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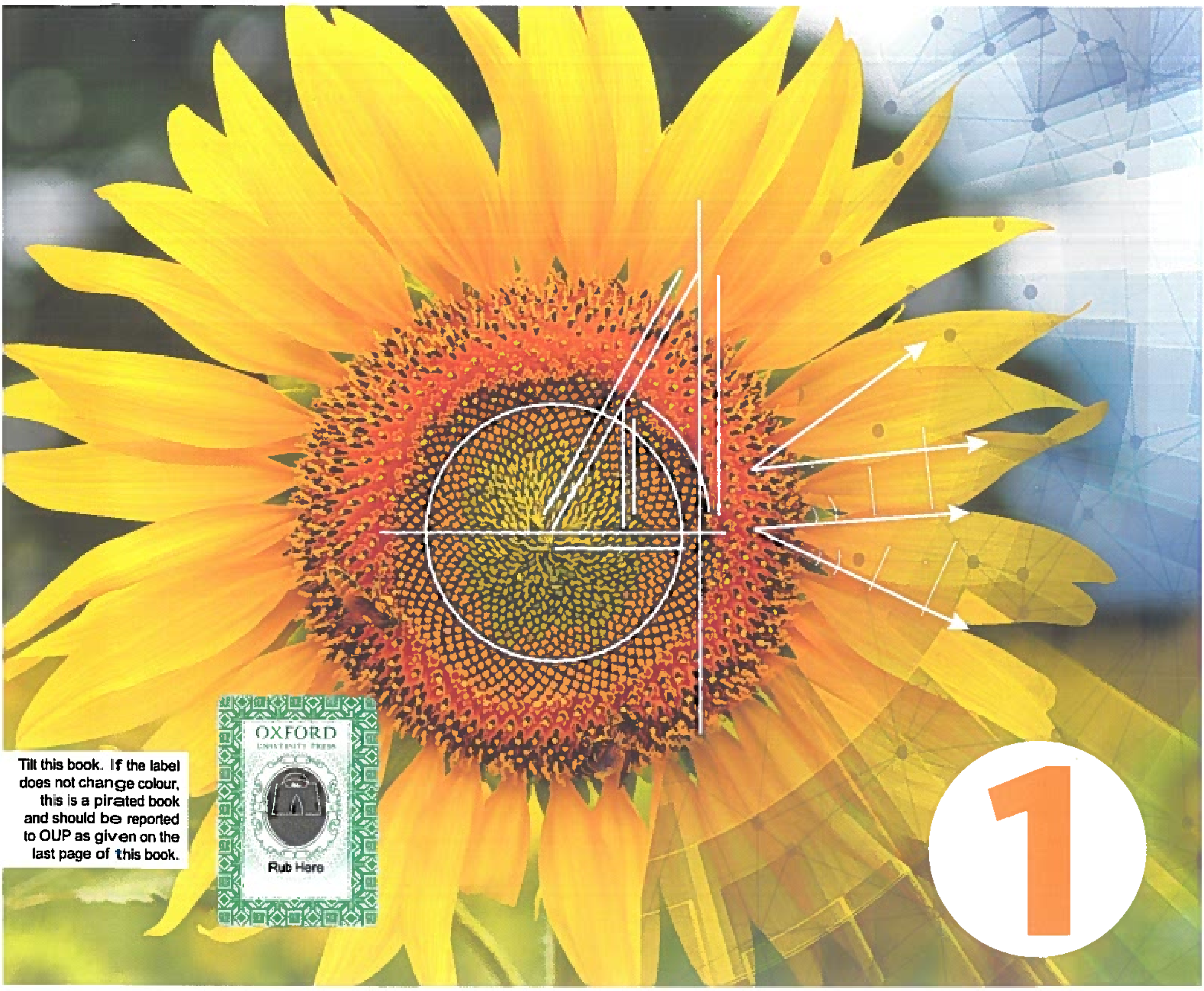
NEW SYLLABUS MATHEMATICS



Consultant • Dr Yeap Ban Har **Authors** • Dr Joseph Yeo • Teh Keng Seng • Loh Cheng Yee • Ivy Chow • Neo Chai Meng • Jacinth Liew

New Syllabus Mathematics (NSM) is a series of textbooks where the inclusion of valuable learning experiences, as well as the integration of real-life applications of learnt concepts serve to engage the hearts and minds of students sitting for the GCE O-level examination in Mathematics. The series covers the new Cambridge O Level Mathematics (Syllabus D) 4024/4029 for examinations from 2018 onwards. The newly formatted questions, which require application of mathematical techniques to solve problems, have been inducted.

Special Features

- Chapter Opener to arouse students' interest and curiosity
- Learning Objectives for students to monitor their own progress
- Investigation, Class Discussion, Thinking Time, Journal Writing, and Performance Task for students to develop requisite skills, knowledge and attitudes
- Worked Examples to show students the application of concepts
- Practise Now for immediate practice
- Similar Questions for teachers to choose questions that require similar application of concepts
- Exercise classified into Basic, Intermediate, and Advanced to cater to students with different learning abilities
- Summary to help students consolidate concepts learnt
- Review Exercise to consolidate the learning of concepts
- Challenge Yourself to challenge high-ability students
- Revision Exercises to help students assess their learning after every few chapters

Components

Textbooks	1 to 4
Workbooks	1 to 4
Teachers' Resource Books	1 to 4
Workbook Solutions	1 to 4

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ISBN 978-0-19-940740-8

9 780199 407408

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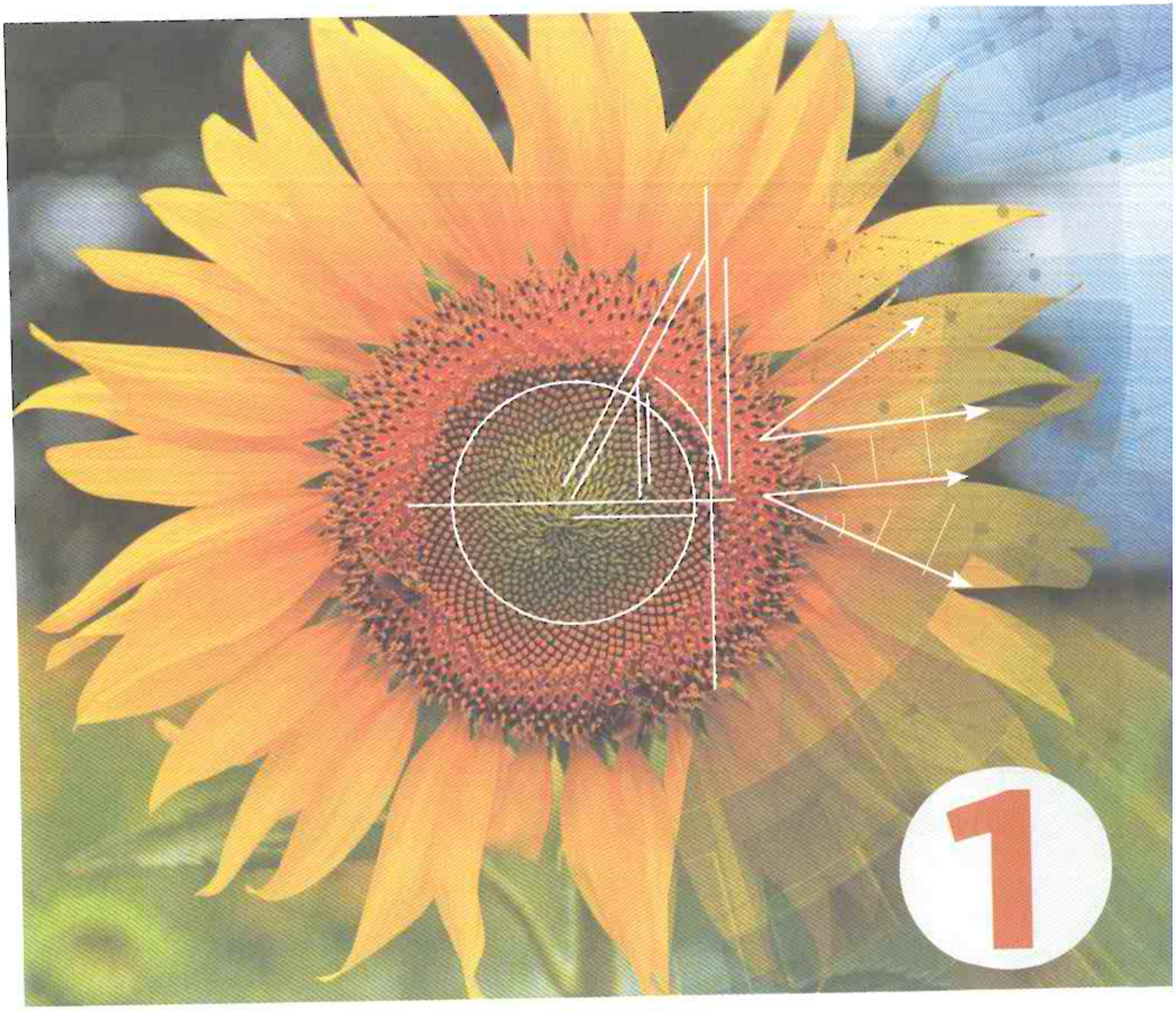

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Oxford University Press is a department of the University of Oxford. It furthers the University's objective of excellence in research, scholarship, and education by publishing worldwide. Oxford is a registered trade mark of Oxford University Press in the UK and in certain other countries

Published in Pakistan by
Ameena Saiyid, Oxford University Press
No. 38, Sector 15, Korangi Industrial Area,
PO Box 8214, Karachi-74900, Pakistan

First Edition published in Singapore in 1992
Sixth Edition published in Singapore in 2008
Third Edition first published in Pakistan by Oxford University Press in 1994
Fourth Edition first published in Pakistan by Oxford University Press in 2000
Fifth Edition first published in Pakistan by Oxford University Press in 2003
Sixth Edition first published in Pakistan by Oxford University Press in 2008
Seventh Edition first published in Pakistan by Oxford University Press in 2018

This Secondary Mathematics Series, adapted from the *New Syllabus Mathematics* Series, is published in collaboration with Shing Lee Publishers Pte Ltd, Singapore

For sale in Pakistan and Pakistani schools in the Middle East only

ISBN 978-0-19-940740-8

Second Impression 2018

Printed on 70gsm wood-free paper

Printed by VVP

Acknowledgements
Editorial Consultant: Shazia Asad

Photograph: cover: Shutterstock

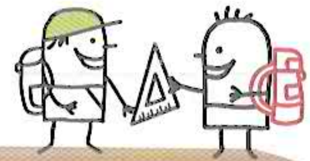
PREFACE

New Syllabus Mathematics (NSM)
is a series of textbooks specially designed to provide valuable learning experiences to engage the hearts and minds of students sitting for the GCE O level examination in Mathematics. Included in the textbooks are **Investigation**, **Class Discussion**, **Thinking Time**, **Journal Writing**, **Performance Task** and **Problems in Real-World Contexts** to support the teaching and learning of Mathematics.

Every chapter begins with a chapter opener which motivates students in learning the topic. Interesting stories about Mathematicians, real-life examples and applications are used to arouse students' interest and curiosity so that they can appreciate the beauty of Mathematics in their surroundings.

The use of ICT helps students to visualise and manipulate mathematical objects more easily, thus making the learning of Mathematics more interactive. Ready-to-use interactive ICT templates are available at http://www.shinglee.com.sg/StudentResources/

KEY FEATURES



CHAPTER OPENER

Each chapter begins with a chapter opener to arouse students' interest and curiosity in learning the topic.

LEARNING OBJECTIVES

Learning objectives help students to be more aware of what they are about to study so that they can monitor their own progress.

RECAP

Relevant prerequisites will be revisited at the beginning of the chapter or at appropriate junctures so that students can build upon their prior knowledge, thus creating meaningful links to their existing schema.

WORKED EXAMPLE

This shows students how to apply what they have learnt to solve related problems and how to present their working clearly. A suitable heading is included in brackets to distinguish between the different Worked Examples.

PRACTISE NOW

At the end of each Worked Example, a similar question will be provided for immediate practice. Where appropriate, this includes further questions of progressive difficulty.

SIMILAR QUESTIONS

A list of similar questions in the Exercise is given here to help teachers choose questions that their students can do on their own.

EXERCISE

The questions are classified into three levels of difficulty – Basic, Intermediate and Advanced.

SUMMARY

At the end of each chapter, a succinct summary of the key concepts is provided to help students consolidate what they have learnt.

REVIEW EXERCISE

This is included at the end of each chapter for the consolidation of learning of concepts.

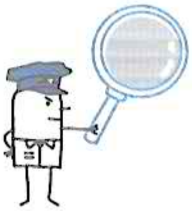
CHALLENGE YOURSELF

Optional problems are included at the end of each chapter to challenge and stretch high-ability students to their fullest potential.

REVISION EXERCISE

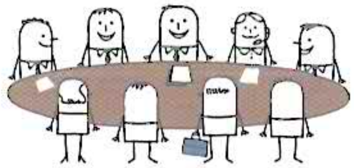
This is included after every few chapters to help students assess their learning.

Learning experiences have been infused into Investigation, Class Discussion, Thinking Time, Journal Writing and Performance Task.



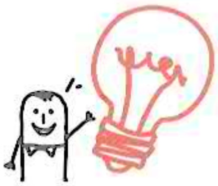
Investigation

Activities are included to guide students to investigate and discover important mathematical concepts so that they can construct their own knowledge meaningfully.



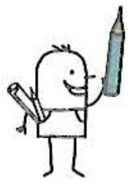
Class Discussion

Questions are provided for students to discuss in class, with the teacher acting as the facilitator. The questions will assist students to learn new knowledge, think mathematically, and enhance their reasoning and oral communication skills.



Thinking Time

Key questions are also included at appropriate junctures to check if students have grasped various concepts and to create opportunities for them to further develop their thinking.

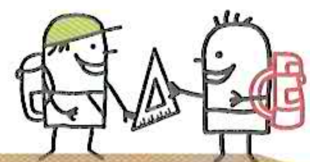


Journal Writing

Opportunities are provided for students to reflect on their learning and to communicate mathematically. It can also be used as a formative assessment to provide feedback to students to improve on their learning.

Performance Task

Mini projects are designed to develop research and presentation skills in the students.



MARGINAL NOTES

ATTENTION

This contains important information that students should know.

Problem Solving Tip

This guides students on how to approach a problem.

INFORMATION

This includes information that may be of interest to students.

RECALL

This contains certain mathematical concepts or rules that students have learnt previously.

Just For Fun

This contains puzzles, fascinating facts and interesting stories about Mathematics as enrichment for students.

Internet Resources

This guides students to search on the Internet for valuable information or interesting online games for their independent and self-directed learning.

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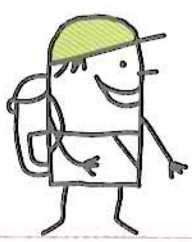

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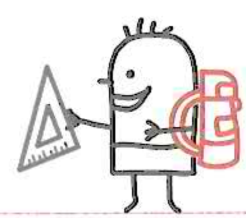

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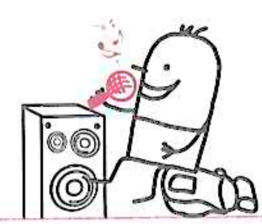
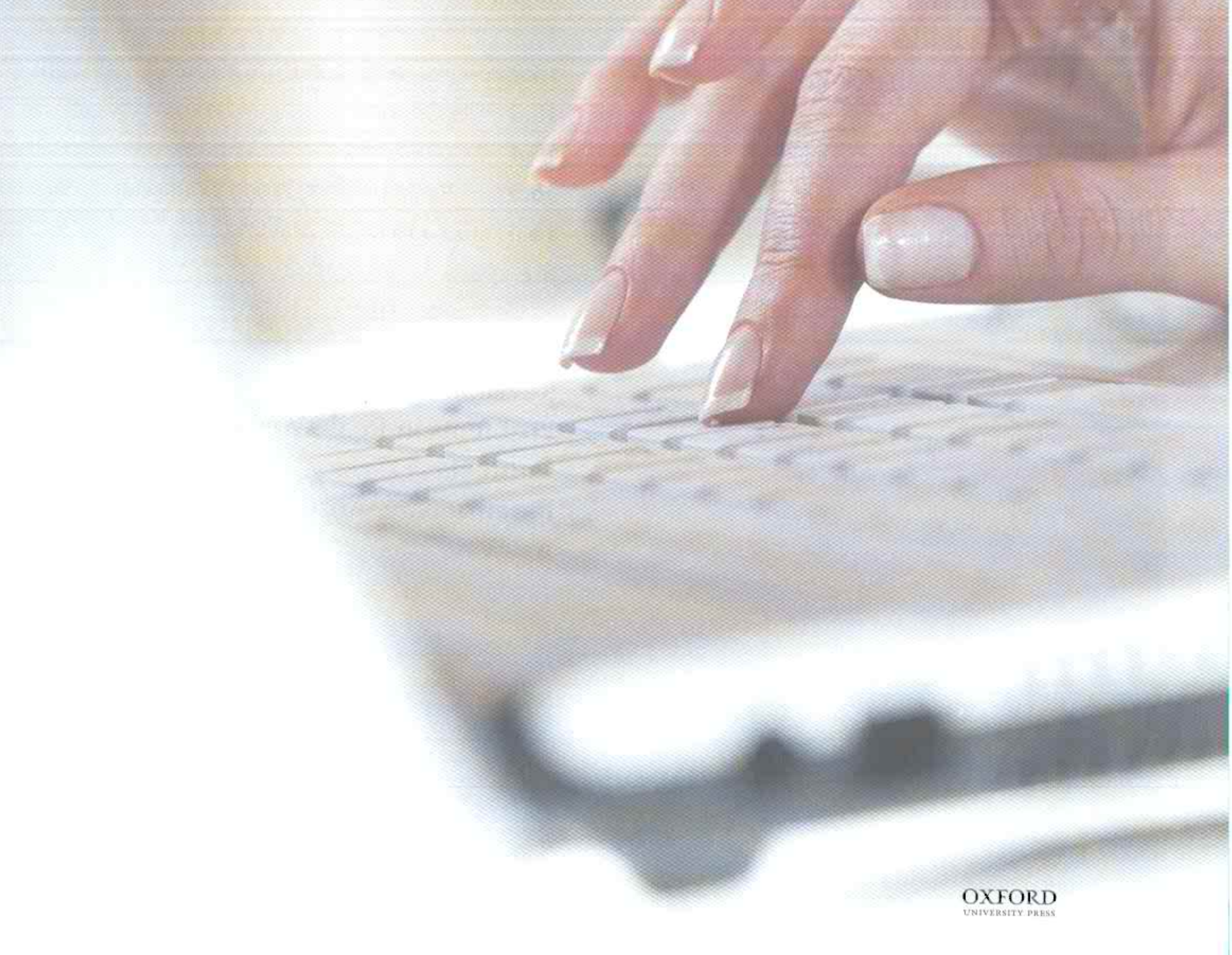


Primes, Highest Common Factor and Lowest Common Multiple

Sensitive data, such as credit card numbers and passwords, that are transferred over the Internet have to be encrypted. What prevents hackers from using sophisticated software to crack the code?

In 1978, Ronald Rivest, Adi Shamir and Leonard Adleman announced the development of the RSA public-key cryptography that is able to encrypt data so securely that it will take even the most advanced computers many years to crack the code. It makes use of a complicated theorem involving a type of numbers called prime numbers.



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Chapter One

LEARNING OBJECTIVES

At the end of this chapter, you should be able to:

- explain what a prime number is,
- determine whether a whole number is prime,
- express a composite number as a product of its prime factors,
- find square roots and cube roots using prime factorisation, mental estimation and calculators,
- find the highest common factor (HCF) and lowest common multiple (LCM) of two or more numbers,
- solve problems involving HCF and LCM in real-world contexts.

1.1 Prime Numbers

Recap (Factors)

In primary school, we have learnt about factors. For example,

$$18 = 1 \times 18$$
$$= 2 \times \_\_\_\_$$
$$= \_\_\_\_ \times 6.$$

Therefore, the **factors** of 18 are 1, 2, ____, 6, ____ and 18.

Notice that 18 is *divisible* by all its factors, i.e. if 18 is divided by each of its factors, there will not be any remainder.

Classifying Whole Numbers

In primary school, we have learnt that we can classify **whole numbers** into two groups:

- **Even numbers** are whole numbers that are divisible by 2, e.g. 0, 2, 4, 6, 8, ...
- **Odd numbers** are whole numbers that are not divisible by 2, e.g. 1, 3, 5, 7, 9, ...

RECALL

Examples of **whole numbers** are 0, 1, 2, 3, ...

Now, we will learn another way to classify whole numbers based on the number of factors they have.

Investigation

Classification of Whole Numbers

1. Find the factors of the numbers in Table 1.1.

Number	Working	Factors
1	1 is divisible by 1 only	1
2	$2 = 1 \times 2$	1, 2
3		
4	$4 = 1 \times 4 = 2 \times 2$	1, 2, 4
5		
6		
7		
8		
9		
10		

Table 1.1

Number	Working	Factors
11		
12		
13		
14		
15		
16		
17		
18	$18 = 1 \times 18 = 2 \times 9 = 3 \times 6$	1, 2, 3, 6, 9, 18
19		
20		

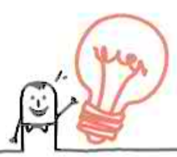
Table 1.1

2. Classify the numbers in Table 1.1 into 3 groups.
   Group A contains a number with exactly *1 factor*: ______________
   Group B contains numbers with exactly *2 different factors*: ______________
   Group C contains numbers with *more than 2 different factors*: ______________
3. Is 0 divisible by 1? 2? 3? 4? How many factors does 0 have?

Prime Numbers and Composite Numbers

The numbers in **Group B** in the investigation are known as **prime numbers** (or **primes**) and the numbers in **Group C** are called **composite numbers**. Composite numbers are *composed*, or *made up*, of the product of at least two primes, e.g. $6 = 2 \times 3$ and $18 = 2 \times 3 \times 3$. The number in **Group A** does not have a name. In fact, 1, as well as 0, is *neither prime nor composite*.

> A **prime number** is a whole number that has *exactly 2 different factors*, 1 and itself.
> A **composite number** is a whole number that has *more than 2 different factors*.



Thinking Time

1. Explain why 0 and 1 are neither prime numbers nor composite numbers.

2. Michael says that if a whole number is not prime, then it must be composite.
   Do you agree? Explain your answer.

In Table 1.1 in the investigation, we have found all the prime numbers less than or equal to 20 by finding all the factors of the numbers from 1 to 20. How do we find all the prime numbers less than or equal to 100? Let us use another method called the **Sieve of Eratosthenes**.

Investigation

Sieve of Eratosthenes

1. In this part, the numbers which will be circled are prime numbers and the numbers which will be crossed out are not prime numbers.
   - **(a)** Cross out 1.
   - **(b)** Circle 2.
     Cross out all the other multiples of 2.
   - **(c)** Circle the next number that has not been crossed out, i.e. 3.
     Cross out all the other multiples of 3.
   - **(d)** Circle the next number that has not been crossed out, i.e. 5 since 4 has been crossed out.
     Cross out all the other multiples of 5.
   - **(e)** Repeat this process until all the numbers have either been circled or crossed out.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2. Answer the following questions.
   - **(a)** What is the smallest prime number?
   - **(b)** What is the largest prime number less than or equal to 100?
   - **(c)** How many prime numbers are less than or equal to 100?
   - **(d)** Is every odd number a prime number? Explain your answer.
   - **(e)** Is every even number a composite number? Explain your answer.
   - **(f)** For a prime number greater than 5, what can the last digit be? Explain your answer.

Journal Writing

Can the product of two prime numbers be

(a) an odd number, **(b)** an even number, **(c)** a prime number?

Worked Example 1

(Test for Prime Numbers)

Determine whether each of the following is a prime or a composite number.

(a) 387 (b) 47

Problem Solving Tip

To determine whether a number is prime or composite, check its divisibility by all the prime numbers before it.

Solution:

(a) 387 is an odd number, so it is not divisible by 2.
Since the sum of the digits of 387 is $3 + 8 + 7 = 18$ which is divisible by 3, therefore 387 is divisible by 3 (divisibility test for 3).
$\therefore$ 387 is a composite number.

(b) 47 is an odd number, so it is not divisible by ____________.
Since the sum of the digits of 47 is $4 + 7 =$ ____________ which is not divisible by 3, then 47 is not divisible by ____________.
The last digit of 47 is neither 0 nor 5, so 47 is not divisible by 5.
A calculator may be used to test whether 47 is divisible by prime numbers more than 5.
Since 47 is not divisible by any prime numbers less than 47, then 47 is a prime number.

ATTENTION

Divisibility Test for 3: A number is divisible by 3 if and only if the sum of the digits of the number is divisible by 3.

PRACTISE NOW 1

SIMILAR QUESTIONS

Exercise 1A Questions 1(a)–(d)

1. Are 537 and 59 prime numbers or composite numbers?
2. In this game, a policeman is chasing a thief. The policeman can only step on tiles with prime numbers. Shade the tiles to trace the route that he can take to catch the thief.

135	49	183	147	93	121	236
201	261	150	11	131	5	89
291	117	153		57	0	61
192	231	27	1	111	100	149
17	103	43	7	127	51	53
83	33	32	105	29	71	37

Just For Fun

Prime numbers that differ by 2, such as 5 and 7, are called **twin primes**. List five other pairs of twin primes.

Worked Example 2

(Problem involving a Prime Number)

If p and q are whole numbers such that $p \times q = 13$, find the value of $p + q$. Explain your answer.

37 and 73 are prime numbers with reversed digits. Name another pair of prime numbers with reversed digits.

Solution:

Since 13 is a prime number, then 1 and 13 are its only two factors.
It does not matter whether p or q is 1 or 13 as we only want to find the value of $p + q$.
$\therefore p + q = 1 + 13 = 14$

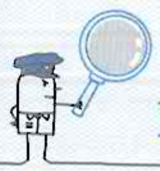
PRACTISE NOW 2

SIMILAR QUESTIONS

Exercise 1A Questions 11–12

1. If p and q are whole numbers such that $p \times q = 31$, find the value of $p + q$. Explain your answer.
2. If n is a whole number such that $n \times (n + 28)$ is a prime number, find the prime number. Explain your answer.

Interesting Facts and Real-Life Applications of Prime Numbers



Investigation

Interesting Facts about Prime Numbers

There are infinitely many primes.

To make sense of the above statement, search the Internet for 'First 1 Million Primes'. What is the 1 000 000th prime number?

Since there are infinitely many primes, there is no largest prime number. However, the *largest known* prime number (at the time of printing) was found by Dr Curtis Cooper on 7 Jan 2016 and it contains *22 338 618* digits. The size of the text file containing this prime (pure text only) is 22 MB. View this prime number by searching for 'Largest Known Prime' on the Internet. What is its last digit?

For many centuries, mathematicians studied prime numbers in a branch of mathematics called *Number Theory* out of interest. Real-life applications of prime numbers came upon the invention of computers when there was a need to encrypt sensitive data transmitted over the Internet.

As explained in the chapter opener, RSA public-key cryptography makes use of a complicated theorem involving prime numbers to encode data securely. If the prime numbers chosen are large, it will take even the most sophisticated computers many years to crack the code.

What would have happened if mathematicians did not study prime numbers out of interest?

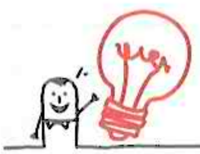
Index Notation

We have learnt in primary school that possible units for area and volume are cm^2 and cm^3 respectively. For example, the area of a square with sides 5 cm is $5 \text{ cm} \times 5 \text{ cm} = 25 \text{ cm}^2$, and the volume of a cube with edges 5 cm is $5 \text{ cm} \times 5 \text{ cm} \times 5 \text{ cm} = 125 \text{ cm}^3$.

5×5 can also be written as 5^2 (read as '5 squared').
Similarly, $5 \times 5 \times 5 = 5^3$ (read as '5 cubed').

What happens if we have $5 \times 5 \times 5 \times 5$? We can write it as 5^4 (read as '5 to the **power** of 4'), where 4 is called the **index** (plural: **indices**). 5^4 is called the **index notation** of $5 \times 5 \times 5 \times 5$.

Write $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ in index notation: ________________



Thinking Time

When is the index notation useful?

Prime Factorisation

Consider a composite number, e.g. 18. We can express 18 as a product of **prime factors**, i.e. factors which are prime numbers.

$$18 = 2 \times 3 \times 3$$
$$= 2 \times 3^2$$

We say that 2 and 3 are the prime factors of 18.

The process of expressing 18 as a product of its prime factors is called the **prime factorisation** of 18.

Do not confuse the prime factorisation of 18 with finding the factors of 18: $18 = 1 \times 18 = 2 \times 9 = 3 \times 6$.

Notice that the factors of 18 are 1, 2, 3, 6, 9 and 18, which are not necessarily its prime factors.

INFORMATION

The **Fundamental Theorem of Arithmetic** states that 'every whole number greater than 1 is either a prime number or it can be expressed as a *unique* product of its prime factors', where 'unique product' means that there is only one product (where the order of the prime factors does not matter). If this theorem is false, many useful mathematical results will also be false.

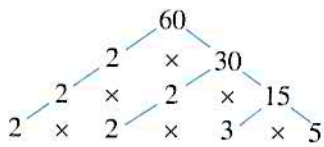
Worked Example 3

(Prime Factorisation)

Find the prime factorisation of 60, leaving your answer in index notation.

Solution:

Method 1:



$\therefore 60 = 2 \times 2 \times 3 \times 5$
$\quad\;\; = 2^2 \times 3 \times 5$

Method 2:

Divide 60 by the *smallest prime factor* and continue the process until we obtain 1.

start with smallest *prime* factor →	2	60	
	2	30	← divide 60 by 2 to get 30
	3	15	
	5	5	
		1	← divide until we obtain 1

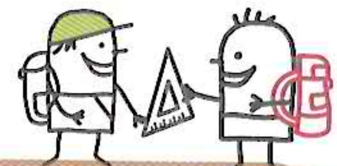
PRACTISE NOW 3

SIMILAR QUESTIONS

Exercise 1A Questions 2(a)–(d)

1. Find the prime factorisation of 126, leaving your answer in index notation.
2. Express 539 as a product of its prime factors.

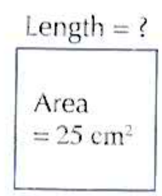
1.2 Square Roots and Cube Roots

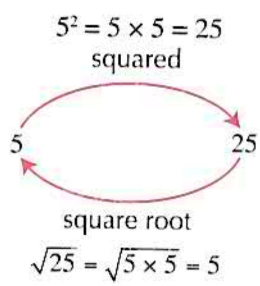


Square Roots

We have learnt in primary school that the area of a square with sides 5 cm is 5 cm × 5 cm = 25 cm².

We have also learnt that if we are given a square with an area of 25 cm², the length of its side is $\sqrt{25} = 5$ cm. We say that the **square root** of 25 is 5.





Similarly,

$0 \times 0 = 0^2 = 0$ implies $\sqrt{0} = \sqrt{0 \times 0} = 0$;
$1 \times 1 = 1^2 = 1$ implies $\sqrt{1} = \sqrt{1 \times 1} = 1$;
$2 \times 2 = 2^2 = 4$ implies $\sqrt{4} = \sqrt{2 \times 2} = 2$;
$3 \times 3 = 3^2 = 9$ implies $\sqrt{9} = \sqrt{3 \times 3} = 3$.

0, 1, 4 and 9 are the squares of whole numbers, and they are called **perfect squares** (or **square numbers**).

List the next three consecutive perfect squares: ________, ________, ________

Worked Example 4 shows how we can find the square root of a perfect square by using prime factorisation.

Worked Example 4

(Finding Square Root using Prime Factorisation)

Find $\sqrt{324}$ by using prime factorisation.

Solution:

2	324
2	162
3	81
3	27
3	9
3	3
	1

$$\begin{aligned} 324 &= 2 \times 2 \times 3 \times 3 \times 3 \times 3 \\ &= (2 \times 3 \times 3) \times (2 \times 3 \times 3) \\ &= (2 \times 3 \times 3)^2 \\ \therefore \sqrt{324} &= 2 \times 3 \times 3 \\ &= 18 \end{aligned}$$

Alternatively,

$$\begin{aligned} 324 &= 2 \times 2 \times 3 \times 3 \times 3 \times 3 \\ &= 2^2 \times 3^4 \\ \therefore \sqrt{324} &= \sqrt{2^2 \times 3^4} \\ &= 2 \times 3^2 \\ &= 18 \end{aligned}$$

ATTENTION

For a number to be a perfect square, the index of each prime factor must be *even*. Why?

Just For Fun

Find the two-digit number where the square of the sum of its digits is equal to the number obtained when its digits are reversed.

PRACTISE NOW 4

SIMILAR QUESTIONS

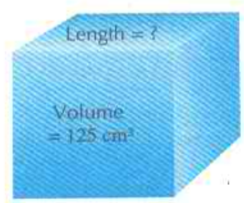
Exercise 1A Questions 3(a)–(b), 4

1. Find $\sqrt{784}$ by using prime factorisation.
2. Given that the prime factorisation of 7056 is $2^4 \times 3^2 \times 7^2$, find $\sqrt{7056}$ without using a calculator.

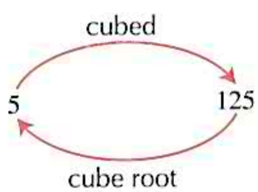
Cube Roots

We have learnt in primary school that the volume of a cube with edges 5 cm is $5 \text{ cm} \times 5 \text{ cm} \times 5 \text{ cm} = 125 \text{ cm}^3$.

We have also learnt that if we are given a cube with a volume of 125 cm^3, the length of its edge is $\sqrt[3]{125} = 5$ cm. We say that the **cube root** of 125 is 5.



$5^3 = 5 \times 5 \times 5 = 125$



$\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$

Similarly,

$0 \times 0 \times 0 = 0^3 = 0$ implies $\sqrt[3]{0} = \sqrt[3]{0 \times 0 \times 0} = 0$;

$1 \times 1 \times 1 = 1^3 = 1$ implies $\sqrt[3]{1} = \sqrt[3]{1 \times 1 \times 1} = 1$;

$2 \times 2 \times 2 = 2^3 = 8$ implies $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$;

$3 \times 3 \times 3 = 3^3 = 27$ implies $\sqrt[3]{27} = \sqrt[3]{3 \times 3 \times 3} = 3$.

0, 1, 8 and 27 are the cubes of whole numbers, and they are called **perfect cubes** (or **cube numbers**).

List the next three consecutive perfect cubes: ________, ________, ________

Worked Example 5 shows how we can find the cube root of a perfect cube by using prime factorisation.

Worked Example 5

(Finding Cube Root using Prime Factorisation)

Find $\sqrt[3]{216}$ by using prime factorisation.

Solution:

2	216
2	108
2	54
3	27
3	9
3	3
	1

$$
\begin{aligned}
216 &= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \\
&= (2 \times 3) \times (2 \times 3) \times (2 \times 3) \\
&= (2 \times 3)^3 \\
\therefore \sqrt[3]{216} &= 2 \times 3 \\
&= 6
\end{aligned}
$$

Alternatively,

$$
\begin{aligned}
216 &= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \\
&= 2^3 \times 3^3 \\
\therefore \sqrt[3]{216} &= \sqrt[3]{2^3 \times 3^3} \\
&= 2 \times 3 \\
&= 6
\end{aligned}
$$

ATTENTION

For a number to be a perfect cube, the index of each prime factor must be a *multiple of 3*. Why?

Just For Fun

Find the two-digit number where the sum of the cubes of its digits is equal to three times of itself.

PRACTISE NOW 5

SIMILAR QUESTIONS

Exercise 1A Questions 3(c)–(d), 5

1. Find $\sqrt[3]{2744}$ by using prime factorisation.
2. Given that the prime factorisation of 9261 is $3^3 \times 7^3$, find $\sqrt[3]{9261}$ without using a calculator.

Mental Estimation of Square Roots and Cube Roots

What are the values of $\sqrt{50}$ and $\sqrt[3]{63}$?

Since $50 = 2 \times 5^2$, the index of the prime factor 2 is not even. Thus, 50 is not a perfect square and $\sqrt{50}$ is not a whole number. Hence, we cannot use the method in Worked Example 4 to find $\sqrt{50}$.

Similarly, since $63 = 3^2 \times 7$, the indices of the prime factors 3 and 7 are not multiples of 3. Thus, 63 is not a perfect cube and $\sqrt[3]{63}$ is not a whole number. Hence, we cannot use the method in Worked Example 5 to find $\sqrt[3]{63}$.

In Worked Example 6, we will learn how to **estimate** the values of numbers such as $\sqrt{50}$ and $\sqrt[3]{63}$ mentally.

Worked Example 6

(Estimation of Square Root and Cube Root)

Estimate the values of $\sqrt{50}$ and $\sqrt[3]{63}$.

Solution:

We observe that 50 is close to 49 which is a perfect square. Thus $\sqrt{50} \approx \sqrt{49} = 7$.

Similarly, 63 is close to 64 which is a perfect cube. Thus $\sqrt[3]{63} \approx \sqrt[3]{64} = 4$.

PRACTISE NOW 6

SIMILAR QUESTIONS

Exercise 1A Questions 6(a)–(d)

Estimate the values of $\sqrt{123}$ and $\sqrt[3]{123}$.

Use of a Calculator to Find Squares, Square Roots, Cubes and Cube Roots

The square, square root, cube and cube root of a number can be found easily using a calculator. The following function keys are used for this purpose.

$\boxed{x^2}$	square key	$\boxed{\sqrt{\ }}$	square root key
$\boxed{x^y}$	power key	$\boxed{\sqrt[x]{\ }}$	x^{th} root key

Some calculators also have the cube root key: $\boxed{\sqrt[3]{\ }}$.

Worked Example 7

(Use of a Calculator to Evaluate Square Root and Cube Root)

Use a calculator to evaluate $\dfrac{8^2 + \sqrt{50}}{7^3 - \sqrt[3]{63}}$, leaving your answer correct to 4 decimal places.

Solution:

Sequence of calculator keys:

$\boxed{(}\ \boxed{8}\ \boxed{x^2}\ \boxed{+}\ \boxed{\sqrt{\ }}\ \boxed{5}\ \boxed{0}\ \boxed{)}\ \boxed{\div}\ \boxed{(}\ \boxed{7}\ \boxed{x^y}\ \boxed{3}\ \boxed{-}\ \boxed{3}\ \boxed{\sqrt[x]{\ }}\ \boxed{6}\ \boxed{3}\ \boxed{)}\ \boxed{=}$

$\therefore\ \dfrac{8^2 + \sqrt{50}}{7^3 - \sqrt[3]{63}} = 0.2096$ (to 4 d.p.)

INFORMATION

For some calculators, you may have to key in the number before pressing the square root key. If you do not want to key in the first pair of brackets, you must press $\boxed{=}$ before pressing $\boxed{\div}$.

PRACTISE NOW 7

SIMILAR QUESTIONS

Exercise 1A Questions 7(a)–(c), 8–9

1. Use a calculator to evaluate each of the following, leaving your answer correct to 4 decimal places where necessary.
   (a) $23^2 + \sqrt{2025} - 7^3$
   (b) $\dfrac{3^2 \times \sqrt{20}}{5^3 - \sqrt[3]{2013}}$
2. The area of a square poster is 987 cm². Find the perimeter of the poster, leaving your answer correct to 1 decimal place.

Thinking Time

If no brackets are used in pressing the sequence of calculator keys in Worked Example 7, what would be the value obtained? Write down the mathematical statement that would have been evaluated.

Prime Numbers Revisited: Trial Division

In Worked Example 1, we have found that 47 is a prime number because it is not divisible by any prime numbers less than 47. Now, what happens if we want to determine whether a big number, such as 997, is a prime? Do we have to test whether 997 is divisible by all the prime numbers less than 997? If so, there are 167 prime numbers (i.e. 2, 3, 5, 7 ..., 991) for us to test!

Since $\sqrt{997} = 31.6$ (to 1 d.p.), the largest prime less than or equal to $\sqrt{997}$ is 31. To determine whether 997 is a prime, it is enough to test whether 997 is divisible by 2, 3, 5, 7, ... or 31 (only 11 prime numbers to test). We do not have to test all the 167 prime numbers. Why?

This method of determining whether a number is a prime by dividing the number by all the prime numbers less than or equal to the square root of itself is called **trial division**.

Just For Fun

Goldbach's Conjecture states that 'every even number greater than 2 can be expressed as the sum of two primes'. For example, $4 = 2 + 2$, $6 = 3 + 3$ and $8 = 3 + 5$. Oliveira e Silva has proven that Goldbach's Conjecture is true for all even numbers up to 3×10^{17} on 30 Dec 2005. Express 18 and 2014 as the sum of two primes.

Worked Example 8

(Test for Prime Number)
Is 997 a prime number?

Solution:

$\sqrt{997} = 31.6$ (to 1 d.p.), so the largest prime less than or equal to $\sqrt{997}$ is 31. Since 997 is not divisible by any of the prime numbers, 2, 3, 5, 7, ..., 31, then 997 is a prime number.

Problem Solving Tip

997 is an odd number, so 997 is not divisible by 2. Since the sum of the digits of 997 is $9 + 9 + 7 = 25$ which is not divisible by 3, 997 is not divisible by 3. The last digit of 997 is neither 0 nor 5, so 997 is not divisible by 5. To test whether 997 is divisible by prime numbers more than 5, it is advisable to use a calculator.

PRACTISE NOW 8

Are the years 2013 and 2017 prime numbers?

SIMILAR QUESTIONS

Exercise 1A Questions 10(a)–(d)

Exercise 1A

BASIC LEVEL

1. Determine whether each of the following is a prime or a composite number.
(a) 87 **(b)** 67
(c) 73 **(d)** 91

2. Find the prime factorisation of each of the following numbers, leaving your answer in index notation.
(a) 72 **(b)** 187
(c) 336 **(d)** 630

3. Find each of the following by using prime factorisation.
(a) $\sqrt{1764}$ **(b)** $\sqrt{576}$
(c) $\sqrt[3]{3375}$ **(d)** $\sqrt[3]{1728}$

4. Given that the prime factorisation of 9801 is $3^4 \times 11^2$, find $\sqrt{9801}$ without using a calculator.

5. Given that the prime factorisation of 21 952 is $2^6 \times 7^3$, find $\sqrt[3]{21\,952}$ without using a calculator.

6. Estimate the value of each of the following.
(a) $\sqrt{66}$ (b) $\sqrt{80}$
(c) $\sqrt[3]{218}$ (d) $\sqrt[3]{730}$

7. Use a calculator to evaluate each of the following, leaving your answer correct to 4 decimal places where necessary.
(a) $7^2 - \sqrt{361} + 21^3$ (b) $\dfrac{\sqrt{555} + 5^2}{2^3 \times \sqrt[3]{222}}$
(c) $\sqrt{4^3 + \sqrt[3]{4913}}$

INTERMEDIATE LEVEL

8. The area of a square photo frame is 250 cm^2. Find the perimeter of the photo frame, leaving your answer correct to 1 decimal place.

9. The volume of a box in the shape of a cube is 2197 cm^3. Find the area of one side of the box.

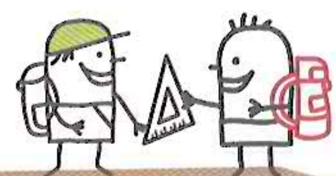
10. Determine whether each of the following is a prime or a composite number.
(a) 667 (b) 677
(c) 2021 (d) 2027

ADVANCED LEVEL

11. If p and q are whole numbers such that $p \times q = 37$, find the value of $p + q$. Explain your answer.

12. If n is a whole number such that $n \times (n + 42)$ is a prime number, find the prime number. Explain your answer.

1.3 Highest Common Factor and Lowest Common Multiple



Highest Common Factor (HCF)

In primary school, we have learnt about factors and common factors. For example,

$$18 = 1 \times 18 \qquad 30 = 1 \times 30$$
$$= 2 \times 9 \qquad = 2 \times 15$$
$$= 3 \times 6, \qquad = 3 \times 10$$
$$= 5 \times 6.$$

Factors of 18: 1 2 3 6 9 18
Factors of 30: 1 2 3 5 6 10 15 30

The common factors of 18 and 30 are 1, 2, 3 and 6.

$\therefore$ The highest common factor (HCF) of 18 and 30 is 6.

This method of finding HCF is called the **listing method**.

What is the HCF of 504 and 588? 504 has 24 factors while 588 has 18 factors. 504 and 588 have 12 common factors. To use the listing method to find the HCF of 504 and 588 is troublesome because it involves many factors. Hence, there is a need for more efficient methods to find the HCF of two or more numbers.

INFORMATION

The lowest common factor of 12 and 18 is 1. There is nothing special about this because the lowest common factor of any two or more whole numbers is always 1.

Let us use two new methods to find the HCF of smaller numbers such as 18 and 30 first.

Worked Example 9

(HCF of Two Numbers)

Find the highest common factor of 18 and 30.

Solution:

Method 1:

Step 1: Express 18 and 30 as products of their prime factors.

Step 2: Extract the common prime factors.

Step 3: The HCF of 18 and 30 is the product of the common prime factors.

common prime factors

$$\begin{aligned} 18 &= \boxed{2} \times \boxed{3} \times 3 \\ 30 &= \boxed{2} \times \boxed{3} \qquad \times 5 \end{aligned}$$

$$\begin{aligned} \text{HCF of 18 and 30} &= 2 \times 3 \\ &= 6 \end{aligned}$$

or

common prime factor

$$\begin{aligned} 18 &= \boxed{2} \times \boxed{3^2} \\ 30 &= \boxed{2} \times \boxed{3} \times 5 \end{aligned}$$

2×3 ← common factor is 3, i.e. choose the power of 3 with the *smaller index*

Method 2:

We can also obtain the common prime factors by division.

common prime factors → $\boxed{2}$	18, 30	← divide 18 and 30 by 2 to get 9 and 15
→ $\boxed{3}$	9, 15	← divide 9 and 15 by 3 to get 3 and 5
	3, 5	← stop dividing when there are no common prime factors

$$\begin{aligned} \text{HCF of 18 and 30} &= 2 \times 3 \\ &= 6 \end{aligned}$$

PRACTISE NOW 9

1. Find the highest common factor of 56 and 84 using both methods.
2. Find the largest whole number which is a factor of both 28 and 70.
3. The numbers 504 and 588, written as the products of their prime factors, are $504 = 2^3 \times 3^2 \times 7$ and $588 = 2^2 \times 3 \times 7^2$. Hence, find the greatest whole number that will divide both 504 and 588 exactly.

SIMILAR QUESTIONS

Exercise 1B Questions 1(a)–(b), 3–4, 9

Problem Solving Tip

For Question 2, the largest whole number, which is a factor of 28 and 70, is the highest common factor of 28 and 70.

Worked Example 10

(HCF of Three Numbers)

Find the HCF of 40, 60 and 100.

Solution:

Method 1:

common prime factors

$$
\begin{aligned}
40 &= \boxed{2} \times \boxed{2} \times 2 \times \boxed{5} \\
60 &= \boxed{2} \times \boxed{2} \times 3 \times \boxed{5} \\
100 &= \boxed{2} \times \boxed{2} \times \boxed{5} \times 5
\end{aligned}
$$

$$
\begin{aligned}
\text{HCF of 40, 60 and 100} &= 2 \times 2 \times 5 \\
&= 20
\end{aligned}
$$

or

common factors are 2^2 and 5, i.e. choose the power of each of the common prime factors with the *smallest index*

$$
\begin{aligned}
40 &= \boxed{2^3} \times \boxed{5} \\
60 &= \boxed{2^2} \times 3 \times \boxed{5} \\
100 &= \boxed{2^2} \times \boxed{5^2}
\end{aligned}
$$

$$2^2 \times 5$$

Method 2:

common prime factors

2	40,	60,	100	← divide 40, 60 and 100 by 2 to get 20, 30 and 50
2	20,	30,	50	← divide 20, 30 and 50 by 2 to get 10, 15 and 25
5	10,	15,	25	← divide 10, 15 and 25 by 5 to get 2, 3 and 5
	2,	3,	5,	← stop dividing when there are no common prime factors

$$
\begin{aligned}
\text{HCF of 40, 60 and 100} &= 2 \times 2 \times 5 \\
&= 20
\end{aligned}
$$

PRACTISE NOW 10

Find the HCF of 90, 135 and 270.

SIMILAR QUESTIONS

Exercise 1B Questions 1(c)–(d)

Lowest Common Multiple (LCM)

In primary school, we have learnt about multiples and common multiples, e.g.

Multiples of 6	:	6		12	18		24	30	36		42	48		54	60, …
Multiples of 10	:		10			20		30		40			50		60, …

The common multiples of 6 and 10 are 30, 60, …

How many common multiples of 6 and 10 are there? Is there a highest common multiple?

Of all the common multiples of 6 and 10, the lowest common multiple (LCM) is 30. This method of finding LCM is called the **listing method**.

The listing method is tedious, especially for large numbers. Are there more efficient methods?

Finding LCM Using Prime Factorisation

Compare the prime factorisation of the numbers 6 and 10, as well as that of their common multiples, 30 and 60.

$$6 = 2 \times 3 \qquad\qquad 10 = 2 \times 5$$

$$30 = \underbrace{3 \times 2}_{\text{factors of 6}} \times 5, \quad 30 = 3 \times \underbrace{2 \times 5}_{\text{factors of 10}} \qquad\qquad 60 = \underbrace{3 \times 2}_{\text{factors of 6}} \times \underbrace{2 \times 5}_{\text{factors of 10}}$$

2 is a common prime factor of 6 and 10. We notice that the common prime factor is only included once in the prime factorisation of the LCM, thus we can use the following method to find the LCM of two numbers:

Step 1: Obtain the prime factorisation of each number.

Step 2: Identify the common prime factors.

Step 3: The LCM of the two numbers is the product of the common prime factors and all the other prime factors.

We can apply the above method to find the LCM of 4 and 6.

common prime factor

$$4 = \boxed{2} \times 2$$
$$6 = \boxed{2} \qquad \times 3$$

$$\text{LCM of 4 and 6} = 2 \times 2 \times 3 = 12$$

Similar to finding the HCF of two numbers, we can also obtain the common prime factors by division.

common prime factor → $\boxed{2}$ | 4, 6 ← divide 4 and 6 by 2 to get 2 and 3

$\boxed{2,\ 3}$ ← stop dividing when there are no common prime factors

$$\text{LCM of 4 and 6} = 2 \times \underbrace{2 \times 3}_{\text{remaining factors}} = 12$$

Worked Example 11

(LCM of Two Numbers)

Find the lowest common multiple of 30 and 36.

Solution:

Method 1:

common prime factors

$$30 = \boxed{2} \times \boxed{3} \times 5$$
$$36 = \boxed{2} \times 2 \times \boxed{3} \times 3$$

LCM of 30 and 36 $= 2 \times 2 \times 3 \times 3 \times 5$
$= 180$

or

choose the power of each of the common prime factors (i.e. 2 and 3) with the *higher index* and the remaining factor (i.e. 5)

$$30 = \boxed{2} \times \boxed{3} \times 5$$
$$36 = \boxed{2^2} \times \boxed{3^2}$$

$$2^2 \times 3^2 \times 5$$

Method 2:

$$\begin{array}{c|cc}
\boxed{2} & 30, & 36 \\ \hline
\boxed{3} & 15, & 18 \\ \hline
 & \boxed{5, \quad 6} &
\end{array}$$

common prime factors

36 ← divide 30 and 36 by 2 to get 15 and 18

18 ← divide 15 and 18 by 3 to get 5 and 6

6 ← stop dividing when there are no common prime factors

LCM of 30 and 36 $= 2 \times 3 \times \underbrace{5 \times 6}_{\text{remaining factors}}$
$= 180$

PRACTISE NOW 11

1. Find the lowest common multiple of 24 and 90 using both methods.
2. The numbers 120 and 126, written as the products of their prime factors, are $120 = 2^3 \times 3 \times 5$ and $126 = 2 \times 3^2 \times 7$. Hence, find the smallest whole number that is divisible by both 120 and 126.
3. Find the smallest value of n such that the LCM of n and 6 is 24.

SIMILAR QUESTIONS

Exercise 1B Questions 2(a)–(b), 5–6, 10–11

Problem Solving Tip

For Question 3, use the prime factorisation method but work backwards.

Worked Example 12

(LCM of Three Numbers)

Find the LCM of 12, 18 and 56.

Solution:

Method 1:

common prime factors

$$12 = \boxed{2} \times \boxed{2} \times \boxed{3}$$
$$18 = \boxed{2} \times \boxed{3} \times 3$$
$$56 = \boxed{2} \times \boxed{2} \times 2 \times 7$$

LCM of 12, 18 and 56 $= 2 \times 2 \times 2 \times 3 \times 3 \times 7$

$= 504$

or

choose the power of each of the common prime factors (i.e. 2 and 3) with the *highest index* and the remaining factor (i.e. 7)

$$12 = \boxed{2^2} \times \boxed{3}$$
$$18 = \boxed{2} \times \boxed{3^2}$$
$$56 = \boxed{2^3} \times 7$$

$$2^3 \times 3^2 \times 7$$

Method 2:

start with the smallest common prime factor

3 is a common prime factor of 3 and 9, so we divide 3 and 9 by 3

2	12,	18,	56
2	6,	9,	28
3	3,	9,	14
	1,	3,	14

these 3 numbers have no common prime factors, but 6 and 28 have a common prime factor 2, so we divide 6 and 28 by 2

stop dividing when there are no common prime factors between *any two numbers*

LCM of 12, 18 and 56 $= 2 \times 2 \times 3 \times \underbrace{3 \times 14}_{\text{remaining factors}}$

$= 504$

PRACTISE NOW 12

Find the LCM of 9, 30 and 108.

SIMILAR QUESTIONS

Exercise 1B Questions 2(c)–(d)

Real-Life Applications of HCF and LCM

One of the applications of prime numbers within mathematics is to find the HCF and LCM of two or more numbers. In this section, we will solve some real-life problems involving HCF and LCM.

Worked Example 13

(Real-life Problem involving LCM)

The lights on three lightships flash at regular intervals. The first light flashes every 18 seconds, the second every 30 seconds and the third every 40 seconds. The three lights flash together at 10.00 p.m. At what time will they next flash together?

Solution:

$18 = 2 \times 3^2$

$30 = 2 \times 3 \times 5$

$40 = 2^3 \times 5$

$\therefore$ LCM of 18, 30 and 40 $= 2^3 \times 3^2 \times 5$

$= 360$

360 seconds = 6 minutes

$\therefore$ The three lights will next flash together at 10.06 p.m.

Problem Solving Tip

The LCM of the three timings is the interval at which the lights flash together.

PRACTISE NOW 13

SIMILAR QUESTIONS

Exercise 1B Questions 7–8, 12–15

1. Three bells toll at regular intervals of 15 minutes, 16 minutes and 36 minutes respectively. Given that they toll together at 2.00 p.m., at what time will they next toll together?

2. Farhan has three pieces of rope with lengths of 140 cm, 168 cm and 210 cm. He wishes to cut all the three pieces of ropes into smaller pieces of equal length such that there is no leftover rope.
   (i) What is the greatest possible length of each of the smaller pieces of rope?
   (ii) How many smaller pieces of rope can he get altogether?

Exercise 1B

BASIC LEVEL

1. Find the highest common factor of each of the following sets of numbers.
   (a) 12 and 30 (b) 84 and 156
   (c) 15, 60 and 75 (d) 77, 91 and 143

2. Find the lowest common multiple of each of the following sets of numbers.
   (a) 24 and 30 (b) 42 and 462
   (c) 12, 18 and 81 (d) 63, 80 and 102

INTERMEDIATE LEVEL

3. Find the largest whole number which is a factor of both 42 and 98.

4. The numbers 792 and 990, written as the products of their prime factors, are $792 = 2^3 \times 3^2 \times 11$ and $990 = 2 \times 3^2 \times 5 \times 11$. Hence, find the greatest whole number that will divide both 792 and 990 exactly.

5. The numbers 176 and 342, written as the products of their prime factors, are $176 = 2^4 \times 11$ and $342 = 2 \times 3^2 \times 19$. Hence, find the smallest whole number that is divisible by both 176 and 342.

6. Find the smallest value of n such that the LCM of n and 15 is 45.

7. Huixian needs to pack 171 pens, 63 pencils and 27 erasers into identical gift bags so that each item is equally distributed among the gift bags. Find
   (i) the largest number of gift bags that can be packed,
   (ii) the number of each item in a gift bag.

8. Two race cars, Car X and Car Y, are at the starting point of a 2-km track at the same time. Car X and Car Y make one lap every 60 s and every 80 s respectively.
   (i) How long, in seconds, will it take for both cars to be back at the starting point at the same time?
   (ii) How long, in minutes, will it take for the faster car to be 5 laps ahead of the slower car?

ADVANCED LEVEL

9. Determine whether each of the following statements is true or false. If it is true, explain your reasoning. If it is false, give a counterexample.
   (a) If 6 is a factor of a number, then 2 and 3 are also factors of that number.
   (b) If 2 and 3 are factors of a number, then 6 is also a factor of that number.
   (c) If 2 and 4 are factors of a number, then 8 is also a factor of that number.
   (d) If f is a factor of n, then $\frac{n}{f}$ is also a factor of n.
   (e) If h is the HCF of p and q, then both p and q are divisible by h.

