Chapter 12.4 Lake Pollution Revisited

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author:

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Wave Motion

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Equation 12.39:

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C(x, t) = \begin{cases}

0, & \text{ if } & t \lt \frac{x-a}{v} \\

P(x-vt), & \text{ if } & t \gt \frac{x-a}{v}.

\end{cases}

$$

This describes \*wave motion\*.

<img src="wave.gif">

Wave Motion

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What happens when we are increasing time, $t$. Notice that $P(a) = P(x-vt)$ for some $x$ and $t$. This implies

$$

a = x - vt, \\

x = a + vt.

$$

Thus, we see that, as $t$ increases, we are moving in the positive $x$ direction. The \*wave\* of pollution is moving in the $+x$-direction with velocity, $v$.

Wave Motion

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Initializing system with a \*wave\* of pollution, $g(t)$, at boundary $a=0$. We assume no pollution in the lake at $t=0$. Graph shows levels of pollution in the lake for $a \leq x \leq b$, with $t = t+nh$, where $n = 1, 2, ...$ and $h = \pi$.

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<img src="fig12.6.png" height=1000px width=250px>

Pollution Reduction

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How long will it take a lake to decrease to $5\%$ of its current level of pollutant with a stream of fresh water as the feeder into the lake, with $g(t) = 0$ and $P(x, 0) \neq 0$, for some $x$ where $a \leq x \leq b$?

1) Concentration to Mass using $C = \frac{M}{V(x)}$, where $V = \Delta x \times \text{width} \times \text{depth}$ is the volume of water at location $x$.

Pollution Reduction

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2) Since we have constant $\text{width}$ and $\text{depth}$, and $C \Delta x$ is a measure of mass, we can state

$$

\begin{align\*}

\int\_a^b C(x, t) dx &= 0.05 \int\_a^b P(x) dx, \\

\int\_a^b P(x-vt) dx &= 0.05 \int\_a^b P(x) dx, \\

\int\_{a-vt}^{b-vt} P(s)ds &= 0.05 \int\_a^b P(x) dx. \\

\end{align\*}

$$