FUTURES AND OPTIONS

Midterm Exam – Thursday, November 16, 2023 – 1:20 p.m. – 4:20 p.m.

Fall 2023 – Instructor Han-Hsing Lee

I. Multiple Choices (34%)

1. D

   Because the company is short, each one cent rise in the price leads to a loss or $0.01\times 50{,}000$ or \$500. A greater than 2 cent rise in the futures price will therefore lead to a margin call.

2. A
3. B
4. D
5. A

   The price received by the trader is the futures price plus the basis. It follows that the trader's position improves when the basis increases.

6. B

   The mining company shorts futures. It gains on the futures when the price decreases and loses when the price increases. It may get margin calls which lead to liquidity problems when the price rises even though the silver in the ground is worth more.

7. C
8. A
9. B

   When inventories decline, the convenience yield increases and the futures price as a percentage of the spot price declines.

10. D

    Intrinsic value = Max (0, 45 - 50) = \$0

    Time value = Call premium - Intrinsic value = \$6.50 - \$0 = \$6.50

11. A
12. C
13. A
14. A
15. C

    As the volatility of the option decreases the time value declines and the option becomes more likely to be exercised early. In the case of A and B, time value increases and the option is less likely to be exercised early.

16. C

    The price of the call has increased by \$1.50. From put-call parity the price of the put must increase by the same amount. Hence the put price will become 4.00 +1.50 = \$5.50.

17. B

II. Calculation (66%)

1.
The open interest went up by 300. We can see this in two ways. First, 1,700 shorts closed out and there were 2,000 new shorts. Second, 1,000 longs closed out and there were 1,300 new longs.

2.
(a) Spot price on Nov. 10 + Gain on Futures = \$75 - \$4 = \$71
Futures price on June 8 + Basis on November 10 = \$68+\$3=\$71
The net cost of oil is therefore (\$71)(20,000)=\$1420,000 or \$71 per barrel.

(b) If the hedge ratio is 0.8, the company takes a long position in 16 December oil futures contracts on June 8 when the futures price is \$68.00. It closes out its position on November 10. The spot price and futures price at this time are \$75 and \$72. The gain on the futures position is

($72-$68.00)(16,000)=$64,000

The effective cost of the oil is therefore

$$20{,}000\times 75 - 64{,}000 = \$1{,}436{,}000$$

or \$71.80 per barrel. (This compares with \$71.00 per barrel when the company is fully hedged.)

3.
(a) the number of contracts the fund manager should short is

$$1.08\times\frac{10{,}000{,}000}{860\times 250} = 50.23$$

Rounding to the nearest whole number, 50 contracts should be shorted.

(b) If the index in two months is 800, the futures price is $800\times 1.0025 = 802$. The gain on the short futures position is therefore

$$(860-802)\times 250\times 50 = \$725{,}000$$

The return on the index is $3\times {}^{2}\!/_{12} = 0.5\%$ in the form of dividend and $-{}^{50}\!/_{850} = -5.882\%$ in the form of capital gains. The total return on the index is therefore -5.382%. The risk-free is 0.5%(=3%/6) per two months. From the capital asset pricing model we expect the return on the portfolio to be

$$E(r_P) = 0.5\% + 1.08(-5.382\% - 0.5\%) = 0.5\% - 6.353\% = -5.853\%$$

The loss on the portfolio is $0.05853\times 10{,}000{,}000 = \$585{,}294$. When this is combined with the gain on the futures the total gain is \$725,000-585,294=\$139,706.

Index in Two Months	800
Futures Price ($)	802
Gain on Futures ($)	725,000
Portfolio Gain (%)	-5.853%
Portfolio Gain ($)	-$585,294
Total Gain ($)	$139,706

4.

Maturity (years)	Zero Rate (%)	Forward Rate (%)	Par Yield (%)
0.5	2.01%	2.01%	
1.0	3.05%	4.08%	3.04%
1.5	4.30%	6.80%	
2.0	4.36%	4.54%	4.33%

(a)
6 months:

$$99e^{R(0.5)} = 100$$

12 months:

$$97e^{R(1)} = 100$$

18 months:

$$(5/2)e^{-(2.01\%)(0.5)} + (5/2)e^{-(3.05\%)(1)} + (2.5+100)e^{-R(1.5)} = 101$$

24 months:

$$(7/2)e^{-(2.01\%)(0.5)} + (3.5)e^{-(3.05\%)(1)} + (3.5)e^{-(4.30\%)(1.5)} + (3.5+100)e^{-R(2)} = 105$$

Note that the zero curve is horizontal prior to the first point and horizontal beyond the last point.