10. The LCM of 9, 12 and n is 252. If n is odd, find all the possible values of n.

11. Determine whether each of the following statements is true or false. If it is true, explain your reasoning. If it is false, give a counterexample.
   (a) If 6 is a multiple of a number, then 12 is also a multiple of that number.
   (b) If 12 is a multiple of a number, then 6 is also a multiple of that number.
   (c) If 18 is a multiple of a number, then 18 is divisible by that number.
   (d) If m is the LCM of p and q, then m is divisible by both p and q.

12. Kate wishes to cut some squares from a vanguard sheet with a length of 64 cm and a breadth of 48 cm. She likes the squares to be as big as possible and she does not want any leftover vanguard sheet.
   (i) What is the length of each square?
   (ii) How many squares can she cut altogether?

13. A class has between 30 to 40 students. Each boy in the class brings 15 chocolate bars for a class party to celebrate Teacher's Day. The chocolate bars are shared equally among the 20 girls of the class and their form teacher with no leftovers.
   (i) How many students are there in the class?
   (ii) How many chocolate bars does their form teacher receive?

14. Michael is an art elective programme student who is working on an assignment. He plans to cover a rectangular sheet of paper of dimensions 126 cm by 108 cm with identical square patterns.
   (i) What is the least number of square patterns that could be formed on the sheet of paper?
   (ii) How do you determine what other shapes can the patterns be if they are to fit the sheet of paper perfectly? Explain your answer.

15. Michael, the art elective programme student, is working on another assignment. He designs a rectangular pattern measuring 45 mm by 42 mm. He is required to use identical rectangular patterns to form a square. The maximum area of the square allowed is 1.6 m^2.
   (i) How many patterns does he need to form the smallest square?
   (ii) What are the dimensions of the largest square that he can form?

1. **Whole Numbers** (0, 1, 2, 3, 4, …)

Neither Prime nor Composite	**Prime Numbers**	**Composite Numbers**
(0 and 1)	has exactly 2 different factors (2, 3, 5, 7, 11, …).	has more than 2 different factors (4, 6, 8, 9, 10, …).

The process of expressing a composite number as a product of its prime factors is known as **prime factorisation**.

2. **Squares and Square Roots:** E.g. since $5^2 = 25$, then $\sqrt{25} = 5$.
   A *perfect square* is a number whose square root is a whole number.

3. **Cubes and Cube Roots:** E.g. since $5^3 = 125$, then $\sqrt[3]{125} = 5$.
   A *perfect cube* is a number whose cube root is a whole number.

4. The **Highest Common Factor (HCF)** of two or more numbers is the largest factor that is common to all the numbers.

5. The **Lowest Common Multiple (LCM)** of two or more numbers is the smallest multiple that is common to all the numbers.

Review Exercise 1

1. Find the value of each of the following by using prime factorisation.
   (a) $\sqrt{1225}$
   (b) $\sqrt[3]{13\,824}$

2. Estimate the value of each of the following.
   (a) $\sqrt{63}$
   (b) $\sqrt[3]{345}$

3. Determine whether each of the following is a prime or a composite number.
   (a) 753
   (b) 757

4. The numbers 840 and 8316, written as the products of their prime factors, are $840 = 2^3 \times 3 \times 5 \times 7$ and $8316 = 2^2 \times 3^3 \times 7 \times 11$. Hence, find
   (i) the greatest whole number that will divide both 840 and 8316 exactly,
   (ii) the smallest whole number that is divisible by both 840 and 8316.

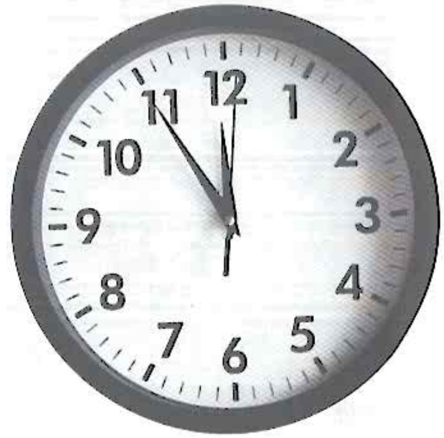
5. The LCM of 6, 12 and n is 660. Find all the possible values of n.

6. Shirley needs to pack 108 stalks of roses, 81 stalks of lilies and 54 stalks of orchids into identical baskets so that each type of flowers is equally distributed among the baskets. Find
   (i) the largest number of baskets that can be packed,
   (ii) the number of each type of flowers in a basket.

7. At 5.45 p.m., Lixin, Khairul and Devi are at the starting point of a 1-km circular path. Lixin takes 18 minutes to walk one round, Khairul needs 360 seconds to run 1 round and Devi cycles 2 rounds in 4 minutes. Find the time when all three of them will next meet.

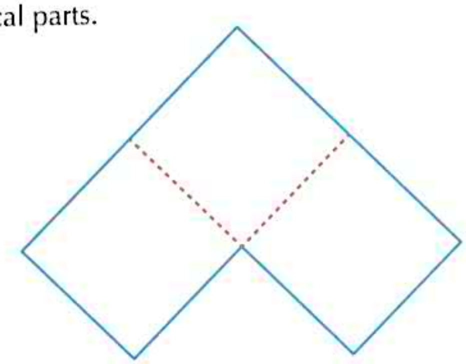
8. Nora and Amirah work in different companies. Nora has a day off every 4 days while Amirah has a day off every 6 days. Nora's last day off was on 29 April while Amirah's was on 1 May.
   (i) When will they next have the same day off?
   (ii) Subsequently, how often will they have the same day off?

Challenge Yourself



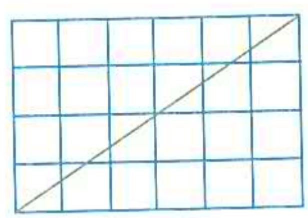
1. The figure shows the face of a clock with the numbers 1 to 12.
   (i) Which are the six adjacent numbers such that the sum of every pair of adjacent numbers for these six numbers is a prime number?
   (ii) Rearrange the other six numbers so that the sum of every pair of adjacent numbers is a prime number. How many ways are there to do this?

2. The figure shows a shape made up of three identical squares. Divide it into four identical parts.

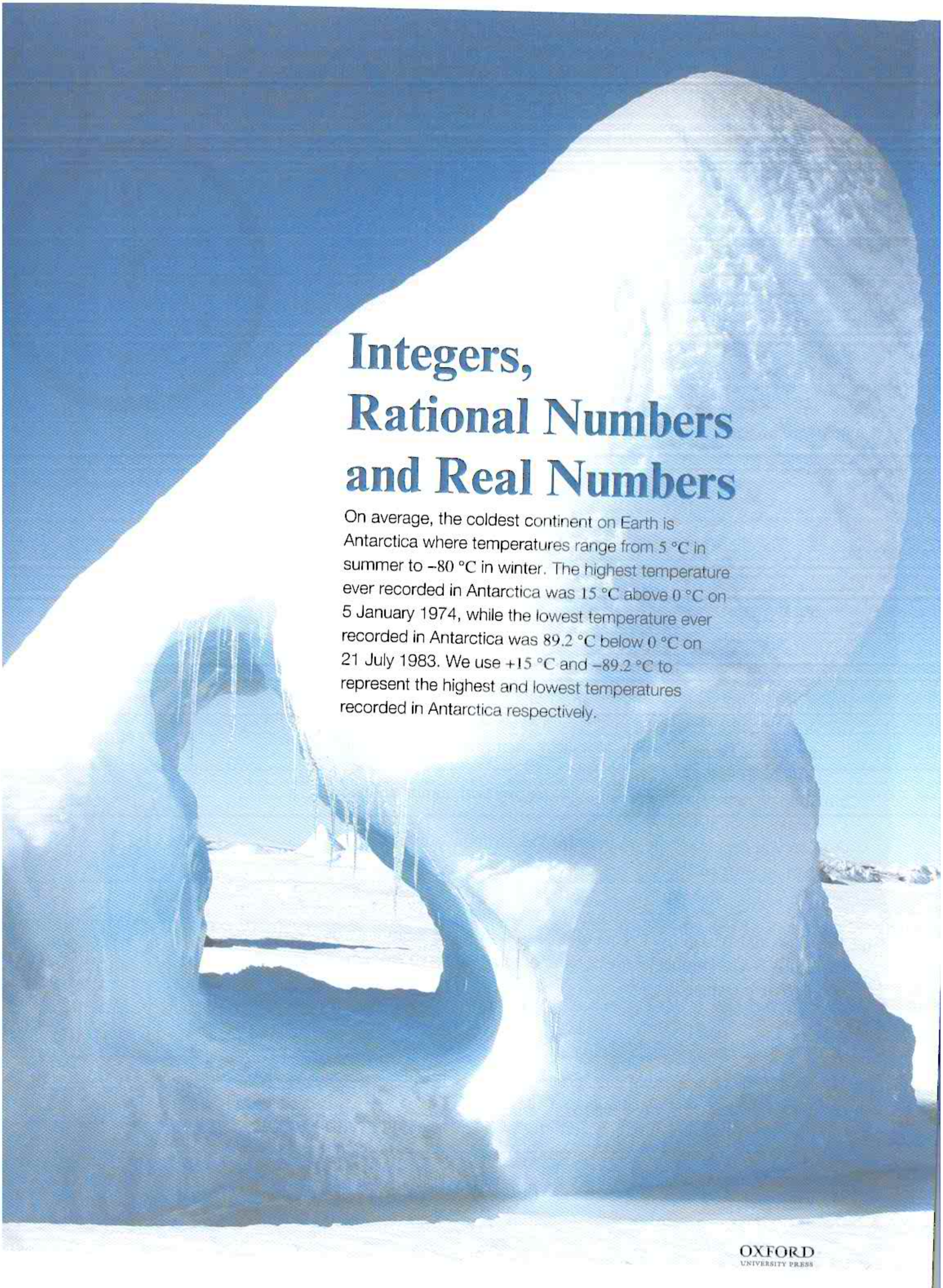


3. (i) Find the HCF and LCM of 120 and 126.
   (ii) Show that the product of the HCF and LCM of 120 and 126 is equal to the product of 120 and 126. By looking at their prime factorisation, explain why this is so.
   (iii) Can you generalise the result in **(ii)** for any two numbers? Explain your answer.
   (iv) Can you generalise the result in **(ii)** for any three numbers? Explain your answer.

4. The diagonal of a 6-by-4 rectangle passes through 8 squares as shown in the figure. Find a formula for the number of squares passed through by a diagonal of a m-by-n rectangle.



5. (i) Find the least number of cuts required to cut 12 identical sausages so that they can be shared equally among 18 people.
   (ii) Find the least number of cuts, in terms of m and n, required to cut m identical sausages so that they can be shared equally among n people.



Integers, Rational Numbers and Real Numbers

On average, the coldest continent on Earth is Antarctica where temperatures range from 5 °C in summer to −80 °C in winter. The highest temperature ever recorded in Antarctica was 15 °C above 0 °C on 5 January 1974, while the lowest temperature ever recorded in Antarctica was 89.2 °C below 0 °C on 21 July 1983. We use +15 °C and −89.2 °C to represent the highest and lowest temperatures recorded in Antarctica respectively.

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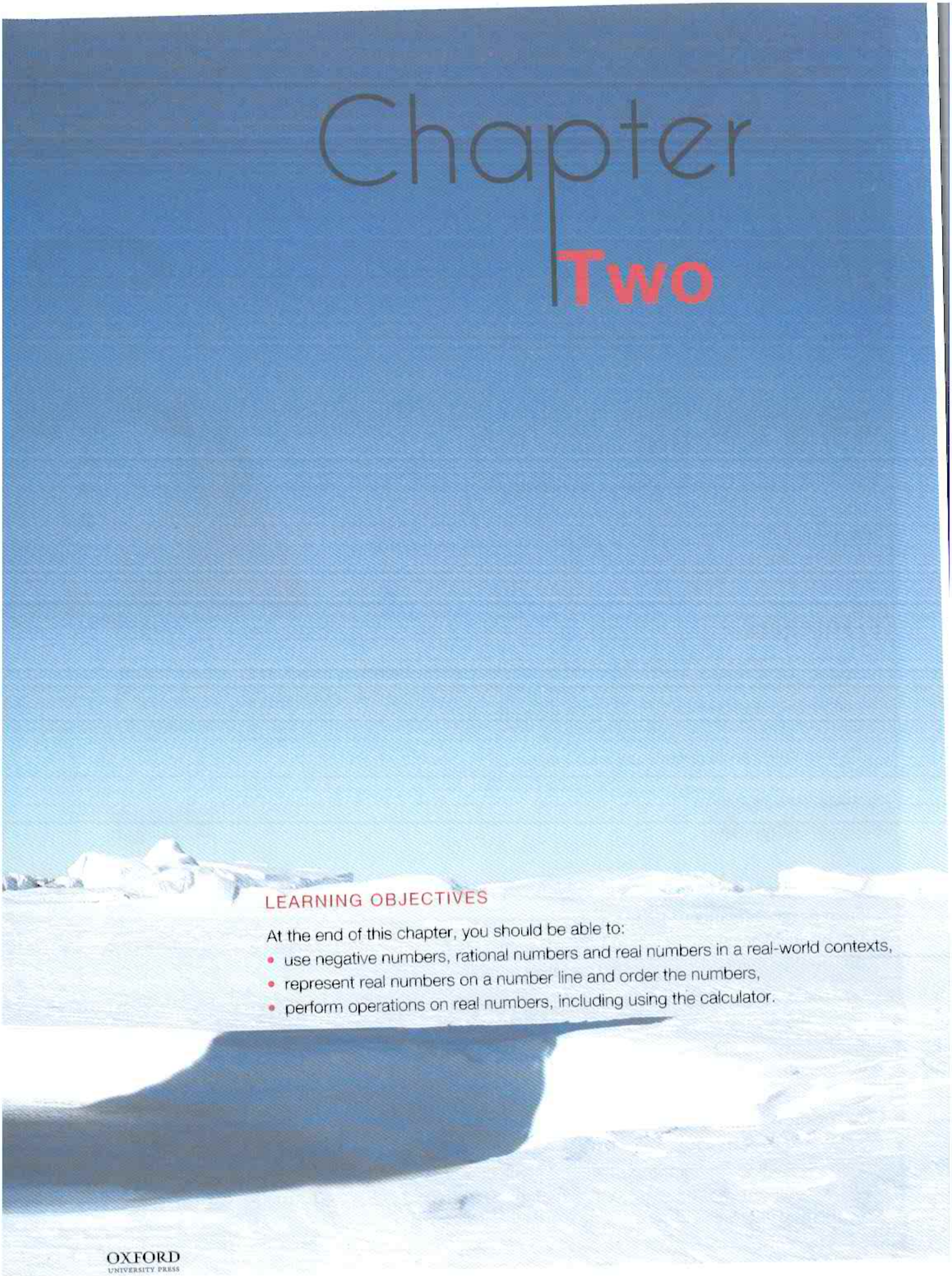


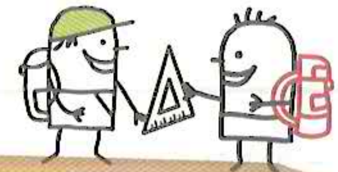
Chapter Two

LEARNING OBJECTIVES

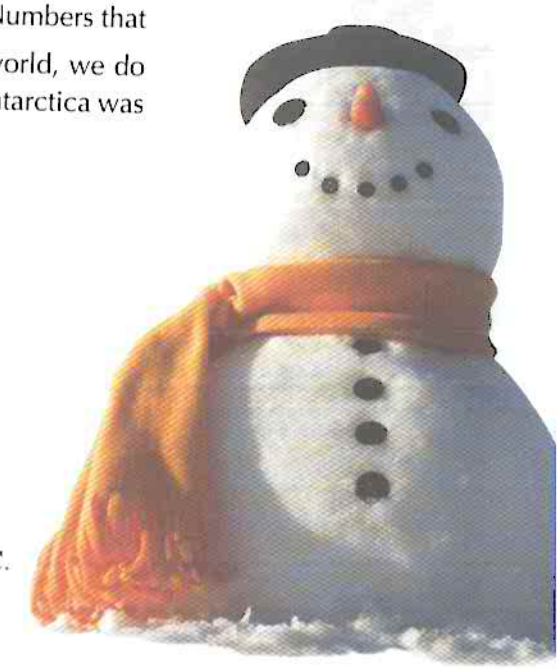
At the end of this chapter, you should be able to:

- use negative numbers, rational numbers and real numbers in a real-world contexts,
- represent real numbers on a number line and order the numbers,
- perform operations on real numbers, including using the calculator.

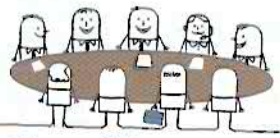
2.1 Negative Numbers



In primary school, we have learnt about whole numbers, decimals and fractions, such as 0, 7, 1.6 and $\frac{1}{2}$. These numbers are greater than or equal to 0. Numbers that are greater than 0 are called **positive numbers**. However, in the real world, we do encounter **negative numbers**, e.g. the lowest temperature recorded in Antarctica was 89.2 °C below 0 °C and is represented by −89.2 °C.



The negative number −89.2 is read as *negative* 89.2.



Class Discussion

Uses of Negative Numbers in the Real World

With your classmates, discuss a few more examples of the use of the negative numbers in the real world. For each example, explain the meaning of the negative number, e.g. −89.2 °C means 89.2 °C below 0 °C.

Integers

In primary school, we have learnt about whole numbers such as 0, 1, 2, 3, etc. The numbers 1, 2, 3, ... are also called positive integers, while examples of negative integers are −1, −2, −3, ... The set of **integers** is ..., −3, −2, −1, 0, 1, 2, 3, ...

0 is neither a positive nor a negative integer.

PRACTISE NOW

SIMILAR QUESTIONS

Exercise 2A Questions 1, 2(a)-(d), 6(a)-(b)

1. $-5, 2013, 0, -\frac{1}{2}, 1.666, -3.8, \frac{3}{4}, -17, 6, -\frac{2}{3}$

   From the given numbers, list the numbers that are

   (i) positive integers, **(ii)** negative integers,

   (iii) positive numbers, **(iv)** negative numbers.

2. **(a)** The coldest temperature ever recorded in Korea was 43.6 °C below 0 °C in the winter of 1933 at Junggangjin. Represent this temperature using a negative number.

   (b) The lowest point on Earth that is on dry land is the shore of the Dead Sea which is 423 m below sea level. Represent this altitude using a negative number.

   (c) In a quiz, 1 point is deducted for each incorrect answer. Represent the deduction of 1 point using a negative number.

   (d) A company suffers a loss or a deficit of $10 000 in the year 2013. Represent this loss using a negative number.

Number Line

Fig. 2.1 shows a thermometer.

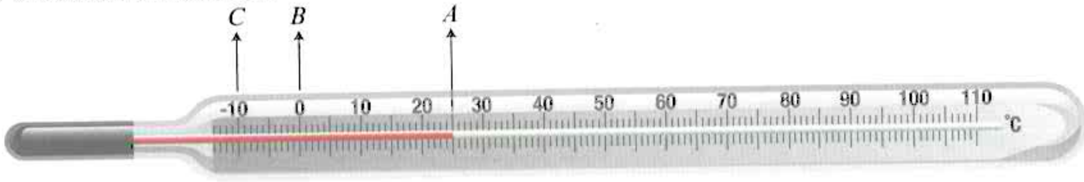


Fig. 2.1

What are the temperatures indicated by each of the points *A*, *B* and *C*?
Which of the points *A*, *B* or *C* shows the highest temperature?
Which of the points *A*, *B* or *C* shows the lowest temperature?

The markings on the thermometer enable us to read the temperature accurately and determine which temperature is higher or lower.

Similarly, we can represent the numbers that we have learnt on a **number line** (see Fig. 2.2). The markings are equally spaced and the arrow indicates the positive direction.

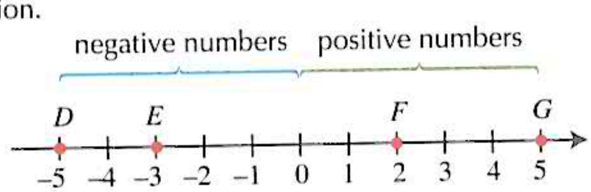


Fig. 2.2

INFORMATION

In some countries, the number line is drawn with an arrow at each end to indicate that the line goes on indefinitely in both directions.

- The numbers on the right of 0 are positive numbers.
- The numbers on the left of 0 are negative numbers.
- A number which is on the left of another number, is less than that number.
- A number which is on the right of another number, is more than that number.

In Fig. 2.2, the numbers –5, –3, 2 and 5 are represented by the points *D*, *E*, *F* and *G* respectively.
'2 is **less than** 5' can be written as '2 < 5'. (*F* is on the left of *G*.)
'5 is **more than** 2' can be written as '5 > 2'. (*G* is on the right of *F*.)
'<' means 'is less than'.
'>' means 'is more than'.
Is 2 more than or less than –5?
Since *F* is on the right of *D*, we say '2 is more than –5' and we write '2 > –5'.

INFORMATION

- '≤' means 'is less than or equal to'.
- '≥' means 'is more than or equal to'.

Thinking Time

By looking at the number line in Fig. 2.2, answer each of the following questions and explain your answer.

(a) Is –3 more or less than 2?
(b) Is –3 more or less than –5?

Use '<' or '>' to represent the relationship between the two numbers in each question.

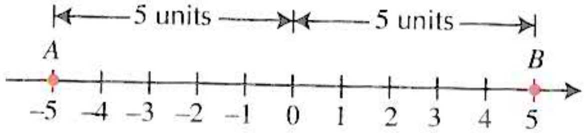
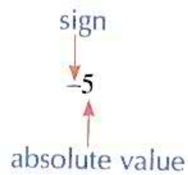


Fig. 2.3

Consider the points A and B on the number line that represent the numbers –5 and 5 respectively. Both points A and B are at the same distance of 5 units from the number 0. The **absolute value** of the negative number –5 is 5, i.e. a number is made up of an absolute value with either a positive or a negative sign in front of it.



Similarly, the positive number 5 can also be written with a positive sign in front of it, i.e. +5 (read as **positive** 5). What is the absolute value of the positive number 5?

Consider the number line in Fig. 2.4. Where should we place a dot to represent the number –3.1?

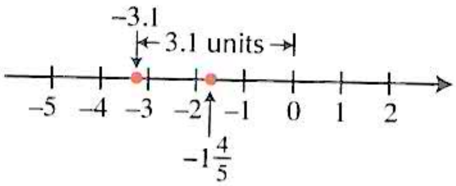


Fig. 2.4

Since –3.1 is negative, it is on the left of 0. Its absolute value is 3.1. Therefore, the point representing –3.1 is at a distance of *3.1 units* to the left of 0.

The number $-1\frac{4}{5}$ is marked out on the number line. Explain how the point on the number line is obtained.

PRACTISE NOW

SIMILAR QUESTIONS

Exercise 2A Questions 3(a)-(f), 4(a)-(d), 5(a)-(b), 7(a)-(d), 8(a)-(d)

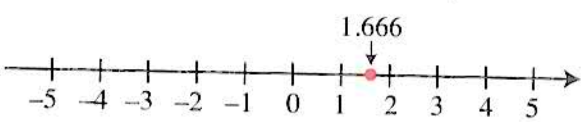
1. Fill in each box with '>' or '<'. The first one has been done for you.

(a) $-3 \boxed{>} -5$ **(b)** $-7 \boxed{} -2$

(c) $4 \boxed{} -4$ **(d)** $-6 \boxed{} -100$

2. Use a dot to represent each of the following numbers on the number line below. An example is given.

$$-5,\ 4,\ 0,\ -1\frac{1}{2},\ 1.666,\ -3.8,\ \frac{3}{4}$$



Hence, arrange the numbers in **ascending order**, i.e. from the smallest to the greatest.

Exercise 2A

BASIC LEVEL

1. $-0.3, \frac{1}{5}, 0, -\frac{5}{7}, 4.33, -12, 10\,001, -1\frac{1}{2}, -2017, 4$
   From the given numbers, list the numbers that are
   (i) positive integers, **(ii)** negative integers,
   (iii) positive numbers, **(iv)** negative numbers.

2. Fill in the blanks below.
   (a) If -6 represents 6 m below sea level, then $+30$ represents ______.
   (b) If $+40$ represents depositing \$40 in the bank, then a withdrawal of \$35 is represented by ______.
   (c) If $-60°$ represents a clockwise rotation of $60°$, then $+30°$ represents ______.
   (d) If $+45$ represents a speed of 45 km/h of a car travelling East, then -45 represents ______.

3. Fill in each box with '>' or '<'.
   (a) $16 \square 60$ **(b)** $3.1 \square 3.2$
   (c) $-6 \square 8$ **(d)** $30 \square -31$
   (e) $-2 \square 0$ **(f)** $9.8 \square -9.9$

4. Use a number line to illustrate each of the following.
   (a) $2\frac{2}{5}, 0, -4, 6, -2.8$
   (b) $-0.55, 4, -\frac{1}{10}, 2, -2$
   (c) integers between -4 to 4
   (d) positive integers less than 10

5. By using a number line, arrange each of the following in ascending order.
   (a) $230, -13, 23, -3, 30$
   (b) $-0.5, 150, 15, -10, -\frac{3}{20}$

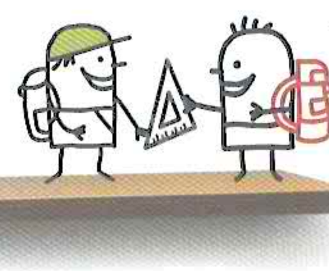
INTERMEDIATE LEVEL

6. **(a)** Absolute zero, defined as 0 Kelvin, is the theoretical lowest possible temperature. This corresponds to a temperature of 273.15 °C below zero. Represent this temperature using a negative number.
   (b) The lowest point in North America is the Badwater Basin which is 86 m below sea level. Represent this altitude using a negative number.

7. Fill in each box with '>' or '<'.
   (a) $-4 \square -6$ **(b)** $-11 \square -11.5$
   (c) $\frac{1}{5} \square \frac{1}{3}$ **(d)** $-\frac{1}{3} \square -\frac{5}{6}$

8. Use a number line to illustrate each of the following.
   (a) $-\frac{1}{3}, 2.5, 1\frac{3}{8}, 1, -0.2, 0.11$
   (b) positive odd integers less than 20
   (c) prime numbers more than or equal to 2 but less than 10
   (d) common factors of 12 and 16

2.2 Addition and Subtraction involving Negative Numbers



In primary school, we have learnt how to add and subtract positive numbers. However, for subtraction, we have only learnt how to subtract a smaller positive number from a greater positive number, e.g. $5 - 2 = 3$. In this section, we will learn how to carry out addition and subtraction that involve negative numbers using algebra discs. Alternatively, you may visit http://www.shinglee.com.sg/StudentResources/ to access the AlgeTools™ software.

The algebra disc shown below has two sides. One side shows the number 1. The other side shows the number –1.

1   –1

front   back

To obtain the *negative* of 1, we flip the disc with the number 1 as shown:

$1 \xrightarrow{\text{flip}} -1$ we write $-(1) = -1$

To obtain the *negative* of –1, we flip the disc with the number –1 as shown:

$-1 \xrightarrow{\text{flip}} 1$ we write $-(-1) = 1$

What happens if we put two discs 1 and –1 together?

$\boxed{1 \quad -1}$ we write $1 + (-1) = 0$

We will get a zero pair.

We can use three 1 discs to represent the number 3.

1 1 1 $3 = 1 + 1 + 1$

We use three –1 discs to represent the number –3.

–1 –1 –1 $-3 = (-1) + (-1) + (-1)$

To obtain the negative of 3, i.e. $-(3)$, we flip the three 1 discs as shown:

$1 \quad 1 \quad 1 \xrightarrow{\text{flip}} -1 \quad -1 \quad -1$ we write $-(3) = -3$

To obtain the negative of –3, i.e. $-(-3)$, we flip the three –1 discs as shown:

$-1 \quad -1 \quad -1 \xrightarrow{\text{flip}} 1 \quad 1 \quad 1$ we write $-(-3) = 3$

What happens if we put three 1 discs and three –1 discs together?

$\boxed{1 \quad -1}$

$\boxed{1 \quad -1}$ we write $3 + (-3) = 0$

$\boxed{1 \quad -1}$

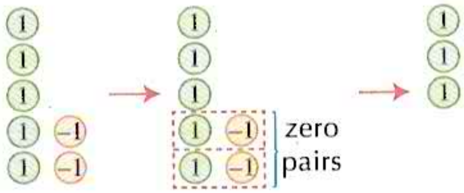
We will get zero pairs.

Can you give another example of a zero pair?

Addition involving Negative Numbers

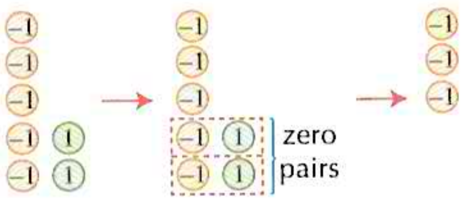
We will show how to carry out addition that involves negative numbers using algebra discs.

Example: $5 + (-2)$



Therefore, $5 + (-2) = 3$.
Since $5 - 2$ is also equal to 3, therefore $5 + (-2) = 5 - 2$.

Example: $(-5) + 2$

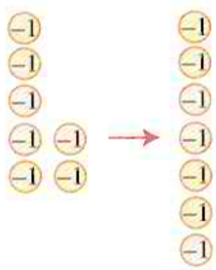


ATTENTION

$(-5) + 2$ can be written as $-5 + 2$.

Therefore, $(-5) + 2 = -3$.

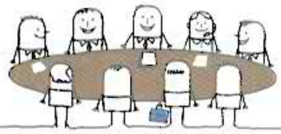
Example: $(-5) + (-2)$



ATTENTION

$(-5) + (-2)$ can be written as $-5 + (-2)$.

Therefore, $(-5) + (-2) = -7$.



Class Discussion

Addition involving Negative Numbers

Part I

Work in pairs to find the value of each of the following by using algebra discs. For each question, discuss with your classmate to write a rule to perform the operation by considering the following questions:

(i) Should the answer be positive or negative?

(ii) Should we find the sum or difference of the absolute values of the two numbers?

1. **(a)** $7 + (-3)$ **(b)** $6 + (-4)$

2. **(a)** $(-7) + 3$ **(b)** $(-6) + 4$

3. **(a)** $(-7) + (-3)$ **(b)** $(-6) + (-4)$

Part II

Challenge each other to add different pairs of positive and negative single-digit integers, just like what you have done in Questions 1, 2 and 3 in **Part I**. You may use algebra discs to help you.

Part III

Now that you have understood the rules of addition involving negative numbers, challenge each other to add different pairs of positive and negative integers without using algebra discs.

PRACTISE NOW

Without using a calculator or algebra discs, find the value of each of the following.

(a) $9 + (-2)$ **(b)** $-7 + 4$ **(c)** $3 + (-5)$

(d) $-6 + (-8)$ **(e)** $27 + (-13)$ **(f)** $-25 + 11$

(g) $14 + (-16)$ **(h)** $-12 + (-15)$

SIMILAR QUESTIONS

Exercise 2B Questions 1(a)–(h), 4(a)–(h)

Worked Example 1

(Addition involving Negative Numbers)

The temperature of a city on a particular night is -5 °C. The next morning, the temperature rises by 3 °C. Find the temperature in the morning.

Solution:

Temperature in the morning $= -5\ ^\circ\text{C} + 3\ ^\circ\text{C}$

$= -2\ ^\circ\text{C}$

PRACTISE NOW 1

The temperature of a town on a particular night is -8 °C. The next morning, the temperature rises by 2 °C. Find the temperature in the morning.

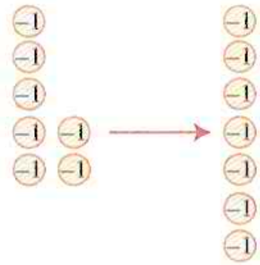
SIMILAR QUESTIONS

Exercise 2B Question 6

Subtraction involving Negative Numbers

We will show how to carry out subtraction that involves negative numbers using algebra discs. Note that $5 - 2 = 5 + (-2) = 3$.

Example: $(-5) - 2 = (-5) + (-2)$

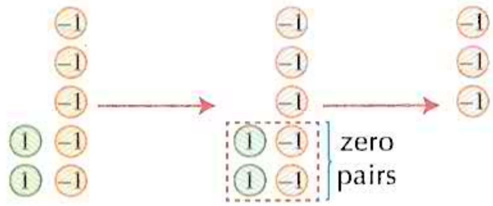


ATTENTION

$(-5) - 2$ can be written as $-5 - 2$.

Therefore, $(-5) - 2 = (-5) + (-2)$
$= -7$.

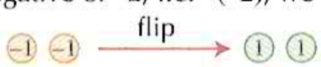
Example: $2 - 5 = 2 + (-5)$



Therefore, $2 - 5 = 2 + (-5)$
$= -3$.

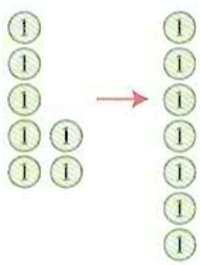
Example: $5 - (-2)$

To obtain the negative of -2, i.e. $-(-2)$, we flip the discs as shown:



we write $-(-2) = 2$

As $-(-2) = 2$, $5 - (-2) = 5 + 2$.



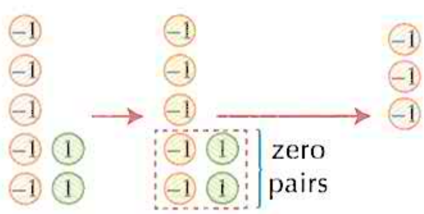
Therefore, $5 - (-2) = 5 + 2$
$= 7$.

ATTENTION

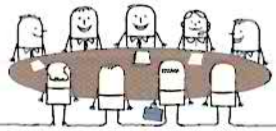
$(-5) - (-2)$ can be written as $-5 - (-2)$.

Example: $(-5) - (-2)$

As $-(-2) = 2$, $(-5) - (-2) = (-5) + 2$.



Therefore, $(-5) - (-2) = (-5) + 2$
$= -3$.



Class Discussion

Subtraction involving Negative Numbers

Part I

Work in pairs to find the value of each of the following by using algebra discs. For each question, discuss with your classmate to write a rule to perform the operation by considering the following questions:

(i) Should the answer be positive or negative?

(ii) Should we find the sum or difference of the absolute values of the two numbers?

1. **(a)** $7 - (-3)$ **(b)** $6 - (-4)$
2. **(a)** $(-7) - 3$ **(b)** $(-6) - 4$
3. **(a)** $(-7) - (-3)$ **(b)** $(-4) - (-6)$
4. **(a)** $3 - 7$ **(b)** $4 - 6$

Part II

Challenge each other to carry out subtraction involving negative integers, just like what you have done in Questions 1, 2, 3 and 4 in **Part I**. You may use algebra discs to help you.

Part III

Now that you have understood the rules of subtraction involving negative numbers, challenge each other to carry out subtraction involving negative integers without using algebra discs.

PRACTISE NOW

Without using a calculator or algebra discs, evaluate each of the following.

(a) $9 - (-2)$ **(b)** $-7 - 4$ **(c)** $-3 - (-5)$ **(d)** $-8 - (-6)$ **(e)** $4 - 8$

(f) $27 - (-13)$ **(g)** $-25 - 11$ **(h)** $-14 - (-16)$ **(i)** $-15 - (-12)$ **(j)** $10 - 28$

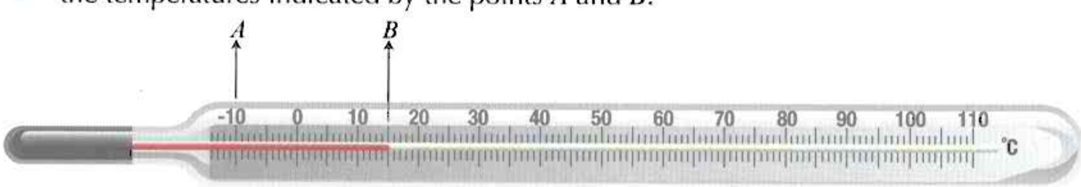
SIMILAR QUESTIONS

Exercise 2B Questions 2(a)–(h), 3(a)–(h), 5(a)–(h)

Worked Example 2

(Subtraction involving Negative Numbers)

The figure shows a thermometer. The readings are in °C. Find the difference between the temperatures indicated by the points A and B.



Solution:

Point A shows -10 °C.

Point B shows 15 °C.

$$\begin{aligned}\text{Difference in temperature} &= 15\ ^\circ\text{C} - (-10\ ^\circ\text{C}) \\ &= 15\ ^\circ\text{C} + 10\ ^\circ\text{C} \\ &= 25\ ^\circ\text{C}\end{aligned}$$

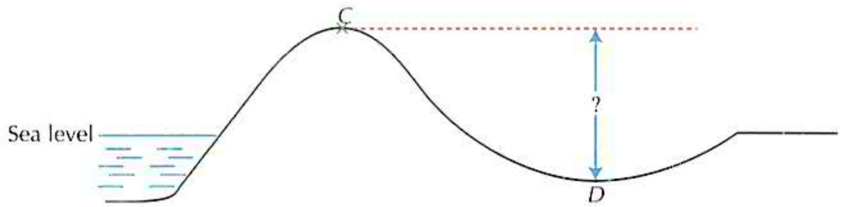
PRACTISE NOW 2

SIMILAR QUESTIONS

Exercise 2B Questions 7–9

1. The figure shows a thermometer. The readings are in °C. Find the difference between the temperatures indicated by the points A and B.

2. A holiday resort C is located at the top of a hill which is 314 m above sea level. A tourist attraction D lies at the bottom of a valley which is 165 m below sea level. Represent the altitude at D using a negative number. Hence, find the difference in altitude between the holiday resort and the tourist attraction.



Puzzle for Consolidation

Why should we not have a conversation near the Merlion? Find the value of each of the following and write the letter in the box above/below the answer to find out.

A $-5-6$	**T** $0+(-8)$	**P** $-47+16$	**O** $5-27$
S $0-(-4)$	**Y** $-88+70$	**N** $-38-10$	**E** $2-9$
D $5+(-11)$	**R** $-6-(-17)$	**U** $9-(-14)$	**H** $8+(-6)$
W $-7-9$			

	−7			−31		−11		4			−11		−7			−22			−18		23			2		−11	
☐	☐		☐	☐	☐	☐	☐	☐		☐	☐	☐	☐	☐		☐	☐		☐	☐	☐	☐		☐	☐	☐	☐
2			4		11		−18			−16		−8		11			−48			−22		11			−7		−6

Exercise 2B

Do not use a calculator for this exercise.

BASIC LEVEL

1. Find the value of each of the following.
   (a) $6+(-2)$ (b) $-5+8$
   (c) $4+(-10)$ (d) $-1+(-7)$
   (e) $9+(-3)$ (f) $-11+(-5)$
   (g) $-10+2$ (h) $1+(-8)$

2. Evaluate each of the following.
   (a) $-(-7)$ (b) $5-(-3)$
   (c) $-4-7$ (d) $-8-(-2)$
   (e) $-1-(-10)$ (f) $6-9$
   (g) $-8-3$ (h) $2-(-7)$

INTERMEDIATE LEVEL

3. Find the value of each of the following.

(a) $4 + (-7) - (-3)$

(b) $-3 - 5 + (-9)$

(c) $1 - 8 - (-8)$

(d) $-2 + (-1) - 6$

(e) $8 - (-9) + 1$

(f) $-5 + (-3) + (-2)$

(g) $6 + (-5) - (-8)$

(h) $2 - (-7) - 8$

4. Find the value of each of the following.

(a) $23 + (-11)$ **(b)** $-19 + 12$

(c) $17 + (-29)$ **(d)** $-21 + (-25)$

(e) $-13 + 18$ **(f)** $-24 + (-13)$

(g) $16 + (-27)$ **(h)** $-26 + 14$

5. Evaluate each of the following.

(a) $22 - (-13)$ **(b)** $-14 - 16$

(c) $-19 - (-11)$ **(d)** $-18 - (-22)$

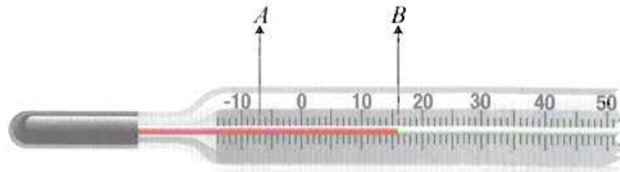
(e) $17 - 23$ **(f)** $-20 - 15$

(g) $12 - (-17)$ **(h)** $-21 - 17$

ADVANCED LEVEL

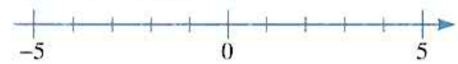
6. The temperature of a village on a particular night is -11 °C. The next morning, the temperature rises by 7 °C. Find the temperature in the morning.

7. The figure shows part of a thermometer. The readings are in °C. Find the difference between the temperatures indicated by the points A and B.



8. A city is located at a height of 138 m above sea level while a town is at a height of 51 m below sea level. Represent the altitude of the town using a negative number. Hence, find the difference in altitude between the city and the town.

9. **(i)** Using the number line, find the difference between -2 and 3.



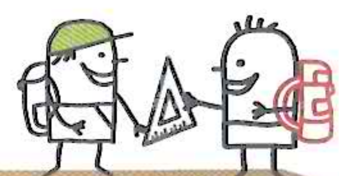
(ii) The figure below shows a timeline for BC and AD. For example, 2 BC stands for 2 years Before Christ and 3 AD stands for 3 years Anno Domini (which means In the year of the Lord, i.e. after Christ was born).

What is the main difference between the timeline for BC and AD, and the number line?

(iii) How many years are there between 2 BC and 3 AD?

(iv) Think of another real-life example that is similar to the timeline in **(ii)** but different from the number line in **(i)**.

2.3 Multiplication and Division involving Negative Numbers



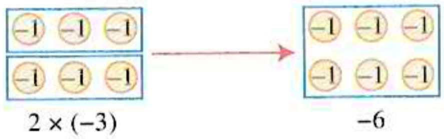
Multiplication involving Negative Numbers

We have learnt in primary school that $2 \times 3 = 3 \times 2 = 6$.
What do you think -2×3 and $-2 \times (-3)$ are equal to?
We will now show how to carry out multiplication that involves negative numbers using algebra discs.

Example: The product 2 × 3 can be represented by:

The model shows '2 groups of 3'.

Example: The product 2 × (–3) can be represented by:

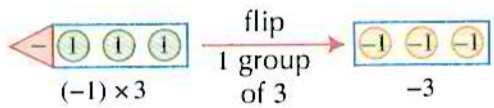


ATTENTION

$2 \times (-3) = -(2 \times 3)$

The model shows '2 groups of –3'.

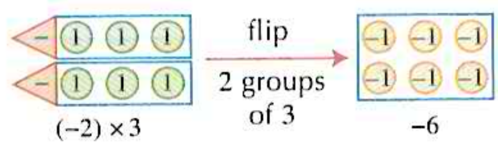
Example: The product (–1) × 3 can be represented by 'the negative of 1 group of 3':



Notice that $(-1) \times 3 = -(1 \times 3) = -3$.

flip 1 group of 3

Note also that (–1) × 3 can be written as –1 × 3.
Similarly, the product (–2) × 3 can be represented by 'the negative of 2 groups of 3':



ATTENTION

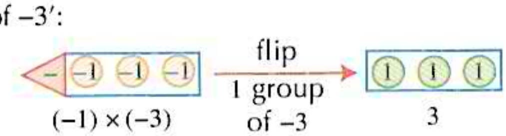
$(-2) \times 3 = 3 \times (-2) = -6$

Notice that $(-2) \times 3 = -(2 \times 3) = -6$.

flip 2 groups of 3

Note also that (–2) × 3 can be written as –2 × 3 or –2(3).

Example: The product (–1) × (–3) can be represented by ' the negative of 1 group of –3':

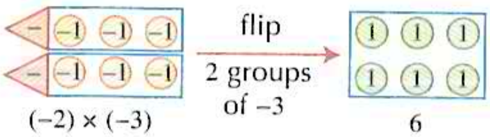


Notice that $(-1) \times (-3) = -[1 \times (-3)] = -(-3) = 3$.

flip 1 group of –3

Note also that (–1) × (–3) can be written as –1 × (–3).

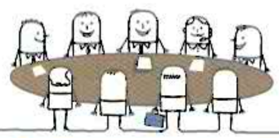
Similarly, the product $(-2) \times (-3)$ can be represented by 'the negative of 2 groups of -3':



Notice that $(-2) \times (-3) = -[2 \times (-3)] = -(-6) = 6$.

flip 2 groups of -3

Note also that $(-2) \times (-3)$ can be written as $(-2)(-3)$ or $-2(-3)$.



Class Discussion

Multiplication involving Negative Numbers

Part I

Work in pairs to find the value of each of the following by using algebra discs. For each question, discuss with your classmate to write a rule to perform the operation by considering whether the answer should be positive or negative.

1. **(a)** $1 \times (-4)$ **(b)** $2 \times (-4)$ **(c)** $3 \times (-4)$

2. **(a)** $(-1) \times 4$ **(b)** $(-2) \times 4$ **(c)** $(-3) \times 4$

3. **(a)** $(-1) \times (-4)$ **(b)** $(-2) \times (-4)$ **(c)** $(-3) \times (-4)$

Part II

Challenge each other to carry out multiplication involving negative integers, just like what you have done in Questions 1, 2 and 3 in **Part I**. You may use algebra discs to help you.

Part III

Now that you have understood the rules of multiplication involving negative numbers, challenge each other to carry out multiplication involving negative integers without using algebra discs.

In general,

positive number × *negative* number = *negative* number, *negative* number × *positive* number = *negative* number, *negative* number × *negative* number = *positive* number.

PRACTISE NOW

SIMILAR QUESTIONS

Exercise 2C Questions 1(a)–(f)

Without using a calculator or algebra discs, find the value of each of the following.

(a) $2 \times (-6)$ **(b)** -5×4 **(c)** $-1 \times (-8)$

(d) $-3 \times (-7)$ **(e)** $-(-10)$ **(f)** $-9(-2)$

(g) $15 \times (-2)$ **(h)** -3×12 **(i)** $-4 \times (-10)$

(j) $-2(-100)$

Division involving Negative Numbers

In primary school, we have learnt that $6 \div 2 = \frac{6}{2} = 6 \times \frac{1}{2} = 3$.

Similarly, $(-6) \div 2 = \frac{-6}{2} = -6 \times \frac{1}{2} = -\left(6 \times \frac{1}{2}\right) = -3$,

$$6 \div (-2) = \frac{6}{-2} = 6 \times \frac{1}{-2} = 6 \times \left(-\frac{1}{2}\right) = -3,$$

$$(-6) \div (-2) = \frac{-6}{-2} = -6 \times \frac{1}{-2} = -6 \times \left(-\frac{1}{2}\right) = 3.$$

In general,

> *positive* number ÷ *negative* number = *negative* number,
> *negative* number ÷ *positive* number = *negative* number,
> *negative* number ÷ *negative* number = *positive* number.

PRACTISE NOW

SIMILAR QUESTIONS

Exercise 2C Questions 2(a)–(f)

Without using a calculator, evaluate each of the following.

(a) $-8 \div 2$ **(b)** $15 \div (-3)$ **(c)** $-8 \div (-4)$

(d) $\frac{-6}{3}$ **(e)** $\frac{20}{-5}$ **(f)** $\frac{-12}{-3}$

Square Roots and Cube Roots Revisited

In Chapter 1, we have learnt that $5^2 = 5 \times 5 = 25$ and thus $\sqrt{25} = 5$. What is $(-5)^2$ or $(-5) \times (-5)$ equal to?

Since $5 \times 5 = 25$ and $(-5) \times (-5) = 25$, then 25 has *two* square roots:

(i) the positive square root of 25, written as $\sqrt{25} = 5$, and

(ii) the negative square root of 25, written as $-\sqrt{25} = -5$.

The square root sign $\sqrt{\ }$ is used to denote the *positive square root* only. We can combine both the positive and negative square roots by writing $\pm\sqrt{25} = \pm 5$.

Worked Example 3

(Finding the Square Roots of a Number)

Find the square roots of 49.

Solution: Square roots of $49 = \pm\sqrt{49}$

$= \pm 7$

PRACTISE NOW 3a

(a) Find the square roots of 64.

(b) Find the negative square root of 9.

(c) Evaluate $\sqrt{36}$.

Thinking Time

Is it possible to obtain the square roots of a negative number, e.g. $\pm\sqrt{-16}$? Explain your answer.

In Chapter 1, we have learnt that $5^3 = 5 \times 5 \times 5 = 125$ and thus $\sqrt[3]{125} = 5$. What is $(-5)^3$ or $(-5) \times (-5) \times (-5)$ equal to?

We realise that the cube root is different from the square root as a number has *only one* cube root and it is *possible* to obtain the cube root of a negative number, e.g.

$$\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5 \text{ and } \sqrt[3]{-125} = \sqrt[3]{(-5) \times (-5) \times (-5)} = -5\,.$$

SIMILAR QUESTIONS

Exercise 2C Questions 3(a)–(d), 4(a)–(d)

PRACTISE NOW 3b

Evaluate each of the following.

(a) $(-3)^3$ **(b)** $(-4)^3$

(c) $\sqrt[3]{216}$ **(d)** $\sqrt[3]{-8}$

SIMILAR QUESTIONS

Exercise 2C Questions 5(a)–(d), 6(a)–(d)

Combined Operations on Numbers

In primary school, we have learnt the 4 basic operations on numbers, +, −, ×, ÷. Previously, we have also learnt square, square root, cube and cube root. Thus in this section, we will learn how to perform all these operations together.

The **order of operations** is as follows:

Rule 1	**Brackets:** Evaluate the expression in the bracket first. If there is more than one pair of brackets, evaluate the expression in the innermost pair first.
Rule 2	**Powers and Roots:** Evaluate the powers and roots.
Rule 3	**Multiplication and Division:** Multiply and divide from the left to the right.
Rule 4	**Addition and Subtraction:** Add and subtract from the left to the right.

Worked Example 4

(Combined Operations on Numbers)

Without using a calculator, find the value of each of the following.

(a) $6 - 7 + 2 \times (4 - 3^2)$ **(b)** $(-2)^3 - 12 \div 2 - \left(\sqrt{25} + 3\right)$

Solution:

(a)

$$\begin{aligned} & 6 - 7 + 2 \times (4 - 3^2) \\ &= 6 - 7 + 2 \times (4 - 9) && \text{(power)} \\ &= 6 - 7 + 2 \times (-5) && \text{(brackets)} \\ &= 6 - 7 + (-10) && \text{(multiplication)} \\ &= -1 + (-10) && \text{(subtraction)} \\ &= -11 && \text{(addition)} \end{aligned}$$

(b)

$$\begin{aligned} & (-2)^3 - 12 \div 2 - \left(\sqrt{25} + 3\right) \\ &= -8 - 12 \div [2 - (5 + 3)] && \text{(power and root)} \\ &= -8 - 12 \div (2 - 8) && \text{(brackets)} \\ &= -8 - 12 \div (-6) \\ &= -8 - (-2) && \text{(division)} \\ &= -8 + 2 && \text{(brackets)} \\ &= -6 && \text{(addition)} \end{aligned}$$

PRACTISE NOW 4a

SIMILAR QUESTIONS

Exercise 2C Questions 7(a)–(j), 9(a)–(h), 11

Without using a calculator, find the value of each of the following.

(a) $-3 \times (15 - 7 + 2)$ **(b)** $4^3 - 7 \times 16 - (\sqrt[3]{64} - 5)$

Use of a Calculator to Evaluate Negative Numbers

Most calculators distinguish between 'minus' and 'negative'. The 'minus' button is $\boxed{-}$ but the 'negative' button is either $\boxed{+/-}$ or $\boxed{(-)}$ depending on the model of the calculator.

For example, to evaluate $2 - (-5)$ using a calculator, press:

$$\boxed{2}\boxed{-}\boxed{(}\boxed{+/-}\boxed{5}\boxed{)}\boxed{=}$$

For some calculators, it is not necessary to key in the brackets.

The answer to Worked Example 4(b) may be evaluated using a calculator. Press:

$$\boxed{(}\boxed{+/-}\boxed{2}\boxed{)}\boxed{x^y}\boxed{3}\boxed{-}\boxed{1}\boxed{2}\boxed{\div}\boxed{(}\boxed{2}\boxed{-}\boxed{(}\boxed{\sqrt{\ }}\boxed{2}\boxed{5}\boxed{+}\boxed{3}\boxed{)}\boxed{)}\boxed{=}$$

PRACTISE NOW 4b

SIMILAR QUESTIONS

Exercise 2C Questions 8, 10

Use a calculator to check your answers in Practise Now 4a.

Exercise 2C

Do not use a calculator for this exercise unless stated otherwise.

BASIC LEVEL

1. Find the value of each of the following.

(a) $3 \times (-9)$ **(b)** -8×4

(c) $-7 \times (-5)$ **(d)** $-1 \times (-6)$

(e) $-2(-7)$ **(f)** -6×0

2. Evaluate each of the following.

(a) $-21 \div 7$ **(b)** $16 \div (-2)$

(c) $-8 \div (-2)$ **(d)** $\frac{-14}{2}$

(e) $\frac{15}{-5}$ **(f)** $\frac{-18}{-3}$

3. Find the square roots of each of the following numbers.

(a) 81 **(b)** 16

(c) 25 **(d)** 100

4. Evaluate each of the following where possible.

(a) $\sqrt{81}$ **(b)** $\sqrt{4}$

(c) $-\sqrt{9}$ **(d)** $\sqrt{-36}$

5. Evaluate each of the following.

(a) $(-2)^3$ **(b)** $(-5)^3$

(c) $(-10)^3$ **(d)** $(-6)^3$

6. Evaluate each of the following.

(a) $\sqrt[3]{27}$ (b) $-\sqrt[3]{64}$

(c) $\sqrt[3]{8}$ (d) $\sqrt[3]{-216}$

7. Find the value of each of the following.

(a) $-55 + (-10) - 10$

(b) $-12 - [(-8) - (-2)] + 3$

(c) $-100 + (-45) + (-5) + 20$

(d) $-2 + 3 \times 15$

(e) $(-5 - 2) \times (-3)$

(f) $-25 \times (-4) \div (-12 + 32)$

(g) $3 \times (-3)^2 - (7 - 2)^2$

(h) $5 \times [3 \times (-2) - 10]$

(i) $-12 \div [2^2 - (-2)]$

(j) $\sqrt{10 - 3 \times (-2)}$

8. Use a calculator to check your answers for Question 7.

INTERMEDIATE LEVEL

9. Find the value of each of the following.

(a) $24 \times (-2) \times 5 \div (-6)$

(b) $4 \times 10 - 13 \times (-5)$

(c) $(16 - 24) - (57 - 77) \div (-2)$

(d) $160 \div (-40) - 20 \div (-5)$

(e) $[(12 - 18) \div 3 - 5] \times (-4)$

(f) $\{[(-15 + 5) \times 2 + 8] - 32 \div 8\} - (-7)$

(g) $(5 - 2)^3 \times 2 + [-4 + (-7)] \div (-2 + 4)^2$

(h) $\{-10 - [12 + (-3)^2] + 3^3\} \div (-3)$

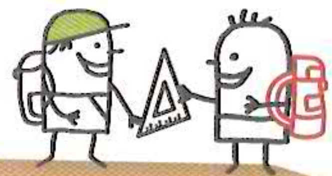
10. Use a calculator to check your answers for Question 9.

ADVANCED LEVEL

11. Find the value of

$$\sqrt[3]{-2 \times (-6.5) - \quad -2 \times \left(-3\right) + 8 \times \left(-2\right) - 8 \times 2 \quad + 5^2}\,.$$

2.4 Rational Numbers and Real Numbers



Fractions and Mixed Numbers

In primary school, we have learnt about **proper fractions** (e.g. $\frac{3}{4}$), **improper fractions** (e.g. $\frac{5}{3}$ and $\frac{2}{2}$) and **mixed numbers** (e.g. $5\frac{1}{4}$). These numbers are positive but they can be extended to include **negative fractions** and **negative mixed numbers**.

The rules for performing the 4 basic operations on negative fractions and negative mixed numbers are the same as those for positive fractions and positive mixed numbers.

To add or subtract fractions with different denominators, i.e. unlike fractions, we must first express the fractions in the *same denominator*, i.e. like fractions, using the idea of equivalent fractions. We may make use of the lowest common multiple (LCM) of the denominators.

Worked Example 5

(Addition and Subtraction of Negative Fractions and Mixed Numbers)

Without using a calculator, find the value of each of the following.

(a) $6\frac{1}{5}+\left(-2\frac{3}{10}\right)$ **(b)** $-\frac{7}{4}-\left(-\frac{5}{6}\right)+\left(-1\frac{1}{3}\right)$

Solution:

(a) $6\frac{1}{5}+\left(-2\frac{3}{10}\right)$

$= 6\frac{1}{5}-2\frac{3}{10}$

$= 4\frac{2}{10}-\frac{3}{10}$ (convert to like fractions: $\frac{1}{5}=\frac{2}{10}$)

$= \left(3+\frac{10}{10}\right)+\frac{2}{10}-\frac{3}{10}$

$= 3\frac{9}{10}$

(b) $-\frac{7}{4}-\left(-\frac{5}{6}\right)+\left(-1\frac{1}{3}\right)$

$= -\frac{7}{4}+\frac{5}{6}-1\frac{1}{3}$

$= -\frac{7}{4}+\frac{5}{6}-\frac{4}{3}$ (change to improper fraction)

$= -\frac{21}{12}+\frac{10}{12}-\frac{16}{12}$ (convert to like fractions: $-\frac{7}{4}=-\frac{21}{12}, \frac{5}{6}=\frac{10}{12}, \frac{4}{3}=\frac{16}{12}$)

$= \frac{-21+10-16}{12}$

$= \frac{-11-16}{12}$

$= \frac{-27}{12}$

$= -\frac{9}{4}$ (reduced to lowest term)

$= -2\frac{1}{4}$

ATTENTION

Always leave your answers in mixed numbers and not improper fractions.

