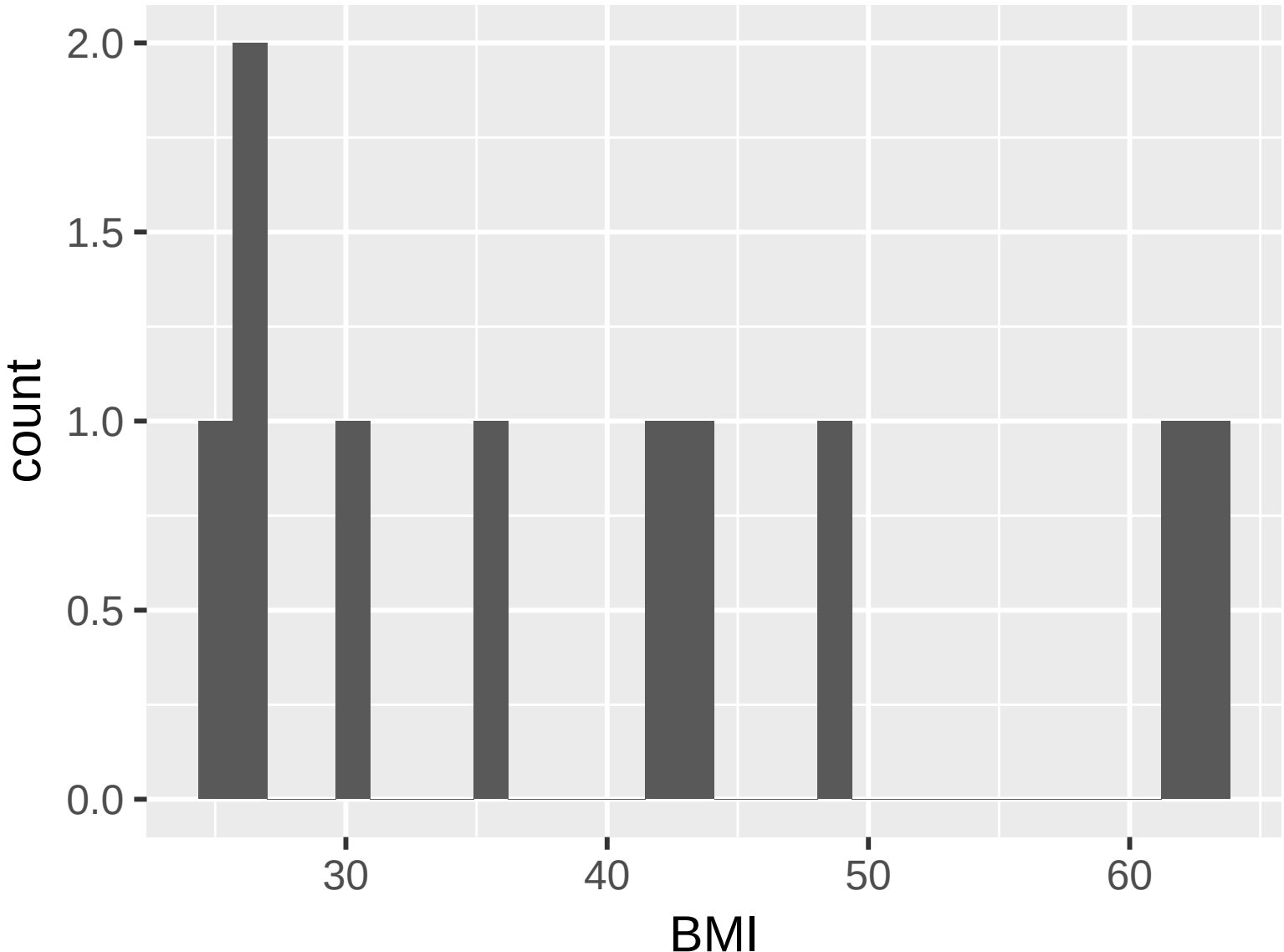
What is Jamie's BMI?

- Figure out Jamie's Body Mass Index, $\frac{kg}{M^2}$.
- Collected these 10 guesses of height and weight in M and kg.

Ht	Wt
1.821870	83.46805
1.386833	83.70864
1.827694	88.13024
1.680432	87.01051
1.419243	84.47353
1.178424	85.92407
1.815408	86.93528
1.570278	87.64317
1.345283	87.76748
1.152229	84.07292

Direct

- Experts say the median BMI is 38.7409177.
- 80% are sure it is between 26.2551993 and 62.0195641



