

## New Environments

# Our Environments

- ▶ Unconstrained Choice: Pick the *acceptable* assets.
- ▶ Exclusive Choice: Pick the *best* asset.
- ▶ *Increasing Cost of Funds*: Pick acceptable assets when you face many interest rates.
- ▶ *Capital Budgeting*: Pick the best group of assets when limited on how much you can spend.

# There is a general statement of the problem

- ▶ Our environments can be interpreted as mixed integer programming problems with various constraints.
- ▶ You get the good stuff in ETM 535/635, Advanced Engineering Economics
- ▶ Except for trivial problems, you need a heuristic.
- ▶ Heuristics are specialized rules and techniques that find a solution. There are some general categories:
  - ▶ Simulated annealing
  - ▶ Ant Colony optimization
  - ▶ Genetic Algorithms

# The optimization algorithms are fun to use

## ► Simulated Annealing

- Random bounces in parameter space.
- More likely to keep those that have smaller evaluations of the objective function than large.
- Sizes of bounces get smaller over time.
- Chances of keeping a worse outcome fall over time.
- Wikipedia has a great graphic  
[https://en.wikipedia.org/wiki/Simulated\\_annealing#/media/File:Hill\\_Climbing\\_with\\_Simulated\\_Annealing.gif](https://en.wikipedia.org/wiki/Simulated_annealing#/media/File:Hill_Climbing_with_Simulated_Annealing.gif)

## ► Genetic Algorithms

- Like simulated annealing but
- Many solutions tried at once.
- They compete and merge with each other
- I used these for stochastic dynamic programming problems.

## Not Doing Any of that

- ▶ We will be using some specialized heuristics to find the best sets of assets in the new environments.
- ▶ The two/three heuristics have some fragile points.
  - ▶ Increasing cost of funds has trouble when an indivisible asset (defined later), that could be funded by a blend of funds from two sources is rejected and the next asset can be funded by a single source of funds.
  - ▶ The capital budgeting problem is exhaustive search and the solution requires  $2^N - 1$  evaluations.

# Increasing Cost of Funds

- ▶ This is an environment similar to unconstrained choice, where you choose as many assets as you wish but no more than one of any given asset.
- ▶ But, you face multiple MARRs from many sources.
  - ▶ A MARR for retained earnings
  - ▶ Another MARR for bank loans
  - ▶ Another MARR for new stock and bond issuance.

# There is a usual order for the sources of funds

The order is based upon uncertainty about results.

- ▶ Less certain results require a higher MARR
- ▶ More certain results require a lower MARR

Put another way, risky investments require a higher risk premium.

# Why Retained Earnings Require the Lowest MARR

- ▶ These are internal funds that are controlled by internal people.
- ▶ They know you. They know the projects you usually work on.
- ▶ They know the details of the business.
- ▶ They have less uncertainty about those details than an outsider.

Less uncertainty about the result, lower MARR.



## Why a lender sees more uncertainty

- ▶ Lenders tend to specialize in industries.
- ▶ They know the industry well but have less knowledge about the insides of your firm.
- ▶ They don't have full details on the team.
- ▶ They have not worked shoulder to shoulder.

They have more uncertainty than insiders and therefore have a higher MARR.

# Why Capital Markets (Stocks and Bonds) have the highest MARR

- ▶ Many of you own stock, even in a mutual fund or retirement fund.
- ▶ Pick a random stock.
- ▶ Where is the headquarters or the nearest location?
- ▶ You don't know much. You have more uncertainty.

You have more uncertainty than a banker because you know less about the firm and the project. Because of that, you will have a higher MARR.