PRACTISE NOW 5

Without using a calculator, find the value of each of the following.

(a) $7\frac{1}{2}+\left(-3\frac{3}{5}\right)$ **(b)** $-2\frac{3}{4}+\left(-\frac{5}{6}\right)-\left(-\frac{2}{3}\right)$

SIMILAR QUESTIONS

Exercise 2D Questions 1(a)–(d), 10(a)–(e), 17

In primary school, we have learnt how to multiply and divide a fraction by a whole number or a proper fraction:

(a) Multiplication of fractions where the numerators and the denominators have no common factors

e.g. $\frac{1}{2}\times\frac{3}{5} = \frac{1\times 3}{2\times 5}$ (multiply the numerators and the denominators respectively)

$= \frac{3}{10}$

(b) Multiplication of fractions where the numerators and the denominators have common factors

e.g. $\frac{1}{\cancel{4}_{2}}\times\frac{\cancel{6}^{3}}{7} = \frac{1\times 3}{2\times 7}$ (divide 4 and 6 by 2 first)

$= \frac{3}{14}$

(c) Division of fractions

Dividing one fraction by another fraction is the same as multiplying the first fraction by the reciprocal of the second fraction.

e.g. $\frac{2}{3} \div \frac{4}{9} = \frac{{}^{1}\cancel{2}}{{}_{1}\cancel{3}} \times \frac{\cancel{9}^{3}}{\cancel{4}_{2}}$

$= \frac{1 \times 3}{1 \times 2}$

$= \frac{3}{2}$

$= 1\frac{1}{2}$

ATTENTION

The reciprocal of a fraction is obtained by interchanging the numerator and the denominator of the fraction, e.g. the reciprocal of $\frac{9}{4}$ is $\frac{4}{9}$.

In this section, we will learn how to multiply and divide a mixed number by a fraction. Then we will extend the operations to negative fractions and mixed numbers.

Worked Example 6

(Multiplication and Division of Positive Fractions and Mixed Numbers)

Without using a calculator, evaluate each of the following.

(a) $2\frac{3}{4} \times \frac{2}{11}$ **(b)** $2\frac{2}{9} \div \frac{5}{3}$

Solution:

(a) $2\frac{3}{4} \times \frac{2}{11} = \frac{{}^{1}\cancel{11}}{{}_{2}\cancel{4}} \times \frac{\cancel{2}^{1}}{\cancel{11}_{1}}$ (change to improper fraction; divide by common factors)

$= \frac{1}{2}$

ATTENTION

In **(a)**, we cancel 2 and 4 to give 1 and 2 respectively because we divide both numbers by the common factor 2:

$\frac{11}{4} \times \frac{2}{11} = \frac{11}{4 \div 2} \times \frac{2 \div 2}{11}$

$= \frac{11}{2} \times \frac{1}{11}$

(b) $2\frac{2}{9} \div \frac{5}{3} = \frac{20}{9} \div \frac{5}{3}$ (change to improper fraction)

$= \frac{{}^{4}\cancel{20}}{{}_{3}\cancel{9}} \times \frac{\cancel{3}^{1}}{\cancel{5}_{1}}$ (multiply by the reciprocal of $\frac{5}{3}$; divide by common factors)

$= \frac{4}{3}$

$= 1\frac{1}{3}$

PRACTISE NOW 6

Without using a calculator, evaluate each of the following.

(a) $2\frac{2}{3} \times \frac{9}{4}$ **(b)** $4\frac{1}{6} \div \frac{5}{2}$

SIMILAR QUESTIONS

Exercise 2D Questions 3(a)–(d), 16

Worked Example 7

(Multiplication and Division of Negative Fractions and Mixed Numbers)

Without using a calculator, find the value of each of the following.

(a) $3\frac{2}{3} \div \left(-2\frac{4}{9}\right)$ **(b)** $-2\frac{4}{5} \times \left[-\frac{5}{2} + \left(-2\frac{4}{3}\right)\right]$

Solution:

(a) $3\frac{2}{3} \div \left(-2\frac{4}{9}\right)$

$= \frac{11}{3} \div \left(-\frac{22}{9}\right)$ (change to improper fractions)

$= \frac{{}^{1}\cancel{11}}{{}_{1}\cancel{3}} \times \left(-\frac{\cancel{9}^{3}}{\cancel{22}_{2}}\right)$ (multiply by the reciprocal of $-\frac{22}{9}$; divide by common factors)

$= -\frac{3}{2}$

$= -1\frac{1}{2}$

(b) $-2\frac{4}{5} \times \left[-\frac{5}{2} + \left(-2\frac{4}{3}\right)\right]$

$= -\frac{14}{5} \times -\frac{5}{2} - \frac{10}{3}$ (change to improper fractions)

$= -\frac{14}{5} \times -\frac{15}{6} - \frac{20}{6}$ (convert to like fractions: $-\frac{5}{2} = -\frac{15}{6}$, $\frac{10}{3} = \frac{20}{6}$)

$= -\frac{{}^{7}\cancel{14}}{{}_{1}\cancel{5}} \times \left(-\frac{\cancel{35}^{7}}{\cancel{6}_{3}}\right)$ (divide by common factors)

$= \frac{49}{3}$

$= 16\frac{1}{3}$

ATTENTION

We only change fractions to like fractions for addition and subtraction, e.g. $-\frac{5}{2} - \frac{10}{3} = -\frac{15}{6} - \frac{20}{6}$.

PRACTISE NOW 7a

SIMILAR QUESTIONS

Exercise 2D Questions 5(a)–(f), 12(a)–(f), 18

Without using a calculator, find the value of each of the following.

(a) $5\frac{1}{4} \div \left(-2\frac{4}{5}\right)$ **(b)** $1\frac{3}{4} \times \left[\frac{6}{5} + \left(-\frac{1}{2}\right)\right]$

Use of a Calculator to Evaluate Fractions and Mixed Numbers

On most calculators, the 'fraction' or 'mixed number' button is $\boxed{a\,^{b}/_{c}}$. For example, to key in $3\frac{4}{5}$, press:

$\boxed{3}\,\boxed{a\,^{b}/_{c}}\,\boxed{4}\,\boxed{a\,^{b}/_{c}}\,\boxed{5}$. Depending on the model of the calculator, $3\frac{4}{5}$ will be displayed as: 3 ┌ 4 ┌ 5 or 3 ┘ 4 ┘ 5.

To convert a mixed number to an improper fraction and vice versa, use the button $\boxed{d\,/\,c}$.

PRACTISE NOW 7b

SIMILAR QUESTIONS

Exercise 2D Questions 2, 4, 6, 11, 13

Use a calculator to check your answers in Practise Now 5, 6 and 7a.

Decimals

In primary school, we have learnt about decimals such as 0.716 and 7.14. We have also learnt how to add and subtract decimals, and how to multiply and divide a decimal by a whole number.

In this section, we will learn how to multiply and divide a decimal by another decimal. Then we will extend the operations to negative decimals.

Worked Example 8

(Multiplication of Decimals)

Without using a calculator, evaluate 12.34 × 5.6.

Solution:

$$\begin{array}{r l}
12.34 & \leftarrow \text{2 decimal places (2 d.p.)} \\
\times \quad 5.6 & \leftarrow \text{1 decimal place (1 d.p.)} \\
\hline
7404 & \\
+\ 6170 & \\
\hline
69.104 & \\
\hline
\end{array}$$

place the decimal point here because 2 d.p. + 1 d.p. = 3 d.p.

$\therefore 12.34 \times 5.6 = 69.104$

Note: $\underbrace{12.34}_{\text{2 d.p.}} \times \underbrace{5.6}_{\text{1 d.p.}} = \frac{1234}{100} \times \frac{56}{10} = \frac{1234 \times 56}{1000}$

$= \frac{69\,104}{1000}$

$= \underbrace{69.104}_{\text{3 d.p.}}$

PRACTISE NOW 8

Without using a calculator, evaluate each of the following.

(a) 13.56×2.4 **(b)** 137.8×0.35

SIMILAR QUESTIONS

Exercise 2D Questions 7(a)–(d)

Worked Example 9

(Division of Decimals)

Without using a calculator, find the value of $0.72 \div 0.3$.

Solution:

$$0.72 \div 0.3 = \frac{0.72}{0.3}$$

$$= \frac{7.2}{3}$$

line up the decimal points

$$\begin{array}{r} 2.4 \\ 3\overline{)\,7.2} \\ -6 \\ \hline 1\ 2 \\ -1\ 2 \\ \hline 0 \\ \hline \end{array}$$

$\therefore 0.72 \div 0.3 = 2.4$

ATTENTION

To convert the denominator to an integer, we multiply both the numerator and the denominator by 10.

$$\frac{0.72}{0.3} = \frac{0.72}{0.3} \times \frac{10}{10}$$

$$= \frac{7.2}{3}$$

PRACTISE NOW 9

Without using a calculator, find the value of each of the following.

(a) $0.92 \div 0.4$ **(b)** $1.845 \div 0.15$

SIMILAR QUESTIONS

Exercise 2D Questions 8(a)–(d)

Worked Example 10

(Negative Decimals)

Without using a calculator, evaluate each of the following.

(a) $2.5 - (-1.3)$ **(b)** $\frac{0.18}{0.3} \times \left(\frac{-0.47}{1.2}\right)$

Solution:

(a) $2.5 - (-1.3) = 2.5 + 1.3$
$= 3.8$

(b) $\frac{0.18}{0.3} \times \left(\frac{-0.47}{1.2}\right) = \frac{1.8}{3} \times \left(\frac{-0.47}{1.2}\right)$

$= 0.6 \times \left(\frac{-0.47}{1.2}\right)$

$= {}^{1}\cancel{6} \times \frac{-0.47}{\cancel{12}_{2}}$

$= -0.235$

ATTENTION

$$\begin{array}{r} 0.6 \\ 3\overline{)\,1.8} \\ -1\ 8 \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{r} 0.235 \\ 2\overline{)\,0.470} \\ -\ 4 \\ \hline 70 \\ -6 \\ \hline 10 \\ -10 \\ \hline 0 \\ \hline \end{array}$$

PRACTISE NOW 10

Without using a calculator, evaluate each of the following.

(a) $32 - (-1.6)$ **(b)** $1.3 + (-3.5)$

(c) $\frac{0.12}{0.4} \times \left(\frac{-0.23}{0.6}\right)$ **(d)** $-0.3^2 \times \frac{4.5}{-2.7} - 0.65$

SIMILAR QUESTIONS

Exercise 2D Questions 9(a)–(d), 14(a)–(d)

Rational Numbers

We have learnt different types of numbers such as whole numbers, integers and fractions. These numbers are called **rational numbers**.

> A **rational** number is a number that can be expressed as the **ratio** of two integers a and b, i.e. in the form $\frac{a}{b}$, where $b \neq 0$.

Fig. 2.5 illustrates the relationships among the different types of numbers.

- Rational Numbers
  - Integers
    - Whole Numbers
      - Positive Integers (e.g. 2, 8)
      - Zero
    - Negative Integers (e.g. -2, -7)
  - Fractions (e.g. $\frac{1}{3}$, $-2\frac{1}{8}$)

Fig. 2.5

INFORMATION

The fractions here are defined to be non-integers, i.e. they exclude *improper fractions* that can be reduced to integers, e.g. $\frac{3}{3}$ and $\frac{8}{2}$.

Thinking Time

How do you express each of the following numbers in the form $\frac{a}{b}$? Is this the only way, i.e. can the values of a and b be other different numbers?

(a) Integers such as 2, 0 and -3.

(b) Decimals such as 0.5 and 0.333...

Real Numbers

We have learnt about π in primary school and about square roots and cube roots, such as $\sqrt{7}$ and $\sqrt[3]{5}$, in Chapter 1. The numbers π, $\sqrt{7}$ and $\sqrt[3]{5}$ are called **irrational numbers** because they cannot be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Real numbers are made up of rational numbers and irrational numbers. Fig. 2.6 illustrates the relationships among these three types of numbers.

- Real Numbers
  - Rational Numbers (e.g. 2, -0.7, $\frac{1}{3}$, $-3\frac{1}{8}$)
  - Irrational Numbers (e.g. π, $\sqrt{7}$, $-\sqrt[3]{5}$)

Fig. 2.6

Investigation

Terminating, Recurring and Non-Recurring Decimals

Use a calculator to evaluate each of the following in Table 2.1 and write down the entire display on the calculator.

Group 1	Group 2	Group 3
$\frac{9}{4} =$	$\frac{1}{3} =$	$\frac{1}{\sqrt{2}} =$
$-3\frac{1}{8} =$	$-\frac{123}{99} =$	$-\sqrt[3]{5} =$
$\frac{63}{64} =$	$\frac{22}{7} =$	$\pi =$

Table 2.1

1. In primary school, we take π to be $\frac{22}{7}$. Based on the above calculator values, is π equal to $\frac{22}{7}$?
2. What do you notice about the decimal representations (i.e. the calculator values) of the numbers in Group 2? Are they rational or irrational numbers?

   (The last digit of $\frac{22}{7}$ in the calculator display may have been rounded up, so you may not see the pattern. The actual value of $\frac{22}{7}$ is 3.142 857 142 857 142 857 ...)
3. What do you notice about the decimal representations of the numbers in Group 1 and in Group 3? Are they rational or irrational numbers?
4. Use a dot to represent each of the numbers in Table 2.1 on the number line in Fig. 2.7. An example is given.

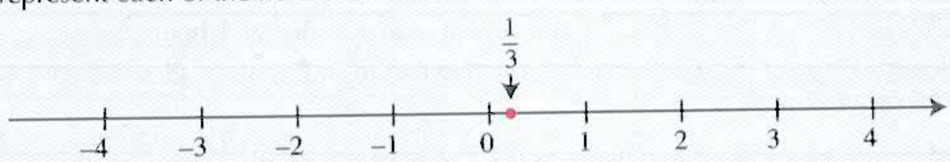


Fig. 2.7

5. Hence, arrange the numbers in **descending order**, i.e. from the greatest to the smallest.

From the investigation, we observe that there are three types of decimals.

Group 1: Terminating Decimals, i.e. the digits after the decimal point terminate.
Group 2: Recurring (or **Repeating**) **Decimals**, i.e. some digits after the decimal point repeat themselves indefinitely.
Group 3: Non-Recurring (and **Non-Terminating**) **Decimals**, i.e. the digits after the decimal point do not repeat but they continue indefinitely.

Terminating decimals and recurring decimals are rational numbers, but non-recurring decimals are irrational numbers. Fig. 2.8 illustrates the relationships among the different types of numbers.

Real Numbers

Rational Numbers | Irrational Numbers

Terminating Decimals (e.g. 2, – 0.7, $\frac{9}{4}$, $-3\frac{1}{8}$)

Recurring Decimals (e.g. $\frac{1}{3}$, $-\frac{123}{99}$, $\frac{22}{7}$)

Non-Recurring (and Non-Terminating) Decimals (e.g. $\frac{1}{\sqrt{2}}$, $-\sqrt[3]{5}$, π)

Fig. 2.8

Worked Example 11

(Use of a Calculator for a More Complicated Calculation)

Use a calculator to evaluate $\dfrac{3.2\pi + 4.3^2}{\sqrt{47.5} - 2\frac{3}{4}}$, leaving your answer correct to 3 decimal places.

Solution:

Sequence of calculator keys:

$\boxed{(}\boxed{3}\boxed{.}\boxed{2}\boxed{\pi}\boxed{+}\boxed{4}\boxed{.}\boxed{3}\boxed{x^2}\boxed{)}\boxed{\div}\boxed{(}\boxed{\sqrt{\ }}\boxed{4}\boxed{7}\boxed{.}\boxed{5}\boxed{-}\boxed{2}\boxed{a^b/_c}\boxed{3}\boxed{a^b/_c}\boxed{4}\boxed{)}\boxed{=}$

$$\therefore \frac{3.2\pi + 4.3^2}{\sqrt{47.5} - 2\frac{3}{4}} = 6.891 \text{ (to 3 d.p.)}$$

PRACTISE NOW 11

Use a calculator to evaluate $\dfrac{\pi \times 0.7^2}{\sqrt[3]{2.4} + 1\frac{3}{10}}$, leaving your answer correct to 3 decimal places.

SIMILAR QUESTIONS

Exercise 2D Questions 15(a)–(d)

Investigation

Some Interesting Facts about the Irrational Number π

1. How many digits does π have?

   To have a sense of what it means by 'the digits of π after the decimal point continue indefinitely', search on the Internet for 'First Million Digits of Pi'. What is the 1 000 000th digit of π?

2. The digits of π do not end after one million. At the time of printing, the record for computing the decimal representation of π was set by Shigeru Kondo, a Japanese systems engineer, and Alexander Yee, an American computer science student, who used a single desktop computer with 20 external hard disks to calculate π to 5 trillion (i.e. 5 000 000 000 000) decimal places on 2 August 2010. Search on the Internet for 'World Record for Pi Calculation'. What is the 5 000 000 000 000th digit of π?

3. How many digits of π can you remember?

   In 2006, Akira Haraguchi, a Japanese mental health counsellor, took more than 16 hours to recite π to *100 000 decimal places* from memory! However, the official record holder in the Guinness Book of Records, at the time of printing, belongs to a graduate student from China, who took 24 hours and 4 minutes to recite π to 67 890 decimal places in 2005. Search on the Internet for 'World Record for Pi Recitation'. What is the name of the graduate student from China?

Exercise 2D

Do not use a calculator for this exercise unless stated otherwise.

BASIC LEVEL

1. Find the value of each of the following.
   (a) $-\frac{1}{2}+\left(-\frac{3}{4}\right)$ (b) $3\frac{1}{8}+\left(-\frac{1}{4}\right)$
   (c) $5\frac{1}{5}-4\frac{1}{2}$ (d) $-3\frac{1}{6}+\left(-4\frac{2}{3}\right)$

2. Use a calculator to check your answers for Question 1.

3. Evaluate each of the following.
   (a) $\frac{15}{8}\times\frac{4}{3}$ (b) $2\frac{3}{5}\times\frac{15}{26}$
   (c) $\frac{15}{4}\div\frac{5}{2}$ (d) $1\frac{7}{9}\div\frac{4}{3}$

4. Use a calculator to check your answers for Question 3.

5. Find the value of each of the following.
   (a) $\frac{64}{15}\times\left(-\frac{3}{8}\right)$ (b) $\frac{4}{15}\div\left(-\frac{10}{3}\right)$
   (c) $-6\frac{1}{8}\times\frac{3}{14}$ (d) $-2\frac{1}{2}\times4\frac{2}{5}$
   (e) $-1\frac{1}{4}\div\frac{3}{8}$ (f) $-\frac{8}{9}\div\left(-1\frac{2}{3}\right)$

6. Use a calculator to check your answers for Question 5.

7. Evaluate each of the following.
   (a) 14.72×1.2 (b) 130.4×0.15
   (c) 0.27×0.08 (d) 0.25×1.96

8. Find the value of each of the following.
   (a) $0.81\div0.3$ (b) $1.32\div0.12$
   (c) $3.426\div0.06$ (d) $4.35\div1.5$

9. Evaluate each of the following.
   (a) $4.3-(-3.9)$ (b) $2.8+(-1.5)$
   (c) $-5.9+2.7$ (d) $-6.7-5.4$

INTERMEDIATE LEVEL

10. Find the value of each of the following.
    (a) $-\frac{8}{5}-\left(-2\frac{1}{4}\right)-\frac{1}{2}$
    (b) $6\frac{1}{5}-\left(-\frac{3}{4}\right)+\left(-4\frac{1}{10}\right)$
    (c) $4\frac{2}{7}+\left(-6\frac{1}{3}\right)-\left(-\frac{4}{21}\right)$
    (d) $-4+\left(-3\frac{1}{8}\right)+\left(-\frac{4}{3}\right)$
    (e) $-\frac{1}{5}+2\frac{1}{4}+\left(-\frac{7}{2}\right)$

11. Use a calculator to check your answers for Question 10.

12. Find the value of each of the following.
    (a) $-\frac{5}{7}\times\left(-\frac{28}{15}+1\frac{2}{3}\right)$
    (b) $\left[-\frac{1}{4}-\left(-\frac{1}{3}\right)\right]\div\left(\frac{1}{4}-\frac{1}{3}\right)$
    (c) $10-\frac{15}{8}\times\left(\frac{3}{2}\div4\frac{1}{2}\right)+\left(-\frac{1}{4}\right)$
    (d) $\left(\frac{1}{2}\right)^3-\left(\frac{3}{4}\right)^2+\left(-\frac{3}{4}\right)$
    (e) $\frac{1}{3}+\frac{4}{9}\times\left(-\frac{1}{2}\right)^2$
    (f) $\left(\frac{3}{2}\right)^2\times\left(\frac{1}{15}-2\frac{1}{3}\right)$

13. Use a calculator to check your answers for Question 12.

14. Evaluate each of the following.

(a) $\frac{0.15}{0.5} \times \left(\frac{-0.16}{1.2}\right)$

(b) $\frac{0.027}{0.03} \times \left(\frac{1.4}{-0.18}\right)$

(c) $-0.4^2 \times \left(\frac{-1.3}{0.8}\right) - 0.62$

(d) $(-0.2)^3 \times \frac{27}{1.6} + 0.105$

15. Use a calculator to evaluate each of the following, leaving your answer correct to 3 decimal places.

(a) $\left(\frac{\pi + 5\frac{1}{2}}{-2.1}\right)^2$ (b) $-\frac{\pi^2 + \sqrt{2}}{7 - \sqrt[3]{4}}$

(c) $\frac{\sqrt[3]{14^2 + 19^2}}{\pi - 4.55}$ (d) $\sqrt{\frac{4.6^2 + 8.3^2 - \left(6\frac{1}{2}\right)^2}{2 \times 4.6 - 8.3}}$

16. Nora spent a total of $8\frac{1}{16}$ hours on community service last year. Given that her visit to the old folks' homes made up $\frac{4}{7}$ of the total time on community service, find the amount of time she spent on visiting the old folks' homes.

ADVANCED LEVEL

17. Find the value of

$$5\frac{3}{4} - 2\frac{5}{6} + \left(-\frac{23}{15}\right) - \left(-4\frac{7}{10}\right).$$

18. Farhan, Khairul, Huixian, Shirley and Jun Wei share a sum of money. Farhan takes $\frac{1}{5}$ of the sum of money. After Farhan has taken his share, Khairul takes $\frac{1}{3}$ of the remaining money. After Khairul has taken his share, Huixian takes $\frac{1}{4}$ of the remaining money. After Huixian has taken her share, Shirley takes $\frac{1}{7}$ of the remaining money. After Shirley has taken her share, Jun Wei takes all of the remaining money. What fraction of the sum of money is Jun Wei's share?

1.

- Real Numbers
  - Rational Numbers
    - Integers
      - Whole Numbers
        - Positive Integers (e.g. 2, 8)
        - Zero
      - Negative Integers (e.g. −2, −7)
    - Fractions (e.g. $\frac{1}{3}$, $-2\frac{1}{8}$)
  - Irrational Numbers
    - Non-Recurring (and Non-Terminating) Decimals (e.g. $\frac{1}{\sqrt{2}}$, $-\sqrt[3]{5}$, π)

2. Addition and Subtraction involving Negative Numbers:

Example	Explanation	Algebra Discs	Answer
$5 + (-2)$	Absolute value of positive number is greater ∴ Answer is positive	More positive discs 1 1 1 1 1 −1 −1	3
$-5 + 2$	Absolute value of negative number is greater ∴ Answer is negative	More negative discs −1 −1 −1 −1 −1 1 1	−3
$-5 + (-2)$ **Note:** $-5 - 2 = -5 + (-2)$	Addition of two negative numbers ∴ Answer is negative	All negative discs −1 −1 −1 −1 −1 −1 −1	−7
$-(-2)$	Concept of negative	−1 −1 $\xrightarrow{\text{flip}}$ 1 1	2

Since $-(-2) = 2$, we have:

- $5 \boxed{-(-2)} = 5 \boxed{+2} = 7$
- $-5 \boxed{-(-2)} = -5 \boxed{+2} = -3$

3. Multiplication and Division of Numbers:

(i) positive × negative = negative **(ii)** negative × negative = positive

(iii) positive ÷ negative = negative **(iv)** negative ÷ negative = positive

Note: You may also think of groups of algebra discs to help you recall the results in multiplication.

4. Square Roots and Cube Roots

(a) A *positive* number (e.g. 64) has *two* square roots (i.e. $\pm\sqrt{64} = \pm 8$) but *only one* cube root (i.e. $\sqrt[3]{64} = 4$).

(b) A *negative* number (e.g. −64) has *no* square root but has *one* cube root (i.e. $\sqrt[3]{-64} = -\sqrt[3]{64} = -4$).

Review Exercise 2

Do not use a calculator for this review exercise unless stated otherwise.

1. Fill in each box with '>' or '<'.

(a) $-7 - 38 \;\square\; 8 + (-55)$ **(b)** $2.36 - 10.58 \;\square\; -11.97 - (-2.69)$

(c) $-5 \times 1.5 \;\square\; 50 \div (-8)$ **(d)** $7\frac{1}{5} - \left(-3\frac{3}{10}\right) \;\square\; 19\frac{2}{5} + \left(-8\frac{1}{10}\right)$

2. By using a number line, arrange each of the following in descending order.

(a) $4, \frac{29}{33}, -2.365, 5.5, -\frac{3}{4}$ **(b)** $\frac{5}{8}, -8, 10\frac{1}{2}, 5.855, -2\pi$

3. Find the value of each of the following.

(a) $13-(-54)$ (b) $(-74)-(-46)$

(c) $11+(-33)-(-7)$ (d) $-13+(-15)+(-8)$

4. Evaluate each of the following.

(a) -12×7 (b) $4\times(-5)\times(-6)$

(c) $-600\div 15$ (d) $50\div(-8)\div(-5)$

5. Find the value of each of the following.

(a) $(-3-5)\times(-3-4)$ (b) $4\times(-5)\div(-2)$

(c) $-5\times 6-18\div(-3)$ (d) $2\times(-3)^2-3\times 4$

(e) $-3\times(-2)\times(2-5)^2$ (f) $(-2)^2-(-2)\times 3+2\times 3^2$

(g) $(-4)^2\div(-8)+3\times(-2)^3$ (h) $4\times 3^2\div(-6)-(-1)^3\times(-3)^2$

(i) $-2\times(-2)^3\times(-2)\times 3+(-2)\times 3\times(-1)^2$ (j) $5-\{12\times[(-5)^2-7]\div 3\}$

6. Evaluate $\dfrac{-18-\sqrt[3]{-3375}-(-6)^2}{\sqrt{4}+9}$.

7. Evaluate each of the following.

(a) $3\frac{4}{7}+1\frac{2}{5}-\left(-\frac{3}{7}\right)$ (b) $\frac{2}{3}-\left(-3\frac{3}{20}\right)+\left(-\frac{2}{5}\right)$

(c) $-6\frac{4}{9}-3\frac{3}{4}-3\frac{5}{9}$ (d) $\left(-\frac{1}{2}+\frac{1}{3}\right)+\left[\frac{1}{4}+\left(-\frac{1}{3}\right)\right]+\left(-\frac{1}{20}\right)$

(e) $-3\frac{1}{4}\times 1\frac{3}{5}\times\left(-1\frac{2}{13}\right)$ (f) $\frac{3}{5}\times\left(-\frac{1}{4}-\frac{1}{6}\right)\div\left(-2\frac{1}{3}+1\frac{1}{4}\right)$

(g) $-3\frac{9}{16}\div 1\frac{3}{16}-\frac{1}{3}\times\left(-1\frac{3}{4}\right)$ (h) $-12\frac{1}{2}+1\frac{2}{3}\div(-4)-\frac{5}{7}\times\left(-2\frac{4}{5}\right)$

8. Use a calculator to evaluate $\dfrac{\left(-\frac{4}{7}\right)^2-\left(-\frac{2}{5}\right)^3}{-\sqrt{\frac{64}{625}}\div\sqrt[3]{-\frac{8}{125}}}$.

9. Find the value of each of the following.

(a) $-12.8-88.2$

(b) $500.3-(-200.2)-210.1$

(c) $1.44\div 1.2\times(-0.4)$

(d) $(-0.3)^2\div(-0.2)+(-2.56)$

Challenge Yourself

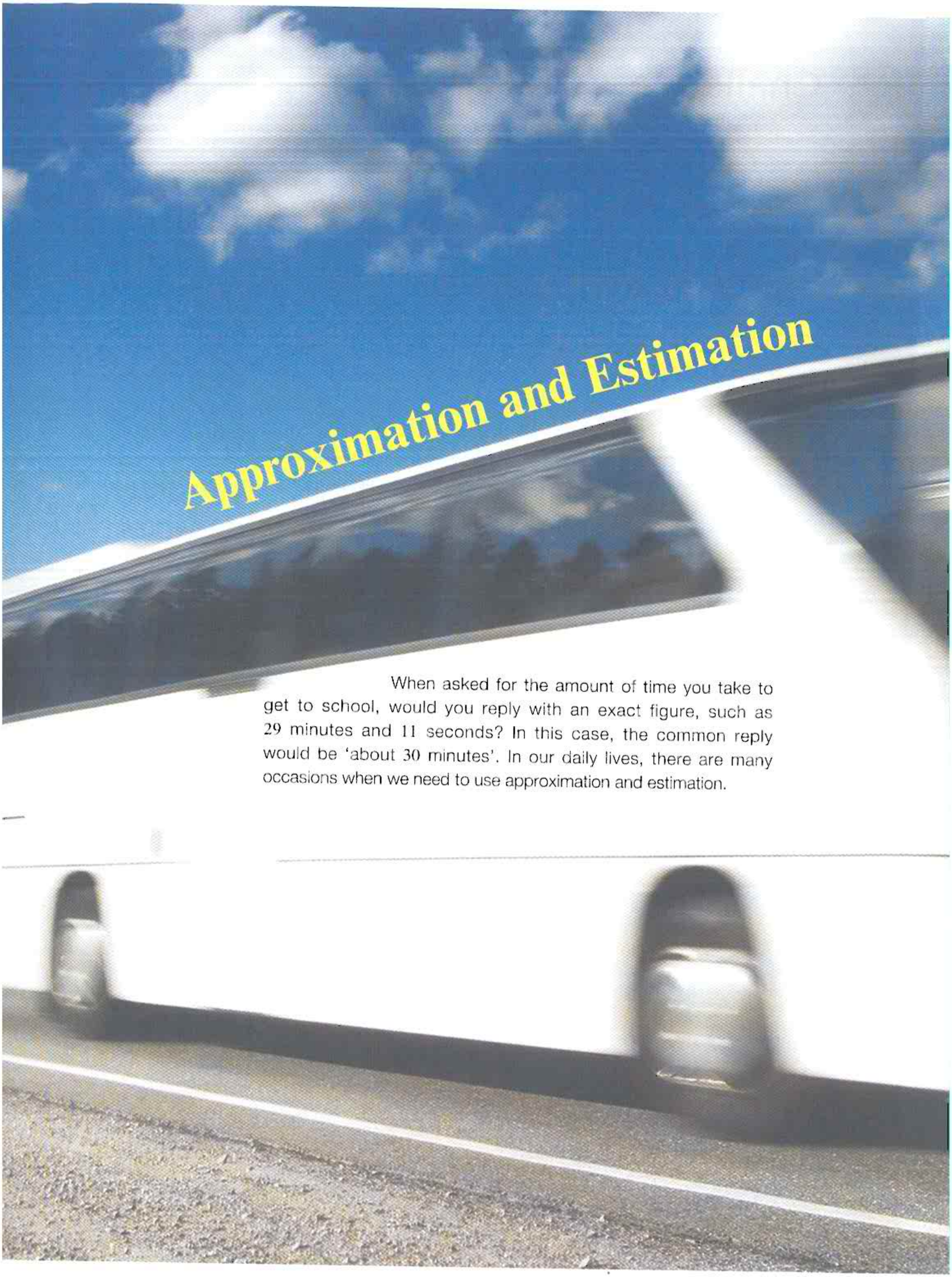
1. Given that $\sqrt{x-3}+(y+2)^2=0$, find the value of x and of y.

2. Complete each of the following.

   (a)
$$\begin{array}{r} \square\,2\,\square \\ \times \quad \square\,7 \\ \hline 2\,2\,\square\,8 \\ \square\,6\,\square\,0 \\ \hline 1\,\square\,4\,6\,\square \\ \hline \end{array}$$

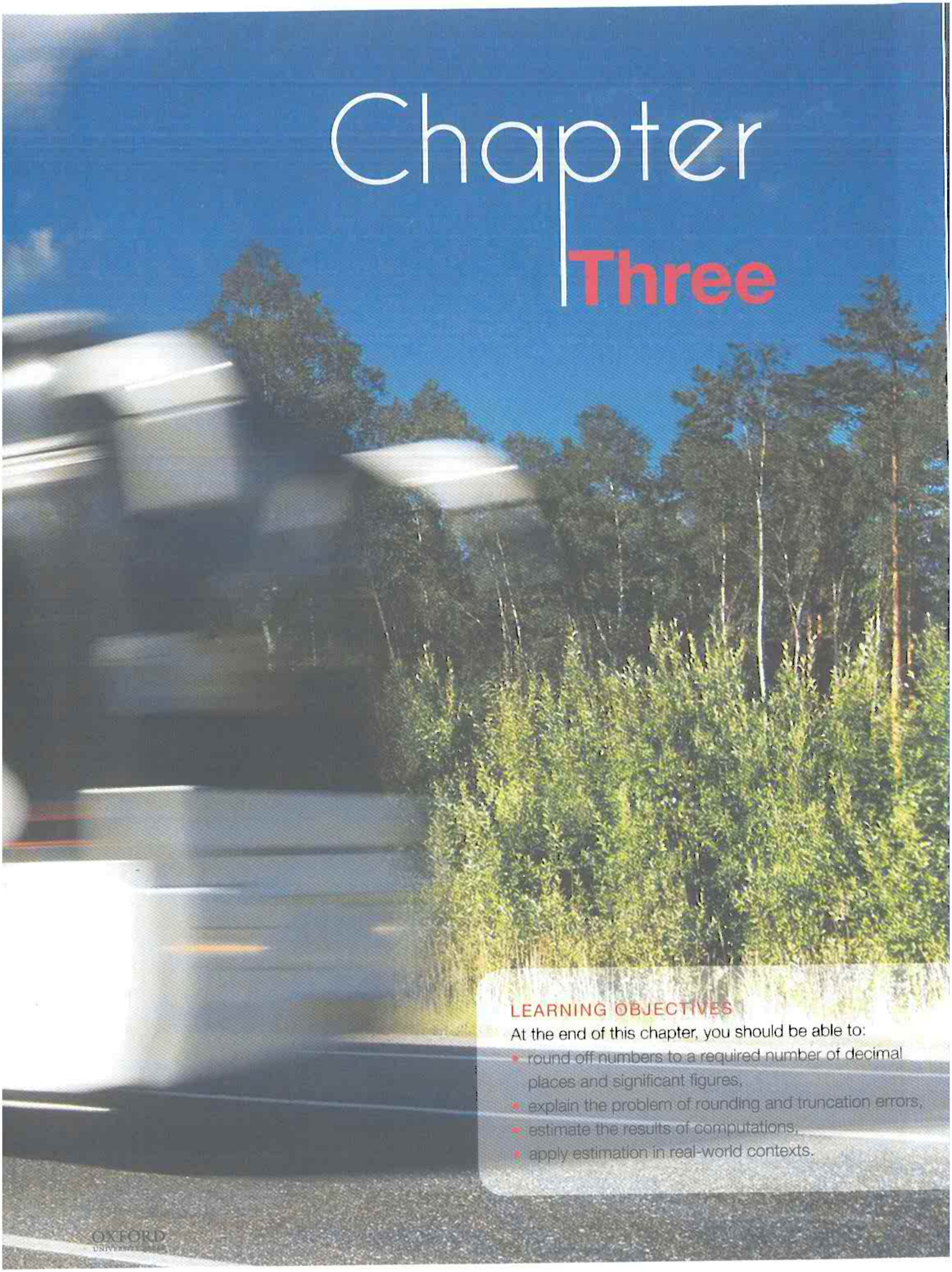
   (b)
$$\begin{array}{r} \square\,\square \\ 28\overline{\smash{)}1\,\square\,\square\,4} \\ 1\,\square\,\square \quad \\ \hline \square\,\square\,4 \\ \square\,\square\,4 \\ \hline 0 \\ \hline \end{array}$$

3. Insert +, −, ×, ÷ or brackets to make each of the following mathematical statements true. The first one has been done for you.
   (a) $(3+3)\div 3-3\div 3=1$
   (b) 3 3 3 3 3 = 2
   (c) 3 3 3 3 3 = 3
   (d) 3 3 3 3 3 = 4
   (e) 3 3 3 3 3 = 5
   (f) 3 3 3 3 3 = 6



Approximation and Estimation

When asked for the amount of time you take to get to school, would you reply with an exact figure, such as 29 minutes and 11 seconds? In this case, the common reply would be 'about 30 minutes'. In our daily lives, there are many occasions when we need to use approximation and estimation.



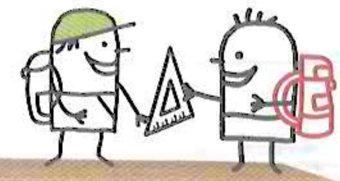
Chapter Three

LEARNING OBJECTIVES

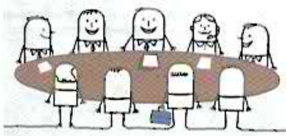
At the end of this chapter, you should be able to:

- round off numbers to a required number of decimal places and significant figures,
- explain the problem of rounding and truncation errors,
- estimate the results of computations,
- apply estimation in real-world contexts.

3.1 Approximation



The chapter opener shows an example when we use an approximate value because the actual value is not necessary. There are other reasons why we need to use approximation and estimation in our daily lives.



Class Discussion

Actual and Approximated Values

Read the article and answer the questions.

NEWS

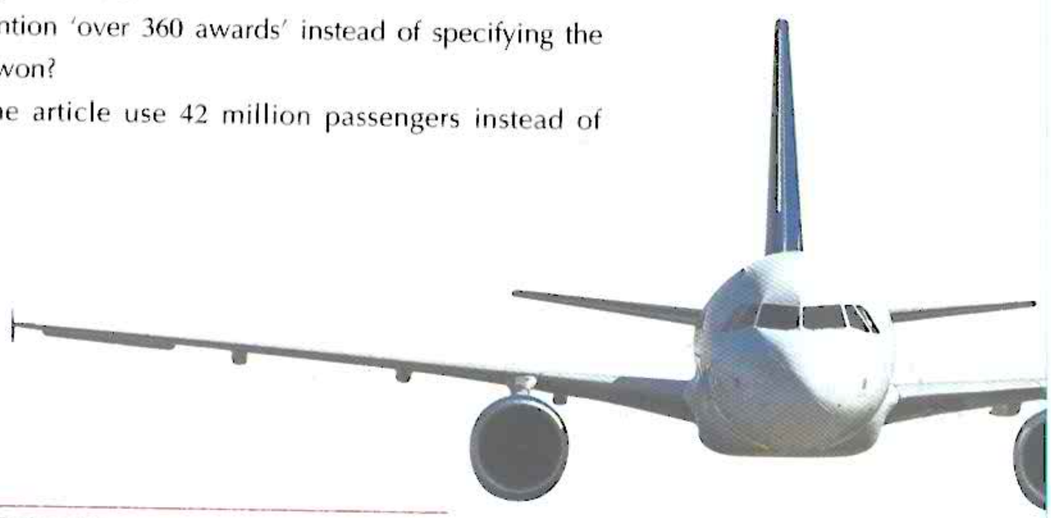
Changi Airport Handles Record 42 Million Passengers

SINGAPORE: Changi Airport handles a record of 42 038 777 passengers in 2010, a 13.0% increase over 2009. This makes the airport the 18th busiest airport in the world by passenger traffic in 2010. The airport has won over 360 awards, with 24 awards won in 2010 alone.

Changi Airport first opened on 1 July 1981 with one terminal. Currently, it has four terminals, including one Budget Terminal, which brings its total annual handling capacity to 73 million passengers.

1. Identify the numbers in the article which are actual values and those which are approximations. How do you decide?
2. **(a)** Why does the article mention 'over 360 awards' instead of specifying the actual number of awards won?

   (b) Why does the title of the article use 42 million passengers instead of 42 038 777 passengers?



Recap (Rounding Off)

We have learnt in primary school how to round off numbers to the nearest 10, 100 and 1000.

Worked Example 1

(Rounding Off)

Round off each of the following numbers to the nearest 10.

(a) 275 **(b)** 273.1

Solution:

(a) 2 <u>7</u> 5 = 2 <u>8</u> 0 (to the nearest 10)

Step 1: Underline the digit in the *tens* place.

Step 2: Circle the next digit on its right. If it is *5 or more*, add 1 to the digit in the tens place.

Step 3: Put a zero in the ones place as a place holder.

ATTENTION

275 is exactly midway between 270 and 280. By convention, it is *rounded up* to 280. If Step 3 is omitted, 275 = 28 (to the nearest 10), which is wrong! Since the degree of accuracy is specified, we use the equal sign: 275 = 280 (to the nearest 10). If the degree of accuracy is not important, we will use the approximation sign: $275 \approx 280$.

(b) 2 <u>7</u> 3 · 1 = 2 <u>7</u> 0 (to the nearest 10)

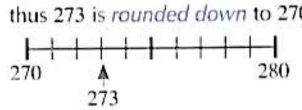
Step 1: Underline the digit in the ______ place.

Step 2: Circle the next digit on its right. If it is *less than 5*, the digit in the ______ place remains the same.

Step 3: Put a ______ in the ______ place as a place holder. Omit all the digits after the decimal point.

ATTENTION

273 is nearer to 270 than to 280, thus 273 is *rounded down* to 270.



PRACTISE NOW 1

SIMILAR QUESTIONS

Exercise 3A Questions 1(a)–(c), 5–6

1. Round off 3 409 725 to the nearest
   (a) 10, **(b)** 100, **(c)** 1000, **(d)** 10 000.

2. In 2010, Singapore welcomed 11 600 000 overseas visitors. This value has been rounded off to the nearest 100 000. What are the largest and smallest possible numbers of overseas visitors?

Worked Example 2

(Rounding off)

Correct 96.482 to
(a) 1 decimal place,
(b) the nearest whole number.

INFORMATION

'Correct 96.482 to 1 decimal place' is the same as 'round off 96.482 to 1 decimal place'.

Solution:

(a) 9 6 · <u>4</u> 8 2 = 9 6 · 5 (to 1 d.p.)

Step 1: Underline the digit in the *tenths* place.

Step 2: Circle the next digit on its right. If it is *5 or more*, add 1 to the digit in the tenths place.

Step 3: Do not add any zero after the first decimal point.

RECALL

9 6 . 4 8 2

tenth (1[st] d.p.) hundredth (2[nd] d.p.) thousandth (3[rd] d.p.)

(b) 9 <u>6</u> · 4 8 2 = 9 6 (to the nearest whole number)

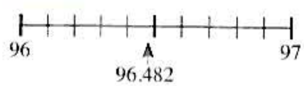
Step 1: Underline the digit in the *ones* place.

Step 2: Circle the next digit on its right. If it is *less than 5*, the digit in the ________ place remains the same.

Step 3: Do not add any zero after the ones place.

ATTENTION

For **(b)**, you cannot use the answer 96.5 in **(a)** to round off to 97 because 96.482 is nearer to 96 than to 97. *You must not round off twice!*



PRACTISE NOW 2

SIMILAR QUESTIONS

Exercise 3A Questions 2(a)–(c), 4(a)–(d), 7

1. Correct 78.4695 to
(a) 1 decimal place, **(b)** the nearest whole number,
(c) the nearest hundredth, **(d)** the nearest 0.001.

2. Jun Wei says that 8.395 is equal to 8.4 when rounded off to 2 decimal places because he thinks that 8.40 is the same as 8.4. Do you agree? Explain your answer.

Worked Example 3

(Application of Rounding Off)

A truck carries 150 ceramic tiles and 225 marble tiles. The mass of a ceramic tile is 1.87 kg and the mass of a marble tile is 4.63 kg. Calculate the total mass of the tiles carried by the truck, giving your answer correct to the nearest 0.1 kg.

Solution:

Mass of 150 ceramic tiles = 150 × 1.87
= 280.5 kg

Mass of 225 marble tiles = 225 × 4.63
= 1041.75 kg

Total mass of the tiles carried by the truck = 280.5 + 1041.75
= 1322.25 kg
= 1322.3 kg (to the nearest 0.1 kg)

PRACTISE NOW 3

SIMILAR QUESTIONS

Exercise 3A Question 3

A household uses 450 kWh of electricity and 38 m^3 of water in a month. The cost of electricity is \$0.29 per kWh and the cost of water usage is \$1.17 per m^3. Find the total amount of money the household has to pay in that month, giving your answer correct to the nearest dollar.

Exercise 3A

BASIC LEVEL

1. Round off 698 352 to the nearest
   (a) 100, **(b)** 1000, **(c)** 10 000.

2. Correct 45.7395 to
   (a) 1 decimal place,
   (b) the nearest whole number,
   (c) 3 decimal places.

INTERMEDIATE LEVEL

3. The dimensions of a rectangular plot of land are 28.3 m by 53.7 m. Find
   (i) the perimeter of the land, correct to the nearest 10 m,
   (ii) the area of grass needed to fill up the entire plot of land, correct to the nearest 100 m^2.

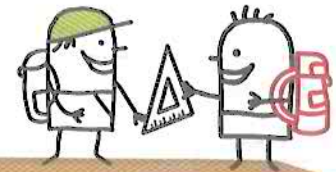
4. Round off
   (a) 4.918 m to the nearest 0.1 m,
   (b) 9.71 cm to the nearest cm,
   (c) \$10.982 to the nearest ten cents,
   (d) 6.489 kg to the nearest $\frac{1}{100}$ kg.

ADVANCED LEVEL

5. Kate says that 5192.3 is equal to 519 when rounded off to the nearest 10. She drops the '2' because it is less than 5. Do you agree with her? Explain your answer.

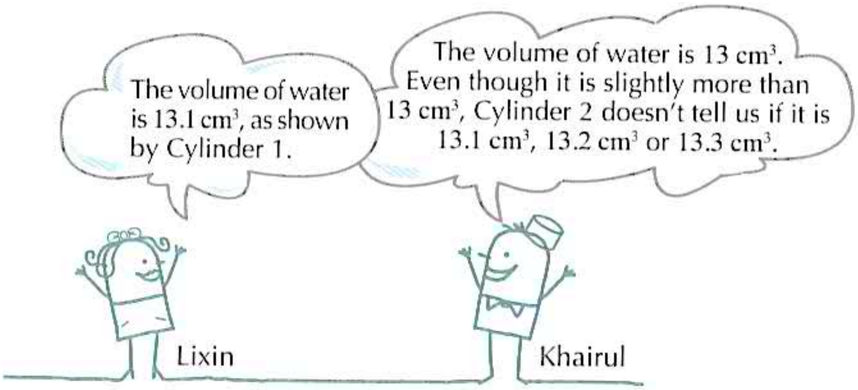
6. Singapore's population was 5 077 000 in 2010. This value has been rounded to the nearest 1000. What are the largest and smallest possible values of Singapore's population in 2010?

7. Farhan says that 26.97 is equal to 27 when rounded off to 1 decimal place because he thinks that 27.0 is the same as 27. Do you agree with him? Explain your answer.



3.2 Significant Figures

Lixin and Khairul are asked to measure the volume of water in a cup using two different measuring cylinders. The readings are shown in Fig. 3.1.



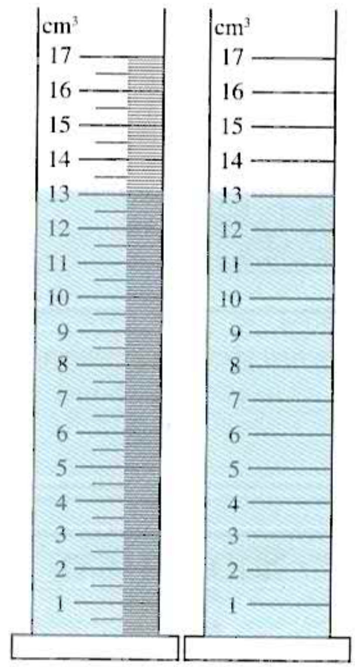


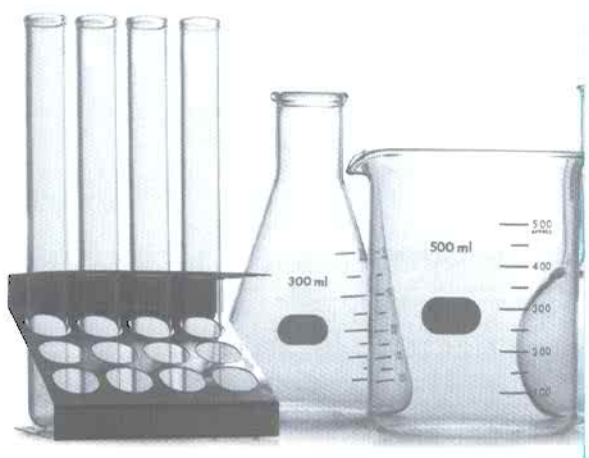
Fig. 3.1

Lixin and Khairul have different answers: 13.1 cm³ and 13 cm³. Which is more *accurate*?

The answer 13.1 cm³ has *3* significant figures, while the answer 13 cm³ has *2* significant figures.

Both answers are approximated to a different degree of accuracy as the values obtained are limited by the accuracy of the measuring instruments. Cylinder 2 can measure up to 1 cm³ while Cylinder 1 can measure up to ______ cm³. Hence, Cylinder 1 provides readings that are more accurate than Cylinder 2.

> **Significant figures** are used to reflect the degree of **accuracy**. A number is more accurate when it is given to a *greater* number of significant figures.



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Five Rules to Identify Digits Which are Significant

Rule 1: All **non-zero** digits are significant.

For example, the number 7258 has 4 significant figures.

PRACTISE NOW

State the number of significant figures in each of the following.

(a) 192 **(b)** 83.76 **(c)** 3 **(d)** 4.5

Rule 2: All **zeros between non-zero digits** are significant.

For example, the number 32.047 has 5 significant figures.

PRACTISE NOW

State the number of significant figures in each of the following.

(a) 506 **(b)** 1.099 **(c)** 3.0021 **(d)** 70.8001

Ethan measures the length of a piece of wood using two different rulers. The lengths are 0.6 m and 0.60 m respectively as shown in Fig. 3.2.

(a) Measured length = 0.6 m **(b)** Measured length = 0.60 m

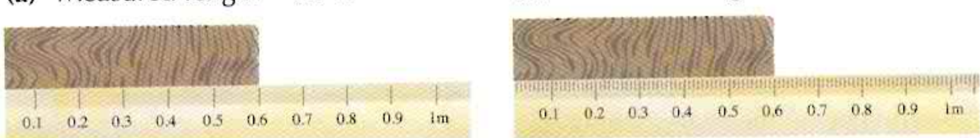


Fig. 3.2

Which measurement is more accurate? Why?

Based on the intervals in both rulers, 0.60 m is more accurate. This is because 0.60 m is measured to 2 significant figures, while 0.6 m is measured to ______ significant figure. Notice that the zero after the digit 6 in 0.60 m is significant.

Rule 3: In a decimal, all **zeros after a non-zero digit** are significant.

PRACTISE NOW

1. State the number of significant figures in each of the following.
   (a) 0.10 **(b)** 0.500 **(c)** 41.0320 **(d)** 6.090
2. A line segment is measured using two different instruments and its length is found to be 4.1 cm and 4.10 cm respectively. Which is more accurate and why?

Raj measures the length of a piece of string and finds it to be 5.7 cm long, correct to 2 significant figures. He then converts it into metres and obtains 0.057 m. How many significant figures does 0.057 m have?

It is not possible to make a measurement more accurate by changing it to a different unit.

Since 0.057 m still has 2 significant figures, this means that the zeros before the number 5 are not significant.

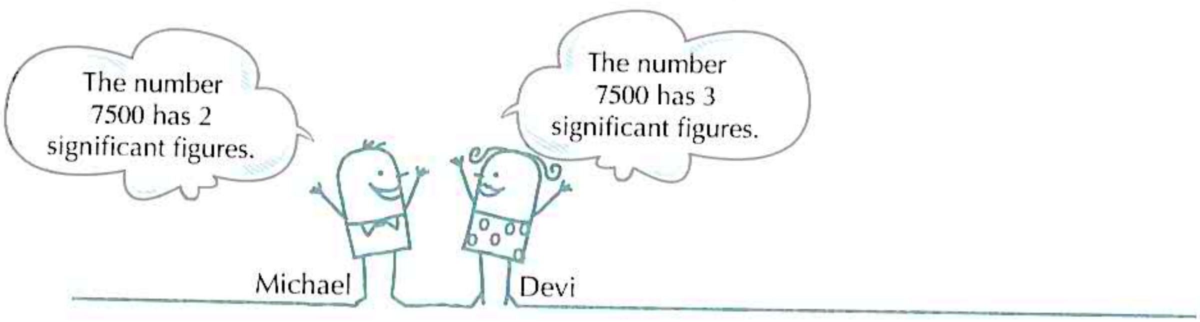
Rule 4: In a decimal, all **zeros before a non-zero digit** are *not* significant.

PRACTISE NOW

State the number of significant figures in each of the following.

(a) 0.021 **(b)** 0.603 **(c)** 0.001 73 **(d)** 0.1090

How many significant figures does 7500 have?



The teacher says that both Michael and Devi are correct. Why is this so?

To answer this question, let us consider the number 7498.
Correct this number to the nearest hundred: ________
Correct this number to the nearest ten: ________
Are both the answers equal?

7500 has ________ significant figures if it is correct to the nearest hundred.
7500 has ________ significant figures if it is correct to the nearest ten.

Rule 5: In whole numbers, the zeros at the end *may or may not* be significant. It depends on how the numbers are approximated.

PRACTISE NOW

SIMILAR QUESTIONS

Exercise 3B Questions 1(a)–(c), 3

State the number of significant figures in each of the following.

(a) 3800 m (to the nearest 10 m)
(b) 25 000 km (to the nearest km)
(c) 100 000 g (to the nearest 10 000 g)

Rounding Off to a Given Number of Significant Figures

Worked Example 4

(Rounding Off to a Required Number of Significant Figures)

Round off each of the following to the number of significant figures as given in brackets.

(a) 8982 (2 s.f.) **(b)** 0.006 019 5 (4 s.f.) **(c)** 0.9999 (3 s.f.)

Solution:

(a) 8 9 (8) 2 = 9 0 0 0 (to 2 s.f.)

Step 1: Determine the s.f. starting from the left (2 s.f.). (Rule 1)

Step 2: Circle the next digit on its right. If it is *5 or more*, add 1 to the previous digit.

Step 3: Put a zero to indicate the 2nd s.f. (Rule 5: This zero is significant.)

Step 4: Put two zeros as place holders. (Rule 5: These two zeros are not significant.)

(b) $\underbrace{0\ .\ 0\ 0}_{\text{Not significant (Rule 4)}}$ 6 0 1 9 (5) = 0.006 020 (to 4 s.f.)

Step 1: Determine the s.f. starting from the left (4 s.f.). (Rules 2 and 4)

Step 2: Circle the next digit on its right. If it is *5 or more*, add 1 to the previous digit.

Step 3: Put a zero to indicate the 4th s.f. (Rule 3)

(c) 0 . 9 9 9 (9) = 1 . 0 0 (to 3 s.f.)

Not significant (Rule 4)

Step 1: Determine the s.f. starting from the left (3 s.f.). (Rule 1)

Step 2: Circle the next digit on its right. If it is *5 or more*, add 1 to the previous digit.

Step 3: Put two zeros to indicate the 2nd and 3rd s.f. (Rule 3). This is a special case where you cannot put a zero at the thousandth place which was previously occupied by the 3rd s.f., or else 1.000 will have 4 s.f.

PRACTISE NOW 4

SIMILAR QUESTIONS

Exercise 3B Questions 2(a)–(f), 4(a)–(d), 8–9

1. Round off each of the following to the number of significant figures as given in brackets.
   (a) 3748 (3 s.f.) **(b)** 0.004 709 89 (4 s.f.)
   (c) 4971 (2 s.f.) **(d)** 0.099 99 (2 s.f. and 3 s.f.)

2. The number 67 0X1 is equal to 67 100, correct to 3 significant figures. Find the value of X if 67 0X1 is a perfect square.

Different Types of Rounding in Real Life

In mathematics, we usually round off a number using the rules in the previous sections. However, in real life, this may not be so.

Investigation

Rounding in Real Life

Suppose 215 students and 5 teachers are going for an excursion by bus and each bus can only carry 30 passengers. How many buses are required? Why?

Suppose you are the designer of a lift that can carry a maximum mass of 897 kg. However, you are told to put the maximum mass correct to the nearest 100 kg. What should you put the maximum mass as? Why?

Sometimes in real life, you have to round off to the nearest 5 cents, but not to the nearest cent. Describe a situation when you have to round off to the nearest 5 cents.