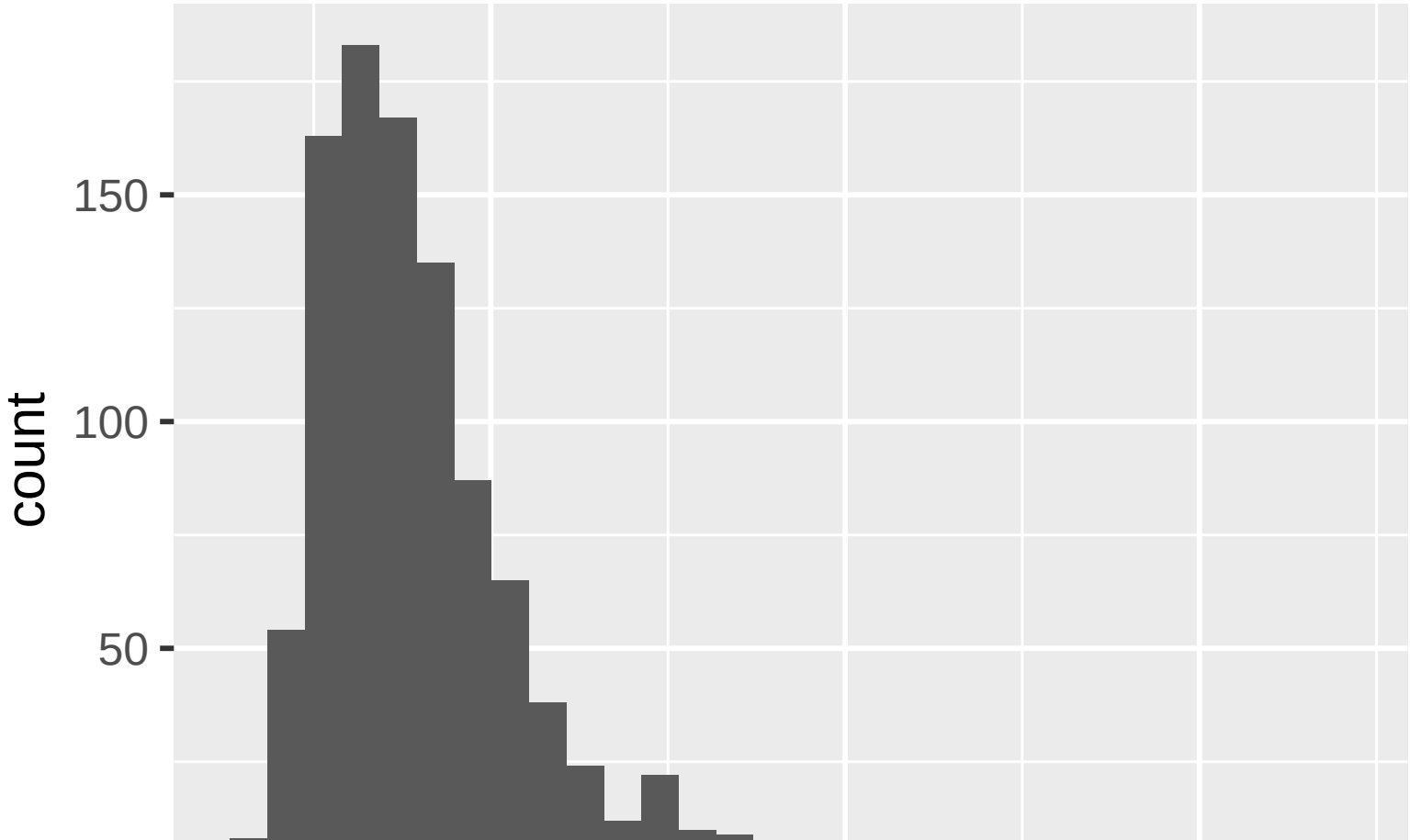
Or Estimate the Joint Normal Distribution

- Means of 1.5197694, 85.9133883
- and var/cov of

0.0679637	0.1440837
0.1440837	3.3214899

And Simulate for 1000 replications

- You can get exact results for Normals added to Normals but the minute you divide, stick to reporting confidence intervals, medians and modes.
- Normal divided by Normal is Cauchy, which has no defined mean and no defined variance.



Problems with "Just ask experts"

- Responses are not always independent
  - The boss is in the room and you don't want to disagree.
  - Everyone uses the same reference document
  - Example, you can tell which field by the number people give for the value of a statistical life.
    - \$9.1 million, Environmental Protection Agency
    - \$7.9 million, Food and Drug Administration
    - \$9.4 million, Transportation Department
- Little thought is put into it

Fixes

- Use incentivized polls like Intel.
- Parimutuel betting, like some electrical utilities.
- Delphi Surveys

Incentivized Polls

- Guess Jamie's BMI. Closest to the actual gets $10.
- Skin in the game gives incentive for more thought.
- Only works when you will eventually find out actual.
- Oddly works better if you don't know how people are betting.
  - Can produce group-think bubbles, if you see how others are betting.
  - Easy to find the mode, but harder to tell if it is real.
- Can get expensive with many winners:
  - everyone guesses the same correct answer.
  - More expensive when there is less certainty.

Parimutuel Betting

- Guess Jamie's BMI. Correct answers split the pot with the others that got it right.
- Racetrack method.
- Less likely to get bubbles, but
  - Harder to see if there is a mode in the guess.
- Cheaper than incentivized poll if high probability of multiple winners.
- Cost does not increase with a priori certainty.

Delphi Surveys

General Pattern:

- Ask each person in private for best guess.
- Compile results.
- Ask outliers why they said what they did.
- Give everyone:
  - The distribution of guesses for each parameter
  - The reasons the outliers gave for the answer the gave.
- Ask for another guess.
- Report the new distribution or repeat if desired.

Interested in More?