Chapter 9.7 - Heat Fins

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Formulating the Differential Equation

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Replacements in word equation 9.28 (below):

$$

\begin{equation}

\begin{Bmatrix}

\mathrm{rate \, heat} \\

\mathrm{conducted} \\

\mathrm{in\, at\, x} \\

\end{Bmatrix}

-

\begin{Bmatrix}

\mathrm{rate \, heat} \\

\mathrm{conducted} \\

\mathrm{out\, at\, x+\Delta x} \\

\end{Bmatrix}

-

\begin{Bmatrix}

\mathrm{rate\, of\, heat} \\

\mathrm{lost\, to} \\

\mathrm{surroundings} \\

\end{Bmatrix}

= 0

\end{equation}

$$

to generate our differential equation.

We want to write this DE in terms of heat flux ($J$) and temperature ($U$), both

with respect to space ($x$), no time.

Formulating the Differential Equation

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Using what we learned in section 9.5 -- Heat Conduction through a Wall -- we can state

$$

\begin{Bmatrix}

\mathrm{rate \, heat} \\

\mathrm{conducted} \\

\mathrm{in\, at\, x} \\

\end{Bmatrix}

= J(x)A(x)

= J(x)bw,

$$

and

$$

\begin{Bmatrix}

\mathrm{rate \, heat} \\

\mathrm{conducted} \\

\mathrm{out\, at\, x+\Delta x} \\

\end{Bmatrix}

= J(x+\Delta x)A(x)

= J(x+\Delta x)bw,

$$

where $bw$ indicates the area through which heat is conducted.

Formulating the Differential Equation

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What we have so far:

$$

J(x)bw - J(x+\Delta x)bw - \begin{Bmatrix}

\mathrm{rate\, of\, heat} \\

\mathrm{lost\, to} \\

\mathrm{surroundings} \\

\end{Bmatrix}

= 0.

$$

Final term from the word equation (9.28):

$$

\begin{Bmatrix}

\mathrm{rate\, of\, heat} \\

\mathrm{lost\, to} \\

\mathrm{surroundings} \\

\end{Bmatrix}

= \text{Newton's Law of Cooling stuff}

$$

Formulating the Differential Equation (Newton Cooling Stuff)

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What we have from section 9.3 -- Model for a Hot Water Heater:

$$

\begin{Bmatrix}

\mathrm{rate\, heat} \\

\mathrm{exchanged\, with} \\

\mathrm{surroundings} \\

\end{Bmatrix}

= \pm hS \Delta U,

$$

where \*\*$h$ is a positive constant of proportionality\*\*, $S$ is the surface area from which heat is lost/gained, and $\Delta U$ is the temperature difference between the object (fin) and surroundings ($u\_s$).

Formulating the Differential Equation

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<img src='./heat\_fin\_dimensions.png' height=300 width=400 align='center'>

Previous slide:

$$

\begin{Bmatrix}

\mathrm{rate\, heat} \\

\mathrm{exchanged\, with} \\

\mathrm{surroundings} \\

\end{Bmatrix}

= hS \Delta U

$$

\*\*\*

$h$ depends on the material,

$S = 2w \Delta x$, since we are ignoring the side faces,

$\Delta U$ is the difference in temperature from $U(x^\*)$ and $u\_s$, where $x \lt x^\* \lt x+\Delta x$.

Formulating the Differential Equation

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Thus, we have

$$

\begin{Bmatrix}

\mathrm{rate\, heat} \\

\mathrm{exchanged\, with} \\

\mathrm{surroundings} \\

\end{Bmatrix}

= hS \Delta U

= 2hw\Delta x[U(x^\*)-u\_s]

$$

and the differential equation is now

$$

J(x)bw - J(x+\Delta x)bw - 2hw\Delta x[U(x^\*)-u\_s] = 0.

$$

or, as the book states:

$$

J(x)bw - J(x+\Delta x)bw - 2hw\Delta x[U(x + \lambda \Delta x)-u\_s] = 0.

$$

where $x^\* = x + \lambda \Delta x$ and $0 \leq \lambda \leq 1$.

Formulating the Differential Equation (Units) (Optional)

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## Units of \*\*$h$\*\*

$$

\begin{align\*}

\big[ hS \Delta U \big] &= \big[J(x)bw \big], \\

\big[ h \big] \times \text{meters}^2 \times \text{Temperature} &= \frac{ \text{Watts} }{\text{meters}^2} \times \text{meters}^2, \\

\big[ h \big] &= \frac{ \text{Watts} }{ \text{Temperature} \times \text{meters}^2 }, \\

\big[ h \big] &= \frac{ \text{Joules} }{ \text{Temperature} \times \text{meters}^2 \times \text{Seconds} }

\end{align\*}

$$