# Distinguishing Divisible and Indivisible Assets

- ▶ Divisible assets

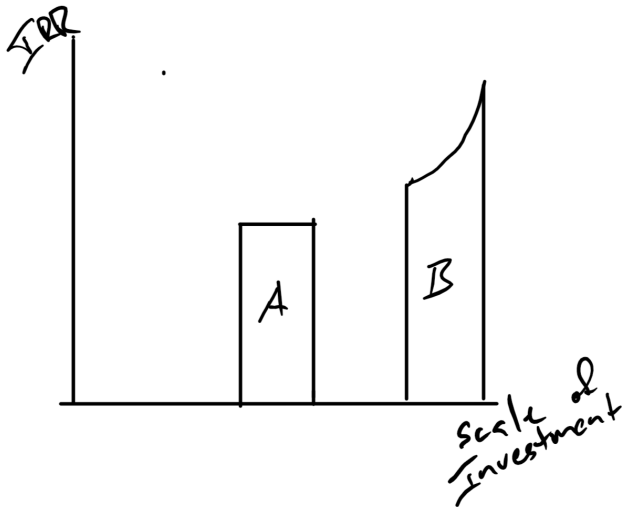
- ▶ You can take some or all.
- ▶ Does not mean any fraction, but close.
- ▶ Think of an apartment complex where you get to choose the number of units.

- ▶ Indivisible Assets

- ▶ You can take all or none.
- ▶ Think of a car.

## Additional Assumption about divisible assets.

No change in IRR dependent on scale, but this often happens in real life.

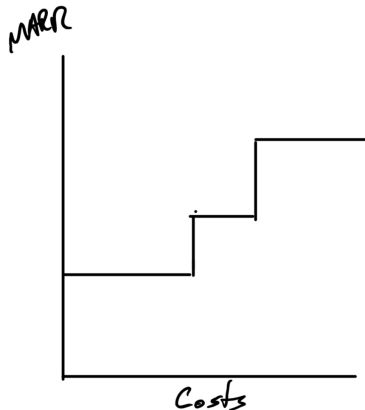
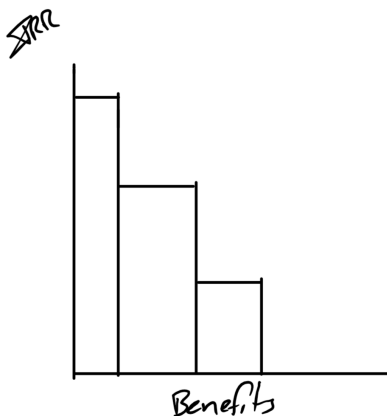


## Logic of the heuristic

- ▶ Not a continuous problem that can be solved by calculus, but you can use calc style logic.
- ▶ Your objective is to maximize *net* benefits, i.e., benefits less costs.
- ▶ In calc land you would set up a net benefit function, differentiate, set it equal to zero and solve.
- ▶ But, that net benefit function has to have the right shape, e.g.,  $NB'' < 0$ , to be sure that your local extrema is a max and the global max.
- ▶ For this problem, move around the funds and the assets to make sure that it has the right shape for a global maximum net benefit.

## Getting the right shape of net benefit function.

- ▶ Order your assets from highest IRR to lowest IRR.
- ▶ Order your funds from lowest MARR to highest MARR.



## Use analogy from maximization to pick assets

- ▶ In calc land you would set up a maximization problem

$$\max_q B(q) - C(q)$$

- ▶ Get the first derivative and set it equal to zero

$$B'(q) = C'(q)$$

- ▶ You just find where marginal benefit is equal to marginal cost.
- ▶ You can also think in terms of incremental costs and incremental benefits.
- ▶ *If incremental benefits is greater than incremental costs, do it.*

