Q1 請推導 Macaulay Duration 公式：

$$MD = \frac{1}{P}\left(\sum_{i=1}^{n} \frac{iC}{(1+r)^i} + \frac{nF}{(1+r)^n}\right)$$

the Macaulay duration, in periods, is equal to

$$MD = -(1+r)\frac{\partial P/P}{\partial r}$$

$$P = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \cdots + \frac{C+F}{(1+r)^n}$$

$$\therefore \frac{\partial P}{\partial r} = \frac{-C}{(1+r)^2} + \frac{-2C}{(1+r)^3} + \cdots + \frac{-n(C+F)}{(1+r)^{n+1}}$$

$$\therefore \frac{\partial P}{\partial r} = -\frac{1}{1+r}\left[\frac{C}{1+r} + \frac{2C}{(1+r)^2} + \cdots + \frac{n(C+F)}{(1+r)^n}\right]$$

$$\therefore \frac{\partial P}{\partial r}\frac{1}{P} = -\frac{1}{1+r}\left[\frac{C}{1+r} + \frac{2C}{(1+r)^2} + \cdots + \frac{n(C+F)}{(1+r)^n}\right] \times \frac{1}{P}$$

$$MD = \frac{\frac{C}{1+r} + \frac{2C}{(1+r)^2} + \cdots + \frac{n(C+F)}{(1+r)^n}}{P} = \frac{\sum_{i=1}^{n} \frac{iC_i}{(1+r)^i} + \frac{nF}{(1+r)^n}}{P}$$ 得証

$$\therefore \frac{\partial P}{\partial y} \times \frac{1}{P} = -\frac{1}{1+y} MD \Rightarrow \frac{\partial P/P}{\partial y/1+y} = -MD$$ ※

Q2:

給定票面價值 100，輸入期數(n)、票面利率(c)、折現率(r)，計算債券的 Macaulay Duration 並 print 出結果

```
4   //計算債券價格，用for迴圈折現
5   double calculate_bond_price(int n, double c, double r) {
6       double bond_price = 0.0;
7       for (int t = 1; t <= n; ++t) {
8           bond_price += (100 * c) / pow(1 + r, t);
9       }
10      bond_price += 100 / pow(1 + r, n);
11      return bond_price;
12  }
```

這個 function 主要是用來先把 p 算出來，如此一來之後再用於計算 Marcaulay duration 才能使用，其中債券價格用 for loop 對殖利率折現

```
//計算marcaulay duration用r折現每一期的利息並加權
double calculate_macaulay_duration(int n, double c, double r) {
    double macaulay_duration = 0.0;
    double bond_price = calculate_bond_price(n, c, r);

    for (int t = 1; t <= n; ++t) {
        macaulay_duration += (t * (100 * c)) / pow(1 + r, t);
    }
    macaulay_duration += n * 100 / pow(1 + r, n);
    macaulay_duration /= bond_price;

    return macaulay_duration;
}
```

這個 function 計算 marcaulay duration 真正的期術，其中我們要先建構出公式的和半段，也就是本金和每期利息的加權折現，算出來之後再除上剛剛上面 function 算出來的債券現值，就能夠得輸期數。