(b)

$$\text{Forward rate: } F_i = \frac{R_{i+1}T_{t+1} - R_i T_t}{T_{i+1} - T_i}$$

(c)
12 months:

$$(c/2)[e^{-(2.01\%)(0.5)} + e^{-(3.05\%)(1)}] + 100e^{-(3.05\%)(1)} = 100 \quad (= \text{par value})$$

24 months:

$$(c/2)[e^{-(2.01\%)(0.5)} + e^{-(3.05\%)(1)} + e^{-(4.30\%)(1.5)} + e^{-(4.36\%)(2)}] + 100e^{-(4.36\%)(2)} = 100$$

(d) \$105 (no calculation needed)

(e) Problem 4.15.
The forward rate is 4.54% with continuous compounding or $2(e^{0.0454/2} - 1) = 0.0459$ with semi-annual compounding. The 2-year interest rate is 4.36% with continuous compounding. From equation (4.10), the value of the FRA is therefore

$$[1{,}000{,}000 \times (0.0459 - 0.05) \times 0.5]\mathrm{e}^{-0.0436\times 2} = -2050\mathrm{e}^{-0.0436\times 2} = -1878.81$$

5.

Suppose that F_0 is the one-year forward price of gold. If F_0 is relatively high, the trader can borrow \$1505 at 3%, buy one ounce of gold and enter into a forward contract to sell gold in one year for F_0.
The profit made in one year is $F_0 - 1505 \times 1.025 = F_0 - 1542.625$

This is profitable if F_0 >1542.625.

If F_0 is relatively low, the trader can sell one ounce of gold for $1500, invest the proceeds at 2%, and enter into a forward contract to buy the gold back for F_0. The profit (relative to the position the trader would be in if the gold were held in the portfolio during the year) is
$1500 \times 1.02 - F_0 = 1530 - F_0$

This shows that there is no arbitrage opportunity if the forward price is between $1530 and $1542.625 per ounce.

6.

(a) the 2-year forward exchange rate should be $0.62e^{(0.05-0.07)2}$=0.5957

(b) An arbitrageur can:

1. Borrow 1,000 USD at 5% per annum for 2 years, convert to 1000/0.620 = 1612.90 AUD and invest the AUD at 7% (both rates are continuous compounded)
2. Enter into a forward contract to sell 1855.28 AUD (=1612.90* $e^{(0.07)2}$)for 1855.28 (0.63)=1168.83 USD

The forward contract has the effect of converting AUD to 1168.83 USD. The amount needed to payoff the USD borrowing is 1000* $e^{(0.05)2}$ = 1105.17

The strategy gives rise to a riskless profit of 1168.83 - 1105.17 = 63.66 USD

7.

$$c + Ke^{-rT} + D = p + S_0$$

or

$$c = p - Ke^{-rT} - D + S_0$$

In this case, the present value of the dividends is $D = 1e^{-0.1\times 2/12} + 1e^{-0.1\times 5/12} = \$1.943$.
Therefore, $c = 2 - 30e^{-0.1\times 6/12} - 1.943 + 31 = 2.520$. In other words, the put price is $3.48

If the call price is $3.00, it is too high relative to the put price. An arbitrageur should sell the call, long the put, buy the stock, and borrow $30.

Future value of dividends in six months is 1.943(exp{(0.1)(6/12)})=2.042

Strategy	*t=0*	*t=T*	
		If $S_T \geq 30$	*If S_T<=30*
sell the call	+3	$-(S_T - 30)$	0
long the put	–2	0	$(30 - S_T)$
buy the stock	–31	$S_T + 2.042$	$S_T + 2.042$
borrow $30	+30	–30exp{(0.1)(6/12)}=31.538	–31.538
Cashflow	0	+0.504	+0.504

8.

The corresponding result is

$$p_2 \le 0.5(p_1 + p_3)$$

where p_1, p_2 and p_3 are the prices of European put option with the same maturities and strike prices K_1, K_2 and K_3 respectively. This can be proved from the result in Problem 10.23 using put-call parity. Alternatively we can consider a portfolio consisting of a long position in a put option with strike price K_1, a long position in a put option with strike price K_3, and a short position in two put options with strike price K_2. The value of this portfolio in different situations is given as follows

$S_T \le K_1$: Portfolio Value $= K_1 - S_T - 2(K_2 - S_T) + K_3 - S_T = K_3 - K_2 - (K_2 - K_1) = 0$

$K_1 < S_T \le K_2$: Portfolio Value $= K_3 - S_T - 2(K_2 - S_T) = K_3 - K_2 - (K_2 - S_T) \ge 0$

$K_2 < S_T \le K_3$: Portfolio Value $= K_3 - S_T$

$S_T > K_3$: Portfolio Value $= 0$

Because the portfolio value is always zero or positive at some future time the same must be true today. Hence

$$p_1 + p_3 - 2p_2 \ge 0$$

or

$$p_2 \le 0.5(p_1 + p_3)$$