# Incremental Benefits and Costs

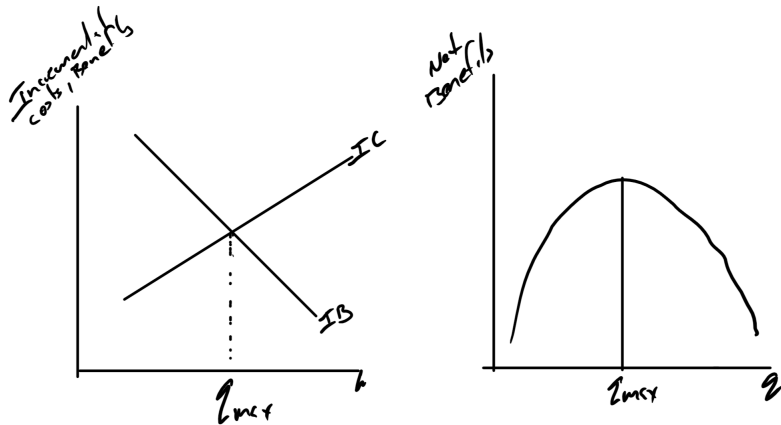


Figure 3



## Why the “ $IB > IC$ Do it” rule

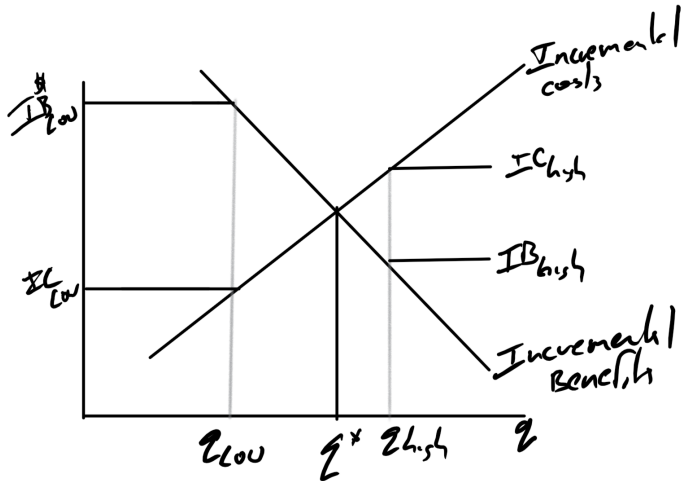


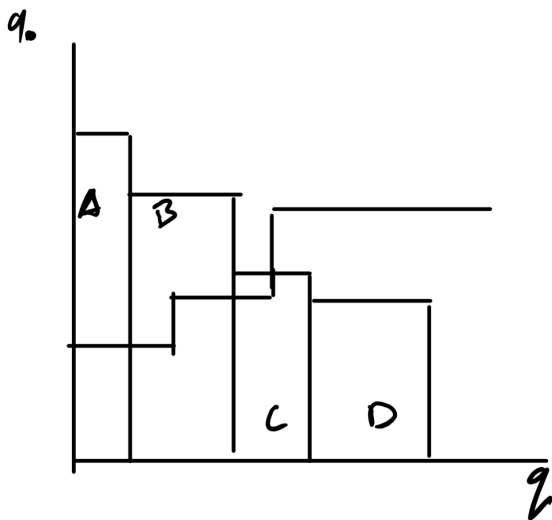
Figure 4

## Application to Divisible Assets

- ▶ This is all dependent on getting the curvature of the net benefit function right.
- ▶ Right means that the local extrema is a global max.
- ▶ Incremental costs should be non-decreasing.
- ▶ Incremental benefits should be non-increasing.
- ▶ Nothing odd happens with divisible assets.

## Divisible asset case

Get all assets and parts of assets where the IRR is greater than MARR.



# Summary of Diagram

- ▶ Vertical is IRR and MARR
- ▶ Horizontal is the scale of investment
- ▶ What gets purchased?
  - ▶ A is purchased because  $IRR > MARR_{Retained\ Earnings}$
  - ▶ B is purchased because  $IRR > MARR_{Retained\ Earnings} > MARR_{Loan\ 1}$
  - ▶ C is partially purchased.
    - ▶ Only the part that can be funded by Loan 1 since  $IRR > MARR_{Loan\ 1}$
    - ▶ Not the part funded by Loan 2.  $IRR < MARR_{Loan\ 2}$

## Try Tabular

Asset	A	B	C	D
IRR	20%	10%	12%	8%
$A_0$	6	5	4	7

$$\text{MARR} = 9\%$$

In these kinds of problems we only deal with investments and not loans. Not tricks on interpreting the IRR.

## Easy Answer

Asset	A	B	C	D
IRR	20%	10%	12%	8%
$A_0$	6	5	4	7

- ▶  $MARR = 9\%$
- ▶ Note that this is just the IRR criteria, “Buy if the IRR is greater than MARR”, i.e., A, B and C.

