Annual Worth

Goals

- Present AW as a frequent client preferred measure of costs and benefits.
- Show details of AW calculation.
- Give warnings about interpretation
- Show a computational advantage in problems of repeated purchases.

AW

$$AW(Asset) = PW(Asset)(A|P, i, Life\ of\ \ Asset)$$

- AW is a transformation of present worth.
- AW is except in one edge case, smaller than PW.
  - The edge case is an asset the is installed at time zero and only lasts through time 1.
  - $(A|P, i, 1) = \frac{i(1+i)}{(1+i)-1} = (1 + i)$
- Many synonyms
  - Levelized cost
  - Capitalized cost (Danger PW is sometimes called this too)
  - Equivalent annual cost

Simple Example

The car costs 10K and will last five years. What is the per-period cost of the car when the MARR is 10%?

$$10K(A|P, i = 10\%, 5) = 2.64K$$

- The purchase price – now – is a present worth.
- Notice that it is not $\frac{10K}{5}$. It does take into account that you paid for the car in time zero.

The AW Criteria

- Annual Worth
  - Per-period costs and benefits ($)
  - Unconstrained: If $AW \geq 0$ get it.
  - Exclusive: If assets have the same life, choose asset with largest AW.

Unconstrained Choice

If $AW \geq 0$ get it.

$$AW(Asset) = PW(Asset)(A|P, i, Life\ of\ Asset)$$

- Note that $(A|P, i, Life\ of\ Asset)$ is always positive.
- That means $PW \geq 0 \Rightarrow AW \geq 0$

Exclusive Choice

If assets have the same life, choose asset with largest AW.

- The "If" is critical
- Example why

Year	A	B
0	0	0
1	10	9
2		9
3		9
4		9
5		9
AW	10	9

- With moderate MARR, B is better but has lower AW.

The If

- It is less restrictive than you think and often useful.
- You can construct assets from other assets.
- Example {Car, Kayak Rack, Kayaks}
  - Doesn't make sense to get a rack without a car or kayaks.
  - Re-frame as exclusive choice:
    - A = {Car}
    - B = {Car, Kayaks}
    - C = {Car, Rack, Kayaks}

How this helps with repeated purchases

- Remember the Roof Example?
  - Metal Roof (Like the old one but 'done correctly') would last 40 years and cost \$1.5M.
  - PVC Roof would last 20 years and cost 800K to install. It also requires 15K in additional inspection and maintenance in years 16-20.
- These have unequal lives but you can use AW by constructing a new asset
  - A = {Metal now}
  - B = {PVC now, PVC in 20 years}

The Metal Roof

What is the levelized cost of the \$1.5M metal roof when the MARR is 10%?

Answer

$$1500K(A|P, i = 1, 40) = 153.3891216K$$

Interpret this as the annual cost of roofing services.

The PVC Roof

What is the levelized cost of the the two PVC roofs when the MARR is 10%?

PW of the first roof is:

$$PW(PVC) = 800K + \frac{15K(P|A, i = .1, 5)}{(1 + .1)^{15}} = 813.61K$$

Trick Question

The answer is the same if you make the calculation with two roofs or one.

- One Roof: \$ 813.6122632K (A|P, i = 10%, 20) = 95.57K\$
- Two Roofs:
  - $PW(PVC|40\,Years) = 813.6122632K + \frac{813.61K}{1.1^{20}} = 934.55K$
  - $AW(PVC|40\,Years) = 934.55K(A|P, i = 10\%, 40) = 95.57K$