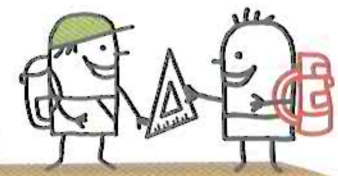
Journal Writing

Describe another two situations where you cannot just round off a number using the rules in the previous sections.

SIMILAR QUESTIONS

Exercise 3B Question 5

3.3 Rounding And Truncation Errors

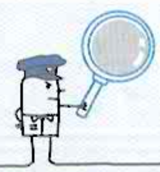


In this section, we will study how approximations can result in rounding and truncation errors.

Non-Exact Answers and Intermediate Steps

If we write all the digits for non-exact answers, e.g. $\sqrt{131} = 11.445\ 523\ 14...$, the working will look messy. In order for the final answer to be accurate to, say, three significant figures, any intermediate working must be correct to *four significant figures*. Can you explain why?

If the intermediate working is correct to three significant figures only, a **rounding error** may occur.



Investigation

The Missing 0.1% Votes

Vishal, Rui Feng and Huixian were nominated to be the President of the Mathematics Society. Table 3.1 shows the votes that they received during the election.

Candidate	Number of Votes	Percentage of Votes
Vishal	187	62.3%
Rui Feng	52	17.3%
Huixian	61	20.3%
Total	**300**	**99.9%**

Table 3.1

INFORMATION

In practice, we will just put 100% for the total percentage of votes in order not to cause confusion.

1. The total percentage of votes is only 99.9%. What has happened to the missing 0.1% of the votes? Explain your answer.
2. Given that there is a new member who voted for Vishal, calculate the percentage of votes each candidate receives from the 301 votes in a similar way. Why is the total percentage of votes more than 100%?

In the investigation, as all the intermediate values, i.e. 62.3%, 17.3% and 20.3%, are correct to 3 significant figures, writing the final answer as 99.9% (to 3 s.f.) is a **follow-through error** due to rounding errors in the intermediate steps. If the final answer is correct to 2 significant figures, we will obtain 100%. Hence, the final answer can only be accurate to 2 significant figures.

If we want the final answer to be correct to 3 significant figures, the values in the intermediate steps should be given to 4 significant figures. Thus we will have:

$$\begin{aligned} 62.33\% + 17.33\% + 20.33\% &= 99.99\% \\ &= 100\% \text{ (to 3 s.f.)} \end{aligned}$$

Thinking Time

1. The population of City A is approximately 2.5 million. Can the exact population size be
   (i) 2.47 million, **(ii)** 2.6 million?

2. The population of City B is equal to 2.5 million (to 2 s.f.). Can the exact population size be
   (i) 2.47 million, **(ii)** 2.6 million?

From the thinking time, we arrive at the conclusion that there is a difference between 'approximately 2.5 million' and 'equal to 2.5 million (to 2 s.f.)'.

Worked Example 5

(Significant Figures in Intermediate Steps)

The area of a square is 131 cm². Find
(i) the length, **(ii)** the perimeter,
of the square.

Solution:

(i) Length of square $= \sqrt{131}$ cm
$= 11.4$ cm (to 3 s.f.)

Do not write $\sqrt{131}$ cm
$= 11.45$ cm
$= 11.4$ cm (to 3 s.f.).

Firstly, $\sqrt{131} \neq 11.45$. Secondly, if $\sqrt{131} \approx 11.45$, it does not mean that it is accurate to 4 s.f. Thirdly, it is confusing to write $11.45 = 11.4$ (to 3 s.f.).

If we write $\sqrt{131}$ cm
$= 11.446$ cm (to 5 s.f.)
$= 11.4$ cm (to 3 s.f),
it defeats the purpose of leaving non-exact answers to 3 s.f. so as not to be too messy.

(ii) Perimeter of square $= 11.45 \text{ cm} \times 4$
$= 45.8$ cm (to 3 s.f.)

We must use *4 s.f.* for the intermediate working. If we use $11.4 \text{ cm} \times 4$, the answer 45.6 cm is only accurate to 2 s.f. because the third s.f. 6 is wrong. Hence, a rounding error will occur. To be more accurate, we can use the calculator value of $\sqrt{131}$ to find the perimeter.

PRACTISE NOW 5

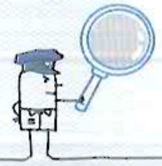
The area of a square is 105 m². Find
(i) the length, **(ii)** the perimeter,
of the square.

SIMILAR QUESTIONS

Exercise 3B Questions 6–7

Rounding and Truncation Errors in Calculators

To truncate means to 'cut off the end'. For example, $\sqrt{162} = 12.727\ 922\ 06\ldots$ If we round off the answer to 3 decimal places, it will be 12.728. However, if we **truncate** the answer at 3 decimal places, it will be 12.727. There is no rounding off.



Investigation

Rounding and Truncation Errors in Calculators

This activity may vary with different calculator models.

Suppose your calculator displays only 10 digits, e.g. if you key in $\sqrt{162}$, the display shows 12.727 922 06. However, your calculator actually stores more than 10 digits.

(a) Do the following to find out how many digits your calculator stores.

1. Subtract 12 from 12.727 922 06. Did you get 0.727 922 06 or 0.727 922 061?

2. Multiply 0.727 922 061 by 10. Did you get 7.279 220 613 or 7.279 220 614 (this depends on your calculator model)?

3. Subtract 7 from the result in Step 2 and multiply by 10 again. Did you get 2.792 206 13 or 2.792 206 14?

Therefore, your 10-digit calculator actually stores __________ digits.

Now, $\sqrt{162} = 12.727\,922\,061\,357\,855\ldots$

If you get 7.279 220 613 in Step 2, this means that your calculator **truncates** the value of $\sqrt{162}$ at the __________th digit to give 12.727 922 061 3, instead of rounding $\sqrt{162}$ to 12.727 922 061 4.

(b) Rounding or truncation errors can result in ridiculous results. Try the following:

4. Take the square root of 7 four times, i.e. $\sqrt{\sqrt{\sqrt{\sqrt{7}}}}$.

5. Take the square of the result in Step 4 four times. Do you get 7 or 6.999 999 999? (If you get 7, you will have to take the square root of 7 more than four times in Step 4 for your calculator model.)

SIMILAR QUESTIONS

Exercise 3B Question 10

Exercise 3B

BASIC LEVEL

1. State the number of significant figures in each of the following.

(a) 39 018 **(b)** 0.028 030

(c) 2900 (to the nearest 10)

2. Round off each of the following to the number of significant figures as given in brackets.

(a) 728 (2 s.f.) **(b)** 503.88 (4 s.f.)

(c) 0.003 018 5 (4 s.f.) **(d)** 6396 (2 s.f. and 3 s.f.)

(e) 9.9999 (3 s.f.) **(f)** 8.076 (3 s.f.)

INTERMEDIATE LEVEL

3. The number 143 200 is correct to x significant figures. Write down the possible values of x.

4. Evaluate each of the following and correct your answers to the number of significant figures as given in brackets.

(a) $\frac{1}{99}$ (4 s.f.) **(b)** 871×234 (2 s.f.)

(c) $\frac{21^2}{0.219}$ (5 s.f.) **(d)** $\frac{3.91^3 - 2.1}{6.41}$ (2 s.f.)

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5. What is the greatest number of sweets that can be bought with \$2 if each sweet costs 30 cents?

6. The area of a square is 264 cm^2. Find
   (i) the length, **(ii)** the perimeter,
   of the square.

7. The circumference of a circle is 136 m. Find
   (i) the radius, **(ii)** the area,
   of the circle.

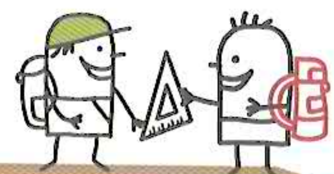
ADVANCED LEVEL

8. The number 21 X09 is equal to 22 000, correct to 2 significant figures. Find the value of X if 21 X09 is a perfect square.

9. The number of people at a concert is stated as 21 200, correct to 3 significant figures. What is the largest and the smallest possible number of people at the concert?

10. **(i)** Without using a calculator, evaluate
    $987\,654\,321 + 0.000\,007 - 987\,654\,321$.
    (ii) Use a calculator to evaluate
    $987\,654\,321 + 0.000\,007 - 987\,654\,321$.
    (iii) Do you get the same answer for **(i)** and **(ii)**? Explain your answer.

3.4 Estimation



There are situations in our daily lives when we do not know the actual value and we have to **estimate** the value. In such cases, it is *not possible* or *not worth the trouble* to obtain the actual value as the actual value is not necessary. For example, the projected world population in 2050 is estimated to be between 7.5 and 10.5 billion.

Estimation is a special case of approximation when *we do not know the actual value*. In the previous sections, we know the actual value but we *approximate* it for various reasons, and the approximated value should be very close to the actual value, e.g. $\sqrt{131} \approx 11.4$.

In this section, we will examine some situations in real-world contexts which require estimation and discuss some methods for estimating to a *sufficient degree of accuracy*.

Estimation of Computations

Sometimes, we may key in the wrong value when using a calculator. There is a need to be *aware* when an answer is *obviously wrong*. Thus we use estimation to check the *reasonableness* of an answer obtained from a calculator.

Worked Example 6

(Using Estimation to Determine Reasonableness of an Answer)

Shirley evaluates $31.5 + 9.87 - 2.1$ using a calculator and she says that the answer is 392.7. Without doing the actual calculation, use estimation to determine whether Shirley's answer is reasonable. Then use a calculator to evaluate $31.5 + 9.87 - 2.1$. Is your estimated value close to the actual value?

Solution:

$$31.5 + 9.87 - 2.1 \approx 32 + 10 - 2$$
$$= 40$$

$\therefore$ Shirley's answer is wrong.

Using a calculator, the actual answer is 39.27.

Hence, the estimated value 40 is close to the actual value 39.27.

ATTENTION

We use the approximation sign $\approx$ when it is not important to specify the degree of accuracy, e.g. $31.5 \approx 32$.

PRACTISE NOW 6

SIMILAR QUESTIONS

Exercise 3C Questions 1, 2(a)–(b), 3–4, 6

1. Nora evaluates 798×195 using a calculator and she says that the answer is 15 561. Without doing the actual calculation, use estimation to determine whether Nora's answer is reasonable.

2. Estimate each of the following without using a calculator. Then use a calculator to evaluate each of the following. Are your estimated values close to the actual values?

   (a) $5712 \div 297$ **(b)** $\sqrt{63} \times \sqrt[3]{129}$

3. The driving distance between Singapore and Malacca is about 250 km. Suppose your father drives at an average speed of 80 km/h and he asks you how long he will take to drive from Singapore to Malacca. Write down a calculation you could do mentally to estimate the number of hours.

INFORMATION

There is another method for Question 1: look at the last digit.

Shopping with a Different Currency

Suppose you are shopping in Thailand and the shopkeeper quotes you a price for a wallet in Thai baht. To know whether the wallet is expensive, you will need to convert to Singapore dollars (S$). How are you going to estimate the price of the wallet in Singapore dollars?

Worked Example 7

(Estimation in Currency Conversion)

A wallet costs 225 Thai baht. The conversion rate is 1 baht = S$0.041 212. Without using a calculator, estimate the price of the wallet in S$.

Problem Solving Tip

1 baht = S$0.041 212 is too small and not easy to remember. Moreover, it is difficult to use this number to estimate the price of the wallet. Thus we need to remember an approximate value for 100 baht, i.e. 100 baht ≈ S$4.

Solution:

100 baht ≈ S$4, so 200 baht ≈ S$8, 50 baht ≈ $2 and 25 baht ≈ $1.

∴ The price of the wallet is 225 baht ≈ $9.

PRACTISE NOW 7

A pair of earrings costs 25 000 Indonesian rupiah (Rp). The conversion rate is Rp 1000 = S$0.145 598. Without using a calculator, estimate the price of the pair of earrings in S$.

SIMILAR QUESTIONS

Exercise 3C Questions 7, 10

Value for Money

Worked Example 8

(Better Value for Money)

Suppose you are in a supermarket with your neighbour and she sees the following options for the same brand of coffee powder.

Option A | Option B

200 g Coffee Powder
$5.80

200 g + 50 g Coffee Powder
$7.45

She cannot decide which option is *better value for money*. Without using a calculator, how do you help her decide which option she should choose?

Solution:

For option A, 200 g costs about \$6.

Thus 100 g will cost about \$3, and 50 g will cost about \$1.50.

$\therefore$ For option A, 250 g will cost about \$6 + \$1.50 = \$7.50.

For option B, 250 g costs \$7.45 which is 5¢ cheaper than option A.

Does it appear that option B is better value for money?

However, for option A, 200 g actually costs \$5.80 which is 20¢ less than \$6.

Thus for option A, 250 g will cost at least 20¢ less than the estimated \$7.50.

$\therefore$ Option A is better value for money.

INFORMATION

There are other ways to estimate. For example:
For option A, 200 g costs \$5.80, which is about \$6.
Thus 100 g will cost about \$3, and 50 g will cost about \$1.50.
$\therefore$ For option A, 250 g will cost about \$5.80 + \$1.50 = \$7.30.
Hence, option A is better value for money.

PRACTISE NOW 8

Without using a calculator, decide which of the options is better value for money.

Option A | Option B

300 ml Olive Oil
$8.80

300 ml + 50 ml Olive Oil
$10.40

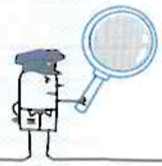
SIMILAR QUESTIONS

Exercise 3C Questions 8–9

Estimation of a Larger Quantity using a Smaller Quantity

The Greek mathematician, Archimedes, estimated that 8×10^{63} grains of sand were required to fill the universe. Did he make a wild guess or did he count the grains one by one?

Archimedes made use of an important **estimation strategy**: use a smaller quantity to estimate a larger quantity. First, he counted the number of grains of sand required to fill a spoon. Next, he estimated the number of spoons of sand required to fill a room, the number of rooms of sand required to fill a stadium, etc.



Investigation

Use of a Smaller Quantity to Estimate a Larger Quantity

In a competition, the public is invited to guess the amount of money in a tank that contains 10¢ coins only. How do we estimate the number of 10¢ coins in the tank?

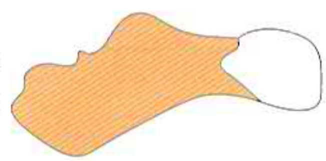
Even if we are able to obtain an identical tank, it is too troublesome to get so many 10¢ coins. One useful strategy is to fill a smaller box with 10¢ coins. As the number of 10¢ coins that can fill the smaller box may vary, we can take the average of three trials.

Suppose we are able to measure the tank in the competition and find that the dimensions are 50 cm by 23 cm by 13 cm. Now, estimate the amount of money in the tank.

Worked Example 9

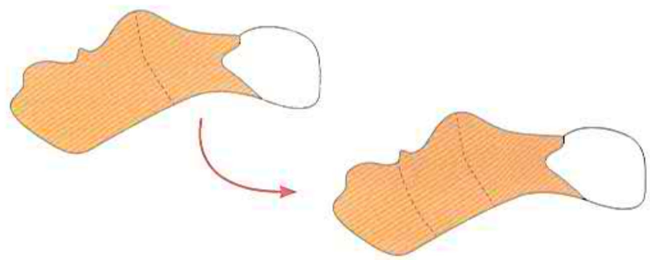
(Estimation of Area)

Estimate the ratio of the area of the shaded region to that of the unshaded region in the figure on the right.



Solution:

To estimate the area, we can divide the shaded region (using dotted lines) into areas approximately equal to the area of the unshaded region. Start from the right side to obtain a more accurate estimate.



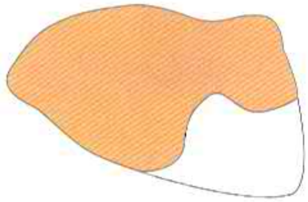
∴ The ratio of the area of the shaded region to that of the unshaded region is estimated to be 3 : 1.

PRACTISE NOW 9

SIMILAR QUESTIONS

Exercise 3C Question 5

Estimate the percentage of the shaded region in the figure on the right.



Performance Task

Estimation in Our Daily Lives

Select *one* task from the list and write a detailed report on how you obtain your estimation.

1. Estimate the total number of hours your classmates spent surfing the Internet in a month.
2. Estimate the amount of pizza needed to feed all the students in your school during an excursion.
3. Estimate the amount of money collected by the drinks stall in your school canteen on a weekday.

Exercise 3C

BASIC LEVEL

1. Priya evaluates $218 \div 31$ using a calculator and she says that the answer is 70.3. Without doing the actual calculation, use estimation to determine whether Priya's answer is reasonable. Then use a calculator to evaluate $218 \div 31$. Is your estimated value close to the actual value?

2. Estimate each of the following without using a calculator. Then use a calculator to evaluate each of the following. Are your estimated values close to the actual values?

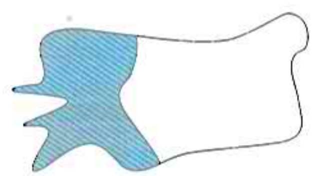
   (a) 2013×39 **(b)** $\sqrt{145.6} \div \sqrt[3]{65.4}$

3. **(i)** Express 3.612 and 29.87 correct to 2 significant figures.

   (ii) Use your answers in part **(i)** to estimate the value of $3.612 \div 29.87$.

4. A car travels 274 km. It travels an average of 9.1 km on a litre of petrol. Write down a calculation you could do mentally to estimate the number of litres used.

5. Estimate the ratio of the area of the shaded region to that of the unshaded region in the figure.



INTERMEDIATE LEVEL

6. A shopkeeper makes the following orders from a wholesaler:

Item	Quantity	Cost per item ($)
Skirts	32	18
Belts	18	8
White Shirts	47	26
Black Blouses	63	23
Grey Leggings	52	9

   Show how you estimate the total amount of money that the shopkeeper has to pay, giving your answer correct to the nearest hundred dollars.

7. A bag costs RM25. The conversion rate is RM1 = S$0.409 608. Without using a calculator, estimate the price of the bag in S$.

8. Without using a calculator, decide which of the options is better value for money.

Option *A*

300 g Jelly Beans
\$5.80

Option *B*

500 g Jelly Beans
\$9.90

9. Shop *A* sells a dress for \$79.50 with a 20% discount while Shop *B* sells the same dress for \$69.50 with a 10% discount. Write down a calculation you could do mentally to help you decide which shop to buy the dress from.

ADVANCED LEVEL

10. A handbag costs 26 700 Korea won (KRW). The conversion rate is S\$1 = KRW 876.333. Without using a calculator, estimate the price of the handbag in S\$.

There are three situations when **approximation** is used:

1. The actual value is known but it is not used for various reasons such as
   - the actual value is not necessary,
   - it is easier to remember an approximate value than the actual value,
   - it is too messy to write a long string of digits for non-exact numbers,
   - it is impossible for calculators to store all the digits of non-exact numbers.
2. The exact value cannot be obtained due to the limitation of the measuring instrument used.
3. The actual value is too troublesome or impossible to obtain, and so we need to use **estimation** to obtain an approximate value.

Five Rules to Identify Digits Which are Significant

Rule 1: All **non-zero** digits are significant.
Rule 2: All **zeros between non-zero** digits are significant.
Rule 3: In a decimal, all **zeros after a non-zero digit** are significant.
Rule 4: In a decimal, all **zeros before a non-zero digit** are *not* significant.
Rule 5: In whole numbers, the zeros at the end *may or may not* be significant. It depends on how the numbers are approximated.

Generally, we count significant figures from the first non-zero number starting from the left.

Review Exercise 3

1. Round off 6479.952 to the nearest
   (a) 100, **(b)** 1000, **(c)** tenth.

2. **(i)** Express 4.793 and 39.51 correct to 2 significant figures.
   (ii) Use your answers in part **(i)** to estimate $4.793 \div 39.51$.

3. The mass of a chocolate truffle is 0.025 kg, correct to 2 significant figures. What is the smallest possible mass of the chocolate truffle?

4. A toy costs 35 000 Indonesian rupiah (Rp). The conversion rate is Rp 1000 = S\$0.145 598. Without using a calculator, estimate the price of the toy in S\$.

5. The masses of 1 packet of sugar, 1 bottle of cooking oil and 1 box of baking powder are 109 g, 148 g and 84 g respectively. Write down a calculation you could do mentally to estimate the total mass of 3 packets of sugar, 2 bottles of cooking oil and 5 boxes of baking powder.

6. A battery-operated device needs to function for 28.2 hours. It functions for an average of 4.03 hours on one battery. Write down a calculation you could do mentally to estimate the number of batteries required.

7. Store *A* sells a 500 GB hard disk for \$85.05 with a 20% discount while Store *B* sells the same hard disk for \$76.05 with a 10% discount. Write down a calculation you could do mentally to help you decide which store to buy the hard disk from.

8. Without using a calculator, decide which of the options is better value for money.

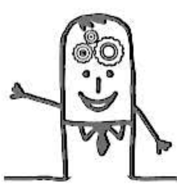
Option *A*

250 m*l*
Hand Lotion
\$15.20

Option *B*

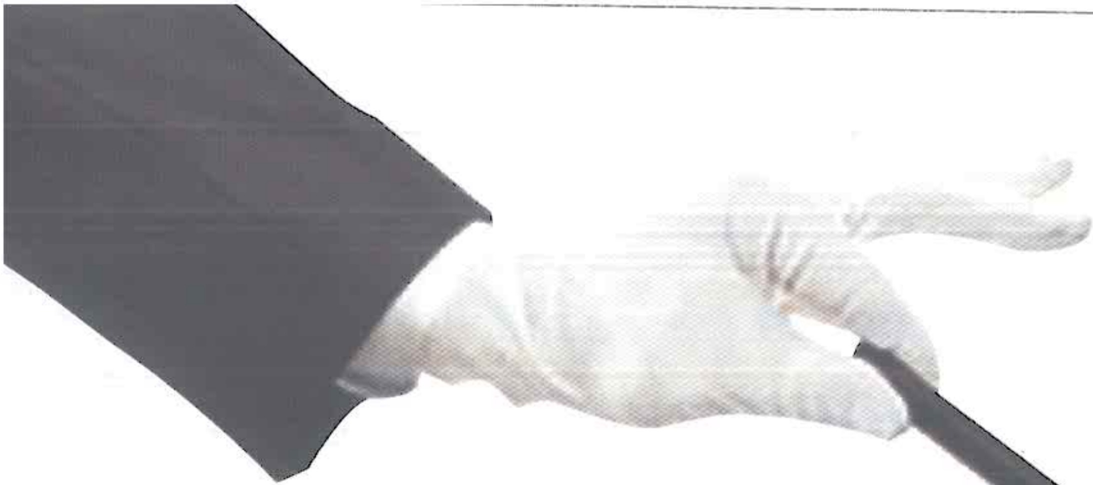
250 m*l* + 50 m*l*
Hand Lotion
\$17.45

Challenge Yourself



1. Without using a calculator, determine whether 987×123 is greater or less than 988×122.

2. Which of the following is likely to be the mass of an ordinary car?
   (a) 20 kg **(b)** 200 kg **(c)** 2000 kg **(d)** 20 000 kg



A teacher is showing a magic trick to his students. He places 12 coins on a table, 5 of which are heads up and 7 are tails up. He then places a blindfold over his eyes and shuffles the coins, keeping the faces up. Next, he separates the coins into two piles of 5 and 7 respectively. He flips over all the coins in the smaller pile. Both piles now have the same number of heads up! Is this really a magic trick? After you have learnt how to manipulate algebraic expressions in this chapter, you will be able to provide an explanation for this.

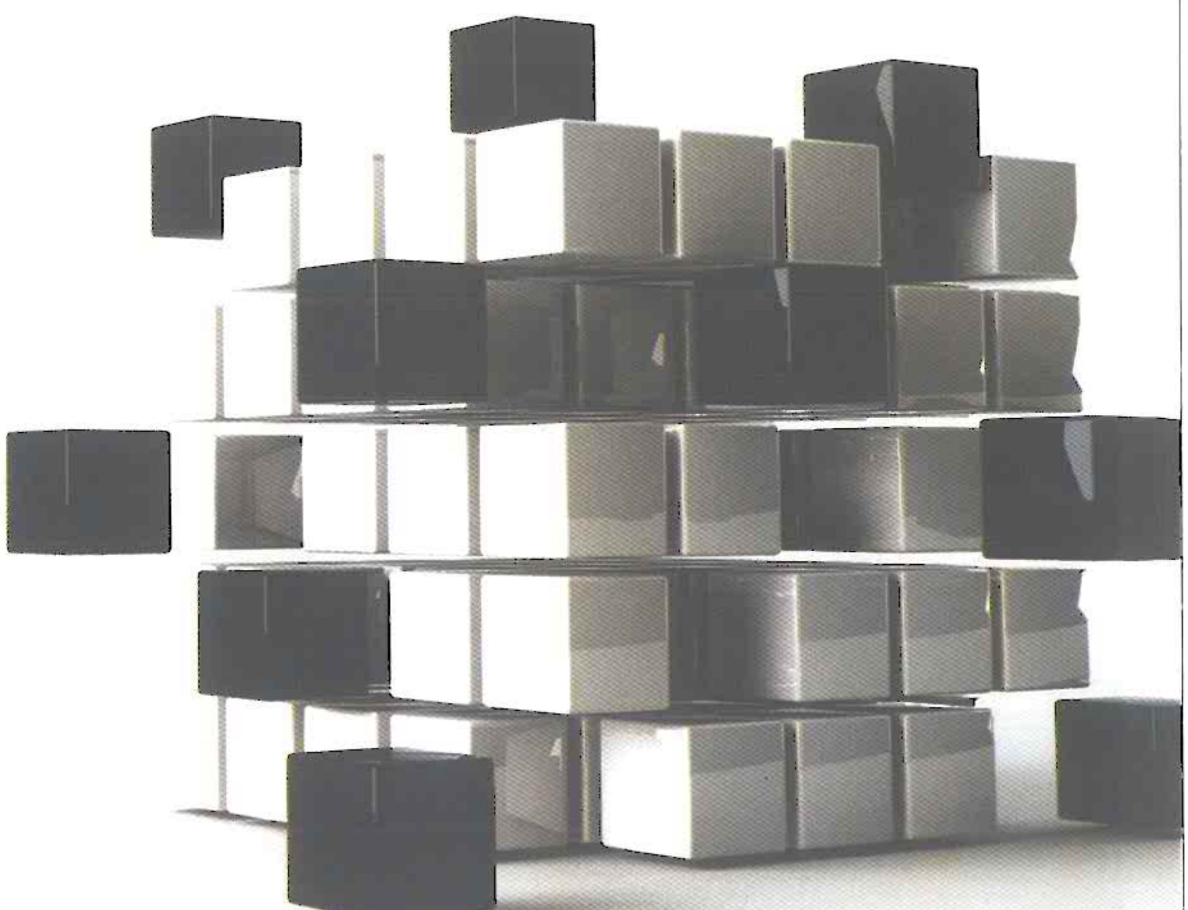
LEARNING OBJECTIVES

At the end of this chapter, you should be able to:

- use letters to represent numbers,
- express basic arithmetical processes algebraically,
- evaluate algebraic expressions,
- add and subtract linear expressions,
- simplify linear expressions,
- factorise algebraic expressions by extracting common factors.

Chapter Four

Basic Algebra and Algebraic Manipulation



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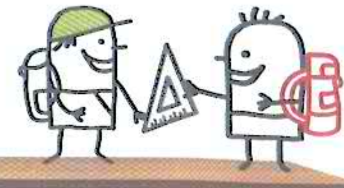
4.1 Fundamental Algebra



In algebra, letters are used to represent numbers. These letters are known as **variables** and they can be used to represent *any* number.

Algebraic Notations

Table 4.1 shows some basic algebraic notations.

Arithmetic		Algebra	
$2 + 3 = 3 + 2 = 5$	The sum of 2 and 3 (or 3 and 2) is 5.	$a + b = b + a = c$	The sum of a and b (or b and a) is c.
$6 - 5 = 1$	The difference between 6 and 5 is 1.	$a - b = c$	The difference between a and b is c, where a is more than b.
$3 \times 5 = 5 \times 3 = 15$	The product of 3 and 5 (or 5 and 3) is 15.	For $a \times b = b \times a = c$, we usually write $ab = c$ or $ba = c$.	The product of a and b (or b and a) is c.
$3 \times 3 = 3^2 = 9$	The square of 3 is 9.	$a \times a = a^2$	The square of a is a^2.
$3 \times 3 \times 3 = 3^3 = 27$	The cube of 3 is 27.	$a \times a \times a = a^3$	The cube of a is a^3.
$24 \div 3 = 24 \times \frac{1}{3} = 8$	The quotient of 24 when divided by 3 is 8.	For $a \div b = a \times \frac{1}{b} = c$, we usually write $\frac{a}{b} = c$.	The quotient of a when divided by b is c, where $b \neq 0$.

Table 4.1

Note:

- $3a = 3 \times a = a + a + a$
- $3a^2b = 3 \times a \times a \times b$
- $3(a + b) = 3 \times (a + b)$
- $\frac{3 + a}{4} = (3 + a) \div 4 = \frac{1}{4} \times (3 + a)$

Story Time

The word **algebra** comes from an Arabic word (*al-jabr*, literally means *restoration*). Its roots can be traced to a mathematician, Muhammed bin Musa al-Khwarizmi (780–850). He wrote 'The Compendious Book on Calculation by Completion and Balancing', which established algebra as a mathematical discipline that is independent of geometry and arithmetic.

ATTENTION

- When we multiply a by 3, we usually write it as $3a$ instead of $a3$.
- When we multiply b by 1, we usually write it as b and *not* $1b$ or $b1$.

Algebraic Expressions

In the **algebraic term** $3a$, the number 3 in front of the variable a is called the **coefficient** of a. Likewise, the coefficient of ab in $18ab$ is 18 and the coefficient of abc in $45abc$ is 45. What is the coefficient of a^3 in $-25a^3$?

An expression that consists of algebraic terms, operation symbols (+, –, ×, ÷) or brackets is known as an **algebraic expression**. An algebraic expression has *no* equal sign.

Table 4.2 shows some examples of linear and non-linear expressions.

Linear expressions	Non-linear expressions
$2x + 3$	$x^2 - 3$
$\frac{1}{4}x - 7$	$\frac{2}{3}x^3 + 1$
$y - 3x + 1$	$xy - 1$
$5x + y - 4z - 3$	$\frac{x}{y} + \sqrt{z}$

Table 4.2

Notice that the index of each of the variables in the **linear expressions** is 1, e.g. the index of the variable x in $2x + 3$ is 1. Also, each of the variables in the linear expressions is not multiplied or divided by another variable. $x^2 - 3$ is not a linear expression as the index of the variable x is 2. $xy - 1$ is also not a linear expression as the variable x is multiplied by another variable y.

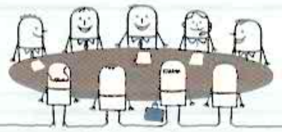
RECALL

5^4 is read as '5 to the power of 4', where 4 is called the *index*. The index of 5 is 1, but we usually write it as 5, instead of 5^1.

In the linear expression $5x + y - 4z - 3$, there are:

- 4 terms: $5x$, y, $-4z$, -3
- 3 variables: x, y, z
- 1 constant term: -3

The coefficients of x, y and z are 5, 1 and -4 respectively.



Class Discussion

Expressing Mathematical Relationships using Algebra

Work in pairs.

1. Complete Table 4.3.

	In words	Algebraic expression
(a)	Sum of $2x$ and $3z$	
(b)	Product of x and $7y$	
(c)	Divide $3ab$ by $2c$	
(d)	Subtract $6q$ from $10z$	
(e)		$(p + q) - xy$
(f)		$\frac{3+y}{5}$
(g)		$\sqrt{b} - 2c$
(h)	There are three times as many girls as boys in a school. Find an expression, in terms of x, for the total number of students in the school, where x represents the number of boys in the school.	It is given that x represents the number of boys. $\therefore$ ________ represents the number of girls. Total number of students = ________
(i)	The age of Nora's father is thrice hers. The age of Nora's brother is 5 years more than hers. Find an expression, in terms of y, for the sum of their ages, where y represents Nora's age.	It is given that y represents Nora's age. $\therefore$ Nora's father is ________ years old. Nora's brother is ________ years old. Sum of their ages = ________ years
(j)	The ____________ is ____________ times as long as the ____________ of the rectangle. Find an expression, in terms of b, for the perimeter and the area of the rectangle, where b represents the breadth of the rectangle.	It is given that b represents the breadth of the rectangle in m. $3b$ represents the length of the rectangle in m. Perimeter of the rectangle = ________ m Area of the rectangle = ________ m^2

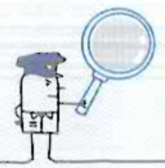
Table 4.3

2. Think of three more different algebraic expressions and get your classmate to interpret the mathematical relationship within each of them.

SIMILAR QUESTIONS

Exercise 4A Questions 1(a)–(f), 6(a)–(b), 13

Evaluation of Algebraic Expressions



Investigation

Comparison between Pairs of Expressions

1. Create a spreadsheet as shown. Key in the formulae in the cells B3, C3, D3, E3 and F3.

	A	B	C	D	E	F
1						
2	n	$2n$	$2+n$	n^2	$2n^2$	$(2n)^2$
3	1	= 2*A3	= 2 + A3	= (A3)^2	= 2*(A3)^2	= (2*A3)^2
4	2					
5	3					
6	4					
7	5					

You should obtain the following results after you enter the formulae in the cells. Do not simply input the values directly!

	A	B	C	D	E	F
1						
2	n	$2n$	$2+n$	n^2	$2n^2$	$(2n)^2$
3	1	2	3	1	2	4
4	2					
5	3					
6	4					
7	5					

2. From B3, extend the formula downwards to cell B7.
   (i) What do you notice about the value of $2n$ as n changes?
   (ii) How do you determine the value of $2n$ when given a value of n?
   (iii) Hence, find the values of $2n$ when the values of n are 8, 9 and 10 respectively.
3. Extend the formulae downwards to cells C7 to F7 respectively.
4. Compare and examine the difference between each of the following pairs of expressions.
   - $2n$ and $2 + n$
   - n^2 and $2n$
   - $2n^2$ and $(2n)^2$

   Comment whether each pair of expressions is equal. Explain your answer.

Consider the linear expression $x - 2$. The expression takes on different values for different values of x.

For example, when $x = 2$, $x - 2 = 2 - 2 = 0$,

when $x = 4$, $x - 2 = 4 - 2 = 2$,

when $x = 6$, $x - 2 = 6 - 2 = 4$, and so on.

Evaluating an algebraic expression means finding the value of the expression when the variables take on certain values.

Worked Example 1

(Evaluation of Algebraic Expressions)

Given that $x = 5$ and $y = -3$, evaluate each of the following expressions.

(a) $3x - 2y$ **(b)** $\frac{2y}{3x}$

Solution:

(a) $3x - 2y = 3(5) - 2(-3)$

$= 15 + 6$

$= 21$

(b) $\frac{2y}{3x} = \frac{2(-3)}{3(5)}$

$= -\frac{6}{15}$

$= -\frac{2}{5}$ (reduced to lowest term)

PRACTISE NOW 1

SIMILAR QUESTIONS

Exercise 4A Questions 2(a)–(d), 3(a)–(d), 7(a)–(d)

1. Given that $x = -2$ and $y = 4$, evaluate each of the following expressions.

   (a) $5y - 4x$ **(b)** $\frac{1}{x} - y + 3$

2. Find the value of $p^2 + 3q^2$ when $p = -\frac{1}{2}$ and $q = -2$.

Journal Writing

Is $5 + n$ or $5n$ greater in value? Explain your answer.

Representation of Linear Expressions

In Chapter 2, we have learnt that an algebra disc has two sides. If one side shows the number 1, the other side shows the number –1.

1 –1

front back

Similarly, for an algebra disc where one side shows the variable x, the other side will show $-x$.

x $-x$

front back

To obtain the negative of x, we flip the disc with x as shown:

$x \xrightarrow{\text{flip}} -x$ we write $-(x) = -x$

To obtain the negative of $-x$, we flip the disc with $-x$ as shown:

$-x \xrightarrow{\text{flip}} x$ we write $-(-x) = x$

If we put two discs x and $-x$ together, we will get a zero pair.

$\boxed{x \quad -x}$ we write $x + (-x) = 0$

We can use algebra discs to represent algebraic terms, e.g. we use three x discs to represent $3x$.

x x x $3x = x + x + x$

We use three $-x$ discs to represent $-3x$.

$-x$ $-x$ $-x$ $-3x = (-x) + (-x) + (-x)$

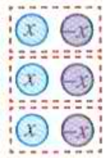
To obtain the negative of $3x$, i.e. $-(3x)$, we flip the three x discs as shown:

$$x \ x \ x \xrightarrow{\text{flip}} -x \ -x \ -x \quad \text{we write } -(3x) = -3x$$

To obtain the negative of $-3x$, i.e. $-(-3x)$, we flip the three $-x$ discs as shown:

$$-x \ -x \ -x \xrightarrow{\text{flip}} x \ x \ x \quad \text{we write } -(-3x) = 3x$$

What happens if we put three x discs and three $-x$ discs together?



we write $3x + (-3x) = 0$

We will get zero pairs.

We can also use algebra discs to represent linear expressions.

Example: $4x + 2$

$x \ x \ x \ x \ 1 \ 1$ $\quad 4x + 2 = x + x + x + x + 1 + 1$

Example: $3x - 1$

$x \ x \ x \ -1$ $\quad 3x - 1 = x + x + x + (-1)$

Example: $-2x + 3$

$-x \ -x \ 1 \ 1 \ 1$ $\quad -2x + 3 = (-x) + (-x) + 1 + 1 + 1$

Example: $x + 2y$

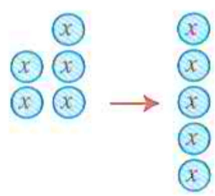
$x \ y \ y$ $\quad x + 2y = x + y + y$

What is the linear expression represented by $x \ x \ -y \ 1 \ 1$?

Addition of Linear Expressions

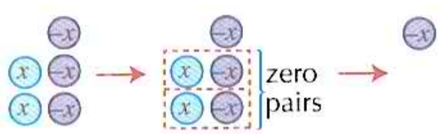
We will show how to carry out addition of linear expressions using algebra discs.

Example: $2x + 3x$



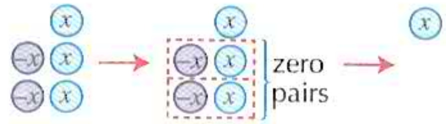
Therefore, $2x + 3x = 5x$.

Example: $2x + (-3x)$



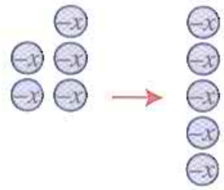
Therefore, $2x + (-3x) = -x$.

Example: $-2x + 3x$



Therefore, $-2x + 3x = x$.

Example: $-2x + (-3x)$



Therefore, $-2x + (-3x) = -5x$.

PRACTISE NOW

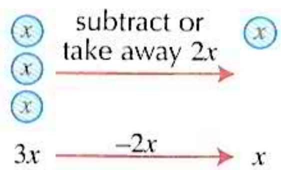
Simplify each of the following expressions by using algebra discs.

(a) $3x + 4x$ **(b)** $3x + (-4x)$ **(c)** $-3x + 4x$ **(d)** $-3x + (-4x)$

Subtraction of Linear Expressions

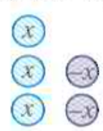
We will show how to carry out subtraction of linear expressions using algebra discs.

Example: $3x - 2x$



Therefore, $3x - 2x = x$.

As $3x + (-2x) = x$, we can write $3x - 2x$ as $3x + (-2x)$ and represent $3x - 2x$ using algebra discs:



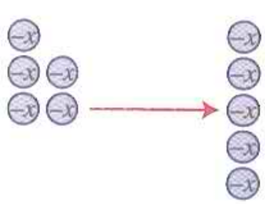
Example: $3x - (-2x)$

To obtain the negative of $-2x$, i.e. $-(-2x)$, we flip the discs as shown:

we write $-(-2x) = 2x$

Therefore, $3x - (-2x) = 3x + 2x$
$= 5x$.

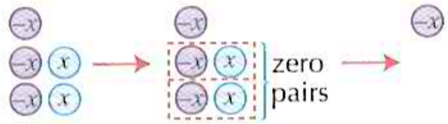
Example: $-3x - 2x = -3x + (-2x)$



Therefore, $-3x - 2x = -3x + (-2x)$
$= -5x$.

Example: $-3x - (-2x)$

As $-(-2x) = 2x$, $-3x - (-2x) = -3x + 2x$.



Therefore, $-3x - (-2x) = -3x + 2x$
$= -x$.

PRACTISE NOW

Simplify each of the following expressions by using algebra discs.

(a) $4x - 3x$ **(b)** $4x - (-3x)$ **(c)** $-4x - 3x$ **(d)** $-4x - (-3x)$

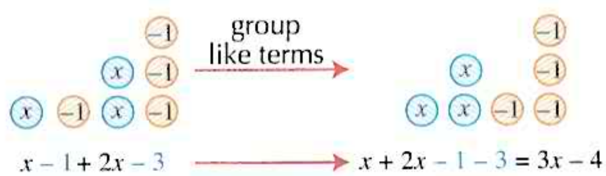
Further Addition and Subtraction of Linear Expressions

In algebra, **like terms** are terms that have the same variable(s) and each variable must have the same index. Examples of like terms are $2x$ and $5x$, $7y^2$ and $-3y^2$, and x^3y and $4x^3y$.

When two terms are not like terms, they are known as **unlike terms**. Some examples of unlike terms are $2x$ and $2y$, and x^2y and xy^2.

Now, let us use algebra discs to show how to add and subtract linear expressions.

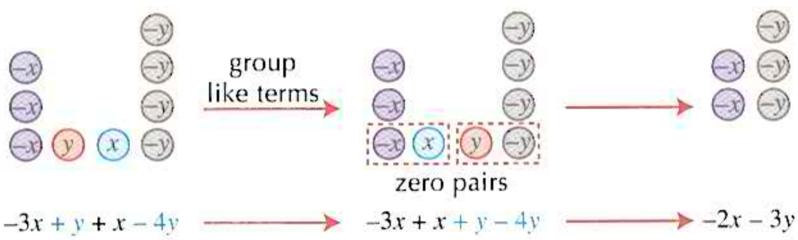
Example: $x - 1 + 2x - 3$



Therefore, $x - 1 + 2x - 3 = x + 2x - 1 - 3$ (group like terms)
$= 3x - 4$.

Example: $-3x + y - (-x) + (-4y)$

As $-(-x) = x$, $-3x + y - (-x) + (-4y)$ can be represented by:



Therefore, $-3x + y \boxed{-(-x)}\boxed{+(-4y)} = -3x + y + x - 4y$
$= -3x + x + y - 4y$ (group like terms)
$= -2x - 3y$.

PRACTISE NOW

Simplify each of the following expressions by using algebra discs. Alternatively, you may visit http://www.shinglee.com.sg/StudentResources/ to access the AlgeTools™ software.

(a) $x + 2 + 5x - 4$ **(b)** $2x + (-3) - 3x + 5$

(c) $-x - y - (-2x) + 4y$ **(d)** $-3x - 7y + (-2y) - (-4x)$

Worked Example 2

(Addition and Subtraction of Linear Expressions)

Simplify each of the following expressions.

(a) $2x - 4y + 4x + 2y$ **(b)** $10x - 7y - 12x - (-8y)$

(c) $-3x + (-5y) + x - (-4y)$ **(d)** $\frac{1}{2}x + \frac{1}{4}x$

(e) $\frac{4}{3}y - \frac{2}{5}y$

Solution:

like terms

(a) $2x - 4y + 4x + 2y = 2x + 4x - 4y + 2y$ (group like terms)

like terms $= 6x - 2y$

like terms

(b) $10x - 7y - 12x - (-8y) = 10x - 7y - 12x + 8y$ $(-(-8y) = 8y)$

like terms

$= 10x - 12x - 7y + 8y$ (group like terms)

$= -2x + y$

like terms

(c) $-3x \boxed{+ (-5y)} + x \boxed{- (-4y)} = -3x - 5y + x + 4y$

like terms

$= -3x + x - 5y + 4y$ (group like terms)

$= -2x - y$

(d) $\frac{1}{2}x + \frac{1}{4}x = \frac{2}{4}x + \frac{1}{4}x$ (convert to like fractions: $\frac{1}{2}x = \frac{2}{4}x$)

$= \frac{3}{4}x$

(e) $\frac{4}{3}y - \frac{2}{5}y = \frac{20}{15}y - \frac{6}{15}y$ (convert to like fractions: $\frac{4}{3}y = \frac{20}{15}y$, $\frac{2}{5}y = \frac{6}{15}y$)

$= \frac{14}{15}y$

Problem Solving Tip

To simplify algebraic expressions that involve more than one variable, we can group the like terms before performing the arithmetic operations. Like terms can be combined into a single term by adding or subtracting from one another.

PRACTISE NOW 2

SIMILAR QUESTIONS

Exercise 4A Questions 4(a)–(d), 5(a)–(d), 8(a)–(d), 9(a)–(d), 10–12

1. Simplify each of the following expressions.
   (a) $2x - 5y + 4y + 8x$
   (b) $11x - (-5y) - 14x - 2y$
   (c) $-9x - (-y) + (-3x) - 7y$
   (d) $\frac{1}{2}x - \frac{1}{3}x$
   (e) $\frac{7}{4}y - \frac{5}{8}y$

2. (i) Simplify the expression $2p - 5q + 7r - 4p + 2q - 3r$.
   (ii) Find the value of the expression when $p = \frac{1}{2}$, $q = -\frac{1}{3}$ and $r = 4$.

Exercise 4A

BASIC LEVEL

1. Write down an algebraic expression for each of the following statements.
   (a) Add $5y$ to the product of a and b.
   (b) Subtract 3 from the cube of f.
   (c) Multiply k by $6q$.
   (d) Divide $2w$ by $3xy$.
   (e) Subtract 4 times the positive square root of z from thrice of x.
   (f) Twice the variable p divided by the product of 5 and q.

2. Given that $x = 6$ and $y = -4$, evaluate each of the following expressions.
   (a) $4x - 7y$
   (b) $\frac{5x}{3y} + x$
   (c) $2x^2 - y^3$
   (d) $3x + \frac{x}{y} - y^2$

3. Given that $a = 3$, $b = -5$ and $c = 6$, evaluate each of the following expressions.
   (a) $a(3c - b)$
   (b) $ab^2 - ac$
   (c) $\frac{b}{a} - \frac{c}{b}$
   (d) $\frac{b+c}{a} + \frac{a+c}{b}$

4. Simplify each of the following expressions.
   (a) $5x + 22 - 6x - 23$
   (b) $x + 3y + 6x + 4y$
   (c) $6xy + 13x - 2yx - 5x$
   (d) $6x - 20y + 7z - 8x + 25y - 11z$

5. Find the sum of each of the following expressions.
   (a) $2x + 4y$, $-5y$
   (b) $-b - 4a$, $7b - 6a$
   (c) $6d - 4c$, $-7c + 6d$
   (d) $3pq - 6hk$, $-3qp + 14kh$

INTERMEDIATE LEVEL

6. Write down an algebraic expression for each of the following statements.
   (a) Subtract the cube root of the product of x and $3y$ from the square of the sum of a and b.
   (b) The total value of x 20-cent coins and y 5-dollar notes in cents.

7. Given that $a = 3$, $b = -4$ and $c = -2$, evaluate each of the following expressions.
   (a) $\frac{3a-b}{2c} + \frac{3a-c}{c-b}$
   (b) $\frac{2c-a}{3c+b} - \frac{5a+4c}{c-a}$
   (c) $\frac{a+b+2c}{3c-a-b} - \frac{5c}{4b}$
   (d) $\frac{b-c}{3c+4b} \div \frac{bc}{a} + \frac{ac}{b}$

8. Simplify each of the following expressions.
   (a) $15x + (-7y) + (-18x) + 4y$
   (b) $-3x + (-5y) - (-10y) - 7x$
   (c) $9x - (-2y) - 8x - (-12y)$
   (d) $-7x - (-15y) - (-2x) + (-6y)$

9. Simplify each of the following expressions.
   (a) $\frac{1}{4}x + \frac{1}{3}x$ (b) $\frac{2}{5}y - \frac{1}{3}y$
   (c) $-\frac{3}{7}a + \frac{3}{5}a$ (d) $\frac{9}{4}b - \frac{4}{3}b$

10. (i) Simplify the expression
   $3p + (-q) - 7r - (-8p) - q + 2r$.
   (ii) Find the value of the expression when $p = 2$, $q = -1\frac{1}{2}$ and $r = -5$.

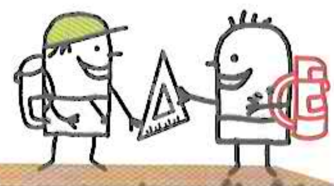
11. Raj is $12m$ years old. His son was born when he was $9m$ years old.
   (i) Find Raj's age 5 years later.
   (ii) Find the sum of their ages in 5 years' time.

12. Huixian bought 8 books at $\$w$ each and 7 pens at $\$m$ each. She had $\$(3w + 5m)$ left. Find the amount of money she had at first.

ADVANCED LEVEL

13. At a famous roti prata shop, for every two people who order egg prata, there are five people who order plain prata.
   (a) If a people order egg prata, how many people order plain prata?
   (b) If b people order plain prata, how many people order egg prata?
   (c) If there are a total of c people in the shop, how many of them order egg prata?

4.2 Expansion and Simplification of Linear Expressions



Negative of a Linear Expression

We have learnt the concept of negative in Chapter 2 and Section 4.1:

$1 \xrightarrow{\text{flip}} -1$ $-(1) = -1$

$-1 \xrightarrow{\text{flip}} 1$ $-(-1) = 1$

$x \xrightarrow{\text{flip}} -x$ $-(x) = -x$

$-x \xrightarrow{\text{flip}} x$ $-(-x) = x$

What about the negative of a linear expression, e.g. $-(x + 2)$, $-(x - 2)$ and $-(-x + 3y - 1)$? How do we simplify them?

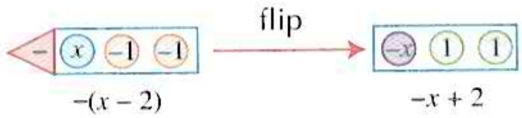
We will show how to find the negative of a linear expression using algebra discs.

Example: $-(x + 2)$

$-(x + 2)$ $-x - 2$

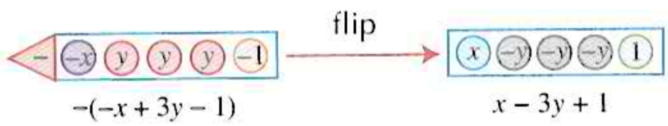
Therefore, $-(x + 2) = -x - 2$.

Example: $-(x-2)$



Therefore, $-(x-2) = -x + 2$.

Example: $-(-x+3y-1)$



Therefore, $-(-x+3y-1) = x - 3y + 1$.

PRACTISE NOW

Simplify each of the following expressions by using algebra discs. Alternatively, you may visit http://www.shinglee.com.sg/StudentResources/ to access the AlgeTools™ software.

(a) $-(3x+2)$ **(b)** $-(3x-2)$ **(c)** $-(-3x-2)$ **(d)** $-(2x+y-4)$

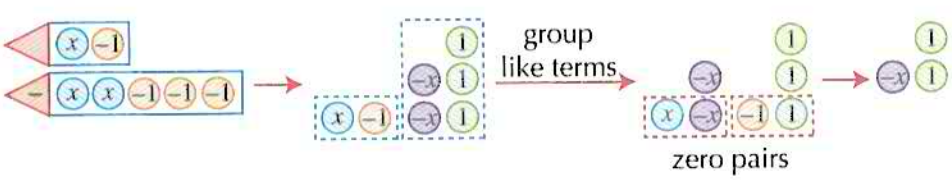
Sums involving the Negative of a Linear Expression

In Section 4.1, we have learnt how to find $x-1+2x-3$, which can be written as $(x-1)+(2x-3)$. This is in fact the sum of the two groups, $(x-1)$ and $(2x-3)$.

What do you think is the sum of the two groups, $(x-1)$ and $-(2x-3)$?

We will show how to find this sum, i.e. $(x-1)+[-(2x-3)]$, using algebra discs.

Example: $(x-1)+[-(2x-3)]$



$(x-1)+[-(2x-3)] \longrightarrow x-1-2x+3 \longrightarrow x-2x-1+3 \longrightarrow -x+2$

ATTENTION

$2x+(-3x)$ can be written as $2x-3x$. Similarly, $(x-1)+[-(2x-3)]$ can be written as $x-1-(2x-3)$.

Therefore, $(x-1)+[-(2x-3)] = x - 1 - 2x + 3$
$= x - 2x - 1 + 3$ (group like terms)
$= -x + 2$.

PRACTISE NOW

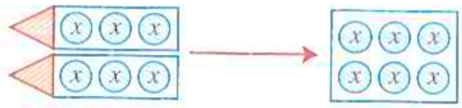
Find the sum of each of the following expressions by using algebra discs. Alternatively, you may visit http://www.shinglee.com.sg/StudentResources/ to access the AlgeTools™ software.

(a) $x+1,\ -(3x-1)$ **(b)** $5x-3,\ -(4x+1)$
(c) $3x+2y,\ -(-y+2x)$ **(d)** $-4x+2y,\ -(-x-5y)$

Expansion of Linear Expressions

We will show how to **expand** linear expressions using algebra discs.

Example: $2(3x)$ can be represented by '2 groups of $3x$':



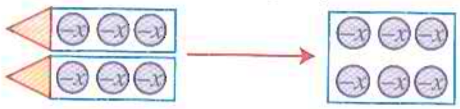
$2(3x)$ $6x$

Therefore, $2(3x) = 6x$.

ATTENTION

$2(3x)$ is the same as $2 \times 3x$.

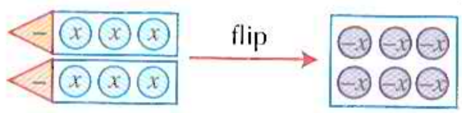
Example: $2(-3x)$ can be represented by '2 groups of $-3x$':



$2(-3x)$ $-6x$

Therefore, $2(-3x) = -6x$.

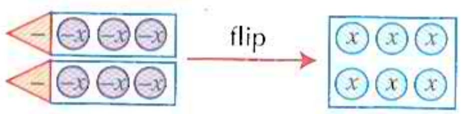
Example: $-2(3x)$ can be represented by 'the negative of 2 groups of $3x$':



$-2(3x)$ $-6x$

Therefore, $-2(3x) = -6x$.

Example: $-2(-3x)$ can be represented by 'the negative of 2 groups of $-3x$':



$-2(-3x)$ $6x$

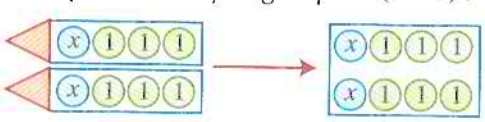
Therefore, $-2(-3x) = 6x$.

PRACTISE NOW

Expand each of the following expressions by using algebra discs.

(a) $3(5x)$ **(b)** $3(-5x)$ **(c)** $-3(5x)$ **(d)** $-3(-5x)$

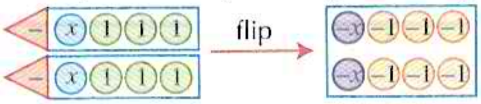
Example: $2(x + 3)$ can be represented by '2 groups of $(x + 3)$':



$2(x + 3)$ $2x + 6$

Therefore, $2(x + 3) = 2x + 6$.

Example: $-2(x+3)$ can be represented by 'the negative of 2 groups of $(x+3)$':



$-2(x+3)$ $\qquad$ $-2x-6$

Therefore, $-2(x+3) = -2x-6$.

Class Discussion

The Distributive Law

Work in pairs.

1. Expand each of the following expressions by using algebra discs.

   (a) $2(-x-4)$ **(b)** $-2(-x-4)$ **(c)** $3(y-2x)$ **(d)** $-3(y-2x)$

2. Expand $a(b+c)$.

From the class discussion, we have:

$$\boxed{a(b+c) = ab + ac}$$

This is known as the **Distributive Law**, where 'a groups of b and c' is the same as 'a groups of b' and 'a groups of c', i.e. 'a times of b' and 'a times of c'.

ATTENTION

If $a = 2$, then we have $2(b+c) = 2b + 2c$.

Worked Example 3

(Expansion of Linear Expressions)

Expand each of the following expressions.

(a) $6(x-2)$ **(b)** $-3(4x-y)$ **(c)** $a(5x+y)$

Solution:

(a) $6(x-2) = 6x - 12$ (Distributive Law; '6 groups of $(x-2)$')

(b) $-3(4x-y) = -12x + 3y$ (Distributive Law; 'the negative of 3 groups of $(4x-y)$')

(c) $a(5x+y) = a(5x) + ay$ (Distributive Law; 'a groups of $(5x+y)$')

$= 5ax + ay$

PRACTISE NOW 3

SIMILAR QUESTIONS

Exercise 4B Questions 1(a)-(h), 3

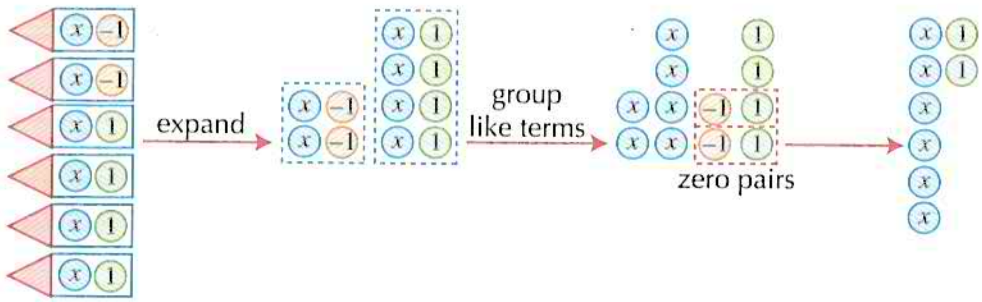
Expand each of the following expressions.