## Limited Retained Earnings

Asset	A	B	C	D
IRR	20%	10%	12%	8%
$A_0$	6	5	4	7

- ▶ \$12 of retained earnings at 9%
- ▶ Line of credit at 12% available.

## Get the Order Right

Asset	A	C	B	D
IRR	20%	12%	10%	8%
$A_0$	6	4	5	7

- ▶ Use \$10 of the retained earnings to buy all of A and C.
- ▶ Use the remaining \$2 to buy  $2/5$  of B, the part where the  $IRR > MARR$ .
- ▶ Don't use the the loan to buy remainder of B or D.

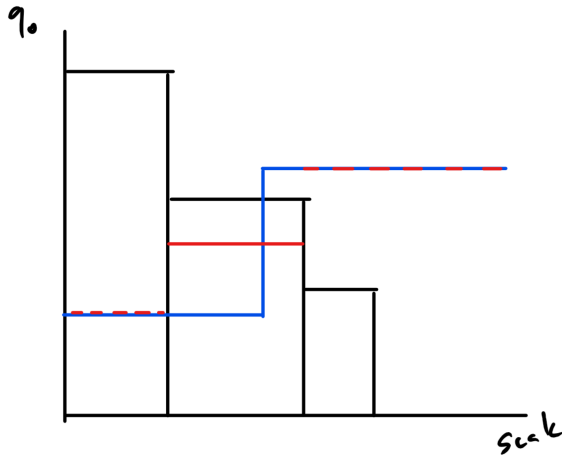


# Indivisible Assets

- ▶ It follows the same rules but cost of funds can get complicated.
- ▶ The cost of funds is the weighted average of the funds used.
  - ▶ Example: \$7 at 3% and \$3 at 10%.
  - ▶  $\frac{7}{10}3\% + \frac{3}{10}10\% = 5.1\%$
- ▶ You need to convert the *marginal* cost of funds to *Weighted average*, sometimes called *blended*, cost of funds.
  - ▶ The curvature of the net benefit function is *usually* OK.
  - ▶ Problems happen when the “IB>IC, Buy It” rule skips a high IRR asset but says to buy less expensive asset with lower IRR. (Picture Later).

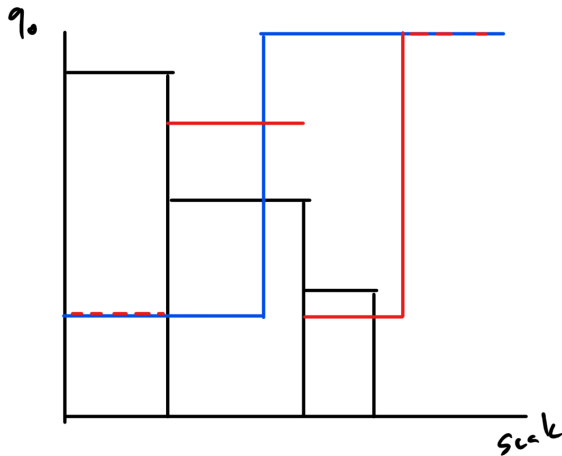
# Incremental Converted to Weighted Average

Note that the weighted average is still monotonically increasing.



## It can go bad

When middle asset is not purchased, IC is not monotonic.



## Indivisible Assets

Asset	A	B	C	D
IRR	20%	10%	12%	8%
$A_0$	6	5	4	7

- ▶ \$14 of retained earnings at 9%
- ▶ Line of credit at 12% available.

## Get the Order Right and then

Asset	A	C	B	D
IRR	20%	12%	10%	8%
$A_0$	6	4	5	7
RE (9%)	6	4	4	0
Loan (12%)	0	0	1	7
IC Funds	9%	9%	9.6%	12%

- ▶ Buy A since  $IRR > MARR_{RE}$
- ▶ Buy C since  $IRR > MARR_{RE}$
- ▶ Buy B since  $IRR > MARR_{\frac{4}{5}RE + \frac{1}{5}Loan}$
- ▶ Do not buy D since  $IRR < MARR_{Loan}$

## Try again

Asset	A	B	C	D
IRR	20%	10%	12%	8%
$A_0$	6	5	4	7

- ▶ \$12 of retained earnings at 9%
- ▶ Line of credit at 12% available.

