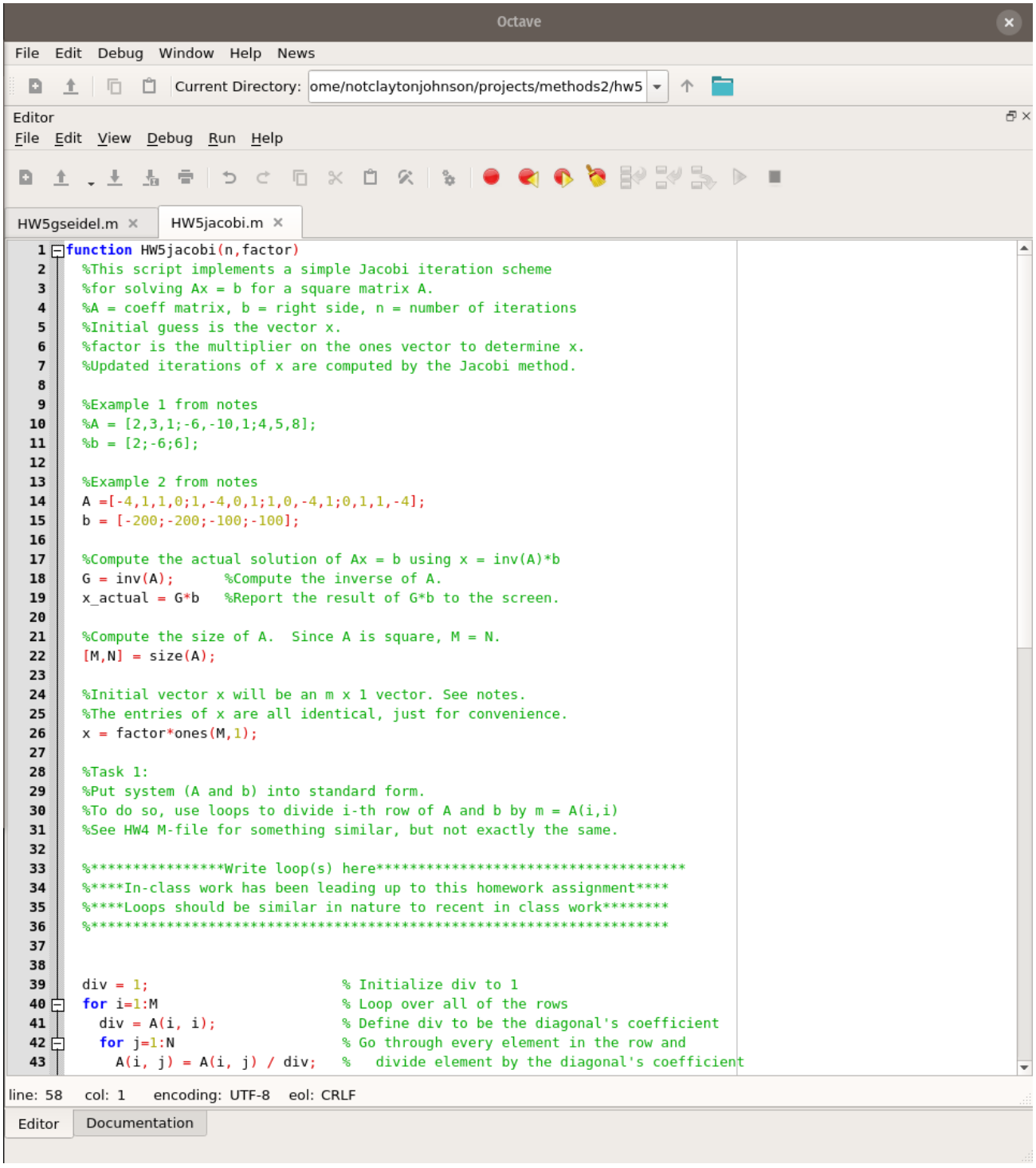
MATH 366 Methods of Applied Mathematics II