(a) $3(x+2)$ **(b)** $-5(x-4y)$ **(c)** $-a(x+2y)$

Expansion and Simplification of Linear Expressions

Now, let us use algebra discs to show how to expand and simplify linear expressions.

Example: $2(x-1)+4(x+1)$ can be represented by '2 groups of $(x-1)$' and '4 groups of $(x+1)$':



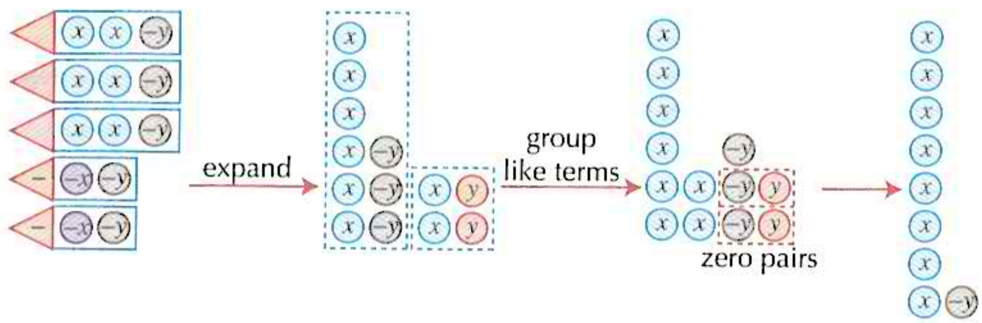
$2(x-1)+4(x+1) \longrightarrow 2x-2+4x+4 \longrightarrow 2x+4x-2+4 \longrightarrow 6x+2$

Therefore, $2(x-1)+4(x+1) = 2x-2+4x+4$ (expand)

$= 2x+4x-2+4$ (group like terms)

$= 6x+2.$

Example: $3(2x-y)-2(-x-y)$ can be represented by '3 groups of $(2x-y)$' and 'the negative of 2 groups of $(-x-y)$':



$3(2x-y)-2(-x-y) \longrightarrow 6x-3y+2x+2y \longrightarrow 6x+2x-3y+2y \longrightarrow 8x-y$

Therefore, $3(2x-y)-2(-x-y) = 6x-3y+2x+2y$ (expand)

$= 6x+2x-3y+2y$ (group like terms)

$= 8x-y.$

PRACTISE NOW

Expand and simplify each of the following expressions by using algebra discs. Alternatively, you may visit http://www.shinglee.com.sg/StudentResources/ to access the AlgeTools™ software.

(a) $x+7+3(x-2)$

(b) $3(x+2)+2(-2x+1)$

(c) $2(-x-y)-(2x-y)$

(d) $-(x+4y)-2(3x-y)$

The rules by which operations are performed when an algebraic expression involves brackets are exactly the same as in arithmetic.

- Simplify the expression *within* the brackets first.
- Use the **Distributive Law** when an expression within a pair of brackets is multiplied by a number.
- When an expression contains more than one pair of brackets, simplify the expression within the *innermost* pair of brackets first. For example, $[a-(b-c)]+2c = [a-b+c]+2c = a-b+c+2c = a-b+3c$.

Worked Example 4

(Expansion and Simplification of Linear Expressions)

Expand and simplify each of the following expressions.

(a) $3(x + 2y) + 4(y - x)$ **(b)** $5y - 2[3x - 2(y - 3x)]$

Solution:

(a) $3(x + 2y) + 4(y - x) = 3x + 6y + 4y - 4x$ (Distributive Law; '3 groups of $(x + 2y)$', '4 groups of $(y - x)$')

$= 3x - 4x + 6y + 4y$ (group like terms)

$= -x + 10y$

(b) $5y - 2[3x - 2(y - 3x)] = 5y - 2(3x - 2y + 6x)$ (remove the innermost brackets by applying the Distributive Law; 'the negative of 2 groups of $(y - 3x)$')

$= 5y - 2(3x + 6x - 2y)$ (group like terms)

$= 5y - 2(9x - 2y)$

$= 5y - 18x + 4y$ (Distributive Law; 'the negative of 2 groups of $(9x - 2y)$')

$= 5y + 4y - 18x$ (group like terms)

$= 9y - 18x$

PRACTISE NOW 4

SIMILAR QUESTIONS

Exercise 4B Questions 2(a)-(d), 4-5, 6(a)-(j), 7(a)-(d), 8(a)-(b), 9

1. Expand and simplify each of the following expressions.
   (a) $6(4x + y) + 2(x - y)$
   (b) $x - [y - 3(2x - y)]$
   (c) $7x - 2[3(x - 2) - 2(x - 5)]$

2. Michael is 5 years older than Rui Feng and Vishal is thrice as old as Michael. If Rui Feng is p years old, find
   (i) Michael's present age,
   (ii) Vishal's present age,
   (iii) the sum of their ages in 6 years' time,
   (iv) the sum of their ages 3 years ago.

Thinking Time

Ethan wants to insert two pairs of brackets into the expression on the left-hand side so that the expression on the right can be obtained. Can you help him to identify where the brackets should be placed?

$$-x - 5 + 6x - 7x - 2 + 12 = -2x + 19$$

Explore two more ways of placing brackets into the expression $-x - 5 + 6x - 7x - 2 + 12$ such that different results will be obtained.

Exercise 4B

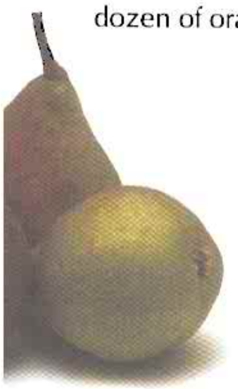
BASIC LEVEL

1. Expand each of the following expressions.
   (a) $-(x + 5)$ (b) $-(4 - x)$
   (c) $2(3y + 7)$ (d) $8(2y - 5)$
   (e) $8(3a - 4b)$ (f) $-3(c + 6)$
   (g) $-4(d - 6)$ (h) $2a(x - y)$

2. Expand and simplify each of the following expressions.
   (a) $5(a + 2b) - 3b$ (b) $7(p + 10q) + 2(6p + 7q)$
   (c) $a + 3b - (5a - 4b)$ (d) $x + 3(2x - 3y + z) + 7z$

3. Khairul is x years old. Khairul's uncle is four times as old as Khairul will be in 5 years . Find the present age of Khairul's uncle.

4. A pear costs x cents. An orange costs y cents less than a pear. Find the cost of 4 pears and half a dozen of oranges.



5. Devi bought 7 skirts at $\$x$ each, n skirts at \$12 each, $(2n + 1)$ skirts at \$15 each and 4 skirts at $\$3x$ each. Find the total cost of the skirts she bought.

INTERMEDIATE LEVEL

6. Expand and simplify each of the following expressions.
   (a) $4u - 3(2u - 5v)$
   (b) $-2a - 3(a - b)$
   (c) $7m - 2n - 2(3n - 2m)$
   (d) $5(2x + 4) - 3(-6 - x)$
   (e) $-4(a - 3b) - 5(a - 3b)$
   (f) $5(3p - 2q) - 2(3p + 2q)$
   (g) $x + y - 2(3x - 4y + 3)$
   (h) $3(p - 2q) - 4(2p - 3q - 5)$
   (i) $9(2a + 4b - 7c) - 4(b - c) - 7(-c - 4b)$
   (j) $-4[5(2x + 3y) - 4(x + 2y)]$

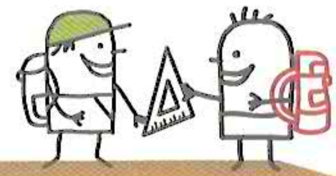
7. Subtract
   (a) $-6x - 3$ from $2x - 5$,
   (b) $6x - y + 5z$ from $10x - 2y + z$,
   (c) $8p + 9q - 5rs$ from $-4p - 4q + 15sr$,
   (d) $8a - 3b + 5c - 4d$ from $10a - b - 4c - 8d$.

ADVANCED LEVEL

8. Expand and simplify each of the following expressions.
   (a) $-2\{3a - 4[a - (2 + a)]\}$
   (b) $5\{3c - [d - 2(c + d)]\}$

9. The average monthly salary of m male employees and f female employees of a company is \$2000. If the average monthly salary of the male employees is $\$(b + 200)$, find the average monthly salary of the female employees.

4.3 Simplification of Linear Expressions with Fractional Coefficients



In the linear expression $\frac{1}{2}x + \frac{3}{4}y$, $\frac{1}{2}$ and $\frac{3}{4}$ are the **fractional coefficients** of the variables x and y respectively. $\frac{x-3}{2}$ and $\frac{2x-5}{3}$, which can be written as $\frac{1}{2}(x-3)$ and $\frac{1}{3}(2x-5)$ respectively, are other examples of linear expressions with fractional coefficients.

The procedure for simplifying linear expressions with fractional coefficients is similar to that of simplifying ordinary numerical fractions.

As seen in Worked Example 2(d) and (e), before we can add or subtract algebraic terms with fractional coefficients, we have to use the idea of equivalent fractions to convert them to like fractions, i.e. fractions that have the same denominator. We may make use of the lowest common multiple (LCM) of the denominators.

In this section, we will look at more examples on the simplification of linear expressions with fractional coefficients.

Worked Example 5

(Simplification of Linear Expressions with Fractional Coefficients)

Expand and/or simplify each of the following expressions.

(a) $\frac{1}{2}x - \frac{1}{9}y - \frac{1}{8}x + \frac{11}{3}y$ **(b)** $\frac{2}{3}[2x - 5(x - 6y)]$

Solution:

(a)
$$\begin{aligned}\frac{1}{2}x - \frac{1}{9}y - \frac{1}{8}x + \frac{11}{3}y &= \frac{1}{2}x - \frac{1}{8}x - \frac{1}{9}y + \frac{11}{3}y \quad \text{(group like terms)}\\ &= \frac{4}{8}x - \frac{1}{8}x - \frac{1}{9}y + \frac{33}{9}y \quad \left(\text{convert to like fractions: } \frac{1}{2}x = \frac{4}{8}x,\ \frac{11}{3}y = \frac{33}{9}y\right)\\ &= \frac{3}{8}x + \frac{32}{9}y\end{aligned}$$

(b)
$$\begin{aligned}\frac{2}{3}[2x - 5(x - 6y)] &= \frac{2}{3}(2x - 5x + 30y) \quad \text{(expand } -5(x - 6y)\text{)}\\ &= \frac{2}{3}(-3x + 30y)\\ &= \frac{2}{\cancel{3}_{1}} \times (\cancel{-3}^{-1}x) + \frac{2}{\cancel{3}_{1}} \times \cancel{30}^{10}y \quad \text{(Distributive Law)}\\ &= -2x + 20y\end{aligned}$$

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PRACTISE NOW 5

SIMILAR QUESTIONS

Exercise 4C Questions 1(a)-(d), 2(a)-(d), 5(a)-(b)

Expand and/or simplify each of the following expressions.

(a) $\frac{1}{2}x+\frac{1}{4}y-\frac{2}{5}y-\frac{1}{3}x$ **(b)** $\frac{1}{8}[-y-3(16x-3y)]$

Worked Example 6

(Simplification of Linear Expressions with Fractional Coefficients)

Express each of the following as a fraction in its simplest form.

(a) $\frac{x}{3}+\frac{2x-5}{7}$ **(b)** $\frac{2x-5}{3}-\frac{3x-2}{5}$

Solution:

(a)

$$\frac{x}{3}+\frac{2x-5}{7}=\frac{7x}{21}+\frac{3(2x-5)}{21} \quad \text{(LCM of 3 and 7 is 21)}$$

$$=\frac{7x+3(2x-5)}{21} \quad \text{(combine into a single fraction)}$$

$$=\frac{7x+6x-15}{21} \quad \text{(Distributive Law)}$$

$$=\frac{13x-15}{21}$$

(b)

$$\frac{2x-5}{3}-\frac{3x-2}{5}=\frac{5(2x-5)}{15}-\frac{3(3x-2)}{15} \quad \text{(LCM of 3 and 5 is 15)}$$

$$=\frac{5(2x-5)-3(3x-2)}{15} \quad \text{(combine into a single fraction)}$$

$$=\frac{10x-25-9x+6}{15} \quad \text{(Distributive Law)}$$

$$=\frac{10x-9x-25+6}{15} \quad \text{(group like terms)}$$

$$=\frac{x-19}{15}$$

ATTENTION

- Always combine the terms into a single fraction before removing the brackets.
- In **(a)**, $\frac{x}{3}$ can be written as $\frac{1}{3}x$.
- For **(b)**, the sign needs to be changed after removing the brackets, i.e. $-3(3x-2)$ $=-9x+6$.

PRACTISE NOW 6

SIMILAR QUESTIONS

Exercise 4C Questions 3(a)–(h), 6(a)–(j), 8(a)–(d)

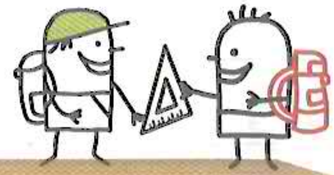
1. Express each of the following as a fraction in its simplest form.

(a) $\frac{x-3}{2}+\frac{2x-5}{3}$ **(b)** $\frac{x-2}{4}-\frac{2x-7}{3}$

2. Express each of the following as a fraction in its simplest form.

(a) $\frac{x-1}{3}+\frac{1}{2}-\frac{2x-3}{4}$ **(b)** $2x+\frac{x-4}{9}-\frac{2x-5}{3}$

4.4 Factorisation



In Section 4.2, we have learnt how to expand linear expressions, e.g. $2(x + y) = 2x + 2y$.

We shall now learn how to carry out the *reverse* process, i.e. express a linear expression as a product of its factors. This process is called **factorisation**.

> **Factorisation** is the process of expressing an algebraic expression as a *product* of two or more algebraic expressions. It is the *reverse* of expansion.

To **factorise** algebraic expressions, we will need to identify the *common factors*, i.e. common numbers or common variables of the terms. For example,
- in $4x + 2 = 2(2x + 1)$, 2 is the common factor of the two terms,
- in $6pq - 3p = 3p(2q - 1)$, 3 and p are the common factors of the two terms.

Worked Example 7

(Factorisation of Algebraic Expressions)

Factorise each of the following expressions completely.

(a) $6x - 12$ **(b)** $4ay - 24az$

ATTENTION

Always check that no other factors can be taken out from the terms within the brackets in your answer. For example, in **(b)**, if we write $4ay - 24az = 4(ay - 6az)$, the factorisation is incomplete as a is also a factor of the expression.

Solution:

(a) $6x - 12$
$= 6(x - 2)$

(b) $4ay - 24az$
$= 4a(y - 6z)$

PRACTISE NOW 7

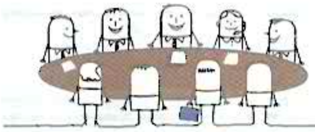
Factorise each of the following expressions completely.

(a) $-10x + 25$ **(b)** $18a - 54ay + 36az$

SIMILAR QUESTIONS

Exercise 4C Questions 4(a)–(e), 7(a)–(e)

Simplify $(x - a)(x - b)(x - c)\ldots(x - z)$.



Class Discussion

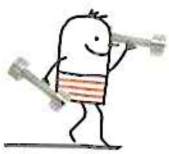
Equivalent Expressions

Work in pairs.

Some algebraic expressions, which consist of five pairs of equivalent expressions, are given in Table 4.4. An example of a pair of equivalent expressions is $3x - 12$ and $3(x - 4)$ as $3x - 12 = 3(x - 4)$. Match and justify each pair of equivalent expressions.

A $\dfrac{1-x}{6}$	B $\dfrac{-23x+75}{12}$	C $7(ay - 7y)$	D $3(x - 2y) - 2(3x - y)$	E $\dfrac{x-3}{2} - \dfrac{2x-5}{3}$
F $-3x - 4y$	G $\dfrac{3(x+3)}{4} - \dfrac{4(2x+3)}{3}$	H $\dfrac{-x-19}{6}$	I $29x - 3y$	J $7y(a - 7)$
K $7ay - 49y$	L $-25x - 9y$	M $2x - 3[5x - y - 2(7x - y)]$	N $\dfrac{-23x-21}{12}$	O $-3x - 8y$

Table 4.4



Exercise 4C

BASIC LEVEL

1. Simplify each of the following expressions.

(a) $\frac{1}{4}x + \frac{1}{5}y - \frac{1}{6}x - \frac{1}{10}y$

(b) $\frac{2}{3}a - \frac{1}{7}b + 2a - \frac{3}{5}b$

(c) $\frac{5}{9}c + \frac{3}{4}d - \frac{7}{8}c - \frac{4}{3}d$

(d) $2f - \frac{5}{3}h + \frac{9}{4}k - \frac{1}{2}f - \frac{28}{5}k + \frac{5}{4}h$

2. Expand and simplify each of the following expressions.

(a) $5a + 4b - 3c - \left(2a - \frac{3}{2}b + \frac{3}{2}c\right)$

(b) $\frac{1}{2}[2x + 2(x - 3)]$

(c) $\frac{2}{5}[12p - (5 + 2p)]$

(d) $\frac{1}{2}[8x + 10 - 6(1 - 4x)]$

3. Express each of the following as a fraction in its simplest form.

(a) $\frac{x}{2} + \frac{2x}{5}$ **(b)** $\frac{a}{3} - \frac{a}{4}$

(c) $\frac{2h}{7} + \frac{h+1}{5}$ **(d)** $\frac{3x}{8} - \frac{x+2}{4}$

(e) $\frac{4x+1}{5} + \frac{3x-1}{2}$ **(f)** $\frac{3y-1}{4} - \frac{2y-3}{6}$

(g) $\frac{a-2}{4} - \frac{a+7}{8}$ **(h)** $\frac{3p-2q}{3} - \frac{4p-5q}{4}$

4. Factorise each of the following expressions completely.

(a) $12x - 9$ **(b)** $-25y - 35$

(c) $27b - 36by$ **(d)** $8ax + 12a - 4az$

(e) $4m - 6my - 18mz$

INTERMEDIATE LEVEL

5. Expand and simplify each of the following expressions.

   (a) $y - \frac{2}{3}(9x - 3y)$

   (b) $-\frac{1}{3}\{6(p + q) - 3[p - 2(p - 3q)]\}$

6. Express each of the following as a fraction in its simplest form.

   (a) $\frac{7(x+3)}{2} + \frac{5(2x-5)}{3}$

   (b) $\frac{3x-4}{5} - \frac{3(x-1)}{2}$

   (c) $\frac{3(z-2)}{4} - \frac{4(2z-3)}{5}$

   (d) $\frac{2(p-4q)}{3} - \frac{3(2p+q)}{2}$

   (e) $-\frac{2b}{3} - \frac{3(a-2b)}{5}$

   (f) $\frac{2(x+3)}{5} - \frac{1}{2} + \frac{3x-4}{4}$

   (g) $\frac{a+1}{2} - \frac{a+3}{3} - \frac{5a-2}{4}$

   (h) $\frac{x+1}{2} + \frac{x+3}{3} - \frac{5x-1}{6}$

   (i) $\frac{2(a-b)}{7} - \frac{2a+3b}{14} + \frac{a+b}{2}$

   (j) $\frac{x+3}{3} + \frac{5(3x+4)}{6} + 1$

ADVANCED LEVEL

7. Factorise each of the following expressions completely.

   (a) $5x + 10x(b + c)$

   (b) $3xy - 6x(y - z)$

   (c) $2x(7 + y) - 14x(y + 2)$

   (d) $-3a(2 + b) + 18a(b - 1)$

   (e) $-4y(x - 2) - 12y(3 - x)$

8. Express each of the following as a fraction in its simplest form.

   (a) $\frac{5(p-q)}{2} - \frac{2q-p}{14} - \frac{2(p+q)}{7}$

   (b) $-\frac{2a+b}{3} - \frac{3(a-3b)}{2} - \frac{4(a+2b)}{5}$

   (c) $\frac{3(f-h)}{4} - \frac{7(h+k)}{6} + \frac{5(k-f)}{2}$

   (d) $4 - \frac{x-y}{3} - \frac{3(y+4z)}{4} + \frac{5(x+3z)}{8}$

1. In algebra, we use symbols, e.g. x, y, a^2 and xy, to represent numbers. The **linear expression** $3x - 4y + 5$ consists of three terms, namely $3x$, $-4y$ and 5.

   → The **constant** is 5.
   → The **variable** is y and its coefficient is -4.
   → The **variable** is x and its coefficient is 3.

2. **Evaluating** an algebraic expression means finding the value of the expression when the variables take on certain values.

3. We add or subtract the **like terms** by adding or subtracting the coefficients. We *do not* add or subtract the coefficients of **unlike terms**, e.g. adding $2x$ and $3y$ gives $2x + 3y$.

4. Simplification of algebraic expressions with different numerical denominators involves finding the LCM of the denominators.

5. **Distributive Law:** $a(b + c) = ab + ac$ ('a groups of b and c' is the same as 'a groups of b' and 'a groups of c', i.e. 'a times of b' and 'a times of c')

6. To simplify an algebraic expression that contains brackets, work with the expressions within the brackets first. If there is more than one pair of brackets, simplify the expression within the innermost pair of brackets first.

7. One way in which **factorisation** of algebraic expressions can be done is by extracting common factors from *all* the terms in the given expressions. It is the *reverse* of expansion.

Review Exercise 4

1. Given that $a = -2$ and $b = 7$, evaluate each of the following expressions.
   (a) $4a + 5b$ (b) $2a^2$ (c) $(2a)^2$
   (d) $a(b - a)$ (e) $b - a^2$ (f) $(b - a)^2$

2. Find the value of $\dfrac{3x - 5y^2 - 2xyz}{\dfrac{x}{y} - \dfrac{y^2}{z}}$ when $x = 3$, $y = -4$ and $z = 2$.

3. Expand and/or simplify each of the following expressions.
   (a) $3ab - 5xy + 4ab + 2yx$
   (b) $4(3p - 5q) + 6(2q - 5p)$
   (c) $2a + 3[a - (b - a)] + 7(2b - a)$
   (d) $-2[3x - (4x - 5y) - 2(3x - 4y)]$
   (e) $4\{h - 3[f - 6(f - h)]\}$
   (f) $5(x + 5y) - \{2x - [3x - 3(x - 2y) + y]\}$

4. Express each of the following as a fraction in its simplest form.
   (a) $\dfrac{2x}{3} + \dfrac{5 - x}{4}$
   (b) $\dfrac{x - y}{8} - \dfrac{3x - 2y}{12}$
   (c) $\dfrac{4(2a - b)}{3} - \dfrac{2(3a + b)}{5}$
   (d) $\dfrac{h + f}{3} - \dfrac{f + k}{2} + \dfrac{4h - k}{5}$
   (e) $3q - \dfrac{4p - 3q}{5} - \dfrac{q - 4p}{6}$
   (f) $\dfrac{4(x - 5)}{7} - \left[\dfrac{5(x - y)}{6} + \dfrac{7x - y}{21}\right]$

5. Factorise each of the following expressions completely.
   (a) $21pq + 14q - 28qr$
   (b) $4x - 8(y - 2z)$

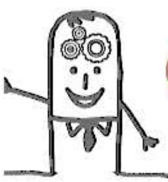
6. A collection of coins contains only 10-cent and 5-cent coins. There are x 5-cent coins in the collection. Find
   (a) the total value of the 5-cent coins,
   (b) the total value of the 10-cent coins if there are three times as many 10-cent as 5-cent coins,
   (c) the total value of the coins if there are seven 5-cent coins for every three 10-cent coins.

7. Farhan cycles x km in 3 hours. If he maintains the same average speed, how far can he cycle in y minutes?

8. (a) Find the difference between $3y$ minutes and $25y$ seconds, giving your answer in seconds.
   (b) Find the sum of $50(3z - 2)$ minutes and $4(z + 1)$ hours, giving your answer in seconds.

9. Shirley and Kate worked during their school holidays. Shirley was paid $\$x$ per hour and Kate was paid $\$y$ per hour. Their overtime rates were 1.5 times their hourly rates. Shirley worked 25 hours, of which 5 were overtime hours, while Kate worked 18 hours, of which 4 were overtime hours.
   (i) Find the total amount they earned.
   (ii) If Shirley was paid \$5.50 per hour and Kate was paid \$0.50 more than her, how much did they earn altogether during the school holidays?

10. An examination consists of 3 papers. The minimum total score required to pass the examination is $(10p + 5q)$ marks. Michael scores $(p - 3q + 13)$ marks and $(3p + 5q - 4)$ marks in the first two papers.
   (i) Find Michael's total score in the first two papers.
   (ii) Given that Michael obtained the minimum total score required to pass the examination, find his score in the third paper.
   (iii) Factorise the result in (ii).



Challenge Yourself

1. A teacher is showing a magic trick to his students. He places 12 coins on a table, 5 of which are heads up and 7 are tails up. He then places a blindfold over his eyes and shuffles the coins, keeping the faces up. Next, he separates the coins into two piles of 5 and 7 respectively. He flips over all the coins in the smaller pile. Show that both piles now have the same number of heads up.

   Note: This is the mathematics behind the chapter opener.

2. If x, y and z are positive integers such that $x < y < z$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, find all the possible values of x, y and z. Prove that no other values of x, y and z are able to satisfy the above conditions.

3. Suppose we wish to multiply a 2-digit number with another 2-digit number. If the digits in the tens place are the same and their ones digits add up to ten, there is a shortcut to it! For example,

multiply the digit in the tens place with its next integer i.e. $7 \times 8 = 56$	$\begin{array}{r} 74 \\ \times\ 76 \\ \hline 56\ 24 \end{array}$	multiply the ones digits together i.e. $4 \times 6 = 24$

   Try this method to find 58×52. Then use a calculator to evaluate 58×52. Do you arrive at the same answer? Explain why this works by using algebra.

A1 Revision Exercise

1. (a) Find the HCF of 42, 66 and 78.
   (b) Find the LCM of 7, 13 and 14.

2. The numbers 405 and 1960, written as the products of their prime factors, are $405 = 3^4 \times 5$ and $1960 = 2^3 \times 5 \times 7^2$. Hence, find
   (i) the greatest whole number which is a factor of both 405 and 1960,
   (ii) the smallest whole number that is divisible by both 405 and 1960.

3. Mr Lee has agreed to sponsor 105 bags of crisps and 126 packets of fruit juice for a class gathering. Each item is to be equally distributed among the students. Find
   (i) the greatest number of students that the refreshment can cater to,
   (ii) the number of bags of crisps each student will receive,
   (iii) the number of packets of fruit juice each student will receive.

4. The numbers −2, −1, 1, 2, 3, 4, 5 and 6 are written on eight separate cards, with one number on each card. List
   (i) the pairs of cards that have a sum of 4,
   (ii) the pairs of cards that have a product of 2,
   (iii) the groups of three cards that have a sum of 10.

5. Estimate each of the following without using a calculator.
   (a) $101 \times \sqrt{80.7}$
   (b) $\sqrt[3]{26} \times 502 \div 49$
   (c) $\sqrt{65} \times \sqrt[3]{63} \div 17$

6. Lixin takes 6.8 minutes to walk from her home to the shopping mall 628 m away. Write down a calculation you could do mentally to estimate the average speed for her journey in m/s.

7. Given that $a = 1$, $b = 2$, $c = 0$ and $d = -3$, evaluate each of the following expressions without the use of a calculator.
   (a) $\dfrac{a^2bd}{3ac - d}$
   (b) $\dfrac{bc + d^2}{a + b}$
   (c) $a^2 + b^2 - c^2 + d^2$
   (d) $-a^3 - b^3 + c^3 - d^3$

8. An apple costs a cents while a pear costs b cents more than an apple. Find the total cost of 10 apples and one dozen pears, giving your answer in dollars.

A2 Revision Exercise

1. **(a)** Find the HCF of 54, 126 and 342.

(b) Find the LCM of 16, 28, 44 and 68.

2. By using prime factorisation, find the value of each of the following.

(a) $-\sqrt{9216}$

(b) $\sqrt[3]{8000}$

3. Given that the HCF and the LCM of 1764 and a number p are 36 and 8820 respectively, find the value of p.

4. At midnight, the temperature of a town in the northern hemisphere is –6 °C. At 8 a.m., the temperature has risen by 8 °C. By 6 p.m., the temperature has dropped by 4 °C. Find

(i) the temperature of the town at 6 p.m.,

(ii) the overall change in the temperature from midnight to 6 p.m., stating whether there is an overall increase or decrease.

5. **(a)** Without using a calculator, find the value of $\frac{2}{3} - \left(-3\frac{1}{20}\right) + \left(-\frac{4}{5}\right)$.

(b) Use a calculator to evaluate each of the following, giving your answer correct to 3 decimal places.

(i) $[-4.749 - 6.558 \times (-2.094)^3] \div \sqrt[3]{-1.999}$

(ii) $\left\{\left(\frac{1}{3}\right)^2 - \sqrt[3]{\frac{8}{33}} \times \left[-\sqrt{\frac{5}{6}} - (-0.375)^3\right]\right\} \times [-\pi \div (-6.5)]$

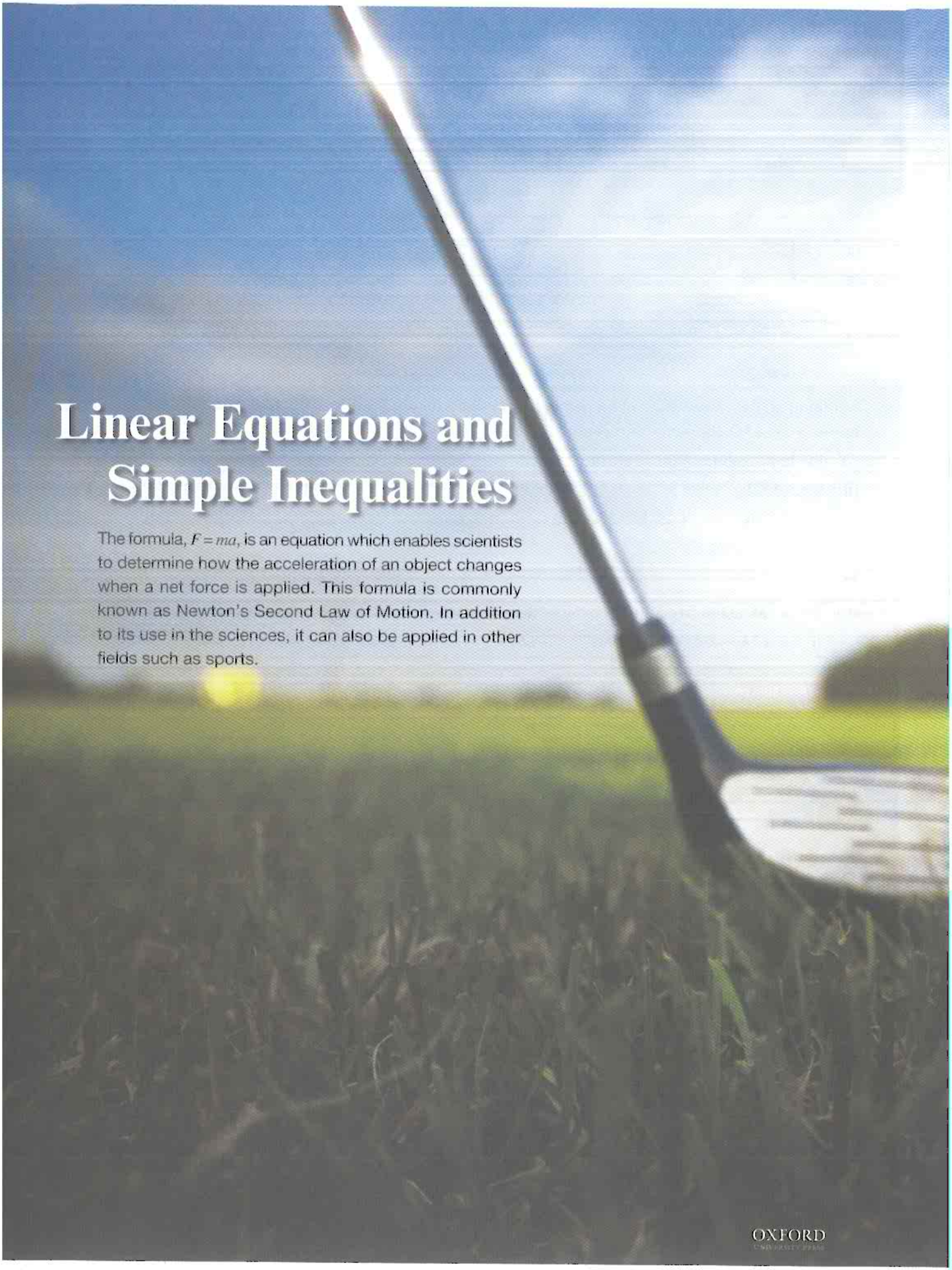
6. Kate has two boxes of buttons. After she transfers 15 buttons from Box A to Box B, the number of buttons in Box B becomes $\frac{5}{7}$ of the number of buttons in Box A. If there are 35 buttons in Box B originally, find the initial number of buttons in Box A.

7. Devi would like to buy a rectangular carpet for her living room. The carpet measures $4\frac{1}{10}$ m by $2\frac{9}{10}$ m.

(i) Write down a calculation you could do mentally to estimate the area of the carpet.

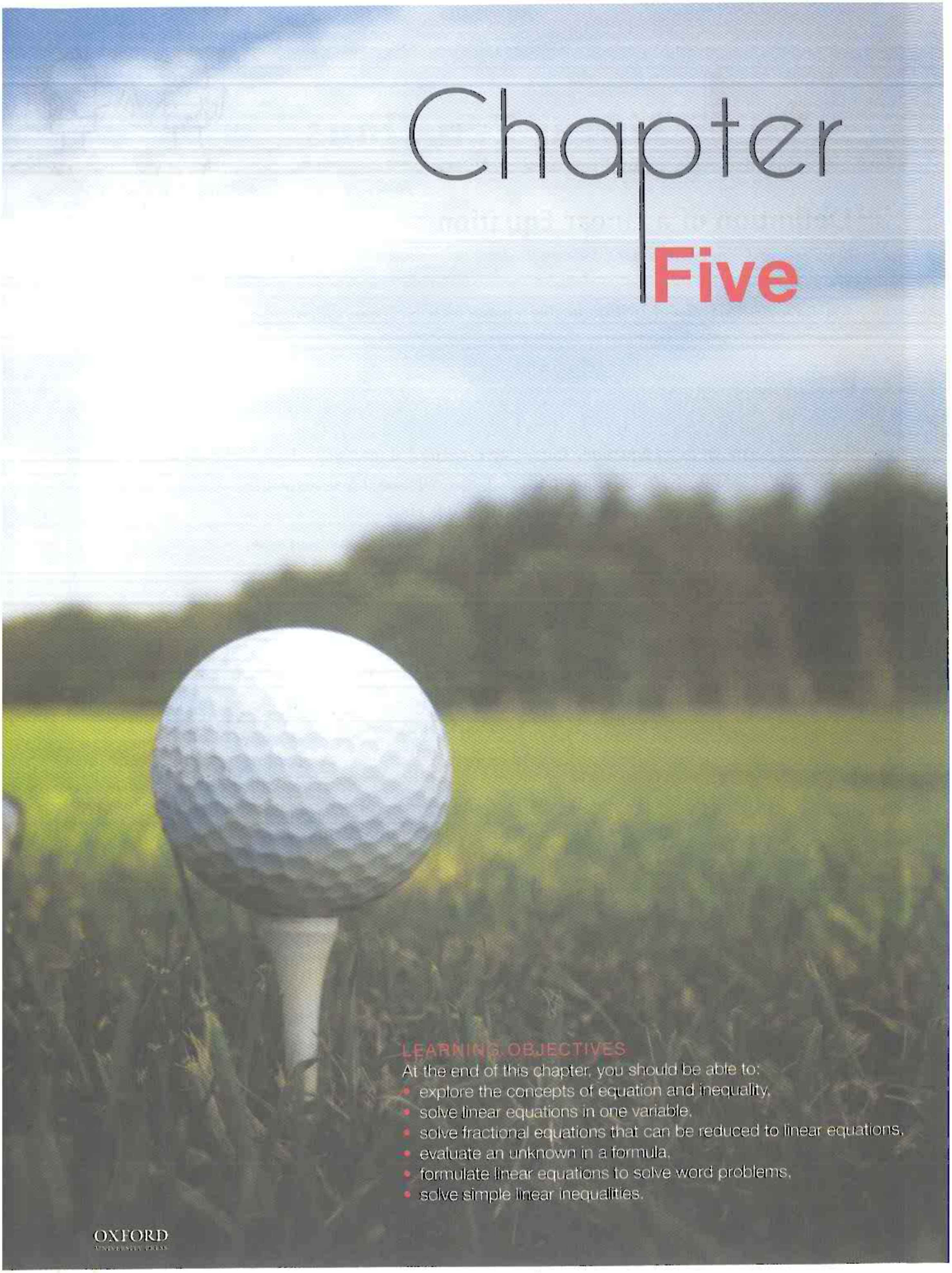
(ii) Given that each square metre of the carpet costs \$89.75, write down a calculation you could do mentally to estimate the cost of the carpet.

8. Subtract the sum of $-7x + 4$ and $5x + 7$ from the sum of $-8x + 9$ and $15 - 4x$.



Linear Equations and Simple Inequalities

The formula, $F = ma$, is an equation which enables scientists to determine how the acceleration of an object changes when a net force is applied. This formula is commonly known as Newton's Second Law of Motion. In addition to its use in the sciences, it can also be applied in other fields such as sports.



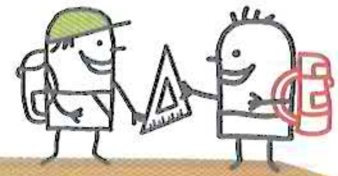
Chapter Five

LEARNING OBJECTIVES

At the end of this chapter, you should be able to:

- explore the concepts of equation and inequality,
- solve linear equations in one variable,
- solve fractional equations that can be reduced to linear equations,
- evaluate an unknown in a formula,
- formulate linear equations to solve word problems,
- solve simple linear inequalities.

5.1 Linear Equations



Definition of a Linear Equation

In primary school, we have come across mathematical sentences such as

$$7 + \square = 13,$$

where we need to replace $\square$ with a suitable number such that the sentence is true. If we replace $\square$ with 6, we have

$$7 + 6 = 13.$$

If we replace $\square$ with x, we have

$$7 + x = 13.$$

$7 + x = 13$ is known as an **equation**. The '=' sign means that the total value on the left-hand side (LHS) of the equation, i.e. $7 + x$, must be the same as the value on the right-hand side (RHS) of the equation, i.e. 13.

To solve $7 + x = 13$ means to find the value of x so that the values on both sides of the equation are equal, i.e. x *satisfies* the equation. For example, if we substitute x with 1, $7 + x = 7 + 1 = 8$, which is not equal to 13. Thus $x = 1$ does not satisfy the equation. However, if we substitute x with 6, $7 + x = 7 + 6 = 13$. Hence, $x = 6$ satisfies the equation. $x = 6$ is called the **solution** of the equation.

Table 5.1 shows some examples of linear and non-linear equations.

Linear equations	Non-linear equations
$x - 1 = 0$	$x^2 - 2 = 0$
$\frac{x}{4} = 12$	$\frac{x^3}{2} = 6$
$6x + 5 = 17$	$4x^3 + 11 = 15$
$y = 3x - 6$	$9\sqrt{x} - y = 3$
$x + y + z = 4$	$xy + \frac{x}{y} = 3$

Table 5.1

ATTENTION

While $x + 1$ is a **linear expression**, $x + 1 = 0$ is a **linear equation**.

From Table 5.1, discuss with your classmate what a linear equation is.

$6x + 5 = 17$ is an example of a **linear equation** in *one* variable. In this chapter, we will learn about linear equations in one variable only.

Solving Linear Equations

We have mentioned that the '=' sign in an equation means the value of the expression on the LHS is the same as that on the RHS. Let us use the idea of a balance to further explore this.

In Fig. 5.1, the number on the left scale pan is equal to the number on the right scale pan, i.e. $2 + 3 = 5$.

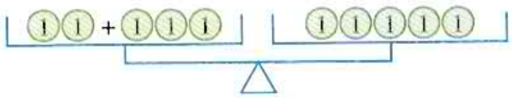


Fig. 5.1

If we add 1 to the number on the right scale pan, the balance will be tilted as the number on the left scale pan is *not* equal to the number on the right scale pan, i.e. $2 + 3 \neq 5 + 1$ (see Fig. 5.2**(a)**). In fact, $2 + 3 < 5 + 1$ (since $2 + 3 = 5$). In order to maintain the balance, we need to add 1 to the number on the left scale pan, such that $2 + 3 + 1 = 5 + 1$ (see Fig. 5.2**(b)**).

(a) **(b)**

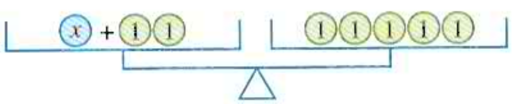
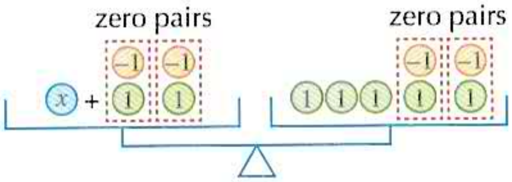
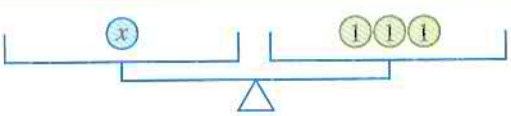
Fig. 5.2

Likewise, we need to subtract the same number from the numbers on both sides, or multiply or divide the numbers on both sides by the same number, in order to maintain the balance.

Now, we shall use the 'Balance Method' to illustrate how to solve linear equations.

Case 1: $x + b = d$, where b and d are constants

Example: $x + 2 = 5$

	$x + 2 = 5$
Add 2 -1 to both scale pans zero pairs zero pairs 	Add -2 to both sides: $x + 2 - 2 = 5 - 2$
	Simplify both sides: $x = 3$

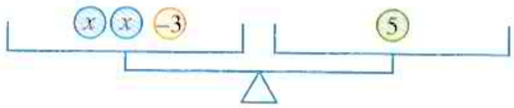
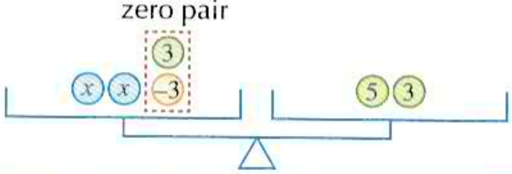
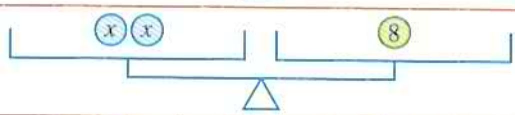
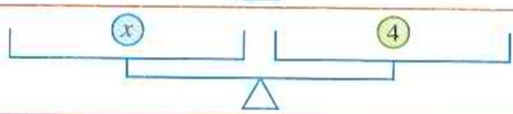
PRACTISE NOW

Solve each of the following equations. You may visit http://www.shinglee.com.sg/StudentResources/ to access the AlgeTools™ software to form each linear equation and solve it.

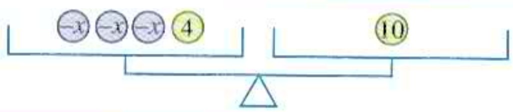
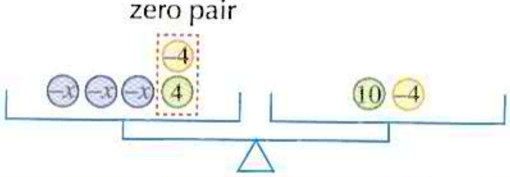
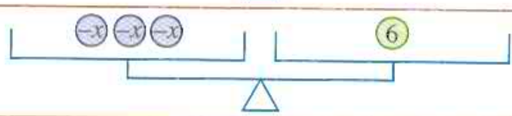
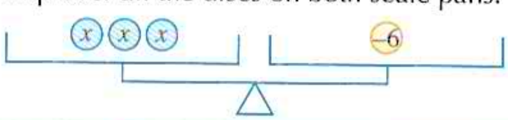
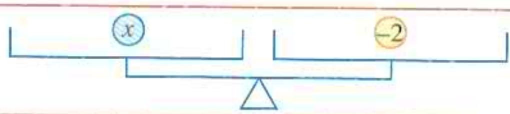
(a) $x + 3 = 7$ **(b)** $x - 7 = 6$ **(c)** $x + 3 = -7$ **(d)** $x - 2 = -3$

Case 2: $ax + b = d$, where a, b and d are constants

Example: $2x - 3 = 5$

<table>
<tr><td></td><td>$2x - 3 = 5$</td></tr>
<tr><td>Add 3 to both scale pans.
zero pair
</td><td>Add 3 to both sides:
$2x - 3 + 3 = 5 + 3$</td></tr>
<tr><td></td><td>Simplify both sides:
$2x = 8$</td></tr>
<tr><td></td><td>Divide by 2 on both sides:
$x = 4$</td></tr>
</table>

Example: $-3x + 4 = 10$

<table>
<tr><td></td><td>$-3x + 4 = 10$</td></tr>
<tr><td>Add –4 to both scale pans.
zero pair
</td><td>Add –4 to both sides:
$-3x + 4 - 4 = 10 - 4$</td></tr>
<tr><td></td><td>Simplify both sides:
$-3x = 6$</td></tr>
<tr><td>Flip over all the discs on both scale pans.
</td><td>Multiply by –1 on both sides (change sign):
$3x = -6$</td></tr>
<tr><td></td><td>Divide by 3 on both sides:
$x = -2$</td></tr>
</table>

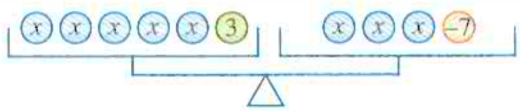
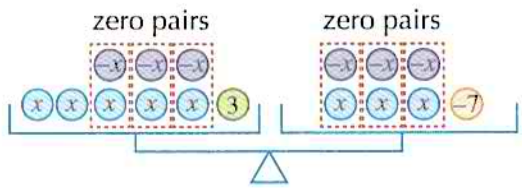
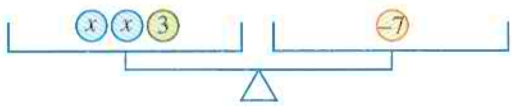
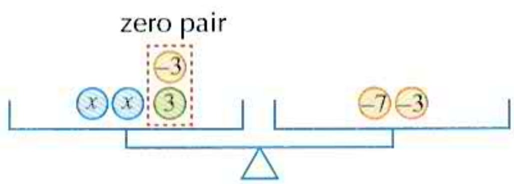
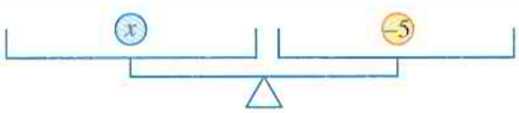
PRACTISE NOW

Solve each of the following equations. You may visit http://www.shinglee.com.sg/StudentResources/ to access the AlgeTools™ software to form each linear equation and solve it.

(a) $2x - 5 = 5$ **(b)** $3x + 4 = 7$ **(c)** $-3x + 3 = 9$ **(d)** $-5x - 2 = 13$

Case 3: $ax + b = cx + d$, where a, b, c and d are constants

Example: $5x + 3 = 3x - 7$

	$5x + 3 = 3x - 7$
Add 3 $-x$ to both scale pans. 	Add $-3x$ to both sides: $5x - 3x + 3 = 3x - 3x - 7$
	Simplify both sides: $2x + 3 = -7$
Add -3 to both scale pans. 	Add -3 to both sides: $2x + 3 - 3 = -7 - 3$
	Simplify both sides: $2x = -10$
	Divide by 2 on both sides: $x = -5$

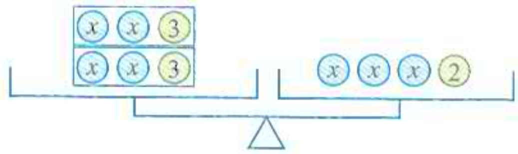
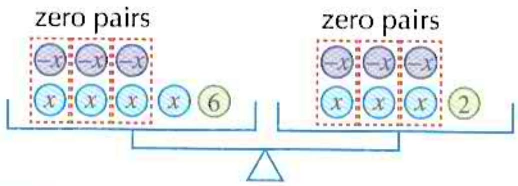
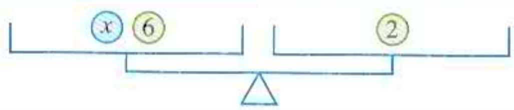
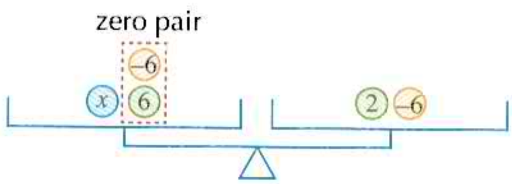
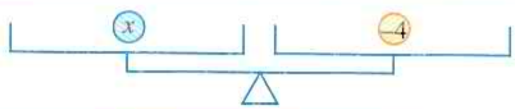
PRACTISE NOW

Solve each of the following equations. You may visit http://www.shinglee.com.sg/StudentResources/ to access the AlgeTools™ software to form each linear equation and solve it.

(a) $3x + 4 = x - 10$ **(b)** $4x - 2 = x + 7$ **(c)** $3x - 2 = -x + 14$ **(d)** $-2x - 5 = 5x - 12$

Case 4: $a(bx + c) = dx + e$**, where** a**,** b**,** c**,** d **and** e **are constants**

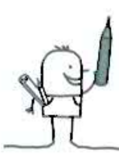
Example: $2(2x + 3) = 3x + 2$

<table>
<tr><td></td><td>$2(2x + 3) = 3x + 2$</td></tr>
<tr><td></td><td>Expand the expression on the LHS:
$4x + 6 = 3x + 2$</td></tr>
<tr><td>Add 3 $-x$ to both scale pans.
</td><td>Add $-3x$ to both sides:
$4x - 3x + 6 = 3x - 3x + 2$</td></tr>
<tr><td></td><td>Simplify both sides:
$x + 6 = 2$</td></tr>
<tr><td>Add -6 to both scale pans.
</td><td>Add -6 to both sides:
$x + 6 - 6 = 2 - 6$</td></tr>
<tr><td></td><td>Simplify both sides:
$x = -4$</td></tr>
</table>

PRACTISE NOW

Solve each of the following equations. You may visit http://www.shinglee.com.sg/StudentResources/ to access the AlgeTools™ software to form each linear equation and solve it.

(a) $2(x - 3) = -3x + 4$ **(b)** $2(x + 3) = 5x - 9$ **(c)** $-2(x + 2) = 3x - 9$ **(d)** $-2(3x - 4) = 4(2x + 5)$



Journal Writing

Based on Cases 3 and 4, reflect on the process of solving a linear equation by considering the following questions:

1. What is meant by solving an equation?
2. How do you determine the sequence of operations?

Worked Example 1

(Solving Linear Equations)

Solve each of the following equations.

(a) $2x + 3 = 9$ **(b)** $5x - 9 = 3x + 3$

(c) $2(2y - 3) = 5(y - 1)$ **(d)** $2(3y + 5) - 7(y - 4) = 13$

Solution:

(a)

$$2x + 3 = 9$$
$$2x + 3 - 3 = 9 - 3 \text{ (subtract 3 from both sides)}$$
$$2x = 9 - 3$$
$$2x = 6 \text{ (simplify the terms on the RHS)}$$
$$\frac{2x}{2} = \frac{6}{2} \text{ (divide by 2 on both sides)}$$
$$\therefore x = \frac{6}{2}$$
$$= 3$$

Problem Solving Tip

It is a good practice to check your solution by substituting the value of the unknown which you have found into the original equation, e.g. in **(a)**, LHS = $2(3) + 3 = 6 + 3 = 9$ = RHS.

(b)

$$5x - 9 = 3x + 3$$
$$5x - 3x - 9 = 3x - 3x + 3 \text{ (subtract } 3x \text{ from both sides)}$$
$$5x - 3x - 9 = 3$$
$$2x - 9 = 3 \text{ (simplify the terms on the LHS)}$$
$$2x - 9 + 9 = 3 + 9 \text{ (add 9 to both sides)}$$
$$2x = 3 + 9$$
$$2x = 12 \text{ (simplify the terms on the RHS)}$$
$$\frac{2x}{2} = \frac{12}{2} \text{ (divide by 2 on both sides)}$$
$$\therefore x = \frac{12}{2}$$
$$= 6$$

(c)

$$2(2y - 3) = 5(y - 1)$$
$$4y - 6 = 5y - 5 \text{ (Distributive Law)}$$
$$4y - 5y - 6 = 5y - 5y - 5 \text{ (subtract } 5y \text{ from both sides)}$$
$$4y - 5y - 6 = -5$$
$$-y - 6 = -5 \text{ (simplify the terms on the LHS)}$$
$$-y - 6 + 6 = -5 + 6 \text{ (add 6 to both sides)}$$
$$-y = -5 + 6$$
$$-y = 1 \text{ (simplify the terms on the RHS)}$$
$$-1 \times (-y) = -1 \times 1 \text{ (multiply by } -1 \text{ on both sides)}$$
$$\therefore y = -1 \times 1$$
$$= -1$$

(d)

$$2(3y + 5) - 7(y - 4) = 13$$
$$6y + 10 - 7y + 28 = 13 \text{ (Distributive Law)}$$
$$6y - 7y + 10 + 28 = 13 \text{ (group like terms)}$$
$$-y + 38 = 13 \text{ (simplify the terms on the LHS)}$$
$$-y + 38 - 38 = 13 - 38 \text{ (subtract 38 from both sides)}$$
$$-y = 13 - 38$$
$$-y = -25 \text{ (simplify the terms on the RHS)}$$
$$-1 \times (-y) = -1 \times (-25) \text{ (multiply by } -1 \text{ on both sides)}$$
$$\therefore y = -1 \times (-25)$$
$$= 25$$

PRACTISE NOW 1

SIMILAR QUESTIONS

Exercise 5A Questions 1(a)–(g), 2(a)–(j), 3(a)–(d), 4(a)–(n), 8(a)–(e)

1. Solve each of the following equations.
   (a) $x + 9 = 4$ **(b)** $3x - 2 = 4$ **(c)** $7x + 2 = 2x - 13$
   (d) $3(3y + 4) = 2(2y + 1)$ **(e)** $2(y - 1) + 3(y - 1) = 4 - 2y$
2. Solve each of the following equations.
   (a) $x + 0.7 = 2.7$ **(b)** $2y - 1.3 = 2.8$

Thinking Time

$x = 3$, $x + 3 = 6$, $2x + 3 = 9$ and $10x - 4 = 5x + 11$ are known as **equivalent equations** as they have the same solution. State four equivalent equations that have the solution $y = -1$.

Solving Fractional Equations

Worked Example 2

(Solving Fractional Equations)

Solve each of the following equations.

(a) $\frac{1}{3}x - 8 = 6$ **(b)** $\frac{5}{2}y + 3\frac{1}{2} = \frac{2}{3}y + 5$ **(c)** $\frac{z+2}{3} = \frac{3z+2}{5}$

Solution:

(a)
$$\frac{1}{3}x - 8 = 6$$
$$\frac{1}{3}x - 8 + 8 = 6 + 8 \text{ (add 8 to both sides)}$$
$$\frac{1}{3}x = 6 + 8$$
$$\frac{1}{3}x = 14 \text{ (simplify the terms on the RHS)}$$
$$3 \times \frac{1}{3}x = 3 \times 14 \text{ (multiply by 3 on both sides)}$$
$$\therefore x = 3 \times 14 = 42$$

(b)
$$\frac{5}{2}y + 3\frac{1}{2} = \frac{2}{3}y + 5$$
$$\frac{5}{2}y - \frac{2}{3}y + 3\frac{1}{2} = \frac{2}{3}y - \frac{2}{3}y + 5 \text{ (subtract } \tfrac{2}{3}y \text{ from both sides)}$$
$$\frac{5}{2}y - \frac{2}{3}y + 3\frac{1}{2} = 5$$
$$\frac{11}{6}y + 3\frac{1}{2} = 5 \text{ (simplify the terms on the LHS)}$$
$$\frac{11}{6}y + 3\frac{1}{2} - 3\frac{1}{2} = 5 - 3\frac{1}{2} \text{ (subtract } 3\tfrac{1}{2} \text{ from both sides)}$$
$$\frac{11}{6}y = 5 - 3\frac{1}{2}$$
$$\frac{11}{6}y = 1\frac{1}{2} \text{ (simplify the terms on the RHS)}$$
$$\frac{11}{6}y \div \frac{11}{6} = 1\frac{1}{2} \div \frac{11}{6} \text{ (divide by } \tfrac{11}{6} \text{ on both sides)}$$
$$\therefore y = 1\frac{1}{2} \div \frac{11}{6}$$
$$= \frac{3}{2} \times \frac{6}{11}$$
$$= \frac{9}{11}$$

(c) $\frac{z+2}{3} = \frac{3z+2}{5}$

$15 \times \frac{z+2}{3} = 15 \times \frac{3z+2}{5}$ (multiply by the lowest common multiple (LCM) of 3 and 5, i.e. 15, on both sides)

$5(z+2) = 3(3z+2)$

$5z + 10 = 9z + 6$ (Distributive Law)

$5z - 9z + 10 = 9z - 9z + 6$ (subtract $9z$ from both sides)

$5z - 9z + 10 = 6$

$-4z + 10 = 6$ (simplify the terms on the LHS)

$-4z + 10 - 10 = 6 - 10$ (subtract 10 from both sides)

$-4z = 6 - 10$

$-4z = -4$ (simplify the terms on the RHS)

$-1 \times (-4z) = -1 \times (-4)$ (multiply by -1 on both sides)

$4z = 4$

$\frac{4z}{4} = \frac{4}{4}$ (divide by 4 on both sides)

$\therefore z = \frac{4}{4}$

$= 1$

PRACTISE NOW 2

Solve each of the following equations.

