Second Example - From SI:

$$f(x) = \frac{9 - x^2}{x + 3} = \frac{3^2 - x^2}{x + 3} = \frac{(3 - x)(3 + x)}{(x + 3)} = 3 - x$$

$$a^2 - b^2 = (a - b)(a + b)$$

$$f(x) = 3 - x \quad \rightarrow \quad y = 3 - x$$

Swap x and y:

$$x = 3 - y$$

Solve for y:

$$x - 3 = -y \quad \rightarrow \quad 3 - x = y$$

Let $y = y^{-1} = f^{-1}(x)$

$$y = 3 - x \quad \rightarrow \quad y^{-1} = f^{-1}(x) = 3 - x$$

Verify $f(f^{-1}(x)) = x$

$$f(f^{-1}(x)) = \frac{9 - (3 - x)^2}{(3 - x) + 3} = \frac{9 - (9 - 6x + x^2)}{3 - x + 3} = \frac{9 - 9 + 6x - x^2}{6 - x}$$

$$f(f^{-1}(x)) = \frac{6x - x^2}{6 - x} = \frac{x(6 - x)}{6 - x} = x$$

Thus $f(f^{-1}(x)) = x$

Verify $f^{-1}(f(x)) = x$

$$f^{-1}(f(x)) = 3 - \frac{9 - x^2}{x + 3} = \frac{3(x + 3)}{x + 3} - \frac{9 - x^2}{x + 3} = \frac{3(x + 3) - (9 - x^2)}{x + 3}$$

$$f^{-1}(f(x)) = \frac{3x + 9 - 9 + x^2}{x + 3} = \frac{3x + x^2}{x + 3} = \frac{x(3 + x)}{x + 3} = x$$

Thus $f^{-1}(f(x)) = x$

Example of Graphs:

`https://www.desmos.com/calculator/8xrhjzjtvs`