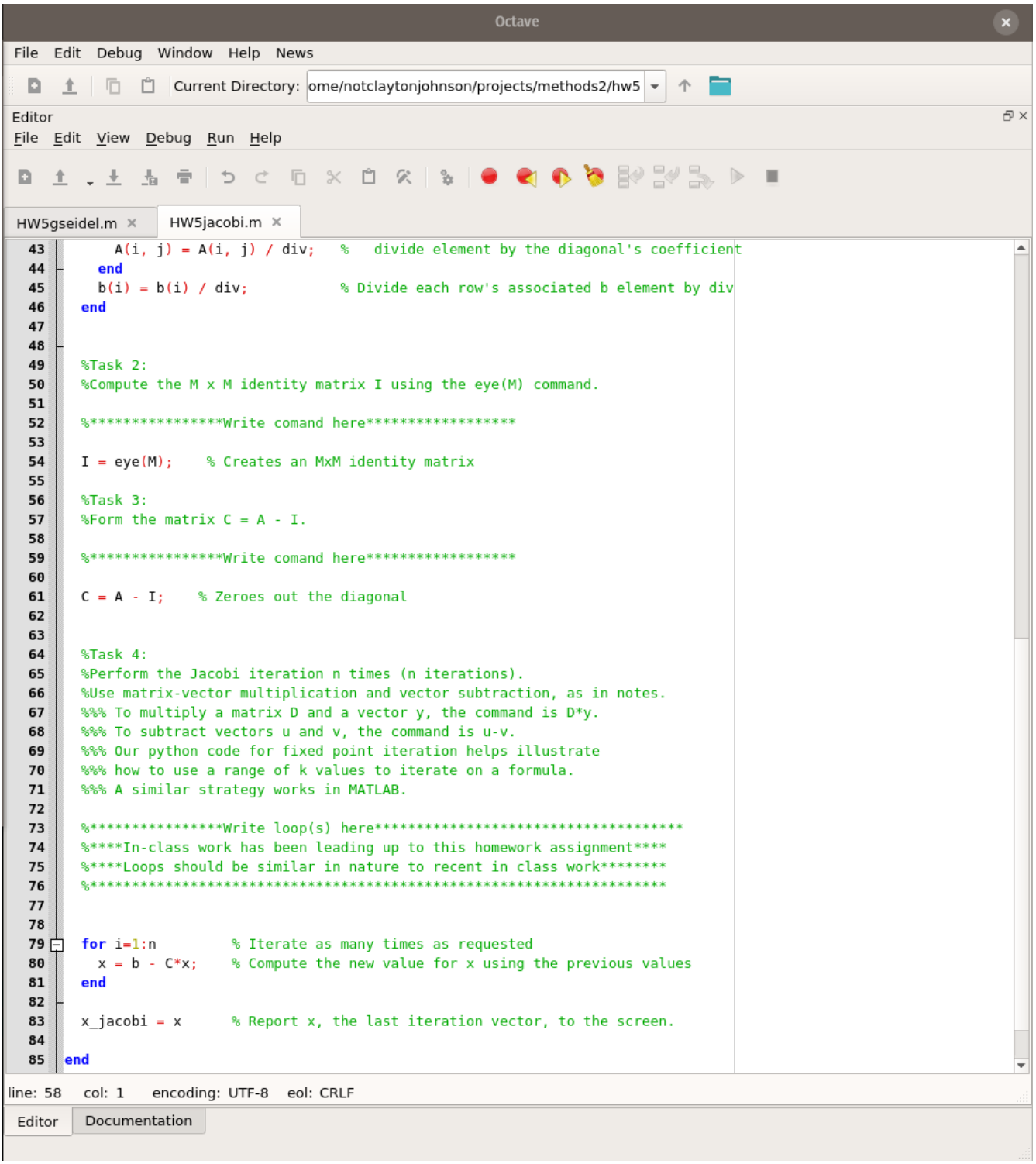
HW5: Ch20.3 Jacobi and Gauss-Seidel Iteration with MATLAB Clayton Johnson

Jacobi Iteration

Code (MATLAB/Octave)




```
function HW5jacobi(n,factor)
%This script implements a simple Jacobi iteration scheme
%for solving Ax = b for a square matrix A.
%A = coeff matrix, b = right side, n = number of iterations
%Initial guess is the vector x.
%factor is the multiplier on the ones vector to determine x.
%Updated iterations of x are computed by the Jacobi method.

%Example 1 from notes
%A = [2,3,1;-6,-10,1;4,5,8];
%b = [2;-6;6];

%Example 2 from notes
A =[-4,1,1,0;1,-4,0,1;1,0,-4,1;0,1,1,-4];
b = [-200;-200;-100;-100];

%Compute the actual solution of Ax = b using x = inv(A)*b
G = inv(A);      %Compute the inverse of A.
x_actual = G*b   %Report the result of G*b to the screen.

%Compute the size of A.  Since A is square, M = N.
[M,N] = size(A);

%Initial vector x will be an m x 1 vector. See notes.
%The entries of x are all identical, just for convenience.
x = factor*ones(M,1);

%Task 1:
%Put system (A and b) into standard form.
%To do so, use loops to divide i-th row of A and b by m = A(i,i)
%See HW4 M-file for something similar, but not exactly the same.

%****************Write loop(s) here************************************
%****In-class work has been leading up to this homework assignment****
%****Loops should be similar in nature to recent in class work********
%********************************************************************



div = 1;                        % Initialize div to 1
for i=1:M                       % Loop over all of the rows
  div = A(i, i);                % Define div to be the diagonal's coefficient
  for j=1:N                     % Go through every element in the row and
    A(i, j) = A(i, j) / div;    %   divide element by the diagonal's coefficient
```