(a) $\frac{x}{2} + 9 = 5$ **(b)** $\frac{5}{7}y + 2 = \frac{1}{2}y + 3\frac{1}{4}$ **(c)** $\frac{3z-1}{2} = \frac{z-4}{3}$

SIMILAR QUESTIONS

Exercise 5A Questions 5(a)–(f), 6(a)–(d), 9(a)–(f), 11(a)–(f), 12

Worked Example 3

(Solving Fractional Equations)

Solve each of the following equations.

(a) $\frac{9}{2x-5} = 3$ **(b)** $\frac{y+4}{2y-3} = \frac{2}{5}$

Solution:

(a) $\frac{9}{2x-5} = 3$

$(2x-5) \times \frac{9}{2x-5} = (2x-5) \times 3$ (multiply by $(2x-5)$ on both sides)

$9 = 3(2x-5)$

$9 = 6x - 15$ (Distributive Law)

$6x - 15 = 9$

$6x - 15 + 15 = 9 + 15$ (add 15 to both sides)

$6x = 9 + 15$

$6x = 24$ (simplify the terms on the RHS)

$\frac{6x}{6} = \frac{24}{6}$ (divide by 6 on both sides)

$\therefore x = \frac{24}{6}$

$= 4$

ATTENTION

Since LHS = RHS in an equation, we can write $9 = 6x - 15$ as $6x - 15 = 9$.

(b) $$\frac{y+4}{2y-3} = \frac{2}{5}$$

$$5(2y-3) \times \frac{y+4}{2y-3} = 5(2y-3) \times \frac{2}{5} \text{ (multiply by } 5(2y-3) \text{ on both sides)}$$

$$5(y+4) = 2(2y-3)$$

$$5y + 20 = 4y - 6 \text{ (Distributive Law)}$$

$$5y - 4y + 20 = 4y - 4y - 6 \text{ (subtract } 4y \text{ from both sides)}$$

$$5y - 4y + 20 = -6$$

$$y + 20 = -6 \text{ (simplify the terms on the LHS)}$$

$$y + 20 - 20 = -6 - 20 \text{ (subtract 20 from both sides)}$$

$$\therefore y = -6 - 20$$

$$= -26$$

PRACTISE NOW 3

SIMILAR QUESTIONS

Exercise 5A Questions 7(a)–(b), 10(a)–(h), 13–14

Solve each of the following equations.

(a) $\frac{8}{2x-3} = 4$ **(b)** $\frac{y-3}{y+4} = \frac{3}{2}$

Exercise 5A

BASIC LEVEL

1. Solve each of the following equations.

(a) $x + 8 = 15$ **(b)** $x + 9 = -5$
(c) $x - 5 = 17$ **(d)** $y - 7 = -3$
(e) $y + 0.4 = 1.6$ **(f)** $y - 2.4 = 3.6$
(g) $-2.7 + a = -6.4$

2. Solve each of the following equations.

(a) $4x = -28$ **(b)** $-24x = -144$
(c) $3x - 4 = 11$ **(d)** $9x + 4 = 31$
(e) $12 - 7x = 5$ **(f)** $3 - 7y = -18$
(g) $4y - 1.9 = 6.3$ **(h)** $-3y - 7.8 = -9.6$
(i) $7y - 2\frac{3}{4} = \frac{1}{2}$ **(j)** $1\frac{1}{2} - 2y = \frac{1}{4}$

3. Solve each of the following equations.

(a) $3x - 7 = 4 - 8x$ **(b)** $4x - 10 = 5x + 7$
(c) $30 + 7y = -2y - 6$ **(d)** $2y - 7 = 7y - 27$

4. Solve each of the following equations.

(a) $2(x + 3) = 8$ **(b)** $5(x - 7) = -15$
(c) $7(-2x + 4) = -4x$ **(d)** $3(2 - 0.4x) = 18$
(e) $2(2x - 2.2) = 4.6$ **(f)** $4(3y + 4.1) = 7.6$
(g) $3(2y + 3) = 4y + 3$ **(h)** $3(y + 1) = 4y - 21$
(i) $3(y + 2) = 2(y + 4)$ **(j)** $5(5y - 6) = 4(y - 7)$
(k) $2(3b - 4) = 5(b + 6)$ **(l)** $3(2c + 5) = 4(c - 3)$
(m) $9(2d + 7) = 11(d + 14)$ **(n)** $5(7f - 3) = 28(f - 1)$

5. Solve each of the following equations.

(a) $\frac{1}{3}x = 7$ **(b)** $\frac{3}{4}x = -6$
(c) $\frac{1}{3}x + 3 = 4$ **(d)** $\frac{y}{4} - 8 = -2$
(e) $3 - \frac{1}{4}y = 2$ **(f)** $15 - \frac{2}{5}y = 11$

6. Solve each of the following equations.

(a) $x = 12 - \frac{1}{3}x$ (b) $\frac{3}{5}x = \frac{1}{2}x + \frac{1}{2}$

(c) $\frac{y}{2} - \frac{1}{5} = 2 - \frac{y}{3}$ (d) $\frac{2}{3}y - \frac{3}{4} = 2y + \frac{5}{8}$

7. Solve each of the following equations.

(a) $\frac{2}{x} = \frac{4}{5}$ (b) $\frac{12}{y-1} = \frac{2}{3}$

INTERMEDIATE LEVEL

8. Solve each of the following equations.

(a) $-3(2 - x) = 6x$

(b) $5 - 3x = -6(x + 2)$

(c) $-3(9y + 2) = 2(-4y - 7)$

(d) $-3(4y - 5) = -7(-5 - 2y)$

(e) $3(5 - h) - 2(h - 2) = -1$

9. Solve each of the following equations.

(a) $\frac{5x+1}{3} = 7$ (b) $\frac{2x-3}{4} = \frac{x-3}{3}$

(c) $\frac{3x-1}{5} = \frac{x-1}{3}$ (d) $\frac{1}{4}(5y + 4) = \frac{1}{3}(2y - 1)$

(e) $\frac{2y-1}{5} - \frac{y+3}{7} = 0$ (f) $\frac{2y+3}{4} + \frac{y-5}{6} = 0$

10. Solve each of the following equations.

(a) $\frac{12}{x+3} = 2$ (b) $\frac{11}{2x-1} = 4$

(c) $\frac{32}{2x-5} - 3 = \frac{1}{4}$ (d) $\frac{1}{2} = \frac{1}{x+2} - 1$

(e) $\frac{y+5}{y-6} = \frac{5}{4}$ (f) $\frac{2y+1}{3y-5} = \frac{4}{7}$

(g) $\frac{2}{y-2} = \frac{3}{y+6}$ (h) $\frac{2}{7y-3} = \frac{3}{9y-5}$

11. Solve each of the following equations.

(a) $10x - \frac{5x+4}{3} = 7$

(b) $\frac{4x}{3} - \frac{x-1}{2} = 1\frac{1}{4}$

(c) $\frac{x-1}{3} - \frac{x+3}{4} = -1$

(d) $1 - \frac{y+5}{3} = \frac{3(y-1)}{4}$

(e) $\frac{6(y-2)}{7} - 12 = \frac{2(y-7)}{3}$

(f) $\frac{7-2y}{2} - \frac{2}{5}(2 - y) = 1\frac{1}{4}$

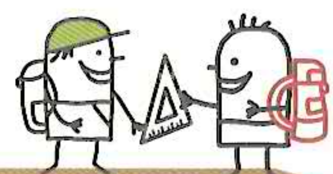
12. By showing your working clearly, verify if $x = \frac{19}{20}$ is the solution of the equation
$2x - \frac{3}{4} = \frac{1}{3}x + \frac{5}{6}$.

ADVANCED LEVEL

13. If $4x + y = 3x + 5y$, find the value of $\frac{3x}{16y}$.

14. If $\frac{3x-5y}{7x-4y} = \frac{3}{4}$, find the value of $\frac{x}{y}$.

5.2 Formulae



We have learnt that: area of a rectangle = length × breadth.

This can be written as a **formula**:

$$A = l \times b \quad \text{or} \quad A = lb,$$

where A, l and b are the area, the length and the breadth of the rectangle respectively.

In general, a formula expresses a rule in algebraic terms. It makes use of variables to write instructions for performing a calculation.

Can you think of other examples of formulae that are commonly used in mathematics or the sciences?

Worked Example 4

(Finding an Unknown in a Formula)

The formula for finding the volume V of a cuboid is given by $V = lbh$, where l, b and h represent the length, the breadth and the height of the cuboid respectively.

(a) If $l = 5$ cm, $b = 2$ cm and $h = 3$ cm, calculate the volume of the cuboid.

(b) If $V = 240$ cm^3, $b = 6$ cm and $h = 5$ cm, calculate the length of the cuboid.

Solution:

(a) $V = lbh$

When $l = 5$, $b = 2$, $h = 3$,

$$V = 5 \times 2 \times 3$$
$$= 30 \text{ cm}^3$$

Volume of the cuboid = 30 cm^3

(b) $V = lbh$

When $V = 240$, $b = 6$, $h = 5$,

$$l \times 6 \times 5 = 240$$
$$30l = 240$$
$$l = \frac{240}{30}$$
$$= 8 \text{ cm}$$

Length of the cuboid = 8 cm

PRACTISE NOW 4

SIMILAR QUESTIONS

Exercise 5B Questions 1–4

Newton's second law states that the net force F acting on a body is given by $F = ma$, where m is the mass and a is the acceleration of the body. The units for F, m and a are the Newton (N), the kilogram (kg) and metre per second squared (m s^{-2}) respectively.

(a) If $m = 1000$ kg and $a = 0.05$ m s^{-2}, find the net force acting on the body.

(b) If $F = 100$ N and $a = 0.1$ m s^{-2}, find the mass of the body.

Worked Example 5

(Finding an Unknown in a More Complicated Formula)

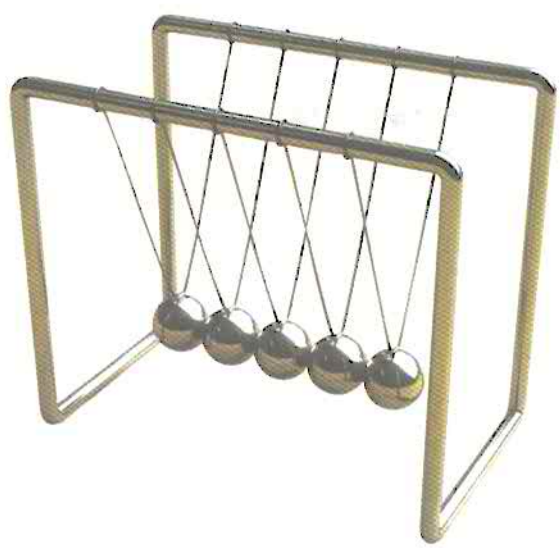
If $y + b = \frac{ay + c}{b}$, calculate the value of c when $y = 12$, $b = 3$ and $a = 14$.

Solution:

$$y + b = \frac{ay + c}{b}$$

When $y = 12$, $b = 3$, $a = 14$,

$$12 + 3 = \frac{14 \times 12 + c}{3}$$
$$15 = \frac{168 + c}{3}$$
$$3 \times 15 = 168 + c$$
$$45 = 168 + c$$
$$45 - 168 = c$$
$$\therefore c = -123$$



PRACTISE NOW 5

SIMILAR QUESTIONS

Exercise 5B Questions 6–14, 17

1. If $\frac{2x + y - 3z}{y + 3x} = \frac{x}{2y}$, find the value of z when $x = 1$ and $y = 4$.
2. If $t = \frac{v - u}{a}$, find the value of a when $t = 3$, $v = 2\frac{1}{2}$ and $u = 1\frac{1}{3}$.

Construction of Formulae

To construct a formula, we choose letters to represent quantities before expressing the rule in algebraic terms. Usually, the first letter of the quantity is used.
For example, the sum, S m, of the heights of two boys, one with a height of x m and the other with a height of y m, is expressed as $S = x + y$.

Worked Example 6

(Construction of a Formula)

(i) Find a formula for the sum S of any three consecutive even numbers.

(ii) Hence, calculate the value of S when the smallest even number is 14.

Solution:

(i) Let the smallest even number be n.
The next even number will be $n + 2$.
The greatest even number will be $(n + 2) + 2 = n + 4$.

$$\begin{aligned} \therefore\ S &= n + (n + 2) + (n + 4) \\ &= n + n + n + 2 + 4 \\ &= 3n + 6 \end{aligned}$$

(ii) When the smallest even number is 14, i.e. $n = 14$,

$$\begin{aligned} S &= 3 \times 14 + 6 \\ &= 48 \end{aligned}$$

PRACTISE NOW 6

SIMILAR QUESTIONS

Exercise 5B Questions 5(a)–(d), 15–16

(i) State the formula for the area A of a semicircle of radius r.

(ii) Hence, find the area of the semicircle that has a radius of 5 cm.
(Take π to be 3.142 when necessary.)

Exercise 5B

BASIC LEVEL

1. If $y = \frac{3}{5}x + 26$, find the value of y when $x = 12$.

2. If $a = \frac{y^2 - xz}{5}$, find the value of a when $x = 2$, $y = -1$ and $z = -3$.

3. If $S = 4\pi r^2$, find
   (i) the value of S when $r = 10\frac{1}{2}$,
   (ii) the positive value of r when $S = 616$.
   (Take π to be $\frac{22}{7}$.)

4. The formula for finding the area A of a triangle is given by $A = \frac{1}{2}bh$, where b and h represent the base and the height of the triangle respectively.
   (i) If b = 20 cm and h = 45 cm, find the area of the triangle.
   (ii) If A = 30 cm^2 and b = 10 cm, find the height of the triangle.

5. Find a formula for each of the following.
   (a) Product P of three numbers x, y and z
   (b) Sum S of the square of p and the cube of q
   (c) Average A of four numbers m, n, p and q
   (d) Time T, in minutes, for a train journey of a hours b minutes

INTERMEDIATE LEVEL

6. If $k = \frac{p + 2q}{3}$, find the value of p when $k = 7$ and $q = 9$.

7. If $U = \pi(r + h)$, find the value of r when $U = 16\frac{1}{2}$ and $h = 2$. (Take π to be $\frac{22}{7}$.)

8. If $v^2 = u^2 + 2gs$, find the value of s when $v = 25$, $u = 12$ and $g = 10$.

9. If $\frac{a}{b} - d = \frac{2c}{b}$, find the value of c when $a = 3$, $b = 4$ and $d = -5$.

10. If $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} + \frac{1}{d}$, find the value of c when $a = \frac{1}{2}$, $b = \frac{1}{4}$ and $d = -\frac{1}{5}$.

11. If $N = \frac{m}{x + q}$, find the value of q when $N = 1\frac{4}{5}$, $m = 9$ and $x = 2$.

12. If $c = \frac{a}{b} - \frac{d - e}{f - d}$, find the value of f when $a = 3$, $b = 4$, $c = -6$, $d = -5$ and $e = 2$.

13. If $a = \frac{b}{c - b}$, find the value of b when $a = 3$ and $c = 10$.

14. If $\frac{m(nx^2 - y)}{z} = 5n$, find the value of n when $m = 6$, $x = -2$, $y = -3$ and $z = -5$.

15. (i) Find a formula for the sum S of any three consecutive odd numbers.
    (ii) Hence, find the value of S when the greatest odd number is -101.

ADVANCED LEVEL

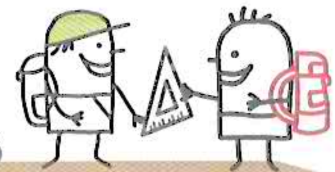
16. (i) Find a formula for the total cost \$$T$ of c pens at \$$d$ each and e pencils at f cents each.
    (ii) If $e = \frac{-145c}{4 - c}$ and $d = \frac{f + 5}{50}$, where $e = 150$ and $d = 3$, find the value of T.

17. In the United States of America, a different unit is used to measure temperature. It is called the degree Fahrenheit (°F). The formula for the conversion of x °F to y degree Celsius (°C) is

$$y = (x - 32) \times \frac{5}{9}.$$

    (i) The highest temperature in the United States of America, recorded in Death Valley in California, is 134 °F. What is this temperature in °C?
    (ii) During winter in the United States of America, it is very common for the temperature to fall below 0 °C. Is it more or less common for the temperature to fall below 0 °F?
    (iii) The lowest temperature in the United States of America, recorded in Prospect Creek in Alaska, is –62.1°C. What is this temperature in °F?

5.3 Applications of Linear Equations in Real-World Contexts



We have learnt in primary school how to solve word problems using the model method. However, not all word problems can be solved using it. For example, you may wish to try solving the following problem using the model method:

> A man is 5 times as old as his son. Four years ago, the product of their ages was 52. Find their present ages.

You will realise that the above problem *cannot* be solved using the model method. Therefore, there is a need to learn a new method called the **algebraic method**. We will only learn how to use the algebraic method to solve the above problem in Book 2 as it does not involve a linear equation.

In this section, we will learn how to solve word problems involving linear equations using the algebraic method.

Linking Model Method to Algebraic Method

Consider the following problem:

> A man is 3 times as old as his son. In 10 years' time, the sum of their ages will be 76. How old was the man when his son was born?

The left column below shows how the problem is solved using the model method. If we *replace* each of the boxes by the unknown x, we will get the algebraic method in the right column. Fill in the blanks below.

Model Method	Algebraic Method
Son $\boxed{x}$ Man $\boxed{x}\boxed{x}\boxed{x}$	Let the age of the son be x years. Then the man is $3x$ years old.
In 10 years' time, $\left.\begin{array}{l}\text{Son } \boxed{x}\boxed{10} \\ \text{Man } \boxed{x}\boxed{x}\boxed{x}\boxed{10}\end{array}\right\\}76$	In 10 years' time, the son will be $(x + 10)$ years old and the man will be _____ years old. $\therefore (x + 10) + (3x + 10) =$ _____ $4x + 20 = 76$
$\therefore$ 4 units $\rightarrow 76 - 20 =$ _____ 1 unit $\rightarrow 56 \div 4 = 14$	$4x = 76 - 20 =$ _____ $x = 56 \div 4 = 14$
When his son was born, the man was $2 \times 14 =$ _____ years old.	When his son was born, the man was $2x = 2 \times 14 =$ _____ years old.

Internet Resources

Visit http://www.shinglee.com.sg/StudentResources/ to access the AlgeTools™ software. Select the AlgeBar™ application to solve word problems on 'Whole Numbers' or 'Fractions'. You may draw models to help you formulate the equations.

Worked Example 7

(Formulating an Equation)

The sum of three consecutive even numbers is 60. Find the numbers.

Solution:

We have found in Worked Example 6 that a formula for the sum S of any three consecutive even numbers is $S = 3n + 6$, where n represents the smallest even number.

Let $3n + 6 = 60$.

$$\begin{aligned} 3n &= 60 - 6 \\ 3n &= 54 \\ n &= \frac{54}{3} \\ &= 18 \end{aligned}$$

$\therefore$ The three consecutive even numbers are 18, 20 and 22.

PRACTISE NOW 7

1. The sum of two numbers, one of which is 5 times as large as the other, is 24. Find the two numbers.
2. In a science test, Devi scores 15 marks more than Lixin. If Devi obtains twice as many marks as Lixin, find the number of marks Lixin obtains.

SIMILAR QUESTIONS

Exercise 5C Questions 1–7, 9–12, 15

Worked Example 8

(Formulating an Equation)

Michael walks at an average speed of 3 km/h for 45 minutes before running for half an hour at a certain average speed. If he travels a total distance of 6 km, calculate his average running speed.

Solution:

Let Michael's average running speed be x km/h.

45 minutes $= \frac{45}{60}$ hour $= \frac{3}{4}$ hour

Total distance he walks $= 3 \times \frac{3}{4} = \frac{9}{4}$ km

Total distance he runs $= x \times \frac{1}{2} = \frac{x}{2}$ km

$$\therefore \quad \frac{9}{4} + \frac{x}{2} = 6$$

$$\frac{9}{4} + \frac{2x}{4} = 6 \text{ (convert to like fractions: } \frac{x}{2} = \frac{2x}{4}\text{)}$$

$$\frac{9 + 2x}{4} = 6 \text{ (combine into a single fraction)}$$

$$\begin{aligned} 9 + 2x &= 6 \times 4 \\ 9 + 2x &= 24 \\ 2x &= 24 - 9 \\ 2x &= 15 \\ x &= \frac{15}{2} \\ &= 7\frac{1}{2} \end{aligned}$$

Michael's average running speed is $7\frac{1}{2}$ km/h.

ATTENTION

We let Michael's average running speed be x km/h, which consists of a letter and the unit, thus x represents a value that has no units.

RECALL

Total distance travelled = average speed × total time taken

PRACTISE NOW 8

SIMILAR QUESTIONS

The sum of one-fifth of a number and $3\frac{7}{10}$ is 7. Find the number.

Exercise 5C Questions 8, 13–14

Exercise 5C

Use the algebraic method to solve each of the following questions. You may draw models to help you formulate the equations.

BASIC LEVEL

1. When loaded with bricks, a lorry has a mass of 11 600 kg. If the mass of the bricks is three times that of the empty lorry, find the mass of the bricks.

2. The sum of 4 consecutive odd numbers is 56. Find the greatest of the 4 numbers.

3. Amirah is 4 years older than Priya and Shirley is 2 years younger than Priya. If the sum of their ages is 47, find their respective ages.

4. The sum of two numbers, one of which is two-thirds of the other, is 45. Find the smaller number.

5. If a number is tripled, it gives the same result as when 28 is added to it. Find the number.

6. A travel agency is planning for a holiday for a group of people. The agency receives quotations from two coach companies, Maya Express and Great Holidays. Maya Express charges \$15 for each person while Great Holidays charges a fixed amount of \$84 and an extra of \$12 per person. If the total amount charged by each company is the same, find the number of people going on the holiday.

INTERMEDIATE LEVEL

7. In a school, the number of boys who play soccer is 3 times as much as the number of boys who play badminton. If 12 boys who play soccer play badminton instead, the number of boys who play each of these sports would be the same. Find the number of boys who play badminton.

8. The sum of half of a number and 49 is $2\frac{1}{4}$ of the number. Find the number.

9. When a number is multiplied by 4 before subtracting from 68, the result obtained is the same as three times the sum of the number and 4. Find the number.

10. A man is six times as old as his son. Twenty years later, the man will be twice as old as his son. Find the age of the man when his son was born.

11. A mooncake with two egg yolks costs \$2 more than a mooncake with one egg yolk. The cost of 6 mooncakes with two egg yolks and 5 mooncakes with one egg yolk is \$130.80. Find the cost of a mooncake with two egg yolks.

12. Jun Wei has 12 more 10-cent coins than 20-cent coins. The total value of all his coins is \$5.40. Find the total number of coins he has.

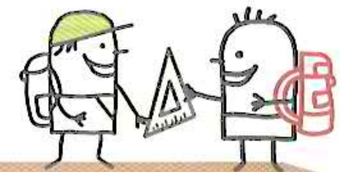
13. Kate cycles the first 350 km of a 470-km journey at a certain average speed and the remaining distance at an average speed that is 15 km/h less than that for the first part of the journey. If the time taken for her to travel each part of her journey is the same, find the average speed for the second part of her journey.

14. The numerator of a fraction is 5 less than its denominator. If 1 is added to the numerator and to the denominator, the new fraction is $\frac{2}{3}$. Find the fraction.

ADVANCED LEVEL

15. A two-digit positive number is such that the ones digit is 2.5 times as much as the tens digit. If the difference between the number and the number obtained when the digits are reversed is 27, find the number.

5.4 Simple Inequalities



We have learnt how to represent numbers on a number line (see Fig. 5.3) in Chapter 2. A number on the number line is *more than* any number on its *left* and *less than* any number on its *right*.

Fig. 5.3

Since the number 10 is to the right of the number 5, 10 is more than 5 (we write $10 > 5$). Similarly, –5 is to the right of –10, so –5 is more than –10 (we write $-5 > -10$). Alternatively, we can say that –10 is less than –5 (we write $-10 < -5$). $10 > 5$ and $-10 < -5$ are known as **inequalities**.

An **inequality** is made up of algebraic expressions together with symbols such as $<$, $>$, $\leqslant$ and $\geqslant$. We can use a number line to illustrate the solutions of the inequality.

The different notations and their meanings are summarised in Table 5.2.

Notation	Meaning
$a > b$	a is more than b.
$a < b$	a is less than b.
$a \geqslant b$	a is more than or equal to b.
$a \leqslant b$	a is less than or equal to b.

Table 5.2

INFORMATION

- $a > b$ and $a < b$ are called strict inequalities as $a \neq b$.
- The **Law of Trichotomy** states that for any two numbers x and y, exactly one of the relations $x > y$, $x = y$ or $x < y$ holds.

Investigation

Properties of Inequalities

In this investigation, we shall explore some properties of inequalities.

1. Consider the inequality $10 > 6$ and complete Table 5.3.

Cases	Working	Inequality	Is the inequality sign reversed?	Conclusion
Multiplication by a *positive* number on both sides of the inequality $10 > 6$	LHS $= 10 \times 5$ $= 50$ RHS $= 6 \times 5$ $= 30$	$50 > 30$	No	If $x > y$ and $c > 0$, then $cx > cy$.
Division by a *positive* number on both sides of the inequality $10 > 6$				If $x > y$ and $c > 0$, then $\frac{x}{c}$ ____ $\frac{y}{c}$.

Table 5.3

2. Do the conclusions which you have drawn from Table 5.3 apply to $10 \geqslant 6$? Can you generalise the conclusions for $x \geqslant y$? What about $x < y$ and $x \leqslant y$?

From the investigation, we can conclude that:

We can multiply or divide both sides of an inequality by a *positive* number *without* having to reverse the inequality sign, i.e.

$$\boxed{\text{if } x \geqslant y \text{ and } c > 0, \text{ then } cx \geqslant cy \text{ and } \frac{x}{c} \geqslant \frac{y}{c}.}$$

SIMILAR QUESTIONS

Exercise 5D Questions 1(a)–(f)

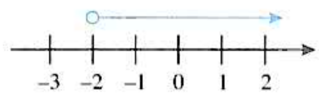
Solving Simple Linear Inequalities

Let x be the number of students attending a workshop.

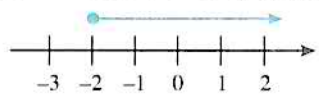
- If there are 100 students, we write $x = 100$.
  This is an **equation**. It has only one answer, i.e. only one value of x satisfies the equation.
- If there are less than 100 students, we write $x < 100$.
  This is an **inequality**. It has many answers, i.e. many values of x satisfy the inequality. x can take on integer values ranging from 0 to 99 inclusive. In this context, non-integer and negative values of x have no meaning. What values can x take if $x \leqslant 100$?

For an inequality with an unknown x, all values of x that satisfy the inequality are called the **solutions** of the inequality. We can use a number line to illustrate these solutions. For example,

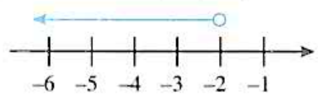
- some of the solutions of the inequality $x > -2$ are $-1, 0, 1, 1.5, \ldots$



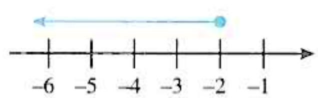
- some of the solutions of the inequality $x \geqslant -2$ are $-2, -1, 0, 1, 1.5, \ldots$



- some of the solutions of the inequality $x < -2$ are $-3, -4, -5, -5.25, \ldots$



- some of the solutions of the inequality $x \leqslant -2$ are $-2, -3, -4, -5, -5.25, \ldots$



ATTENTION

On a number line, a circle ○ is used to indicate that x cannot take on a particular value whereas a dot ● is used to indicate that x can take on the particular value.

Worked Example 9

(Solving Linear Inequalities)

Solve each of the following inequalities and illustrate the solutions on a number line.

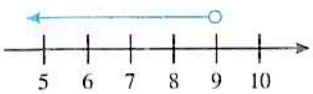
(a) $3x < 27$ **(b)** $5x \geqslant -60$

Solution:

(a) $3x < 27$

$x < \frac{27}{3}$ (since $3 > 0$, no change in the inequality sign)

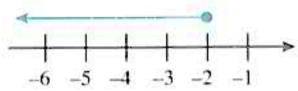
$\therefore x < 9$



(b) $5x \geqslant -60$

$x \geqslant -\frac{60}{5}$ (since $5 > 0$, no change in the inequality sign)

$\therefore x \geqslant -12$



PRACTISE NOW 9

SIMILAR QUESTIONS

Exercise 5D Questions 2(a)–(d), 4(a)–(d), 5–8

1. Solve each of the following inequalities and illustrate the solutions on a number line.

   (a) $15x > 75$ **(b)** $4x \leqslant -16$

2. Find the smallest integer value of x that satisfies the inequality $6x > 7$.

Worked Example 10

(Problem involving Inequalities)

A curry puff costs 90 cents. By setting up an inequality, find the maximum number of curry puffs that can be bought with \$20.

Solution:

Let the number of curry puffs that can be bought with \$20 be x.

Then $90x \leqslant 2000$ (\$20 = 2000 cents)

$x \leqslant \frac{2000}{90}$ (since $90 > 0$, no change in the inequality sign)

$x \leqslant 22\frac{2}{9}$

$\therefore$ The maximum number of curry puffs that can be bought with \$20 is 22.

PRACTISE NOW 10

SIMILAR QUESTIONS

Exercise 5D Question 3

A bus can ferry a maximum of 45 students. By setting up an inequality, find the minimum number of buses that are needed to ferry 520 students.

Journal Writing

Think of two real-life examples of inequalities. Formulate an inequality in each situation.

Exercise 5D

BASIC LEVEL

1. Fill in each box with '<', '>', '≤' or '≥'.
   (a) If $x > y$, then $5x \,\square\, 5y$.
   (b) If $x < y$, then $\frac{x}{20} \,\square\, \frac{y}{20}$.
   (c) If $x \geqslant y$, then $3x \,\square\, 3y$.
   (d) If $x \leqslant y$, then $\frac{x}{10} \,\square\, \frac{y}{10}$.
   (e) If $15 > 5$ and $5 > x$, then $15 \,\square\, x$.
   (f) If $x < 50$ and $50 < y$, then $x \,\square\, y$.

2. Solve each of the following inequalities and illustrate the solutions on a number line.
   (a) $3x \leqslant 18$ (b) $4x \geqslant 62$
   (c) $3y < -36$ (d) $5y > -24$
   (e) $4x < 28$ (f) $12x \geqslant 126$
   (g) $2y \leqslant -5$ (h) $9y > -20$

3. A van can ferry a maximum of 12 people. By setting up an inequality, find the minimum number of vans that are needed to ferry 80 people.

INTERMEDIATE LEVEL

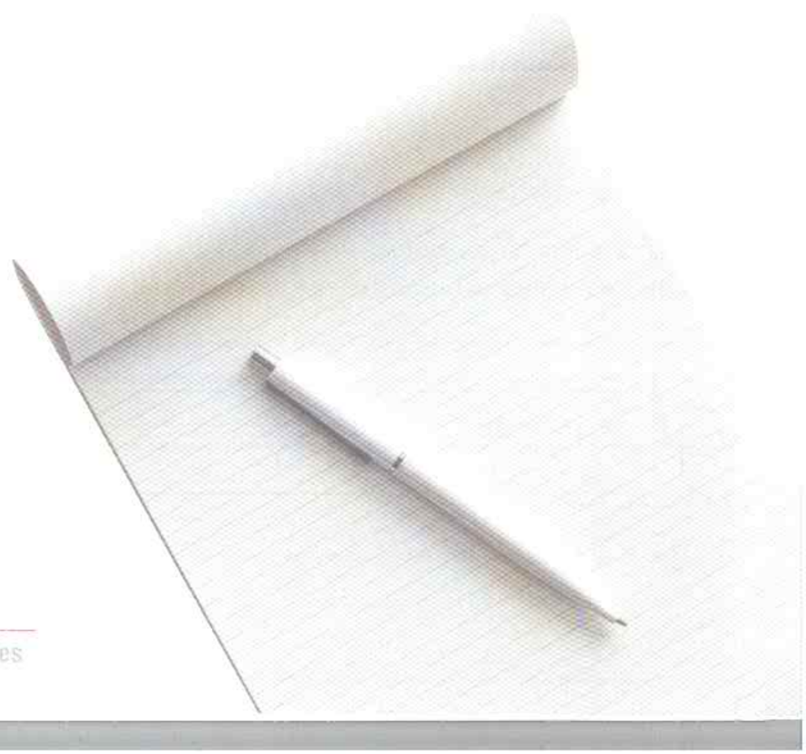
4. Find the smallest rational value of y that satisfies the inequality $8 \leqslant 7y$.

5. Given that x satisfies the inequality $20x > 33$, find the smallest value of x if x is a prime number.

6. Find the greatest odd integer value of x that satisfies the inequality $3x < -105$.

ADVANCED LEVEL

7. If y is an integer, and it satisfies the inequalities $5y < 20$ and $2y \geqslant -6$, find all the possible values of y.



1. To solve an **equation**, we
   - add the same number to both sides,
     e.g. $x - 2 = 7$
     $x - 2 + 2 = 7 + 2$
     $\therefore x = 9$
   - subtract the same number from both sides,
     e.g. $x + 2 = 7$
     $x + 2 - 2 = 7 - 2$
     $\therefore x = 5$
   - multiply by the same number on both sides,
     e.g. $\frac{1}{2}x = 7$
     $\frac{1}{2}x \times 2 = 7 \times 2$
     $\therefore x = 14$
   - divide by the same number on both sides.
     e.g. $2x = 14$
     $\frac{2x}{2} = \frac{14}{2}$
     $\therefore x = 7$

2. A **formula** expresses a rule in algebraic terms. It makes use of variables to write instructions for performing a calculation.

3. An **inequality** is made up of algebraic expressions together with symbols such as <, >, ⩽ and ⩾. We can use a number line to illustrate the solutions of the inequality.

4.

Case	Multiplying a positive number c to both sides of the inequality	Dividing a positive number c from both sides of the inequality
$x > y$	$cx > cy$	$\frac{x}{c} > \frac{y}{c}$
$x \geqslant y$	$cx \geqslant cy$	$\frac{y}{c} \geqslant \frac{y}{c}$
$x < y$	$cx < cy$	$\frac{y}{c} < \frac{y}{c}$
$x \leqslant y$	$cx \leqslant cy$	$\frac{y}{c} \leqslant \frac{y}{c}$

Review Exercise 5

1. Solve each of the following equations.
   (a) $x - 1 = \frac{1}{2}x$
   (b) $2(x - 1) + 3(x + 1) = 4(x + 4)$
   (c) $2y - [7 - (5y - 4)] = 6$
   (d) $\frac{3}{4}x - 5 = 0.5x$
   (e) $\frac{2y + 7}{4} = 12$
   (f) $\frac{4y - 1}{5y + 1} = \frac{5}{7}$
   (g) $\frac{a + 1}{4} + \frac{a - 1}{3} = 4$
   (h) $\frac{b - 4}{3} - \frac{2b + 1}{6} = \frac{5b - 1}{2}$
   (i) $\frac{2c}{9} - \frac{c - 1}{6} = \frac{c + 3}{12}$
   (j) $\frac{2(3 - 4d)}{3} - \frac{3(d + 7)}{2} = 5d + \frac{1}{6}$

2. Solve each of the following inequalities.
   (a) $18x < -25$
   (b) $10y \geqslant -24$

3. If $3(x - 1) - 5(x - 4) = 8$, find the value of $x - 5\frac{1}{2}$.

4. Find the smallest integer value of x that satisfies the inequality $4x \geqslant 11$.

5. Find the greatest integer value of y that satisfies the inequality $3y < -24$.

6. Given that x satisfies the inequality $5x < 125$, find the greatest value of x if x is divisible by 12.

7. Given that y satisfies the inequality $5y \geqslant 84$, find the smallest value of y if y is a prime number.

8. If $V = \frac{4}{3}\pi r^3$, find
   (i) the value of V when $r = 7$,
   (ii) the value of r when $V = 113\frac{1}{7}$.
   (Take π to be $\frac{22}{7}$.)

9. If $n - 2y = \frac{3y - n}{m}$, find the value of n when $y = 5$ and $m = -3$.

10. Find two consecutive odd numbers such that the sum of the greater number and 5 times the smaller number is 92.

11. Object A is 5 kg heavier than Object B and the mass of Object C is twice the mass of Object A. If the total mass of the three objects is 255 kg, find the mass of Object C.

12. The sum of the ages of Farhan and his cousin is 38. Seven years ago, Farhan was thrice as old as his cousin. Find Farhan's present age.

13. Nora is twice as old as Raj and half as old as Ethan. In 22 years' time, Ethan will be twice as old as Raj. Find Nora's present age.

14. A man has 25 sweets and his son has 55 sweets. Find the number of sweets that the man has to give to his son so that his son would have 4 times as many sweets as him.

15. Rui Feng has enough money to buy 24 apples. If the price of each apple is reduced by 5 cents, he will be able to buy an extra 6 apples with the same amount of money. Find the original price of each apple.

16. A man travels from Town A to Town B at an average speed of 4 km/h and from Town B to Town A at an average speed of 6 km/h. If he takes 45 minutes to complete the entire journey, find his total distance travelled.

17. The numerator of a fraction is 2 less than its denominator. If 3 is subtracted from the numerator and from the denominator, the new fraction obtained is $\frac{3}{4}$. Find the fraction.

18. A set of multimedia equipment costs \$1900. By setting up an inequality, find the maximum number of sets of multimedia equipment that can be bought with \$35 000.

19. Admission to a school play costs \$12.50. Form an inequality and solve it to find the maximum number of tickets Jun Wei can buy with \$250.

20. If the sum of two consecutive integers is less than 42, find the square of the largest possible integer.

21. Nora is 4 years older than Kate. If the sum of their ages is at most 45 years, find the maximum possible age of Kate.

22. A ship can carry a maximum of 60 passengers. By setting up an inequality, find the minimum number of ships that are needed to carry 400 passengers

23. A pencil costs \$2.50. By setting up an inequality, find the maximum number of pencils that can be bought with \$27.

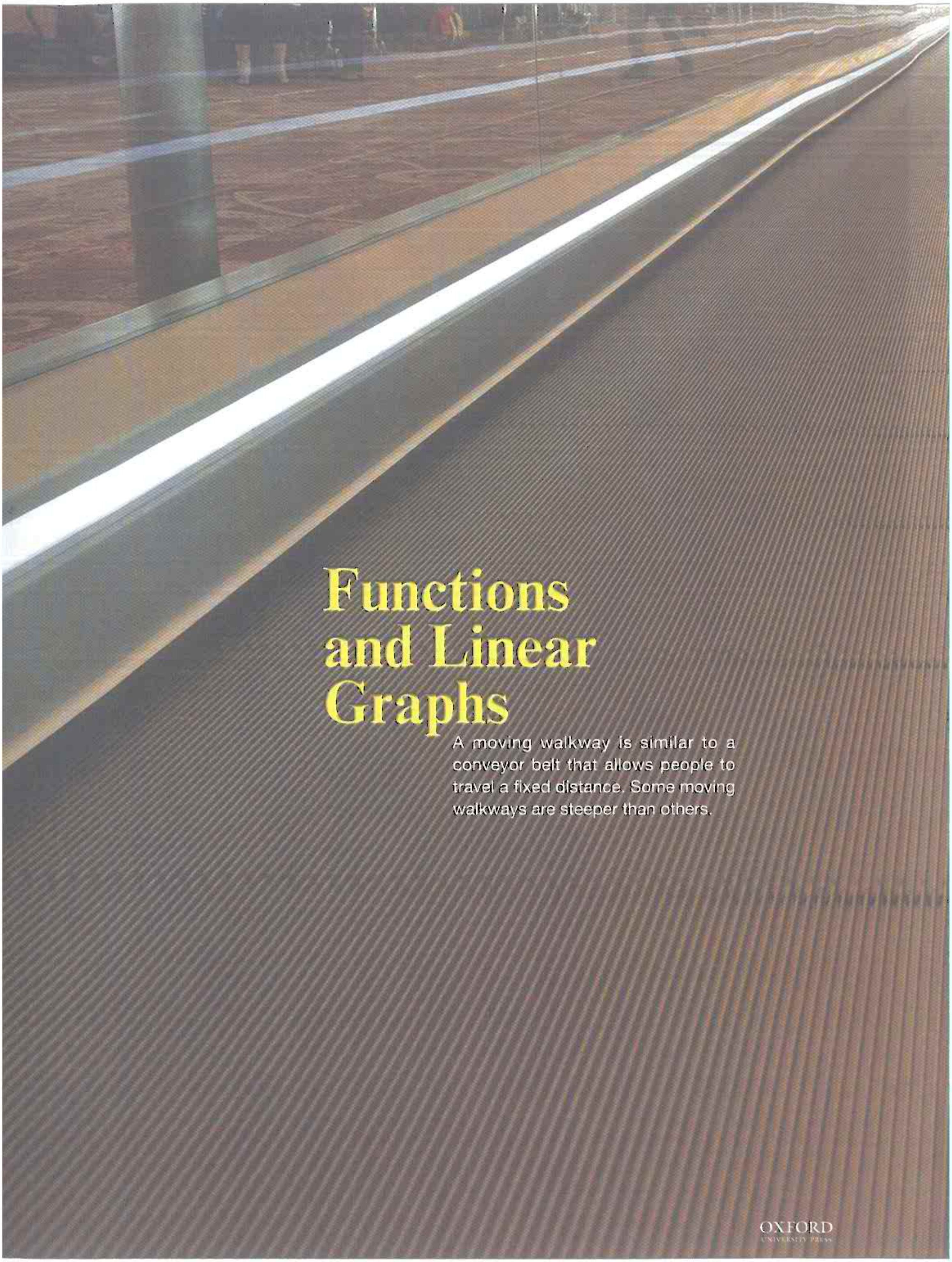
Challenge Yourself

1. Solve $\sqrt{x} + 2 = 0$.

2. If $(x + 2)^2 + (y - 3)^2 = 0$, find the value of $x + y$.

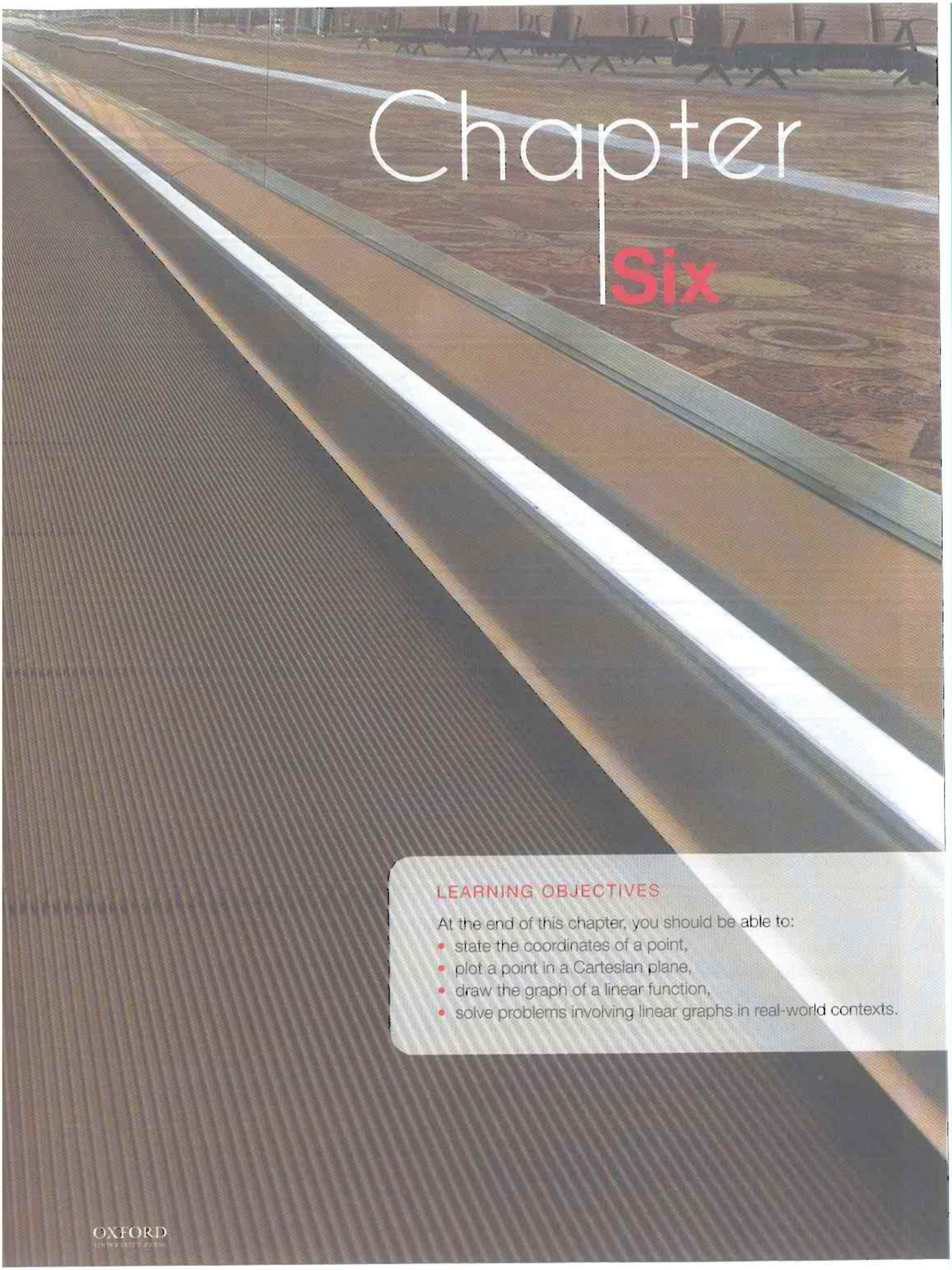
3. If A, B, C and D are whole numbers such that $A + B = 8$, $B + C = 11$, $B + D = 13$ and $C + D = 14$, find the values of A, B, C and D.

4. If A, B, C and D are whole numbers such that $A \times B = 8$, $B \times C = 28$, $C \times D = 63$ and $B \times D = 36$, find the values of A, B, C and D.



Functions and Linear Graphs

A moving walkway is similar to a conveyor belt that allows people to travel a fixed distance. Some moving walkways are steeper than others.



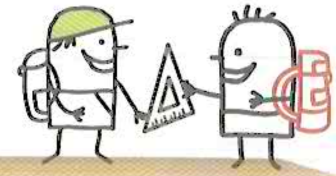
Chapter Six

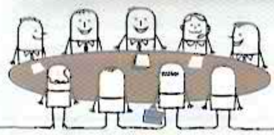
LEARNING OBJECTIVES

At the end of this chapter, you should be able to:

- state the coordinates of a point,
- plot a point in a Cartesian plane,
- draw the graph of a linear function,
- solve problems involving linear graphs in real-world contexts.

6.1 Cartesian Coordinates





Class Discussion

Battleship Game (Two Players)

Sink your opponent's battleships before your opponent sinks all your ships.

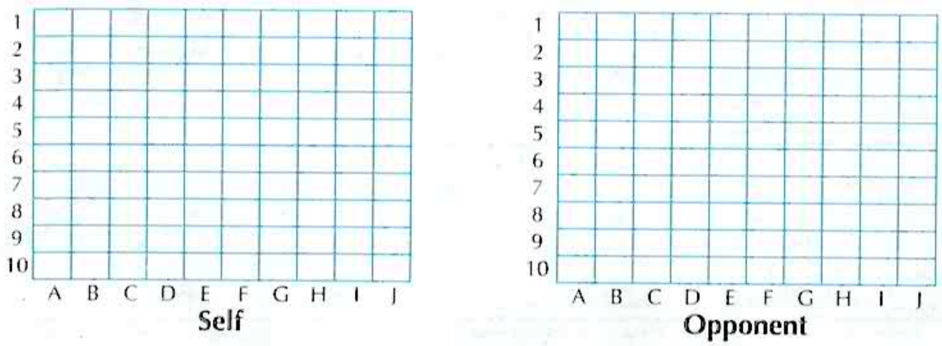


Fig. 6.1

Internet Resources

Search on the Internet for free online battleship games.

1. On the grid labelled 'Self', arrange the following five ships:
   - An aircraft carrier A A A A A
   - A battleship B B B B
   - A submarine S S S
   - A cruiser C C C
   - A destroyer D D

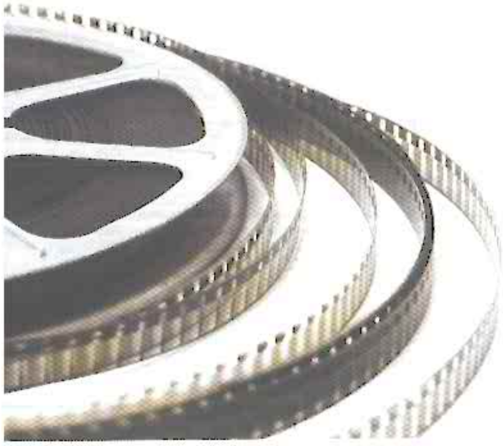
   The ships must be placed horizontally or vertically; none of the ships can be placed diagonally. Note that players are not allowed to see each other's grids.

2. Take turns to try to sink your opponent's ships.
   - Call out a location on the grid, e.g. D7, F4 and G10, to hit your opponent's ship.
   - If a ship is found at the location called out, the other player says 'hit'. Otherwise, the player says 'missed'.
   - On the 'Opponent' grid, record each location you have called out by shading it for a 'hit' and drawing a cross for a 'miss'.
   - Similarly, on the 'Self' grid, record the locations that your opponent has called out.

3. A ship is sunk when all the spaces it occupies have been called out. The player whose ship is sunk says 'My ship has been sunk.'.

4. A player wins the game when all his opponent's ships have been sunk.

In the battleship game, we use the labels of the columns and the rows to call out locations on the grids. We shall use the same idea to locate a student in a classroom where the students are seated at desks that are arranged neatly in rows and columns as shown in Fig. 6.2.



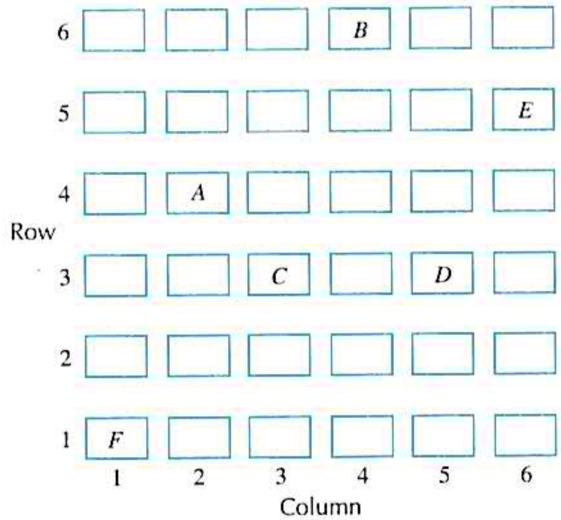


Fig. 6.2

To locate a student, the teacher can associate the student with the column and the row his or her seat is at. The teacher can write a pair of numbers against the name of a student in the class list as follows:

$$A(2, 4) \qquad B(4, 6) \qquad C(3, 3)$$

From the pair of numbers (2, 4), we will be able to know that student A is seated at column 2 and in row 4. The pairs of numbers (4, 6) and (3, 3) tell us that student B is seated at column 4 and in row 6, and student C is seated at column _____ and in row _____ respectively.

Are you able to write down the pairs of numbers which correspond to students D, E and F?

Class Discussion

Ordered Pairs

Discuss each of the following questions with your classmates.

1. Is a single number sufficient to describe the exact position of a student in the classroom seating plan? Can the location of a seat in a cinema be represented by a single number?
2. Is the order in which the two numbers are written important, i.e. do (5, 3) and (3, 5) indicate the same position? The pairs of numbers (2, 4), (4, 6), (3, 3) and so on are examples of **ordered pairs**. Do you know why they are called ordered pairs?

Journal Writing

1. Using a horizontal scale of 1 to 10 and a vertical scale of A to J, design a map that includes the locations of your house, a bus stop and a shopping mall in your neighbourhood.
2. Using a horizontal scale of 1 to 12 and a vertical scale of A to K, design a ground floor map for a shopping mall.
3. Use suitable horizontal and vertical scales to design the seating plan of a 100-seat cinema.

Now let us display the same classroom plan in Fig. 6.2 with horizontal and vertical lines drawn through the centres of the boxes, showing the positions of the students. The horizontal lines and vertical lines are numbered as shown in Fig. 6.3.

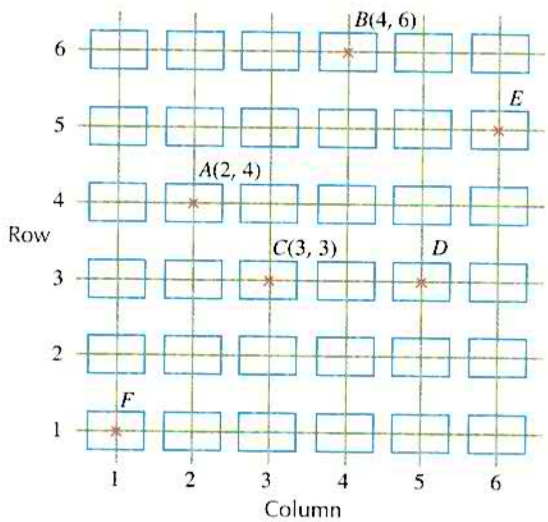


Fig. 6.3

In the battleship game, each location is called out by the column labels followed by the row labels. Similarly, the first number in each ordered pair is with reference to the horizontal scale while the second number is with reference to the vertical scale. To further simplify it, we can use a point (indicated by a cross) to show the position of each student. This gives us an idea of locating a point in a plan.

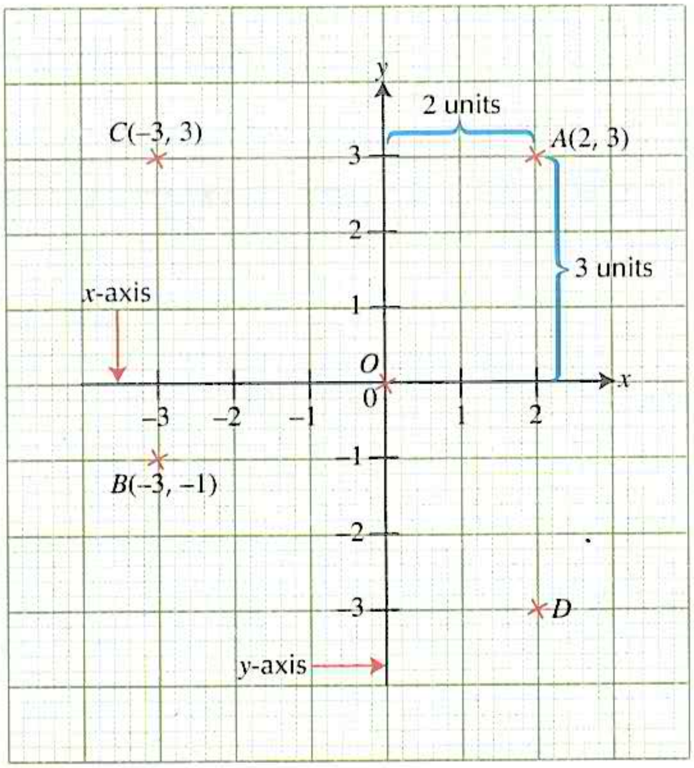


Fig. 6.4

Fig. 6.4 shows a rectangular or **Cartesian** plane which consists of two number lines intersecting at right angles at the point O, known as the **origin**. The horizontal and vertical axes are called the **x-axis** and the **y-axis** respectively.

The position of any point in the plane can be determined by its distance from each of the axes. In Fig. 6.4, point A is 2 units to the *right* of the y-axis and 3 units *above* the x-axis. Thus its position is given by the ordered pair (2, 3). Similarly, the ordered pair (–3, –1) determines point B and (–3, 3) represents point C.

Name the ordered pair that determines point D. What is the origin, O, represented by?

In general, each point P in the plane is located by an ordered pair (x, y). We call x the **x-coordinate** (or abscissa) of P and y the **y-coordinate** (or ordinate) of P, i.e. P has **coordinates** (x, y).

PRACTISE NOW

SIMILAR QUESTIONS

Exercise 6A Questions 1–2, 5–7,

On a sheet of graph paper, use a scale of 1 cm to represent 1 unit to draw the x-axis for values of x from –2 to 3 and the y-axis for values of y from –2 to 3. Plot the points $A(2, 2)$, $B(-2, 3)$, $C(-1, -2)$ and $D(3, -1)$.