## Solution

Asset	A	C	B	D
IRR	20%	12%	10%	8%
$A_0$	6	4	5	7
RE (9%)	6	4	2	2
Loan (12%)	0	0	3	5
IC Funds	9%	9%	10.8%	11.14%

You can't calculate the IC funds until you know if the previous asset was purchased.

- ▶ Buy A since  $IRR > MARR_{RE}$
- ▶ Buy C since  $IRR > MARR_{RE}$
- ▶ Do not Buy B since  $IRR < MARR_{\frac{2}{5}RE + \frac{3}{5}Loan}$
- ▶ Do not buy D since  $IRR < MARR_{\frac{2}{7}RE + \frac{5}{7}Loan}$

## Increasing Costs of Funds Summary

- ▶ Common environment when funding comes from many sources.
- ▶ In the divisible asset case, take the fractions of assets with  $IRR > MARR$ .
- ▶ The described heuristic is generally useful but can break in the divisible asset case.
- ▶ The problem is that the weighted average cost of funds is often dependent on which assets were accepted before.



# Introduction Capital Budgeting

“Capital Budgeting” refers to a problem where you are strictly limited to the amount that can be spent in period zero, but have a defined MARR.

- ▶ Commonly arises in small businesses when you don't have access to lending.
  - ▶ The MARR is frequently based on the return on common assets, or
  - ▶ the interest rate on your savings.
- ▶ Or, in large organizations when you have separate operating and capital budgets with no ability to transfer.
  - ▶ Does not apply in a use it or lose it environment (UIOLI).
  - ▶ Very common, but assumes that higher ups understand long-term needs more than lower downs.
  - ▶ In UIOLI you buy dumb things

# The Heuristic

It is the stupidest one you can use – “exhaustive search”

- ▶ Form all combinations of your assets.
- ▶ Eliminate all assets that are not feasible, i.e., cost more than you can spend in period zero.
- ▶ Choose the combination with the highest PW.

# Why is it stupid?

Because computation cost grows so quickly. There are  $2^N - 1$  proper subsets of a set.

- ▶ Three elements (A, B, C)
  - ▶ Pairs ((A,B),(A,C), (B,C))
  - ▶ One Trio (A, B, C)
- ▶ 10 Asset Choices, 1023, combinations.
- ▶ Typical maintenance list for Parkrose had 150-200 items.
  - ▶  $1.42 \times 10^{45}$  to  $1.6 \times 10^{60}$  combinations.
  - ▶ Only  $9.6 \times 10^{56}$  atoms in the solar system.

## Example

Asset	A	B	C
$A_0$	2	4	3
PW(10%)	2	3	1

With a \$7 capital budget. Note ABC is not feasible.

Combo	PW	Combo	PW
A	2	AB	5
B	3	AC	3
C	1	BC	4

AB with \$1 left over is the best combination.

# Observations

- ▶ The MARR does not account for the full opportunity cost of an asset
  - ▶ It accounts for the cost of funds.
  - ▶ Does not capture the net benefits of assets that are forced out.
    - ▶ \$10 available and two assets, one that costs \$8 and another that costs \$3
    - ▶ Buying the \$3 assets makes it so you can't get the \$8 asset.
    - ▶ The opportunity cost is not just \$3 but the net benefits of the \$8 asset.
    - ▶ If you had Calc IV, Lagrange multiplier type effect.
- ▶ Often have money left over because of the Lagrange type effect.

## A Frequent Problem

You often have capital budgeting/ combinatorial optimization problems in government budgets.

- ▶ Your high IRR project is not funded, but other larger – lower IRR – projects are funded.
- ▶ Your project would have 'bumped' one of the larger projects.
- ▶ Don't argue against the other projects/programs, argue for a larger overall budget.

You should notice this in many budget processes. There is a strong tendency to commit one-time money, or reduced contingency, to fund a few high IRR programs.