Octave
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Editor
File Edit View Debug Run Help
HW5gseidel.m
HW5jacobi.m
43 A(i, j) = A(i, j) / div; % divide element by the diagonal's coefficient
44 end
45 b(i) = b(i) / div; % Divide each row's associated b element by div
46 end
47
48
49 %Task 2:
50 %Compute the M x M identity matrix I using the eye(M) command.
51
52 %***************Write comand here*****************
53
54 I = eye(M); % Creates an MxM identity matrix
55
56 %Task 3:
57 %Form the matrix C = A - I.
58
59 %***************Write comand here*****************
60
61 C = A - I; % Zeroes out the diagonal
62
63
64 %Task 4:
65 %Perform the Jacobi iteration n times (n iterations).
66 %Use matrix-vector multiplication and vector subtraction, as in notes.
67 %%% To multiply a matrix D and a vector y, the command is D*y.
68 %%% To subtract vectors u and v, the command is u-v.
69 %%% Our python code for fixed point iteration helps illustrate
70 %%% how to use a range of k values to iterate on a formula.
71 %%% A similar strategy works in MATLAB.
72
73 %***************Write loop(s) here************************************
74 %****In-class work has been leading up to this homework assignment****
75 %****Loops should be similar in nature to recent in class work********
76 %***
77
78
79 for i=1:n % Iterate as many times as requested
80 x = b - C*x; % Compute the new value for x using the previous values
81 end
82
83 x_jacobi = x % Report x, the last iteration vector, to the screen.
84
85 end
line: 58 col: 1 encoding: UTF-8 eol: CRLF
Editor
Documentation