Story Time

Cartesian coordinates were invented by René Descartes (1596 – 1650) when he tried to describe the path of a fly crawling along crisscrossed beams on the ceiling while he lay on his bed. Due to his poor health, he had developed a lifetime habit of sleeping until 11 a.m. every morning. He broke this habit only when Queen Christina of Sweden persuaded him to go to Stockholm to teach her how to draw tangents at 5 a.m.! After a few months of walking in the cold climate to the palace at 5 a.m., he died of pneumonia. Even up till his death, Descartes displayed a strong sense of diligence.

6.2 Functions

A function performs one or more operations on the inputs, i.e. the values it takes in, to produce outputs, i.e. the results. The operations performed on the inputs are known as the rule of the function. In this section, we will take a look at functions.

Internet Resources

Search on the Internet for an interactive 'Function Machine' which you can key in an input and the machine will give you an output. Then you can guess the equation of the function.

Investigation

Function Machine

In this investigation, we shall explore the concept of a function by looking at how a **function machine** works.

Fig. 6.5 shows a function machine whose function is to *'add 3'* to any input x to produce an output y. For example, if you input $x = 2$, the output will be $y = 2 + 3 = 5$.

INFORMATION

We can represent a function using words.

Fig. 6.5

INFORMATION

We can represent a function using an equation.

1. Write down an equation that shows the relationship between the output y and the input x.

   $y =$ ________

2. Write down the output y for each of the following inputs x.

   (a) Input $x = 4 \rightarrow$ Output $y =$ _____

   (b) Input $x = -7 \rightarrow$ Output $y =$ _____

3. Write down the input x for each of the following outputs y.

(a) Input $x =$ _____ $\rightarrow$ Output $y = 9$

(b) Input $x =$ _____ $\rightarrow$ Output $y = 0$

4. The above data can be represented by Table 6.1. Complete the table.

x	-7		2	4	
y		0	5		9

Table 6.1

INFORMATION

We can represent a function using a table.

5. In Fig. 6.6, the point (2, 5) is shown. Plot the rest of the points based on Table 6.1 and draw a straight line that passes through all the points.

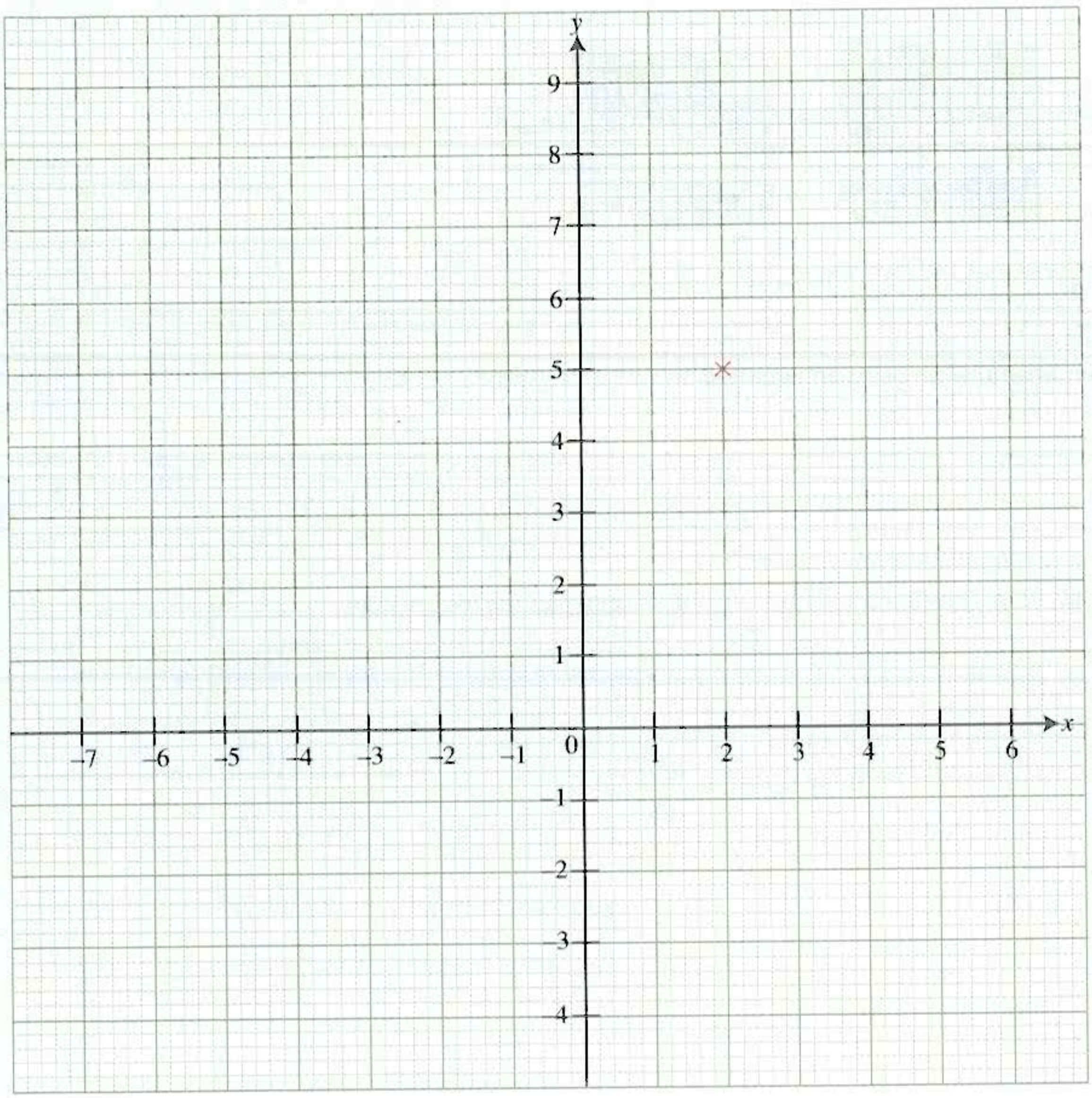


Fig. 6.6

INFORMATION

We can represent a function using a graph.

Select any point on the straight line. Do the coordinates satisfy the equation which you have written down in Question 1?

6. Based on Table 6.1 and Fig. 6.6, state the number of output(s) y for each input x.

From the above, we can see that a function is such that every input produces only one output. The input x and the output y of a function can be written as an ordered pair (x, y). A function can be represented using words, an equation, a table or a graph.

$y = x + 3$ is called the **equation** of the function.

Fig. 6.7 shows another function machine whose function is to *'multiply −2'* to any input x before *'subtracting 1'* from the result to produce an output y. For example, if you input $x = 3$, the output will be $y = 3 \times (-2) - 1 = -7$.

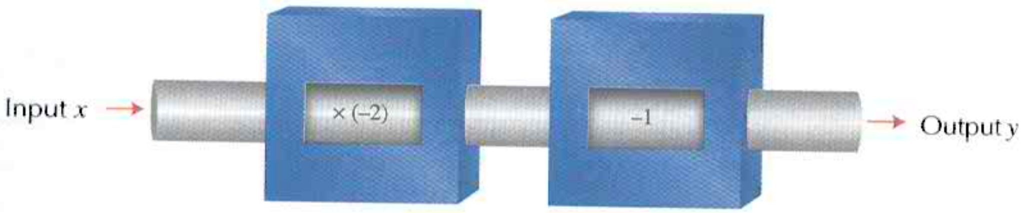


Fig. 6.7

7. Representation of a function using an **equation**
Write down the equation of the function.
$y =$ ________

8. Representation of a function using a **table**
Complete Table 6.2 to show the corresponding output values for the input values.

x	−1			2	3
y		0	−1		−7

Table 6.2

9. Representation of a function using a **graph**

In Fig. 6.8, the point $(3, -7)$ is shown. Plot the rest of the points based on Table 6.2 and draw a straight line that passes through all the points.

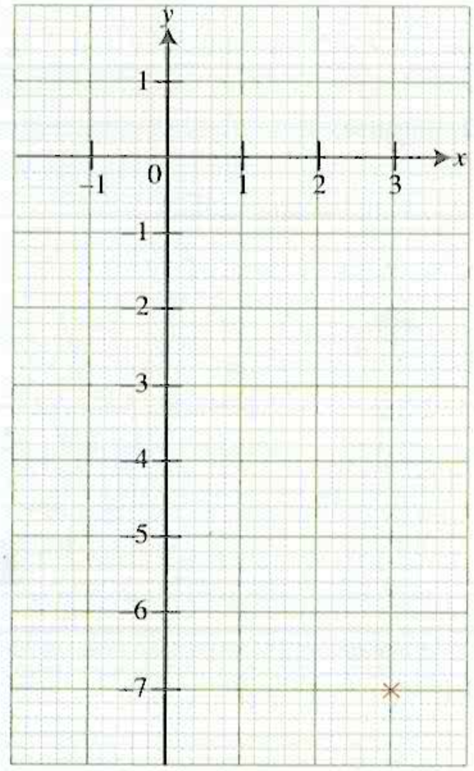


Fig. 6.8

10. Based on Table 6.2 and Fig. 6.8, state the number of output(s) y for each input x.

From the investigation, in general,

> A **function** is a relationship between two variables x and y such that every input x produces *exactly one* output y.

Thinking Time

1. $y^2 = x$ is *not* the equation of a function because
   - there are two values of y for every positive value of x, e.g. if the input $x = 9$, then the output $y = \pm$ ____,
   - there is no value for the output y if the input x is negative.

2. Is it possible for a function to have *two* input values x with the same output value y?
   Hint: Consider the equation of the function $y = x^2$.

PRACTISE NOW

SIMILAR QUESTIONS

Exercise 6A Questions 3–4, 8

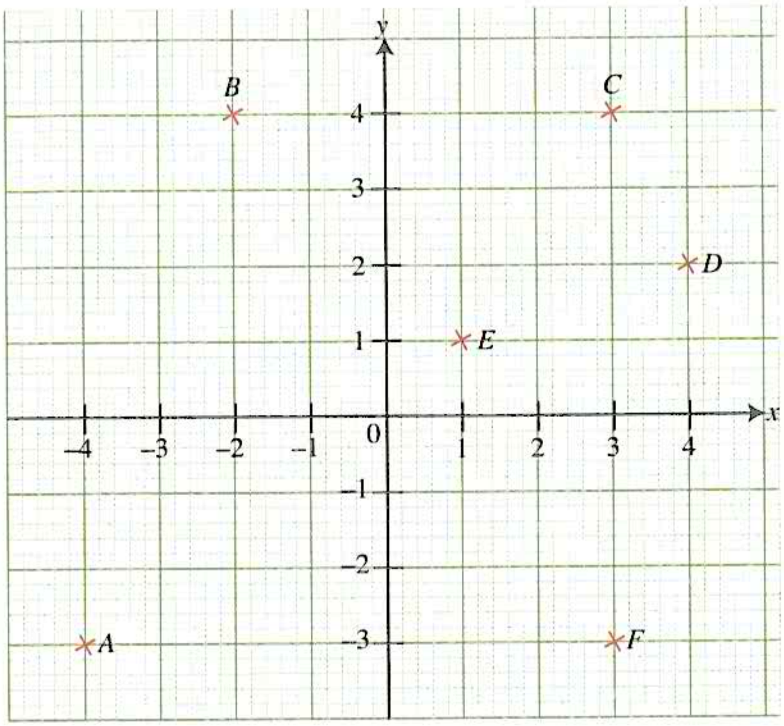
1. The equation of a function is $y = 2x - 3$. Find
   (i) the value of y when $x = 4$, **(ii)** the value of x when $y = -5$.

2. The equation of a function is $y = -\frac{1}{3}x - \frac{2}{5}$. Find
   (i) the value of y when $x = 0$, **(ii)** the value of x when $y = -\frac{2}{3}$.

Exercise 6A

BASIC LEVEL

1. Write down the coordinates of each point shown in the figure.



2. On a sheet of graph paper, use a scale of 1 cm to represent 1 unit to draw the x-axis for values of x from -3 to 6 and the y-axis for values of y from -2 to 5. Plot the points $A(2, 5)$, $B(1, 2)$, $C(-2, -1)$, $D(6, -2)$, $E(3, -2)$ and $F(-1, 2)$.

3. The equation of a function is $y = 4x + 5$. Find the value of y when
   (i) $x = 3$, **(ii)** $x = -2$.

4. The equation of a function is $y = 25 - 3x$. Find the value of x when
   (i) $y = 34$, **(ii)** $y = -5$.

INTERMEDIATE LEVEL

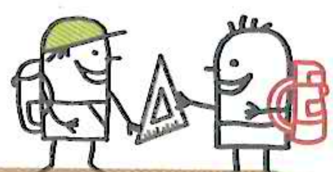
5. Plot each set of the given points on a sheet of graph paper. Join the points in order with straight lines and identify each geometrical shape obtained.
   (a) (6, 4), (–6, 4), (–6, –4), (6, –4)
   (b) (0, 5), (–6, 0), (0, –5), (6, 0)
   (c) (0, 0), (0, 8), (5, 4)
   (d) (1, 0), (0, 3), (–1, 4), (–5, –2)
   (e) (5, 2), (–1, 3), (–1, –3), (5, –2)

6. The vertices of a right-angled triangle are $A(1, 0)$, $B(7, 0)$ and $C(1, 8)$. Plot the points A, B and C on a sheet of graph paper. Hence, find the area of $\triangle ABC$.

7. Plot each of the following points on a sheet of graph paper.
   (3, –5), (2, –3), (1, –1), (0, 1), (–1, 3), (–2, 5), (–3, 7)
   Do you notice that the points lie in a special pattern? Describe the pattern.

8. The equation of a function is $y = \frac{2}{3}x + \frac{1}{3}$. Find
   (a) the value of y when
   (i) $x = -3$, **(ii)** $x = 1\frac{1}{2}$.
   (b) the value of x when
   (i) $y = 1$, **(ii)** $y = -\frac{1}{6}$.

6.3 Graphs of Linear Functions



Consider the equation of the function $y = 2x$. We shall look at all pairs of values x and y that satisfy the equation.

When $x = 1$, $y = 2 \times 1 = 2$;
$x = 2$, $y = 2 \times 2 = 4$;
$x = 2.5$, $y = 2 \times 2.5 = 5$;
$x = 3$, $y = 2 \times 3 = 6$;
$x = 3.1$, $y = 2 \times 3.1 = 6.2$, etc.

Since we are not able to list all the values of x and y that satisfy the equation of the function $y = 2x$, we can use a graph to display the function as illustrated in Worked Example 1.

Worked Example 1

(Drawing the Graph of a Linear Function)

(i) On a sheet of graph paper, using a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of the function $y = 2x$ for values of x from 0 to 4.

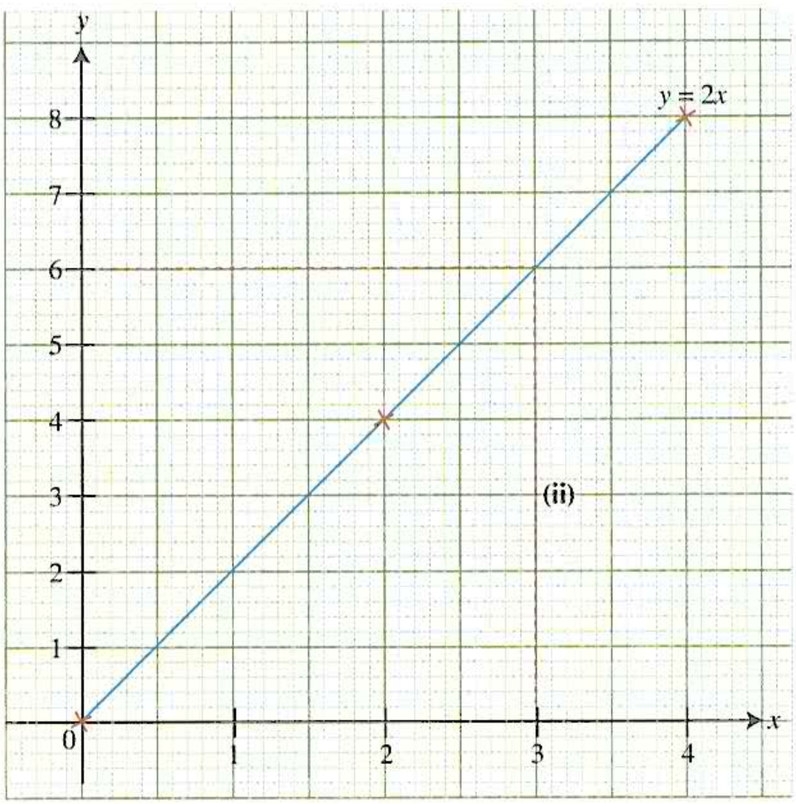
(ii) The point $(3, p)$ lies on the graph in **(i)**. Find the value of p.

Solution:

(i) We first set up a table of values for x and y. These pairs of values for x and y satisfy the equation of the function $y = 2x$.

x	0	2	4
$y = 2x$	0	4	8

Using the scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, the three pairs of values are plotted as points in the Cartesian plane and a straight line is drawn to pass through these three points.



(ii) From the graph in **(i)**,

when $x = 3$, $p = y = 6$

Problem Solving Tip

We only need to plot 3 points to obtain the graph of a linear function. In fact, a straight line can be determined by plotting 2 points. We use the 3rd point to check for mistakes in the graph.

ATTENTION

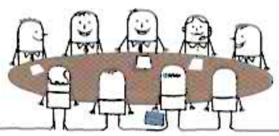
- Ensure that the graph is only drawn for values of x from 0 to 4.
- The graph must be labelled with $y = 2x$.

PRACTISE NOW 1

SIMILAR QUESTIONS

Exercise 6B Questions 1–6

1. (i) On a sheet of graph paper, using a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of the function $y = 2x + 1$ for values of x from 0 to 4.
   (ii) The point $(q, 6)$ lies on the graph in (i). Find the value of q.
2. On a sheet of graph paper, using a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graphs of the functions $y = 3x$ and $y = 2 - 2x$ for values of x from -2 to 2.

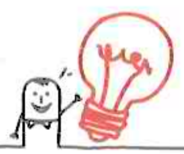


Class Discussion

Equation of a Function

Refer to Worked Example 1.

1. The coordinates of the points A and B are (1, 2) and (3, 7) respectively. Do the coordinates of each of the points satisfy the equation of the function $y = 2x$? Explain your answers.
2. Using the graph drawn, state the coordinates of two points that satisfy the equation of the function $y = 2x$.
3. Amirah says that 'the coordinates of every point on the line satisfy the equation of the function $y = 2x$.' Discuss with your classmates whether she is right.



Thinking Time

The line drawn in Worked Example 1 is said to be the graph of the *linear* function $y = 2x$ because the graph is a *straight line*.

(i) What can you say about the coordinates of the points that lie on the line $y = 2x$?

(ii) Lixin says that the graphs of the functions $y = x + 3$ and $y = -2x - 1$ are linear. Do you agree with her? Explain your answer.

Exercise 6B

BASIC LEVEL

1. (a) On a sheet of graph paper, using a scale of 1 cm to represent 1 unit on the x-axis and 1 cm to represent 2 units on the y-axis, draw the graph of each of the following functions for values of x from 0 to 4.
   (i) $y = 2x + 8$ (ii) $y = 2x + 2$
   (iii) $y = 2x - 3$ (iv) $y = 2x - 6$
   (b) What do you notice about the lines you have drawn in (a)?

2. (a) On a sheet of graph paper, using a scale of 1 cm to represent 1 unit on the x-axis and 1 cm to represent 2 units on the y-axis, draw the graph of each of the following functions for values of x from –4 to 4.
   (i) $y = 3x + 7$ (ii) $y = 3x + 5$
   (iii) $y = 3x - 3$ (iv) $y = 3x - 6$
   (b) What do you notice about the lines you have drawn in (a)?

3. (a) On a sheet of graph paper, using a scale of 1 cm to represent 1 unit on the x-axis and 1 cm to represent 2 units on the y-axis, draw the graph of each of the following functions for values of x from –4 to 4.
   (i) $y = -2x + 5$ (ii) $y = -2x + 3$
   (iii) $y = -2x - 4$ (iv) $y = -2x - 7$
   (b) What do you notice about the lines you have drawn in (a)?

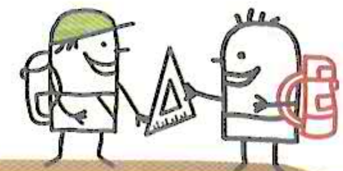
INTERMEDIATE LEVEL

4. (a) On a sheet of graph paper, using a scale of 1 cm to represent 1 unit on the x-axis and 1 cm to represent 2 units on the y-axis, draw the graph of each of the following functions for values of x from –4 to 4.
   (i) $y = -4x + 8$ (ii) $y = -4x + 2$
   (iii) $y = -4x - 3$ (iv) $y = -4x - 6$
   (b) Write down another set of four linear functions whose graphs are parallel to each other.

5. (i) On a sheet of graph paper, using a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of the function $y = 6 - 3x$ for values of x from –3 to 3.
   (ii) The points $(a, 0)$, $(-2, b)$ and $(c, 1.5)$ lie on the graph in (i). Find the values of a, b and c.

6. On a sheet of graph paper, using a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graphs of the functions $y = 2x + 4$ and $y = 2 - 3x$ for values of x from –2 to 2.

6.4 Applications of Linear Graphs in Real-World Contexts



Linear graphs are used in many daily situations. For example, we can use a graph to convert between different currencies, to convert between units of volume and to show the journey of a moving vehicle.

Worked Example 2

(Expenditure)

Nora's father gives her \$100 as her monthly allowance.

(a) If she spends an average of \$3 each day, find the amount of money that she is left with after

(i) 2 days, **(ii)** 5 days, **(iii)** 10 days.

(b) Given that \$$y$ represents the amount of money Nora has left after x days, copy and complete the table.

x	2	5	10
y			

(c) On a sheet of graph paper, using a scale of 2 cm to represent 5 days on the horizontal axis and 1 cm to represent \$10 on the vertical axis, plot the pairs of values of (x, y).

Solution:

(a) (i) Amount of money left after 2 days $= \$100 - 2 \times \$3$
$= \$100 - \$6$
$= \$94$

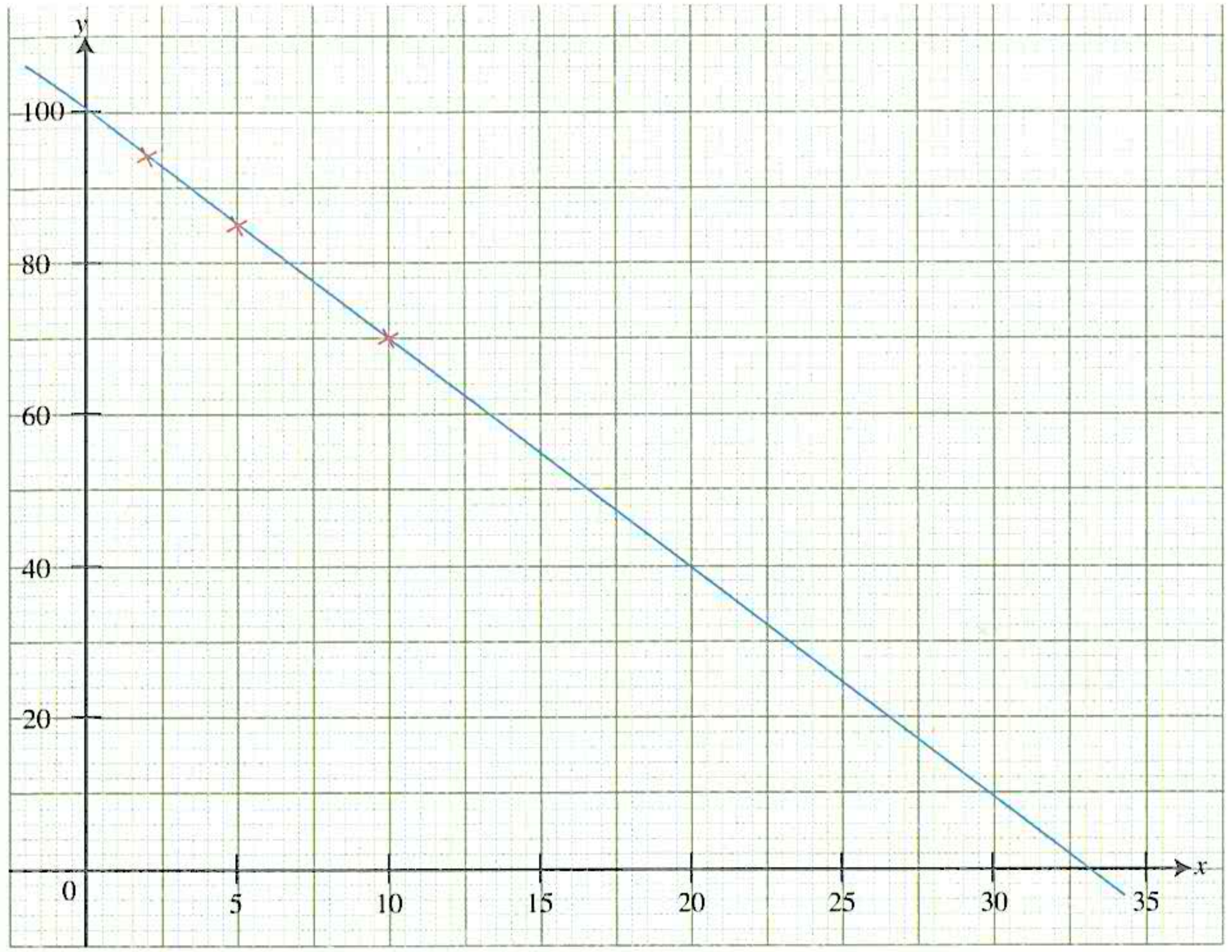
(ii) Amount of money left after 5 days $= \$100 - 5 \times \$3$
$= \$100 - \$15$
$= \$85$

(iii) Amount of money left after 10 days $= \$100 - 10 \times \$3$
$= \$100 - \$30$
$= \$70$

(b)

x	2	5	10
y	94	85	70

(c)



PRACTISE NOW 2

SIMILAR QUESTIONS

Exercise 6C Questions 1–3

The flag-down fare of a taxi is $3.

(a) Given that a passenger is charged $0.50 for each kilometre the taxi travels, find the amount of money the passenger has to pay if the taxi covers a distance of

(i) 3 km, **(ii)** 6 km, **(iii)** 10 km.

(b) Given that $$y$ represents the amount of money a passenger has to pay if the taxi travels x km, copy and complete the table.

x	3	6	10
y			

(c) On a sheet of graph paper, using a scale of 1 cm to represent 1 km on the horizontal axis and 2 cm to represent $1 on the vertical axis, plot the pairs of values of (x, y).

Thinking Time

In Worked Example 2, what is the value of y when

(i) $x = -2$? **(ii)** $x = 35$?

Explain the meaning of each of the values of y in the context of the question.

Exercise 6C

BASIC LEVEL

1. Huixian's father gives her \$120 as her monthly allowance.

(a) If she spends an average of \$5 each day, find the amount of money that she is left with after

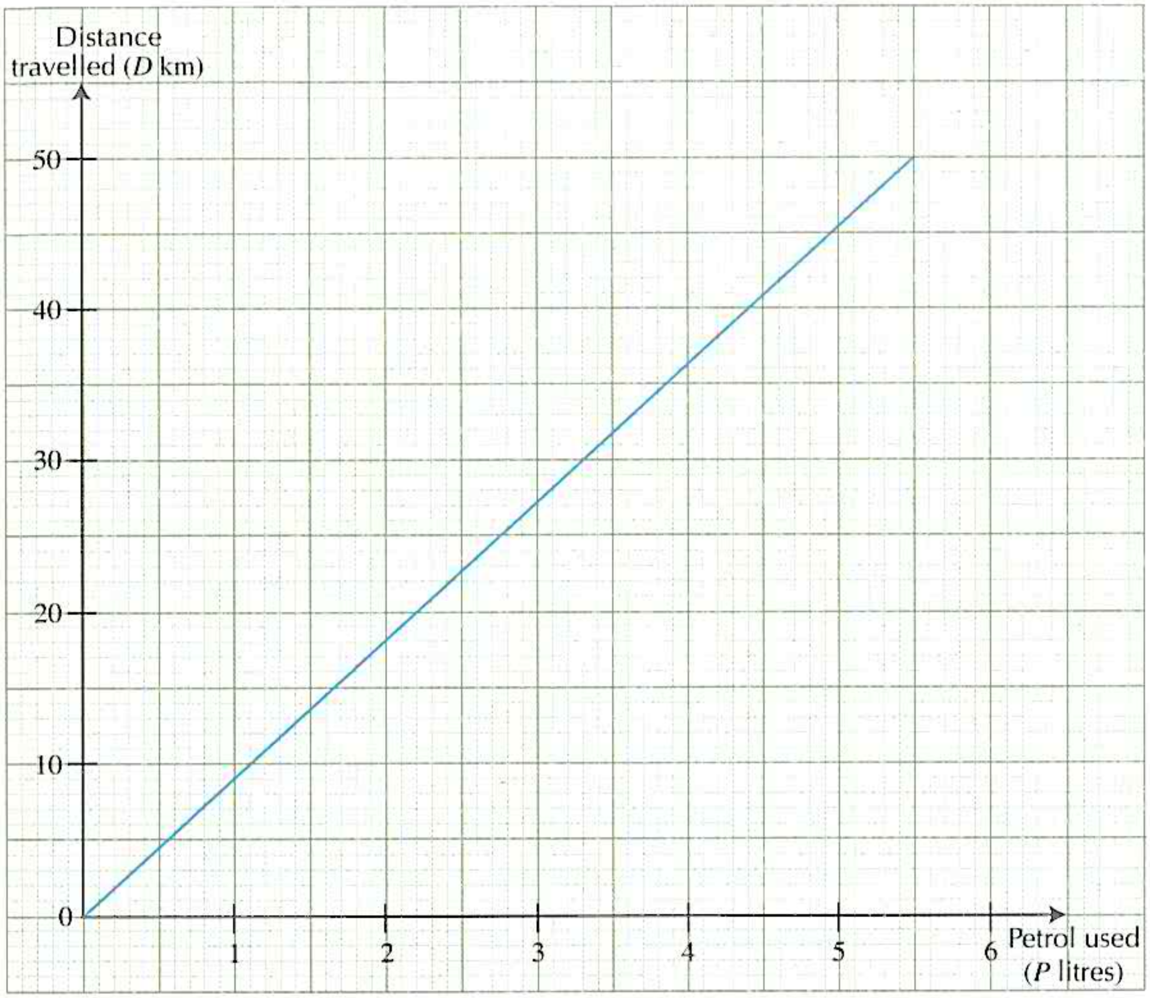
(i) 3 days, **(ii)** 6 days, **(iii)** 10 days.

(b) Given that \$$y$ represents the amount of money Huixian has left after x days, copy and complete the table.

x	3	6	10
y			

(c) On a sheet of graph paper, using a scale of 1 cm to represent 1 day on the horizontal axis and 1 cm to represent \$10 on the vertical axis, plot the pairs of values of (x, y).

2. The graph shows that P litres of petrol are consumed by a car to travel D km.



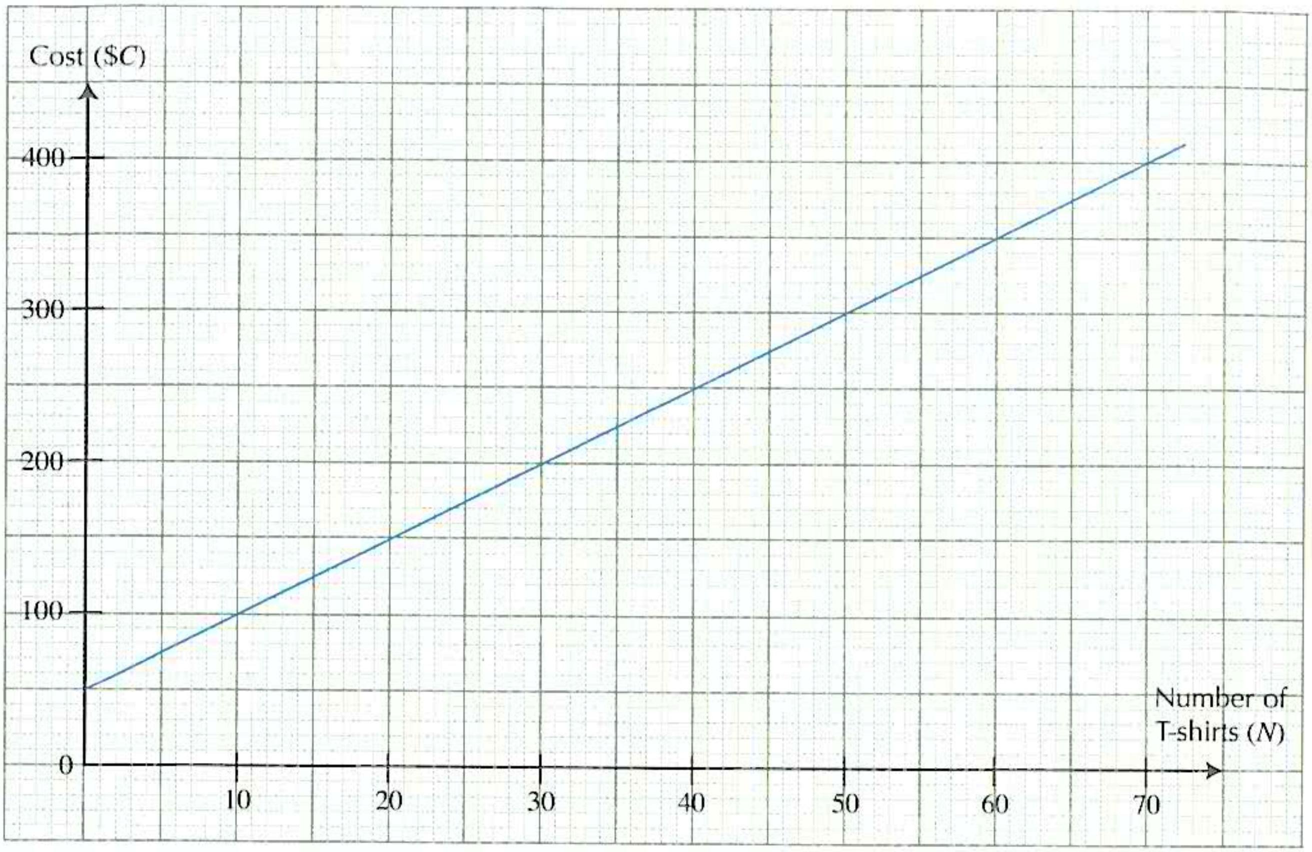
Use the graph to find

(a) how far the car can travel if it has

(i) 3 litres of petrol, **(ii)** 5.2 litres of petrol,

(b) the cost of petrol required to travel 36 km, given that 1 litre of petrol costs $1.40.

3. Devi, the president of the school photography club, wishes to order T-shirts with the club's logo for its members. She goes to Mr Lee's shop to find out the cost of ordering the T-shirts. Mr Lee, a mathematics enthusiast, shows her a graph displaying the cost ($\$C$) of making N T-shirts.



(i) Copy and complete the table.

N	10	30	50	70
C				

(ii) Devi notices that the cost per T-shirt decreases when more T-shirts are ordered and is puzzled by the observation from the graph that '0 T-shirt costs \$50'. Provide a possible explanation to this problem.

(iii) Find the amount of money Devi has to pay for 68 T-shirts.

(iv) If Devi has a budget of \$410, state the number of T-shirts she can order.

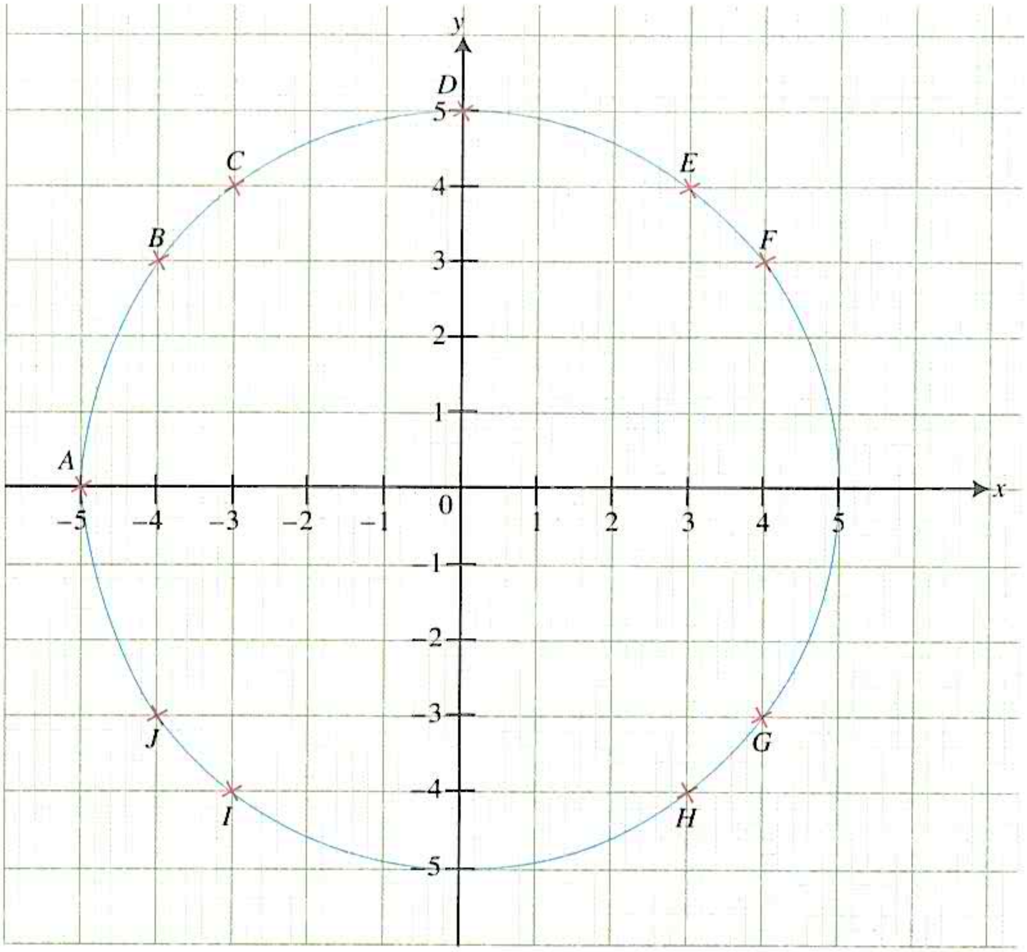
Summary

1. A **Cartesian** plane consists of two axes – the x-axis and the y-axis – intersecting at right angles at the origin $O(0, 0)$.
2. The **coordinates** of a point P in the Cartesian plane are (x, y), where x is the **x-coordinate** and y is the **y-coordinate** of the point.
3. A **function** is a relationship between two variables x and y such that every input x produces exactly one output y.
4. Every pair of values (x, y) that satisfies the equation of a function appears as a point on the graph of the function. Conversely, every point on the graph of the function has coordinates that satisfy the equation of the function.

Review Exercise 6

1. Plot each set of the given points on a sheet of graph paper. Join the points in order with straight lines and identify each geometrical shape obtained.
   (a) $(-2, 2)$, $(-2, 6)$, $(4, 6)$, $(4, 2)$
   (b) $(2, -2)$, $(6, 2)$, $(2, 6)$, $(-2, 2)$
   (c) $(2, -4)$, $(8, 4)$, $(6, 8)$, $(-2, 4)$
   (d) $(0, 7)$, $(2, 7)$, $(2, 5)$, $(-4, 1)$

2. The figure shows a circle.



(a) Write down the coordinates of each of the points shown in the figure.

(b) State the point on the circle that has

(i) the same x-coordinate as E,

(ii) the same y-coordinate as J.

3. The equation of a function is $y = 4x - 1\frac{1}{2}$. Find the value of y when

(i) $x = 12$, (ii) $x = 2\frac{1}{2}$, (iii) $x = -\frac{1}{2}$.

4. The equation of a function is $y = 250 - 20x$. Find the value of x when

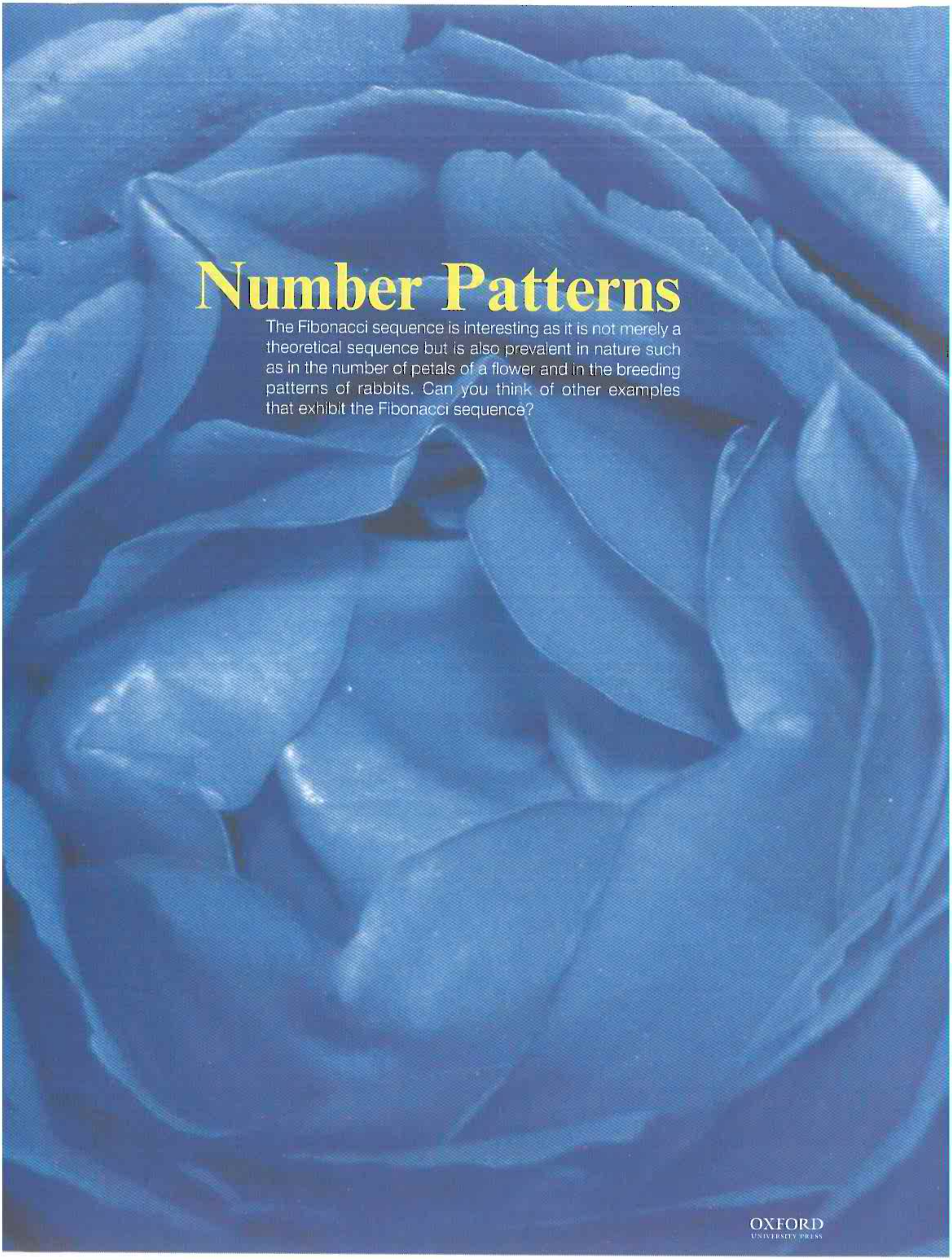
(i) $y = 150$, (ii) $y = 450$, (iii) $y = -1150$.

5. (i) On a sheet of graph paper, using a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of the linear function $y = 2\frac{1}{2}x + 3$ for values of x from -3 to 3.

(ii) The points $(-2, a)$ and $(b, 3)$ lie on the graph in (i). Find the value of a and of b.

Challenge Yourself

Two of the vertices of a triangle *ABC* are *A*(1, 1) and *B*(5, 5). The area of $\triangle ABC$ is 12 units^2 and the *y*-coordinate of the point *C* is 1. By plotting the points *A* and *B* on a sheet of graph paper, determine the possible *x*-coordinates of *C*.



Number Patterns

The Fibonacci sequence is interesting as it is not merely a theoretical sequence but is also prevalent in nature such as in the number of petals of a flower and in the breeding patterns of rabbits. Can you think of other examples that exhibit the Fibonacci sequence?

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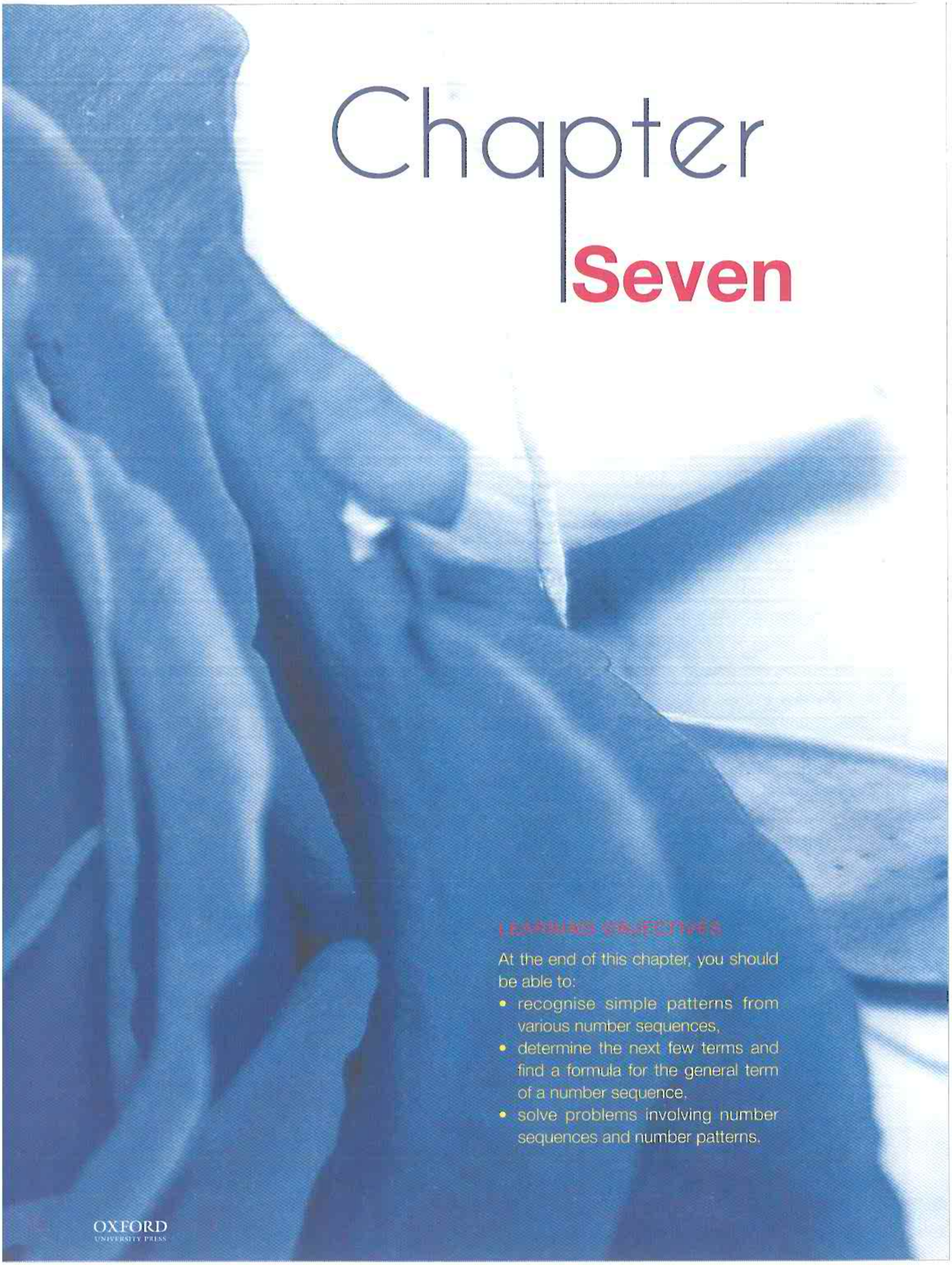

Chapter Seven

LEARNING OBJECTIVES

At the end of this chapter, you should be able to:

- recognise simple patterns from various number sequences,
- determine the next few terms and find a formula for the general term of a number sequence,
- solve problems involving number sequences and number patterns.

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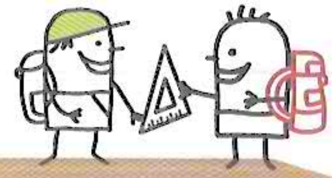
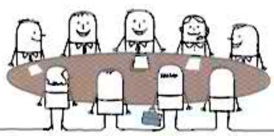
7.1 Number Sequences



Consider the following whole numbers:

1st term		2nd term		3rd term		4th term		5th term
2,	$\xrightarrow{+3}$	5,	$\xrightarrow{+3}$	8,	$\xrightarrow{+3}$	11,	$\xrightarrow{+3}$	14,...

2, 5, 8, 11, 14, ... forms a **number sequence**. The numbers in the number sequence are known as the **terms** of the sequence. The numbers are governed by a *specific rule*, i.e. start with 2, then add 3 to each term to get the next term.



Class Discussion

Number Sequences

Work in pairs.

1. Table 7.1 shows a few examples of number sequences. Complete Table 7.1.

	Sequence	Rule
Positive even numbers	2, 4, 6, 8, 10, ____, ____, ... +2 +2 +2 +2 ____ ____	Start with _____, then add _____ to each term to get the next term.
Positive odd numbers	1, 3, 5, 7, 9, ____, ____, ... +2 +2 +2 +2 ____ ____	Start with _____, then add _____ to each term to get the next term.
Multiples of 3	3, 6, 9, 12, 15, ____, ____, ... +3 +3 +3 +3 ____ ____	Start with _____, then add _____ to each term to get the next term.
Powers of 2	1, 2, 4, 8, 16, ____, ____, ... ×2 ×2 ×2 ×2 ____ ____	Start with _____, then multiply each term by _____ to get the next term.
Powers of 3	1, 3, 9, 27, 81, ____, ____, ... ×3 ×3 ×3 ×3 ____ ____	Start with _____, then multiply each term by _____ to get the next term.

Table 7.1

2. The sequence of positive even numbers can also be obtained by multiplying each term of the sequence 1, 2, 3, 4, 5, ... by 2. Can you think of a rule, different from that in Table 7.1, to obtain the sequence of positive odd numbers?

3. For each of the following sequences, state a rule and write down the next two terms.

(a) **Perfect squares:** 1, 4, 9, 16, 25, ... **(b)** **Perfect cubes:** 1, 8, 27, 64, 125, ...

Worked Example 1

(Rules of Sequences)

For each of the following sequences, state a rule and write down the next two terms.

(a) 42, 39, 36, 33, 30, …

(b) –22, –18, –14, –10, –6, …

(c) 256, 128, 64, 32, 16, …

(d) –1, 1, –1, 1, –1, …

Solution:

(a) $42 \xrightarrow{-3} 39 \xrightarrow{-3} 36 \xrightarrow{-3} 33 \xrightarrow{-3} 30, \ldots$

Rule: Subtract 3 from each term to get the next term. The next two terms are 27 and 24.

(b) $-22 \xrightarrow{+4} -18 \xrightarrow{+4} -14 \xrightarrow{+4} -10 \xrightarrow{+4} -6, \ldots$

Rule: Add 4 to each term to get the next term. The next two terms are –2 and 2.

(c) $256 \xrightarrow{\div 2} 128 \xrightarrow{\div 2} 64 \xrightarrow{\div 2} 32 \xrightarrow{\div 2} 16, \ldots$

Rule: Divide each term by 2 to get the next term. The next two terms are 8 and 4.

(d) $-1 \xrightarrow{\times(-1)} 1 \xrightarrow{\times(-1)} -1 \xrightarrow{\times(-1)} 1 \xrightarrow{\times(-1)} -1, \ldots$

Rule: Multiply each term by (–1) to get the next term. The next two terms are 1 and –1.

Problem Solving Tip

Check if
- the sequence is a common one that you recognise,
- two consecutive terms in the sequence are related by a constant value,
- you can add/subtract/multiply/divide consecutive terms to get the next term.

PRACTISE NOW 1

SIMILAR QUESTIONS

Exercise 7A Questions 1(a)–(k), 2(a)–(e), 3(a)–(e)

1. For each of the following sequences, state a rule and write down the next two terms.

(a) 3, 8, 13, 18, 23, …
(b) –20, –26, –32, –38, –44, …
(c) 5, 15, 45, 135, 405, …
(d) 4374, –1458, 486, –162, 54, …

2. Write down the next two terms of each of the following sequences.

(a) 1, 2, 4, 7, 11, 16, …
(b) 26, 25, 21, 20, 16, …

Exercise 7A

BASIC LEVEL

1. For each of the following sequences, state a rule and write down the next two terms.
   (a) 14, 19, 24, 29, 34, …
   (b) 80, 72, 64, 56, 48, …
   (c) 6, 12, 24, 48, 96, 192, …
   (d) 1600, 800, 400, 200, 100, …
   (e) –16 384, 4096, –1024, 256, –64, …
   (f) 9, –18, 36, –72, 144, …
   (g) –52, –59, –66, –73, –80, …
   (h) –100, –90, –80, –70, –60, …
   (i) 0, 10, 20, 30, 40, …
   (j) 52, 59, 66, 73, …
   (k) 4, 12, 36, 108

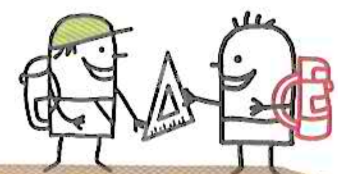
INTERMEDIATE LEVEL

2. Write down the next two terms of each of the following sequences.
   (a) –6, –5, –3, 0, 4, …
   (b) 47, 38, 30, 23, 17, …
   (c) –50, –45, –44, –39, –38, …
   (d) 100, 98, 95, 93, 90, …
   (e) 1, 3, 6, 10, 15, …

ADVANCED LEVEL

3. Write down the next two terms of each of the following sequences.
   (a) –5, –7, –11, –19, –35, …
   (b) 1, 1, 2, 3, 5, …
   (c) 4, 16, 36, 64, 100, …
   (d) 1, –8, 27, –64, 125, …
   (e) 1, 3, 9, 27, …

7.2 General Term of a Number Sequence



Simple Sequences

Consider the sequence of positive even numbers: 2, 4, 6, 8, 10, …
The terms of the sequence are denoted by $T_1, T_2, T_3, T_4, T_5, \ldots, T_n$, where

$$T_1 = 1^{\text{st}} \text{ term} = 2,$$
$$T_2 = 2^{\text{nd}} \text{ term} = 4,$$
$$T_3 = 3^{\text{rd}} \text{ term} = 6,$$
$$T_4 = 4^{\text{th}} \text{ term} = 8,$$
$$T_5 = 5^{\text{th}} \text{ term} = 10,$$
$$\vdots$$
$$T_n = n^{\text{th}} \text{ term } \textbf{(general term)}.$$

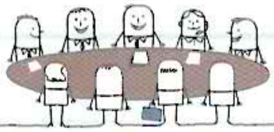
From Table 7.2, observe that each term in the sequence can be obtained by multiplying its position n by 2.

Position n	1	2	3	4	5	…	n
Term T_n	$2 \times 1 = 2$	$2 \times 2 = 4$	$2 \times 3 = 6$	$2 \times 4 = 8$	$2 \times 5 = 10$	…	$2n$

Table 7.2

Hence, the general term of the sequence is $T_n = 2n$.

Note: n is a variable, i.e. by substituting different values of n, we are able to generate the terms of the sequence. For example, the 68th term of the sequence is given by $T_{68} = 2(68) = 136$.



Class Discussion

Generalising Simple Sequences

Work in pairs.
For each of the following sequences, use the table provided to find a formula for the general term and hence, state the 100^{th} term, T_{100}.

(a) Multiples of 3: 3, 6, 9, 12, 15, …
From Table 7.3, observe that each term in the sequence can be obtained by multiplying its position n by 3.

Position n	1	2	3	4	5	…	n
Term T_n	$3 \times 1 = 3$	$3 \times 2 = 6$	$3 \times 3 = 9$	$3 \times 4 = 12$	$3 \times 5 = 15$	…	

Table 7.3

Hence, $T_n =$ ______.
100^{th} term, $T_{100} =$ ______

(b) Perfect squares: 1, 4, 9, 16, 25, …
From Table 7.4, observe that each term in the sequence can be obtained by squaring its position n .

Position n	1	2	3	4	5	…	n
Term T_n	$1^2 = 1$	$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$	…	

Table 7.4

Hence, $T_n =$ ______.
100^{th} term, $T_{100} =$ ______

(c) Perfect cubes: 1, 8, 27, 64, 125, …
From Table 7.5, observe that each term in the sequence can be obtained by cubing its position n.

Position n	1	2	3	4	5	…	n
Term T_n	$1^3 = 1$	$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$	…	

Table 7.5

Hence, $T_n =$ ______.
100^{th} term, $T_{100} =$ ______

Worked Example 2

(Finding a Specific Term)
Given that the n^{th} term, T_n, of a sequence is $T_n = 5n - 3$, find
(i) the 3^{rd} term, **(ii)** the difference between the 3^{rd} term and the 5^{th} term,
of the sequence.

