Insulating a Water Pipe - Results and Interpretation

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Numerical Investigation

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$$

Q = \frac{ 2k \pi l(u\_a - u\_b) }{ \ln \left(\frac{ b }{a} \right) + \frac{k}{bh}}

$$

\*\*\*

Let's use some values to help us see how the heat flow changes as we vary the outer-radius ($b$):

- $l$ = $1 \text{ meter}$

- $a$ = $15 \text{ millimeters}$

- $u\_a$ = $60 \text{ °C}$

- $u\_b$ = $15 \text{ °C}$

- $h$ = $5 \frac{ \text{W} }{ \text{m}^2 \text{ °C} }$

Numerical Investigation (Cont.)

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#### Total Rate of Heat Loss ($Q$) with respect to $b$ and $k$

Insulation thickness $(b-a)$ | Rate of heat loss $k = 0.05$ | Rate of heat loss $k = 0.17$

------------ | ------------- | -------------

$0$ | $21.2$ | $21.2$

$1$ | $20.5$ | $22.0$

$2$ | $19.8$ | $22.6$

$5$ | $18.0$ | $24.2$

$10$ | $15.5$ | $25.7$

#### Observations

- $(k=0.05 \text{ and } \uparrow b) \downarrow Q$

- $(k=0.17 \text{ and } \uparrow b) \uparrow Q$

Numerical Investigation (Cont.)

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```{r, echo=FALSE, fig.width = 16, fig.height = 10}

q <- function(k, a, b) {

l <- 1

u\_a <- 60

u\_b <- 15

h <- 5

return( ( 2\*k\*pi\*l\*( u\_a - u\_b ) )/( log(b / a) + k/(b\*h) ) )

}

figure116 <- function() {

# Set up our constants

N <- 1000

# Min/Max values for b

b\_min <- 0.005

b\_max <- 0.05

# Step size on x-axis

h <- (b\_max - b\_min) / N

# Set the minimum radius

a <- 0.015

# Initialize the b vector to zero

b <- rep(0, N)

# Set up all the values of b we want to talk about

for(i in 1:N) {

b[i] <- b[1] + i\*h

}

# Initialize the plot so we can talk to it

plot(0, 0, main = "Rate of Heat Loss with Changing Insulation Width",

xlab = "b (meters)", #x-axis label

ylab = "Rate of Heat Loss (Q)", #y-axis label

type = "l",col="blue", #Line type and color

xlim = c(b\_min, b\_max), #x-axis range vector: c(0,T) = [0,T]

ylim = c(0,30), #y-axis range vector: c = "combine")

cex.main=2,

cex.lab=2,

cex.axis=2

)

# Generate the graphs at all steps of k values

k\_high <- 0.20

k\_low <- 0.03

k\_step <- 0.01

# Use to generate all the colors for the graph (these are just kind of random

# values because I didn't want to hand-pick every color and I wanted to keep

# the code dynamic!)

colors\_list <- colors()

color\_counter <- 20

# Create these empty vectors for the labeling of the plots

labels <- c()

label\_colors <- c()

label\_types <- c()

for(k in seq(k\_low, k\_high, by=k\_step)) {

# Initialize the q values as zero

q <- rep(0, N)

# Acquire the values for q

for(i in 1:N) {

q[i] <- q(k, a, b[i])

}

# Find the point with the highest q-value and graph it

b\_coord = b[which.max(q)]

q\_coord = max(q)

points(b\_coord, q\_coord)

# Graph the values with some color

col = colors\_list[ 4\*color\_counter ]

lines(b, q, col=col, lwd=3)

# Keep track of the color used for the labels

labels <- c( labels, paste( c("k=", k) , collapse="") )

label\_colors <- c( label\_colors, col )

label\_types <- c( label\_types, 1 )

color\_counter <- color\_counter + 1

}

# Add the vertical line of what a is

abline(v=a)

a\_label <- paste( c("a=", a), collapse="")

text(a, 0, a\_label, adj = c(-0.1, 1.0), cex=1.75)

legend("bottomright",

legend = labels, #Vector of legend items

col = label\_colors, #vector of colors for legend items

lty = label\_types, #Vector of line types for legend items

lwd=3,

cex=2

)

}

figure116()

```

Numerical Investigation (Cont.)

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##### What are we seeing?

- These 'critical values' ($\frac{k}{h}$) need to be less than the inner radius ($a$) for the insulation to be effective (depending on how much you're willing to put on!)

- Competing effects: with more insulation...

- there is more volume the heat needs to flow through to escape (decreasing heat flow),

- the higher to surface area heat can be lost through (increasing heat flow).