Output

Example 1

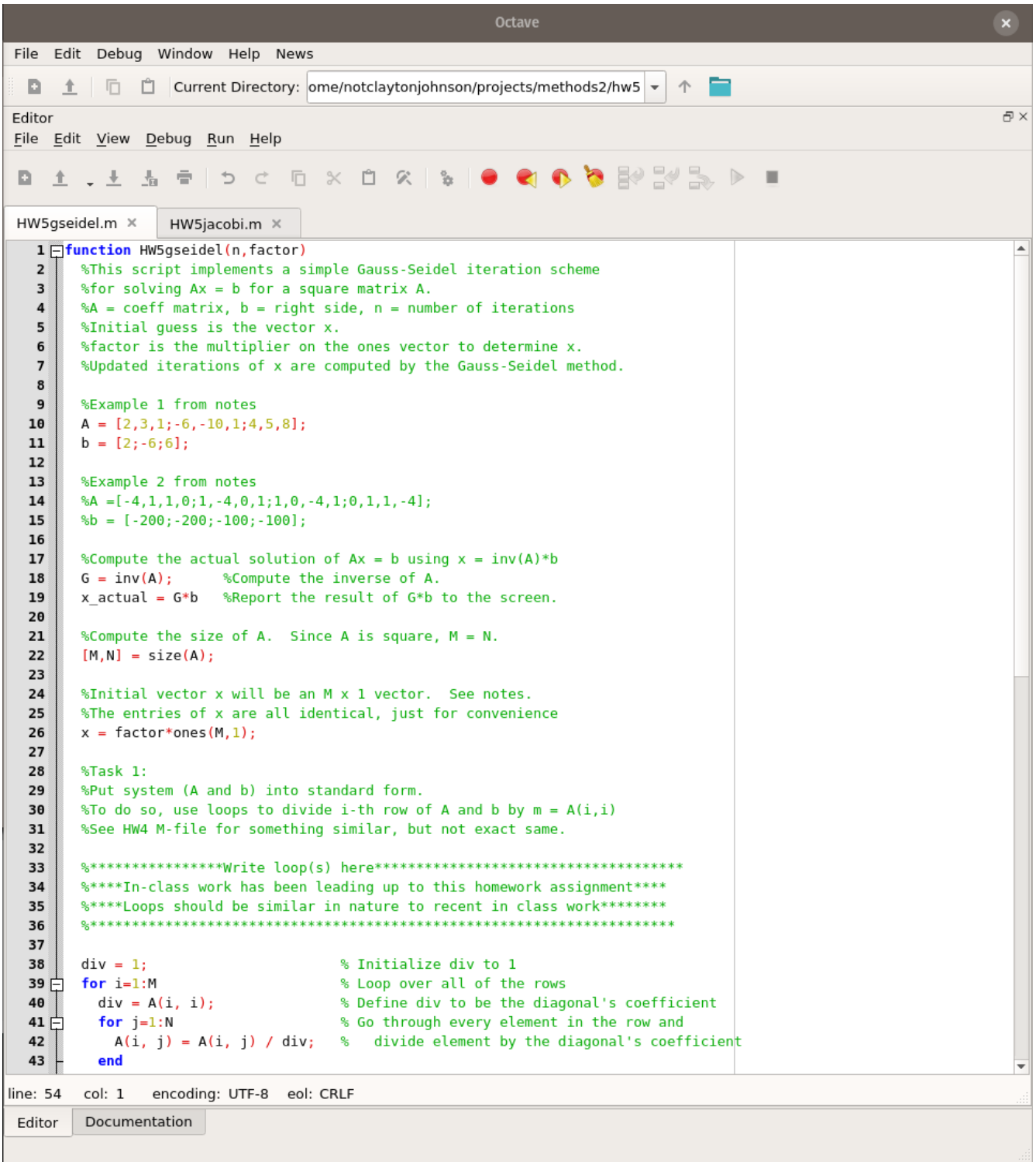
```
Command Window
>> HW5jacobi(1,1)
x_actual =

  -5.5000
   4.0000
   1.0000

x_jacobi =

  -1.00000
   0.10000
  -0.37500

>> HW5jacobi(100,1)
x_actual =

  -5.5000
   4.0000
   1.0000

x_jacobi =

   5676.5
   2744.3
   4174.0

>>
```

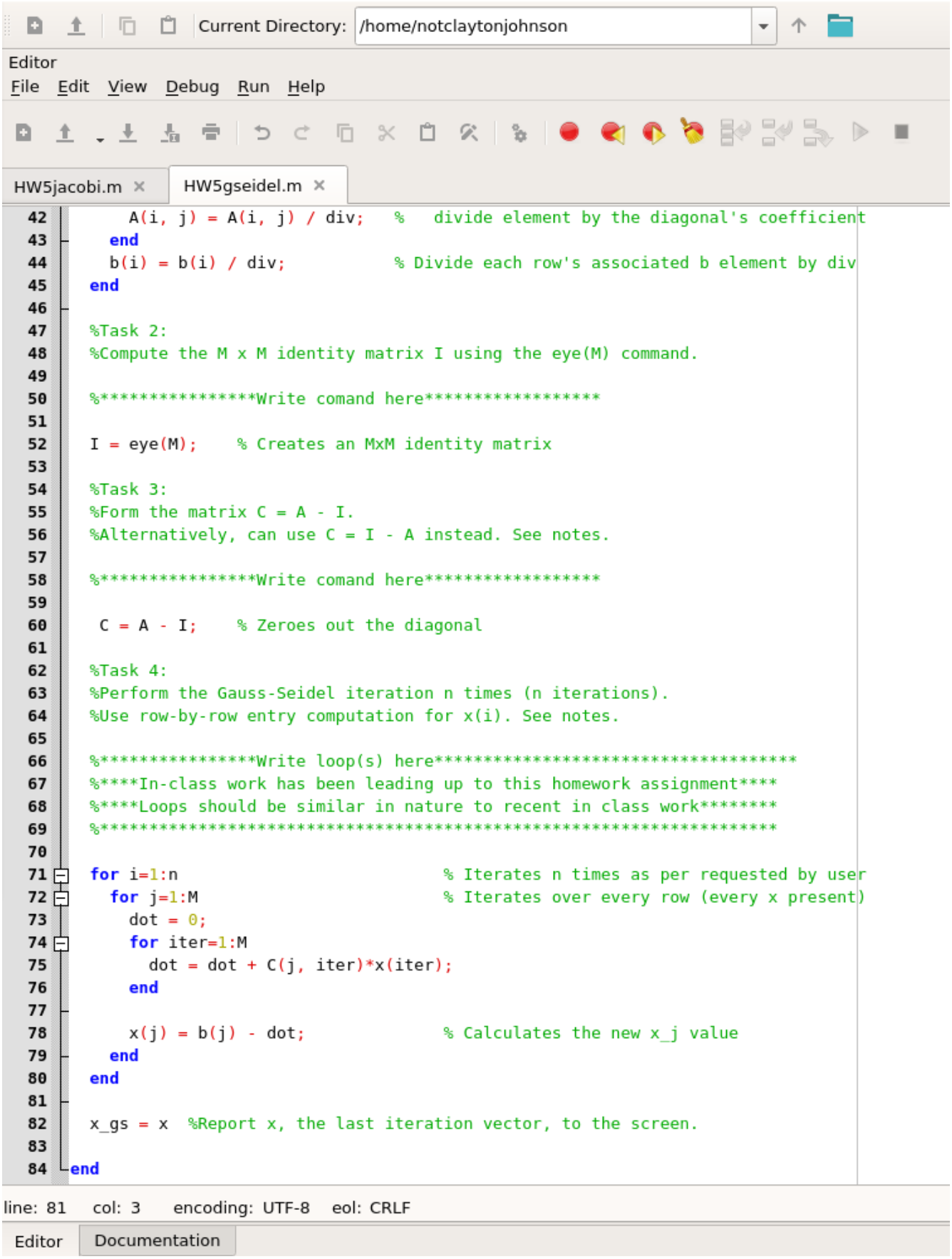
Example 2

```
Command Window
>> HW5jacobi(1,100)
x_actual =

   87.500
   87.500
   62.500
   62.500

x_jacobi =

   100
   100
    75
    75

>> HW5jacobi(2,100)
x_actual =

   87.500
   87.500
   62.500
   62.500

x_jacobi =

   93.750
   93.750
   68.750
   68.750

>> HW5jacobi(10,100)
x_actual =

   87.500
   87.500
   62.500
   62.500

x_jacobi =

   87.524
   87.524
   62.524
   62.524

>>
```

