

# problem formulation

hongzhe.yu

November 2019

## 1 Problem formulation

The collaborative transportation problem is divided into several sub-problems. From top layer to the bottom layer are:

- Estimation and decision-making on distribution proportion
- Payload distribution
- Trajectory generation
- Feedback control

### 1.1 off line calculation of the reference trajectories for the payload

For agent  $i$  in the group, assuming that it knows the destined states (position  $p_0^T$  and velocity  $v_0^T$ ) for the payload at the end of transportation, or the set of destined states (way points  $[p_0^0, p_0^1, \dots, p_0^T]$ ,  $[v_0^0, v_0^1, \dots, v_0^T]$ ) of the payload along the transportation process.

The agent  $i$  then calculates the reference polynomial trajectories for the center of mass of the payload as the result of an minimum-snap optimization problem:

$$P(t) = \sum_{n=0}^N p_n t^n \quad (1)$$

which minimize the objectives:

$$J = \int_0^T c_0 P(t)^2 + c_1 P'(t)^2 + c_2 P''(t)^2 + \dots + c_N P^{(N)}(t)^2 dt \quad (2)$$

Such that the trajectory and its derivatives pass the way points  $[p_0^0, p_0^1, \dots, p_0^T]$ ,  $[v_0^0, v_0^1, \dots, v_0^T]$ .

## 1.2 Transform the reference trajectories for the agent

Once the agent  $i$  calculates the payload's trajectories off-line, and assuming that it knows its relative position to the center of mass of the payload  $r_i$ :

$$p_i = p_0 + r_i \quad (3)$$

then it can get immediately its own trajectories.

In our case now, we consider only the simplest case: the payload only has transitional movements. In this case, the reference trajectory of the agents are just the translation of the payload's reference trajectory.

## 1.3 Feedback control

Once the agent knows its reference trajectories, it can feed it as the reference to the control part. It can by the PID controller on force, it can get the desired force  $F_{des}$  to follow the trajectories:

$$F_{des} = -K_p e_P - K_v e_v + mgZ_W + [\ddot{\sigma}_{des1}, \ddot{\sigma}_{des2}, \ddot{\sigma}_{des3}]^T \quad (4)$$

For the same, the agent can get its desired moments  $M_{des}$  to follow the trajectories:

$$M_{des} = -K_R e_R - K_\omega e_\omega \quad (5)$$

## 1.4 Considering the payload

Until now, the agent  $i$  has not taken into consideration the payload dynamics and the results are calculated as if the agent  $i$  is flying independently. However, there is the payload dynamics to be considered. The payload distribution methods helps us to decouple the payload dynamics from the UAV dynamics. From the distribution proportions  $[c_0, c_1, c_2, c_3]$ , the agent  $i$  can calculate a additional Force and Moment  $\delta F, \delta M$  which is to be added to the already got  $F_{des}$  and  $M_{des}$ :

$$\begin{bmatrix} \delta F \\ \delta M \end{bmatrix} = \begin{bmatrix} c_i I_3 & -c_i S(r_i) \Pi^{-1} \\ 0_3 & c_i \Pi^{-1} \end{bmatrix} \begin{bmatrix} F_0 \\ M_0 \end{bmatrix}, \quad (6)$$

with

$$\Pi = I_3 + \sum c_i S(r_i) S^T(r_i) \quad (7)$$

## 1.5 Add payload distribution to the UAV's results

Now that the agent has got the  $\delta F, \delta M$ , it adds them to the  $F_{des}$  and  $M_{des}$ :

$$\begin{aligned} F_{des} &= F_{des} + \delta F \\ M_{des} &= M_{des} + \delta M \end{aligned}$$

### 1.6 On the top: MARL agent to predict the correct distribution

On the top layer, the MARL agent predict the distribution of all the agents from partial observations of the states.

## 2 Possible problem encountered

Now the trajectories are calculated with the method mentioned in section 1.1, offline. However, the distribution method mentioned in section 1.4 will ensure that the dynamics of the center of mass of the payload is satisfied, but not ensure the dynamics of each agent being satisfied.

Especially when the distribution is unequal, which is also our case, the agent that has larger  $c$  will be distributed with larger  $\delta F$  and  $\delta M$ , which will make its trajectories ahead of other agents, (which will also cause the payload to rotate). However, this difference in trajectory caused by the payload distribution is not considered ahead when the trajectory was calculated off-line. The PID controller will force the agent to follow the original trajectory, while the additional force  $\delta F$  forces the agent to follow the true trajectory.

## 3 Possible solution

One of the possible solution to this problem will be designing a mechanism where agents modifies its trajectory according to the distribution  $c$ , and let the PID do the trajectory following, instead of adding directly the  $\delta F$  and  $\delta M$ .