Gauss-Seidel Iteration

Code (MATLAB/Octave)

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Editor
File Edit View Debug Run Help
HW5jacobi.m
HW5gseidel.m
A(i, j) = A(i, j) / div; % divide element by the diagonal's coefficient
end
b(i) = b(i) / div; % Divide each row's associated b element by div
end
%Task 2:
%Compute the M x M identity matrix I using the eye(M) command.
%****************Write comand here*****************
I = eye(M); % Creates an MxM identity matrix
%Task 3:
%Form the matrix C = A - I.
%Alternatively, can use C = I - A instead. See notes.
%****************Write comand here*****************
C = A - I; % Zeroes out the diagonal
%Task 4:
%Perform the Gauss-Seidel iteration n times (n iterations).
%Use row-by-row entry computation for x(i). See notes.
%****************Write loop(s) here************************************
%****In-class work has been leading up to this homework assignment****
%****Loops should be similar in nature to recent in class work********
%***
for i=1:n % Iterates n times as per requested by user
for j=1:M % Iterates over every row (every x present)
dot = 0;
for iter=1:M
dot = dot + C(j, iter)*x(iter);
end
x(j) = b(j) - dot; % Calculates the new x_j value
end
end
x_gs = x %Report x, the last iteration vector, to the screen.
end
line: 81 col: 3 encoding: UTF-8 eol: CRLF
Editor
Documentation

Output

Example 1

```
Command Window
>> HW5gseidel(1,1)
x_actual =

  -5.5000
   4.0000
   1.0000

x_gs =

  -1.00000
   1.30000
   0.43750

>> HW5gseidel(100,1)
x_actual =

  -5.5000
   4.0000
   1.0000

x_gs =

  -5.06621
   3.73451
   0.94904

>>
```

Example 2

```
Command Window
>> HW5gseidel(1,100)
x_actual =

   87.500
   87.500
   62.500
   62.500

x_gs =

   100.000
   100.000
    75.000
    68.750

>> HW5gseidel(2,100)
x_actual =

   87.500
   87.500
   62.500
   62.500

x_gs =

   93.750
   90.625
   65.625
   64.062

>> HW5gseidel(100,100)
x_actual =

   87.500
   87.500
   62.500
   62.500

x_gs =

   87.500
   87.500
   62.500
   62.500

>>
```