Solution:

(i) $T_3 = 5(3) - 3$
$= 15 - 3$
$= 12$

(ii) $T_5 = 5(5) - 3$
$= 25 - 3$
$= 22$

Difference between the 3^{rd} term and the 5^{th} term of the sequence $= T_5 - T_3$
$= 22 - 12$
$= 10$

PRACTISE NOW 2

SIMILAR QUESTIONS

Exercise 7B Question 2

Given that the n^{th} term, T_n, of a sequence is $T_n = 4n + 7$, find

(i) the 4^{th} term, **(ii)** the sum of the 4^{th} term and the 7^{th} term,

of the sequence.

More Complicated Sequences

Consider the sequence 2, 5, 8, 11, 14, …

How do we find a formula for the general term?

Looking for a Pattern

Notice that the differences between consecutive terms are all equal to a constant. Thus we say that the **common difference** of this sequence is equal to 3.

Position n	1	2	3	4	5	
Term T_n	2,	5,	8,	11,	14,	…
	+3	+3	+3	+3	+3	

Therefore, we can express each term as follows:

$$\begin{aligned}
T_1 &= 2 = 2 &&= 2 + 0 \times 3\\
T_2 &= 5 = 2 + 3 &&= 2 + 1 \times 3\\
T_3 &= 8 = 2 + 3 + 3 &&= 2 + 2 \times 3\\
T_4 &= 11 = 2 + 3 + 3 + 3 &&= 2 + 3 \times 3\\
T_5 &= 14 = 2 + 3 + 3 + 3 + 3 &&= 2 + 4 \times 3\\
&\vdots\\
T_n &= 2 + \underbrace{3 + 3 + 3 + \ldots + 3}_{(n-1)\text{ terms}} &&= 2 + (n-1) \times 3
\end{aligned}$$

By looking at the above pattern, we can infer that

$$\begin{aligned}
T_n &= 2 + (n - 1) \times 3\\
&= 2 + 3n - 3\\
&= 3n - 1.
\end{aligned}$$

Transformation to Another Sequence

Let us try another method to find a formula for the general term of the same sequence 2, 5, 8, 11, 14, …

Since the *common difference is equal to 3*, we can transform this sequence to another sequence which consists of terms that are *multiples of 3* (*the first term must be 3* so that we know that the general term is given by $3n$ immediately):

Position n	1	2	3	4	5	n
Term T_n	2,	5,	8,	11,	14, …	$3n - 1$
	↓ +1	↓ +1	↓ +1	↓ +1	↓ +1	↑ −1
	3,	6,	9,	12,	15, …	$3n$

Since we add 1 to each term of the first sequence to obtain the corresponding term in the second sequence, we will have to subtract 1 from $3n$ (as we are moving *backwards*) to get $3n - 1$.

Therefore, the general term of the original sequence is $T_n = 3n - 1$.

Connecting the General Term to a Graph

The explanation for this method is long, but after understanding how it works, we will be able to write down the formula for the general term of a sequence straightaway *without drawing the graph*. This is by far the *fastest* method.

Let us consider the same sequence 2, 5, 8, 11, 14, …

If we plot T_n against n, we will get a straight line as shown in Fig. 7.1(a).
The equation of the straight line is $T_n = mn + c$ (in the form of $y = mx + c$).

From Fig. 7.1(b), gradient $m = \frac{\text{rise}}{\text{run}} = \frac{3}{1} = 3 = d$, where d is the common difference and T_n-intercept $c = 2 - 3 = -1$.

There is another way to find c without looking at the graph.

When $n = 0$, $T_0 = c$, where T_0 is the term just before the first term T_1.

Thus we can find $c = T_0 = 2 - 3 = -1$ *mentally* as follows:

T_0	T_1	T_2	T_3	T_4	T_5	
?,	2,	5,	8,	11,	14,	…
-3	$+3$	$+3$	$+3$	$+3$	$+3$	

Hence, the general term is $T_n = 3n - 1$.

Worked Example 3 shows how to use the above concept to find a formula for the general term of a sequence without drawing the graph.

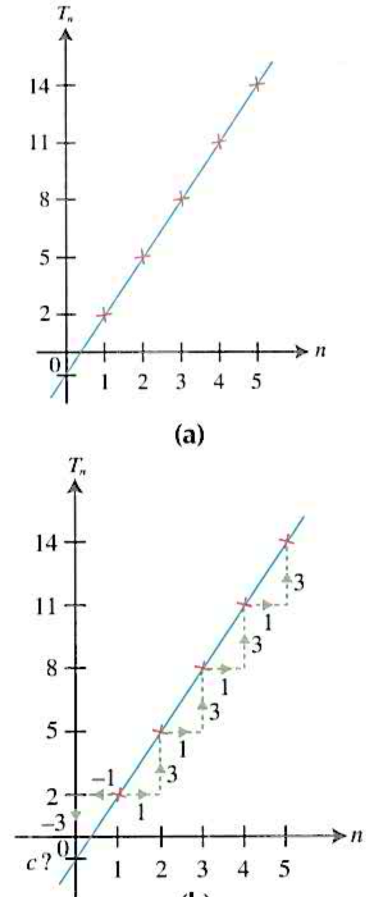


Fig. 7.1

Worked Example 3

(Finding the General Term by Observation)

Find a formula for the general term of the sequence 8, 12, 16, 20, 24, …

Solution:

T_0	T_1	T_2	T_3	T_4	T_5	
?,	8,	12,	16,	20,	24,	…
-4	$+4$	$+4$	$+4$	$+4$	$+4$	

Since the common difference is 4, $T_n = 4n + ?$.

The term before T_1 is $c = T_0$

$= 8 - 4$

$= 4$.

$\therefore$ General term of the sequence, $T_n = 4n + 4$

Problem Solving Tip

You can work out the solution *mentally* and write down the formula straightaway.

PRACTISE NOW 3

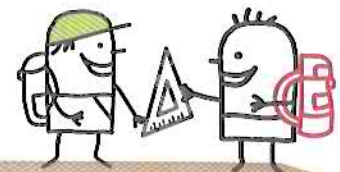
SIMILAR QUESTIONS

Exercise 7B Questions 1(a)–(d), 3–4, 7, 16(a)

1. Find a formula for the general term of each of the following sequences.
   (a) 5, 9, 13, 17, 21, … **(b)** 7, 12, 17, 22, 27, …
   (c) 2, 8, 14, 20, 26, … **(d)** 1, 4, 7, 10, 13, …

2. Consider the sequence 3, 7, 11, 15, 19, …
   (i) Write down the next two terms of the sequence.
   (ii) Find, in terms of n, a formula for the n^{th} term of the sequence.
   (iii) Hence, find the 50^{th} term.

7.3 Number Patterns



In this section, we shall apply what we have learnt for number sequences on **number patterns**.

Worked Example 4

(Number Pattern)

The first four figures of a sequence are as shown.

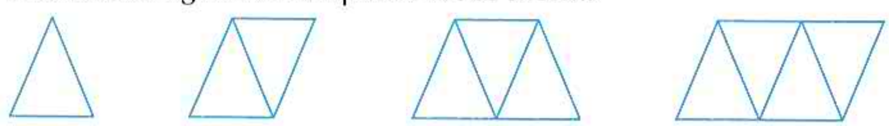


Figure 1 Figure 2 Figure 3 Figure 4

(i) Draw the next two figures of the sequence.

(ii) Complete the table.

Figure Number	Number of Triangles	Number of Lines
1	1	$1 + 1 \times 2 = 3$
2	2	$1 + 2 \times 2 = 5$
3	3	$1 + 3 \times 2 = 7$
4	4	$1 + 4 \times 2 = 9$
5		
6		
⋮	⋮	⋮
n		

(iii) Calculate the number of triangles and lines in the 121^{st} figure.

(iv) Write down a formula connecting the number of triangles, T, and the number of lines, L, in the sequence shown above.

Solution:

(i) The next two figures of the sequence are:

Figure 5 Figure 6

(ii)

Figure Number	Number of Triangles	Number of Lines
1	1	$1 + 1 \times 2 = 3$
2	2	$1 + 2 \times 2 = 5$
3	3	$1 + 3 \times 2 = 7$
4	4	$1 + 4 \times 2 = 9$
5	5	$1 + 5 \times 2 = 11$
6	6	$1 + 6 \times 2 = 13$
⋮	⋮	⋮
n	n	$1 + n \times 2 = 2n + 1$

(iii) In the 121st figure, i.e. $n = 121$,

Number of triangles $= n$
$= 121$

Number of lines $= 2n + 1$
$= 2(121) + 1$
$= 243$

(iv) From **(ii)**, number of triangles, $T = n$
number of lines, $L = 2n + 1$

Since $T = n$, we substitute n with T in $L = 2n + 1$ to get $L = 2T + 1$ as the formula connecting T and L.

PRACTISE NOW 4

SIMILAR QUESTIONS

Exercise 7B Questions 6, 8–15, 16(b)

1. The first four figures of a sequence are as shown.

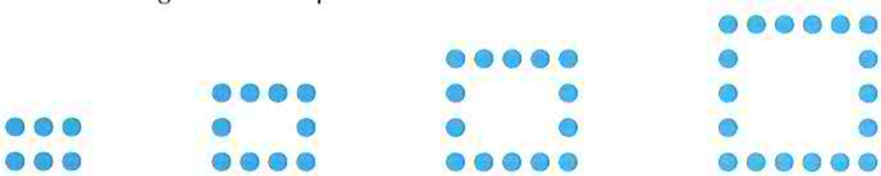


Figure 1 Figure 2 Figure 3 Figure 4

(i) Draw the next two figures of the sequence.
(ii) Complete the table.

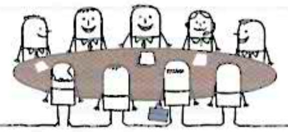
Figure Number	Number of Dots
1	$2 + 1 \times 4 = 6$
2	$2 + 2 \times 4 = 10$
3	$2 + 3 \times 4 = 14$
4	$2 + 4 \times 4 = 18$
5	
6	
⋮	⋮
n	

(iii) Find the number of dots in the 2013th figure.

2. Consider the following number pattern:

$$\begin{aligned} 2 &= 1 \times 2 \\ 6 &= 2 \times 3 \\ 12 &= 3 \times 4 \\ 20 &= 4 \times 5 \\ &\vdots \\ 110 &= k(k + 1) \\ &\vdots \end{aligned}$$

(i) Write down the 8th line in the pattern.
(ii) Deduce the value of k.



Class Discussion

The Triangular Number Sequence

Work in pairs.

Fig. 7.2 shows the first 4 figures of the triangular number sequence.

A triangular number is the number of equally-spaced objects in a triangle.

Fig. 7.2

1. Use coins to make the next two figures of the sequence shown in Fig. 7.2.

2. Complete Table 7.6.

Figure Number, n	Number of Coins at the Base of the Triangle, n	Total Number of Coins, T_n
1	1	$1 = 1 = \frac{1 \times 2}{2}$
2	2	$1 + 2 = 3 = \frac{2 \times 3}{2}$
3	3	$1 + 2 + 3 = 6 = \frac{3 \times 4}{2}$
4	4	$1 + 2 + 3 + 4 = 10 = \frac{4 \times 5}{2}$
5		
6		
⋮	⋮	⋮
n		

Table 7.6

3. Find the total number of coins needed to form a triangle with a base that has 100 coins.

7.4 Number Patterns in Real-World Contexts

In this section, we shall take a look at number patterns that can be found in the real world.

Investigation

Fibonacci Sequence

As mentioned at the start of this chapter, the number of petals of a flower follows a special type of sequence known as the **Fibonacci sequence**. In this investigation, we shall explore this by taking a look at some flowers.

1. Write down the number of petals for each of the following flowers on the line beside its name. Some of them have been done for you.

Picture A: White Calla Lily ____

Picture B: Euphorbia 2
(Note: There are 8 flowers in the picture.)

Picture C: Mariposa Lily 3

Picture D: Madagascar Periwinkle ____

Picture E: Moonbeam Coreopsis 8

Picture F: Black-eyed Susan ____

Picture G: Shasta Daisy ____

Picture H: Sunflower 34

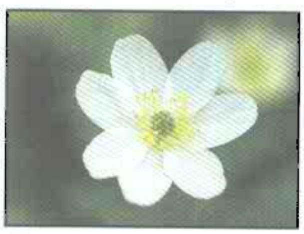
2. The number of petals for each of the flowers forms a sequence.
Fill in the next 6 terms of the sequence.
1, 1, 2, _____, _____, _____, _____, _____, _____.
This is known as the Fibonacci sequence.

3. A sunflower has 34 petals. Using your answer in Question 2, predict the number of petals for the flower next in the sequence. The name of this flower is Michaelmas Daisy (see photo below).

4. Note that there are four common exceptions to the Fibonacci sequence. Write down the number of petals for each of the following flowers on the line beside its name.

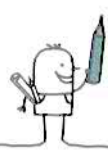
Picture I: Ixora _____

Picture J: Daylily _____



Picture K: Anemone Nemorosa _____

Picture L: Passion Flower _____



Journal Writing

Find out more about Pascal's Triangle. Illustrate clearly how the Fibonacci sequence is found in Pascal's Triangle.

Worked Example 5

(Number Patterns in Chemistry)

The members in a family of chemical compounds are made up of carbon atoms and hydrogen atoms.

(i) The number of carbon atom(s) and hydrogen atoms of the first four members in the family are given in the table. Complete the table.

Member Number	Number of carbon atom(s)	Number of hydrogen atoms
1	1	4
2	2	6
3	3	8
4	4	10
5		
6		
⋮	⋮	⋮
n		

(ii) If a member of the family has 30 carbon atoms, how many hydrogen atoms does it have?

(iii) If a member of the family has 52 hydrogen atoms, how many carbon atoms does it have?

Solution:

(i)

Member Number	Number of carbon atom(s)	Number of hydrogen atoms
1	1	4
2	2	6
3	3	8
4	4	10
5	5	12
6	6	14
⋮	⋮	⋮
n	n	$2n + 2$

To find a formula for the general term of the number of hydrogen atoms, consider the sequence 4, 6, 8, 10, 12, 14, …

$$\begin{array}{ccccccc} T_0 & T_1 & T_2 & T_3 & T_4 & T_5 & T_6 \\ ?, & 4, & 6, & 8, & 10, & 12, & 14, \ldots \end{array}$$

$$-2 \quad +2 \quad +2 \quad +2 \quad +2 \quad +2$$

Since the common difference is 2, $T_n = 2n + ?$.

The term before T_1 is $c = T_0$

$= 4 - 2$

$= 2.$

$\therefore$ General term of the sequence, $T_n = 2n + 2$

(ii) When $n = 30$, $T_{30} = 2(30) + 2$

$= 62$

The member of the family has 62 hydrogen atoms.

(iii) Let $2n + 2 = 52$.

$2n = 52 - 2$

$= 50$

$\therefore n = 25$

The member of the family has 25 carbon atoms.

PRACTISE NOW 5

SIMILAR QUESTIONS

Exercise 7B Questions 17–18

The members in a family of chemical compounds are made up of carbon atoms and hydrogen atoms.

(i) The number of carbon atom(s) and hydrogen atoms of the first four members in the family are given in the table. Complete the table.

Member Number	Number of carbon atoms	Number of hydrogen atoms
1	2	4
2	3	6
3	4	8
4	5	10
5		
6		
⋮	⋮	⋮
n		

(ii) If the h^{th} member of the family has 55 carbon atoms, find the value of h. Hence, find the number of hydrogen atoms the member has.

(iii) If the k^{th} member of the family has 120 hydrogen atoms, find the value of k. Hence, find the number of carbon atoms the member has.

Exercise 7B

BASIC LEVEL

1. Find a formula for the general term of each of the following sequences.

(a) 7, 13, 19, 25, 31, … **(b)** –4, –1, 2, 5, 8, …

(c) 60, 67, 74, 81, 88, … **(d)** 14, 11, 8, 5, 2, …

2. Given that the n^{th} term, T_n, of a sequence is $T_n = 2n + 5$, find

(i) the 5^{th} term,

(ii) the 8^{th} term,

(iii) the lowest common multiple of the 5^{th} term and the 8^{th} term,

of the sequence.

3. Consider the sequence 3, 6, 9, 12, 15, …

(i) Write down the next two terms of the sequence.

(ii) Find, in terms of n, a formula for the n^{th} term of the sequence.

(iii) Hence, find the 105^{th} term.

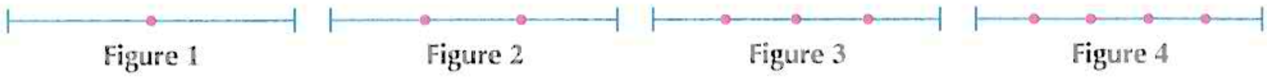
4. Consider the sequence 10, 14, 18, 22, 26, …

(i) Write down the next two terms of the sequence.

(ii) Find, in terms of n, a formula for the n^{th} term of the sequence.

(iii) Hence, find the 200^{th} term.

5. The first four figures of sequence are as shown.



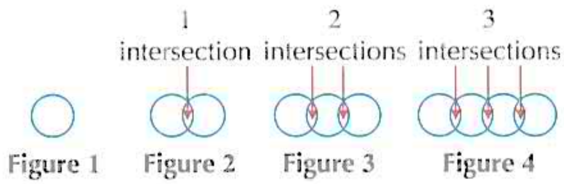
(i) Complete the table.

Number of points	1	2	3	4	5	6
Number of segments	$1 + 1 = 2$	$2 + 1 = 3$				

(ii) Hence, write down the number of segments there will be when 49 points divide the given line segment.

(iii) How many points are needed to divide a given line segment into 101 segments?

6. The first four figures of a sequence are as shown.



(i) Draw the next two figures of the sequence.

(ii) Complete the table.

Figure Number	Number of Intersection(s) between the Circles
1	0
2	1
3	
4	
5	
6	
⋮	⋮
n	

(iii) Find the value of n for which the circles in the figure have 28 intersections.

INTERMEDIATE LEVEL

7. (a) The n^{th} term of a sequence is given by $2n^2 + 1$. Write down the first four terms of the sequence.

(b) The first four terms of another sequence are 1, 7, 17, 31.

(i) By comparing this sequence with the sequence in (a), write down, in terms of n, a formula for the n^{th} term of the sequence.

(ii) Hence, find the 388th term.

8. The first four figures of a sequence are as shown.

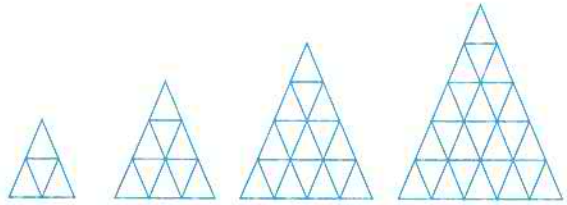


Figure 1 Figure 2 Figure 3 Figure 4

(i) Draw the next two figures of the sequence.

(ii) Complete the table.

Figure Number	Number of Small Triangles
1	4
2	9
3	
4	
5	
6	
⋮	⋮
n	

(iii) Find the number of triangles in the 20th figure.

(iv) Find the value of n for which the figure has 121 small triangles.

9. Consider the following number pattern:

$$4 = 1 \times 4$$
$$10 = 2 \times 5$$
$$18 = 3 \times 6$$
$$28 = 4 \times 7$$
$$\vdots$$
$$208 = k(k + 3)$$
$$\vdots$$

(i) Write down the 6^{th} line in the pattern.

(ii) Deduce the value of k.

10. Consider the following number pattern:

$$1 + 3 = 4 = 2^2 = (1 + 1)^2$$
$$1 + 3 + 5 = 9 = 3^2 = (2 + 1)^2$$
$$1 + 3 + 5 + 7 = 16 = 4^2 = (3 + 1)^2$$
$$1 + 3 + 5 + 7 + 9 = 25 = 5^2 = (4 + 1)^2$$
$$\vdots$$
$$1 + 3 + 5 + 7 + \ldots + a = b = c^2 = (d + 1)^2$$
$$\vdots$$

(i) Write down the 5^{th} line in the pattern.

(ii) Given that $b = 169$, find the values of a, c and d.

11. A restaurant has only small square tables that can be joined end to end to form a large long table. Each small square table can seat only one person on each side.

(a) Study the diagram below and complete the tables **(i)** and **(ii)** that follow.

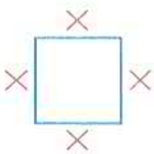


Figure 1

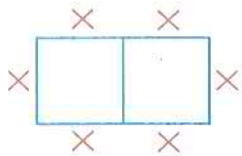


Figure 2

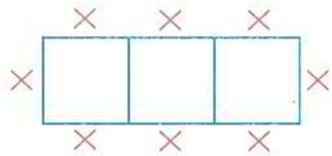


Figure 3

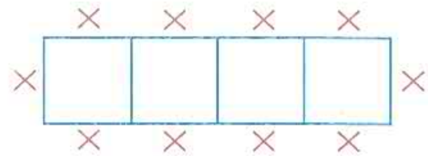


Figure 4

(i)

Number of people	4	6	8	10	12	14
Number of tables	$\frac{4-2}{2}=1$	$\frac{6-2}{2}=2$				

(ii)

Number of tables	1	2	3	4	5	6
Number of people	$2(1)+2=4$	$2(2)+2=6$				

(b) How many tables will be needed to seat

(i) 20 people, **(ii)** 30 people?

(c) How many people can be seated if there are

(i) 22 tables, **(ii)** 36 tables?

12. The diagram shows a line segment, AB, on which 18 points (P_1, P_2, ..., P_{18}) are marked.

Figure 1

(i) Study the diagram below and complete the table that follows.

Figure 2

Figure 3

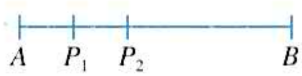


Figure 4

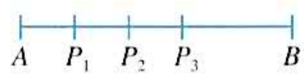


Figure 5

Number of points on the line segments AB (including the points A and B)	2	3	4	5	6	7
Number of possible line segments	$\frac{2\times(2-1)}{2}=1$	$\frac{3\times(3-1)}{2}=3$*				

*The three line segments are AP_1, P_1B and AB.

(ii) What is the total number of possible line segments in Figure 1?

ADVANCED LEVEL

13. The first five rows of Pascal's Triangle are as shown.

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

(i) Write down the next line in the pattern.

(ii) Complete the table.

Row	Sum
1	$1 = 1 = 2^0$
2	$1 + 1 = 2 = 2^1$
3	$1 + 2 + 1 = 4 = 2^2$
4	$1 + 3 + 3 + 1 = 8 = 2^3$
5	$1 + 4 + 6 + 4 + 1 = 16 = 2^4$
6	
⋮	⋮
n	$1 + (n - 1) + \ldots + (n - 1) + 1 =$ ______

14. The following diagrams show a sequence of tiling patterns. The shape of each tile is a 1 cm by 1 cm square.

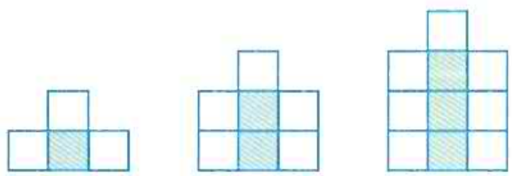


Figure 1 Figure 2 Figure 3

(a) Copy and complete the table below.

Figure	1	2	3	4	5	6
Number of black squares (b)						
Number of white squares (w)						
Area of the whole figure ($b + w$)						
Perimeter of the whole figure (cm)						

(b) Find

(i) the number of white squares in Figure 9,

(ii) the perimeter of Figure 9,

(iii) the number of white squares in Figure n, giving your answer in terms of n,

(iv) the perimeter of the whole figure in Figure n, giving your answer in terms of n.

15. Consider the pattern:

$$\frac{2}{1\times2\times3} = \frac{1}{1} - \frac{2}{2} + \frac{1}{3}$$
$$\frac{2}{2\times3\times4} = \frac{1}{2} - \frac{2}{3} + \frac{1}{4}$$
$$\frac{2}{3\times4\times5} = \frac{1}{3} - \frac{2}{4} + \frac{1}{5}$$
$$\vdots$$
$$\frac{2}{p(p+1)(p+2)} = \frac{1}{p} - \frac{2}{p+1} + \frac{1}{p+2}$$
$$\vdots$$

(i) Write down the 8th line in the pattern.

(ii) Use the above pattern to find the value of $\frac{1}{10} - \frac{2}{11} + \frac{1}{12}$.

(iii) Find the value of p such that $\frac{2}{7980} = \frac{1}{p} - \frac{2}{p+1} + \frac{1}{p+2}$.

16. (a) Write down the next two terms of the sequence.

(i) 3, 5, 7, 9, …

(ii) 8, 12, 16, 20, …

(iii) 4, 12, 24, 40, 60, …

(iv) 5, 13, 25, 41, 61, …

(b) Consider the following pattern carefully.

$3^2 + 4^2 = 5^2$
$5^2 + 12^2 = 13^2$
$7^2 + 24^2 = 25^2$
$9^2 + 40^2 = 41^2$
$11^2 + 60^2 = 61^2$
$\vdots$

Write down the next two lines in the pattern with the help of the number sequence in **(a)**.

17. The members in a family of chemical compounds are made up of carbon atoms and hydrogen atoms.

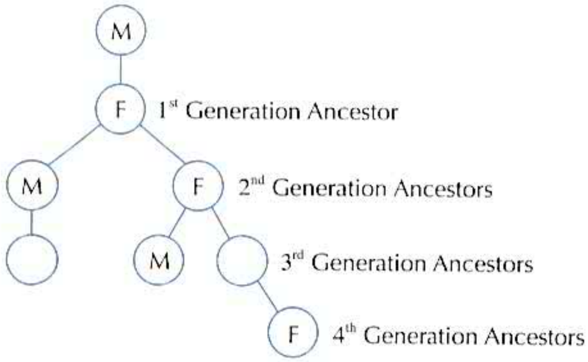
(i) The number of carbon atom(s) and hydrogen atoms of the first four members in the family are given in the table. Complete the table.

Member Number	Number of carbon atoms	Number of hydrogen atoms
1	3	4
2	4	6
3	5	8
4	6	10
5		
6		
⋮	⋮	⋮
n		

(ii) If the h^{th} member of the family has 25 carbon atoms, find the value of h. Hence, find the number of hydrogen atoms the member has.

(iii) If the k^{th} member of the family has 64 hydrogen atoms, find the value of k. Hence, find the number of carbon atoms the member has.

18. It is a curious biological fact that a male bee (M) has only one parent (the mother) while a female bee (F) has two parents (both mother and father). The figure shows *part of* the bee ancestry model for a male bee up to his 4^{th} generation ancestors.



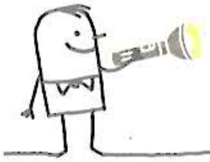
(i) Complete the model and show all the 4^{th} generation ancestors. How many 4^{th} generation ancestors does a male bee have?

(ii) The number of n^{th} generation ancestors forms a sequence with an interesting pattern. How do you obtain the next term in the sequence?

(iii) Predict the number of 5^{th} generation ancestors that a male bee has. Verify your answer by drawing the 5^{th} generation ancestors in the above model.

(iv) Predict the number of 10^{th} generation ancestors that a male bee has.

1. A **number sequence** is formed by a set of numbers. These numbers, known as the **terms** of the sequence, are governed by a *specific rule*.
2. The **general term** T_n of a number sequence can be represented by an algebraic expression.



Review Exercise 7

1. Write down the next two terms of each of the following sequences.
   (a) 98, 89, 80, 71, 62, ...
   (b) −2, 0, 4, 10, 18, ...
   (c) $9, 3, 1, \frac{1}{3}, \frac{1}{9}, \ldots$
   (d) 1, 9, 25, 49, 81, ...

2. Consider the sequence 9, 16, 25, 36, 49, …
   (i) Write down the next two terms of the sequence.
   (ii) Find, in terms of n, a formula for the n^{th} term of the sequence.
   (iii) Hence, find the 25^{th} term.

3. Kate uses buttons to make a sequence of figures. The first four figures are as shown.

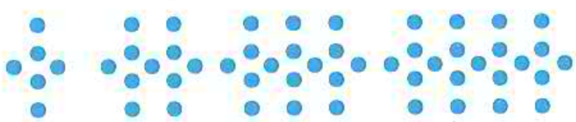


Figure 1 Figure 2 Figure 3 Figure 4

   (i) Draw the 5^{th} figure.
   (ii) Complete the table.

Figure Number	Number of Buttons
1	$5 \times 1 + 1 = 6$
2	$5 \times 2 + 1 = 11$
3	$5 \times 3 + 1 = 16$
4	$5 \times 4 + 1 = 21$
5	
⋮	⋮
n	

   (iii) Find the number of buttons in the 56^{th} figure.
   (iv) Is it possible for a figure in the sequence to be made up of 583 buttons? Explain your answer.

4. The first four figures of a sequence are as shown.

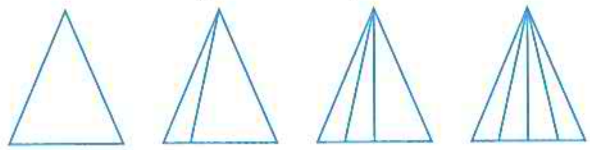


Figure 1 Figure 2 Figure 3 Figure 4

   (i) Draw the next figure of the sequence.
   (ii) Complete the table.

Figure Number	Number of Triangles
1	$1 = \frac{1 \times 2}{2}$
2	$3 = \frac{2 \times 3}{2}$
3	$6 = \frac{3 \times 4}{2}$
4	$10 = \frac{4 \times 5}{2}$
5	
⋮	⋮
n	

   (iii) Find the number of triangles in the 77^{th} figure.
   (iv) Find the value of n for which the figure has 66 triangles.

5. Consider the following number pattern:

$$1^3 = 1 = 1^2$$
$$1^3 + 2^3 = 9 = (1 + 2)^2$$
$$1^3 + 2^3 + 3^3 = 36 = (1 + 2 + 3)^2$$
$$1^3 + 2^3 + 3^3 + 4^3 = 100 = (1 + 2 + 3 + 4)^2$$
$$\vdots$$

(i) Write down the 7^{th} line in the pattern.

(ii) Find the value of $1^3 + 2^3 + 3^3 + 4^3 + \ldots + 15^3$.

(iii) Given that $1^3 + 2^3 + 3^3 + 4^3 + \ldots + k^3 = 1296$, find the value of k.

6. Three different sequences are shown in the table.

Sequence	1^{st} term	2^{nd} term	3^{rd} term	4^{th} term	5^{th} term
A	4	6	8	10	12
B	3	8	15	24	35
C	5	10	17	26	a

(i) Find the value of a.

Hint: The first term of sequence C is obtained from the first terms of sequences A and B as follows: $\sqrt{4^2 + 3^2} = 5$.

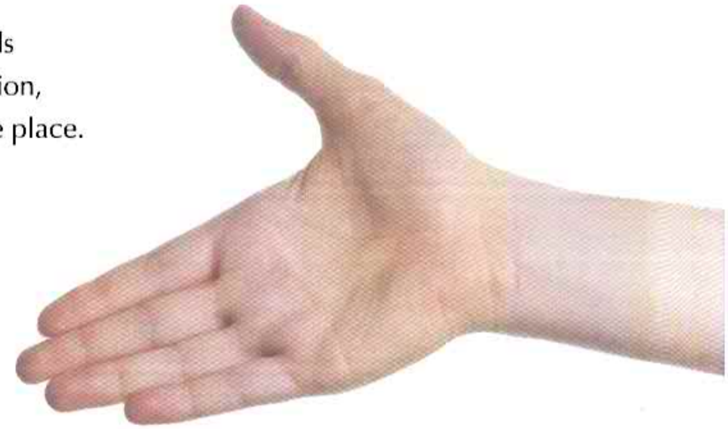
(ii) Find, in terms of n, a formula for the n^{th} term of sequence A.

(iii) If the n^{th} term of sequence B is given by $n^2 + 2n$, find the 18^{th} term of sequence C.

Challenge Yourself

1. Determine the last digit of 3^{2015}.

2. There are n people at a party. If each person shakes hands with each of the other people only once, find an expression, in terms of n, for the number of handshakes that will take place.



3. Consider the sequence 1, 4, 9, 7, 7, 9, 4, 1, 9, 1, …
   (i) Write down the next two terms of the sequence.
   (ii) State the rule of the sequence.

4. Consider the sequence 2, 1, 3, 4, 7, …
   (i) Write down the next two terms of the sequence.
   (ii) State the rule of the sequence.
   (iii) There is a name for this sequence. Search on the Internet to find out its name.

5. Consider the sequence 3, 0, 2, 3, 2, 5, 5, 7, …
   (i) Write down the next two terms of the sequence.
   (ii) State the rule of the sequence.
   (iii) There is a name for this sequence. Search on the Internet to find out its name.

B1 Revision Exercise

1. Solve each of the following equations.

   (a) $0.15x + 2.35(x - 2) = 1.3$

   (b) $\dfrac{5}{1 - y} - \dfrac{7}{2 - 2y} = 4$

2. Solve each of the following inequalities.

   (a) $12x > 60$

   (b) $15y \leqslant -24$

3. If $\dfrac{x - 4y}{5x + y} = \dfrac{3}{5}$, find the value of $\dfrac{x}{3y}$.

4. A box contains 54 coins which are either 20-cent coins or 50-cent coins. If the total value of all the coins is \$20.70, find the number of 20-cent coins in the box.

5. A motorist travels part of a 375-km journey on an expressway at 95 km/h and the rest along a stretch of road at 65 km/h. The time he spends on the stretch of road at 65 km/h is twice that of which he spends on the expressway. Find the time taken for his entire journey.

6. On a sheet of graph paper, using a scale of 1 cm to represent 1 unit on both axes, draw the graphs of the functions $y = \dfrac{1}{2}x + 3$ and $y = -x + 6$ for values of x from -4 to 6.

7. Consider the sequence 6, 15, 24, 33, 42, …

   (i) Find, in terms of n, a formula for the n^{th} term of the sequence.

   (ii) Given that the k^{th} term of the sequence is 159, find the value of k.

8. Consider the following number pattern:

$$\begin{aligned} 1 &= 1^2 \\ 1 + 3 &= 2^2 \\ 1 + 3 + 5 &= 3^2 \\ 1 + 3 + 5 + 7 &= 4^2 \\ &\vdots \\ 1 + 3 + 5 + \ldots + (2k - 1) &= 144 \\ &\vdots \end{aligned}$$

   (i) Write down the 8^{th} line in the pattern.

   (ii) Find the value of k.

B2 Revision Exercise

1. Solve each of the following equations.

   (a) $\frac{1}{3}(x-3)-x+5=3(x-1)$

   (b) $\frac{2}{y}-\frac{3}{y}+1=3$

2. Solve each of the following inequalities.

   (a) $14x \geqslant -110$

   (b) $-18 < 3y$

3. The sum of 7 consecutive even numbers is 336. Find the smallest of the 7 numbers.

4. A meat-seller sells x kg of duck at \$8.50 per kg and $(2x + 5)$ kg of chicken at \$3.60 per kg. If he collects a total of \$206.40, find the value of x.

5. (i) The vertices of a triangle are $A(0, -4)$, $B(4, -2)$ and $C(2, 2)$. Plot the points A, B and C on a sheet of graph paper.

   (ii) Plot the point D such that $ABCD$ is a square. Hence, write down the coordinates of D.

6. (a) On a sheet of graph paper, using a scale of 1 cm to represent 1 unit on both axes, draw the graph of each of the following functions.

   (i) $y = x + 2$

   (ii) $y = x - 3$

   (iii) $y = 2$

   (iv) $y = -3$

   (b) Find the area enclosed by the four lines.

7. Consider the sequence 44, 41, 38, 35, 32, …

   (i) Find, in terms of n, a formula for the n^{th} term of the sequence.

   (ii) Given that the k^{th} term of the sequence is -13, find the value of k.

8. Consider the following number pattern:

$$1^2 - 2 \times 1 = -1$$
$$2^2 - 2 \times 2 = 0$$
$$3^2 - 2 \times 3 = 3$$
$$4^2 - 2 \times 4 = 8$$
$$\vdots$$
$$k^2 - 2k = 63$$
$$\vdots$$

   (i) Write down the 6th line in the pattern.

   (ii) Find the value of k.

Percentage

Expressions such as '20 p 100' were first used by Italians to indicate 20 percent. Later on, 'xx. per .c.', first used by Giorgio Chiarino in 1481, was used to represent 20 percent. As a result, percentage was widely used in business to calculate profit and loss, and taxation since the 15th century. What are some other uses of percentage?

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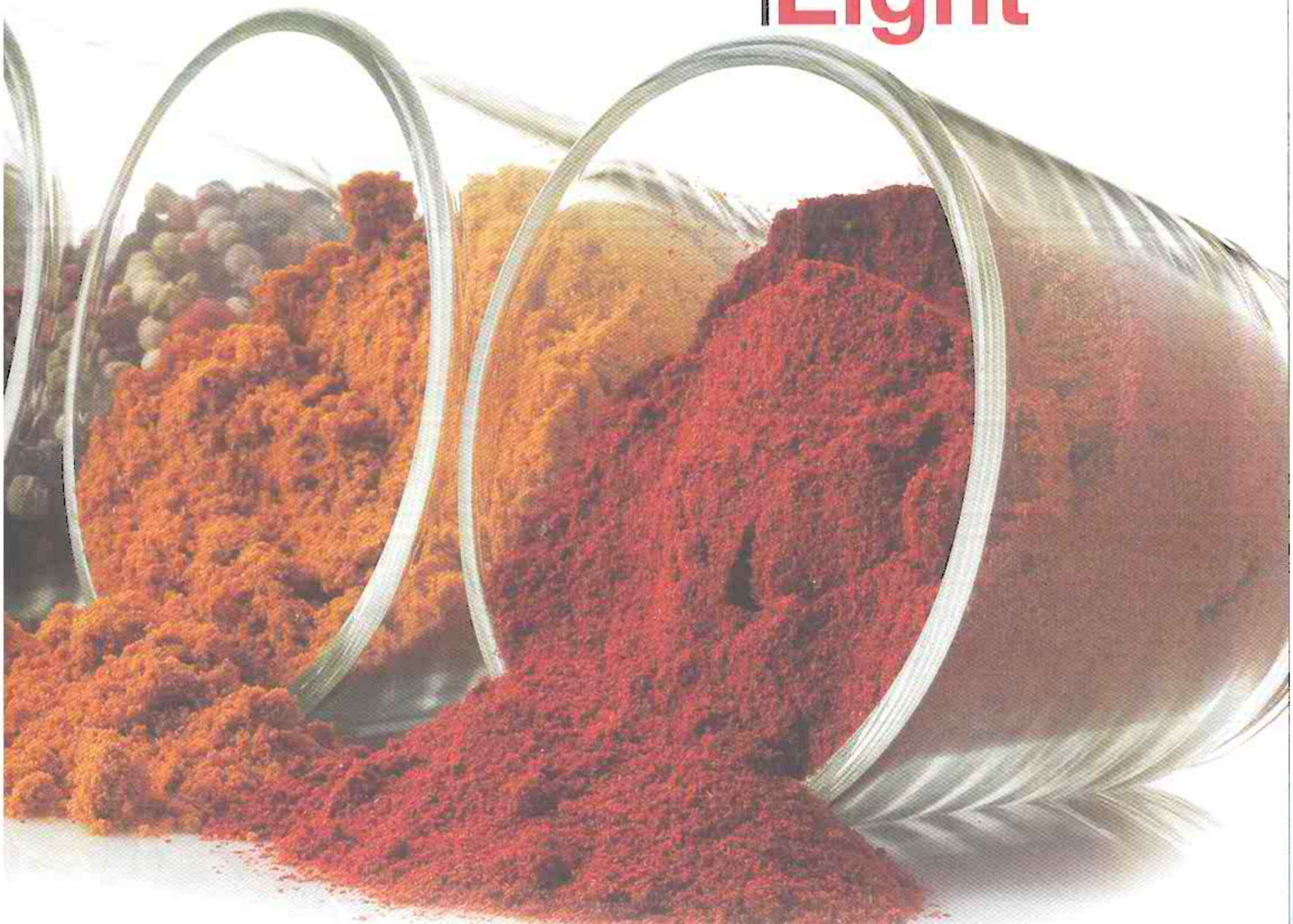
Chapter Eight

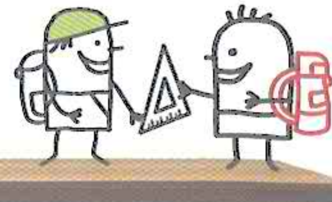


LEARNING OBJECTIVES

At the end of this chapter, you should be able to:

- express a percentage as a fraction and vice versa,
- express a percentage as a decimal and vice versa,
- express one quantity as a percentage of another,
- compare two quantities by percentage,
- solve problems involving percentage change and reverse percentage.

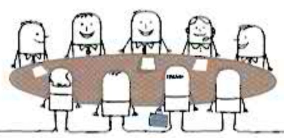
8.1 Introduction to Percentage



Percentages are useful in conveying information in everyday life. We often see advertisements or newspaper reports which carry phrases like 'Warehouse Sale at 50% off' and 'Gross Domestic Product is up by 0.5%'. 50% and 0.5% are examples of percentages. The symbol, %, is used to represent '**percent**'. Thus 50% is read as '50 percent'.

INFORMATION

The word '**percent**' originated from the Latin phrase 'per centum', which means 'per hundred' or 'out of every hundred'.



Class Discussion

Percentage in Real Life

Work in pairs.

1. Cut out an advertisement/article from newspapers/magazines in which percentage(s) can be found. Tell your classmate the meaning of the percentage(s) found in the context.
2. Phrases like 'Discount up to 80% on All Items' are commonly found during the period of the Great Singapore Sale. Does this mean that there is an 80% discount on all items? Discuss with your classmate.
3. Are there instances when percentages are more than 100%? Consider the phrase 'this year's sales is 200% of last year's sales'. What does this mean? Discuss with your classmate.

Percentages, Fractions and Decimals

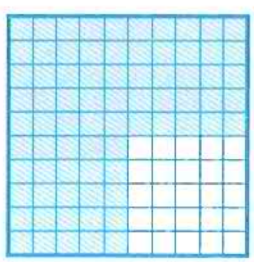


Fig. 8.1

Fig. 8.1 shows a square grid divided into 100 equal parts. 75 parts are shaded. Since there are 100 parts altogether, 75 out of 100 parts or $\frac{75}{100}$ of the grid is shaded.

The fraction $\frac{75}{100}$ can be expressed as 75%. Conversely, 75% can be expressed as $\frac{75}{100}$.

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In general, x percent is defined as x parts per hundred, i.e.

$$\boxed{x\% = \frac{x}{100}\ .}$$

ATTENTION

$100\% = \frac{100}{100} = 1$

A percentage can be greater than 100%.

After the class discussion on the previous page, do you understand the phrase 'this year's sales is 200% of last year's sales'?

Suppose the sales of a company last year was \$100 million and the sales of the company this year is \$200 million. The sales of the company this year is $\frac{200}{100} = 200\%$ of its sales last year.

Worked Example 1

(Expressing a Percentage as a Fraction and vice versa)

(a) Express each of the following percentages as a fraction.

(i) 88% **(ii)** 128% **(iii)** 0.5% **(iv)** $9\frac{2}{3}\%$

(b) Express each of the following fractions as a percentage.

(i) $\frac{1}{2}$ **(ii)** $1\frac{1}{8}$

Solution:

(a) (i)
$$\begin{aligned} 88\% &= \frac{88}{100} \\ &= \frac{22}{25} \quad \text{(reduced to lowest term)} \end{aligned}$$

(ii)
$$\begin{aligned} 128\% &= \frac{128}{100} \\ &= \frac{32}{25} \quad \text{(reduced to lowest term)} \\ &= 1\frac{7}{25} \end{aligned}$$

(iii)
$$\begin{aligned} 0.5\% &= \frac{0.5}{100} \\ &= \frac{0.5 \times 10}{100 \times 10} \\ &= \frac{5}{1000} \\ &= \frac{1}{200} \quad \text{(reduced to lowest term)} \end{aligned}$$

(iv)
$$\begin{aligned} 9\frac{2}{3}\% &= \frac{29}{3}\% \quad \text{(change to an improper fraction)} \\ &= \frac{29}{3} \div 100 \\ &= \frac{29}{3} \times \frac{1}{100} \\ &= \frac{29}{300} \end{aligned}$$

ATTENTION

- To express $x\%$ as a fraction, we divide x by 100.
- To express a fraction as a percentage, we multiply it by 100%.

(b) (i)
$$\begin{aligned} \frac{1}{2} &= \frac{1}{2} \times 100\% \\ &= 50\% \end{aligned}$$

(ii)
$$\begin{aligned} 1\frac{1}{8} &= \frac{9}{8} \quad \text{(change to an improper fraction)} \\ &= \frac{9}{8} \times 100\% \\ &= \frac{900}{8}\% \\ &= 112.5\% \end{aligned}$$

ATTENTION

$\frac{9}{8}$ is different from $\frac{9}{8}\%$ as $\frac{9}{8} = 112.5\% \neq \frac{9}{8}\%$.

PRACTISE NOW 1

SIMILAR QUESTIONS

Exercise 8A Questions 1(a)–(d), 3(a)–(f)

(a) Express each of the following percentages as a fraction.

(i) 45% **(ii)** 305% **(iii)** 5.5% **(iv)** $8\frac{5}{7}\%$

(b) Express each of the following fractions as a percentage.

(i) $\frac{17}{20}$ **(ii)** $23\frac{1}{5}$

Worked Example 2

(Expressing a Percentage as a Decimal and vice versa)

(a) Express each of the following percentages as a decimal.

(i) 85% **(ii)** 228%

(iii) 0.7% **(iv)** $7\frac{1}{8}\%$

(b) Express each of the following decimals as a percentage.

(i) 0.16 **(ii)** 1.456

Solution:

(a) (i) $85\% = \frac{85}{100}$
$= 0.85$

(ii) $228\% = \frac{228}{100}$
$= 2.28$

(iii) $0.7\% = \frac{0.7}{100}$
$= 0.007$

(iv) $7\frac{1}{8}\% = \frac{57}{8}\%$ (change to an improper fraction)

$= \frac{57}{8} \div 100$

$= \frac{57}{8} \times \frac{1}{100}$

$= \frac{57}{800}$

$= 0.071\ 25$

Alternatively,

$7\frac{1}{8}\% = 7.125\%$

$= \frac{7.125}{100}$

$= 0.071\ 25$

ATTENTION

- To express $x\%$ as a decimal, we divide x by 100.
- To express a decimal as a percentage, we multiply it by 100%.

(b) (i) $0.16 = 0.16 \times 100\%$
$= 16\%$

(ii) $1.456 = 1.456 \times 100\%$
$= 145.6\%$

ATTENTION

1.456 is different from 1.456% as $1.456 = 145.6\% \neq 1.456\%$.

PRACTISE NOW 2

SIMILAR QUESTIONS

Exercise 8A Questions 2(a)–(d), 4(a)–(f)

(a) Express each of the following percentages as a decimal.

(i) 12% **(ii)** 413% **(iii)** 23.6% **(iv)** $6\frac{1}{4}\%$

(b) Express each of the following decimals as a percentage.

(i) 0.76 **(ii)** 2.789

Expressing One Quantity as a Percentage of Another

We can use percentage to gauge how large/small a quantity is with respect to another quantity. For example, in 2010, the total population in Singapore was 5 076 700, of which 3 230 700 were Singaporeans. We can say that approximately 64% of the total population in Singapore were Singaporeans. The percentage 64% is obtained by expressing the number of Singaporeans as a percentage of the total population in Singapore.

> To express one quantity, a, as a percentage of another quantity, b, we write a as a fraction of b before converting the fraction $\frac{a}{b}$ into a percentage.

ATTENTION

Both a and b must be of the *same* unit.

Worked Example 3

(Expressing One Quantity as a Percentage of Another)

There are 90 teachers in a school, of which 40 are male. Calculate the percentage of

(i) male teachers,

(ii) female teachers,

in the school.

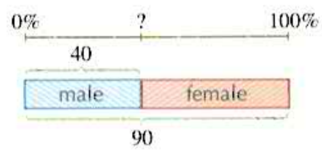
Just For Fun

99 boys and 1 girl are in a lecture theatre. How many boys must leave the theatre so that the percentage of boys becomes 98%?

Solution:

(i) Percentage of male teachers in the school $= \frac{40}{90} \times 100\%$

$= 44\frac{4}{9}\%$

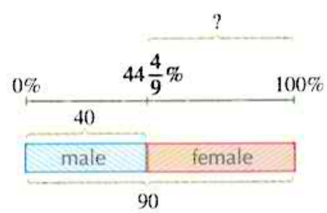


(ii) Method 1: Number of female teachers in the school $= 90 - 40$

$= 50$

Percentage of female teachers in the school $= \frac{50}{90} \times 100\%$

$= 55\frac{5}{9}\%$



Method 2: Percentage of female teachers in the school $= 100\% - 44\frac{4}{9}\%$

$= 55\frac{5}{9}\%$

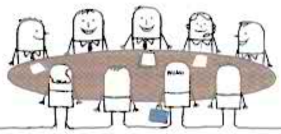
PRACTISE NOW 3

SIMILAR QUESTIONS

Exercise 8A Questions 6, 9(a)–(h), 11

1. There are 45 male teachers and 75 female teachers in a school. Find the percentage of
   (i) male teachers,
   (ii) female teachers,
   in the school.
2. Express 1400 ml as a percentage of 2.1 l.

Finding the Percentage of a Quantity



Class Discussion

Expressing Two Quantities in Equivalent Forms

1. **(a)** Refer to Worked Example 3 to fill in the blanks below.

   (i) Express the number of male teachers as a percentage of that of female teachers.

   Required percentage = ______ (fraction) × 100% = ______%

   We can conclude that:

   There are ______% as many male teachers as female teachers.

   The following statements are equivalent to the above statement:

   - The number of male teachers is ______% of the number of female teachers.
   - The number of male teachers is ______ (fraction) of the number of female teachers.

   (ii) Express the number of female teachers as a percentage of that of male teachers.

   Required percentage = ______ (fraction) × 100% = ______%

   We can conclude that:

   There are ______% as many female teachers as male teachers.

   The following statements are equivalent to the above statement:

   - The number of female teachers is ______% of the number of male teachers.
   - The number of female teachers is ______ (fraction) of the number of male teachers.

   (b) Given that A and B represent the number of male teachers and female teachers respectively, complete Table 8.1.

In words	A is ______% of B.	B is ______% of A.
Percentage	A = ______% × B	B = ______% × A
Fraction	A = ______ (fraction) × B	B = ______ (fraction) × A
Decimal	A = ______ × B	B = ______ × A

Table 8.1

As shown in Table 8.1, the relationship between A and B can be expressed in various equivalent forms.

2. **(i)** Complete Table 8.2.

In words	P is ______% of Q.	R is ______% of S.	T is ______% of U.
Percentage	$P = 20\% \times Q$	R = ______% × S	T = ______% × U
Fraction	P = ______ (fraction) × Q	$R = \frac{1}{2} \times S$	T = ______ (fraction) × U
Decimal	P = ______ × Q	R = ______ × S	$T = 1.25 \times U$

Table 8.2

   (ii) Draw a model to illustrate each of the relationships, i.e. P and Q, R and S, and T and U.

   (iii) Create a question similar to one of the three columns in Table 8.2 and challenge your classmate.

PRACTISE NOW

SIMILAR QUESTIONS

1. Find the value of each of the following.
   (a) 20% of \$13.25 (b) $15\frac{3}{4}\%$ of \$640
2. Find the value of 2500% of \$4.60.

Exercise 8A Questions 5(a)–(b), 10(a)–(d), 13

Worked Example 4

(Finding the Percentage of a Quantity)

A class has 40 students. Given that 75% of them passed a Mathematics test, calculate the number of students who failed the test.

Solution:

Method 1:

Number of students who passed the test = 75% of 40

$= \frac{75}{100} \times 40$ (express 75% as a fraction)

$= 30$

Number of students who failed the test = 40 – 30

$= 10$

Method 2:

Percentage of students who failed the test = 100% – 75%

$= 25\%$

Number of students who failed the test = 25% × 40

$= \frac{25}{100} \times 40$ (express 25% as a fraction)

$= 10$

PRACTISE NOW 4

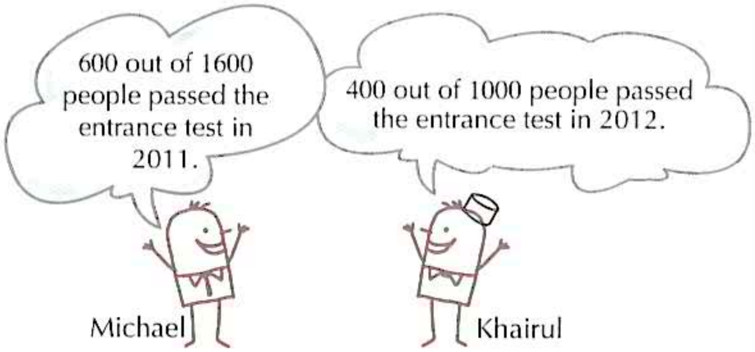
SIMILAR QUESTIONS

1. A school has 1500 students. Given that 3% of them were late for school on a particular day, find the number of students who were punctual for school.
2. 1800 people attended the National Day dinner in a certain constituency. Given that 35.5% of them were men, 40% of them were women and the rest were children, find the number of children who attended the dinner.

Exercise 8A Questions 7, 12–14

Comparing Two Quantities by Percentage

A school conducts an entrance test on a yearly basis.



Since 600 > 400, does it mean that a higher proportion of people passed the entrance test of the school in 2011 than in 2012? Worked Example 5 shows how a comparison can be done.

Worked Example 5

(Comparing Two Quantities by Percentage)

In 2011, 600 out of 1600 people passed the entrance test of a school. 400 out of 1000 people passed the entrance test of the same school in 2012. In which year did a higher percentage of people pass the entrance test of the school?

Solution:

Percentage of people who passed the entrance test in 2011 $= \frac{600}{1600} \times 100\%$

$= 37.5\%$

Percentage of people who passed the entrance test in 2012 $= \frac{400}{1000} \times 100\%$

$= 40\%$

$\therefore$ A higher percentage of people passed the entrance test of the school in 2012.

PRACTISE NOW 5

There were 30 000 people in Village A and 4000 people attended its New Year party. There were 25 000 people in Village B and 2800 people attended its New Year party. Which village had a higher percentage of people who attended its party?

SIMILAR QUESTIONS

Exercise 8A Question 8

Exercise 8A

BASIC LEVEL

1. Express each of the following percentages as a fraction.
   (a) 28% (b) 158%
   (c) 12.4% (d) $6\frac{3}{5}\%$

2. Express each of the following percentages as a decimal.
   (a) 4% (b) 633%
   (c) 0.02% (d) $33\frac{2}{3}\%$

3. Express each of the following fractions as a percentage.
   (a) $\frac{3}{5}$ (b) $\frac{9}{10}$
   (c) $\frac{6}{125}$ (d) $\frac{6}{5}$
   (e) $\frac{12}{25}$ (f) $1\frac{6}{25}$

4. Express each of the following decimals as a percentage.
   (a) 0.78 (b) 0.25
   (c) 0.07 (d) 0.095
   (e) 1.35 (f) 2.6

5. Find the value of each of the following.
   (a) 50% of \$70 (b) 80% of 4.5 m

6. There are 20 boys and 18 girls in a class. Find the percentage of
   (i) boys, (ii) girls,
   in the class.

7. There are 120 cars in a multi-storey car park. Given that 30% of them are blue, find the number of cars which are not blue.

8. Jun Wei is a bus driver who earns \$1600 per month while Lixin is a manager who earns \$6800 per month. In 2012, Jun Wei donated a total of \$1200 to charitable organisations while Lixin donated a total of \$4500 to charitable organisations. Who donated a higher percentage of his or her annual income to charitable organisations?

INTERMEDIATE LEVEL

9. For each of the following, express the first quantity as a percentage of the second quantity.
   (a) 25 seconds, 3.5 minutes
   (b) 45 minutes, 1 hour
   (c) 1 year, 4 months
   (d) 15 mm, 1 m
   (e) 335 cm, 5 m
   (f) 1 kg, 800 g
   (g) 60°, 360°
   (h) 63 cents, \$2.10

10. Find the value of each of the following.
    (a) $6\frac{1}{5}\%$ of 1.35 ml (b) $56\frac{7}{8}\%$ of 810 m
    (c) 0.56% of 15 000 l (d) 2000% of 5¢

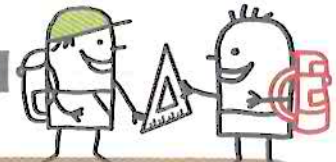
11. The maximum number of marks attainable at a Mathematics competition is 60. Kate obtains 40 marks, Priya obtains 46 marks and Nora obtains 49 marks. The examination board decides that those who score 80% and above will get a gold award, those who score 70% to 79% inclusive will get a silver award and those who score 60% to 69% inclusive will get a bronze award. Determine the type of award each girl gets.

12. A company had 12 000 employees. During the financial crisis in 2008, 2.5% of them were retrenched, 50.75% of them had a pay cut and the rest were unaffected. Find the number of employees who were unaffected by the financial crisis.

13. Ethan's monthly salary is \$1850. In a particular month, he spent 20.5% of his salary on room rental, \$690 on food and \$940 on other expenses. Express the amount that he overspent as a percentage of his monthly salary, giving your answer correct to 2 decimal places.

14. There are 600 pages in a novel. Rui Feng reads 150 pages of the novel on Friday and 40% of the remaining pages on Sunday. Express the number of pages that remains to be read as a percentage of the total number of pages in the novel.

8.2 Percentage Change and Reverse Percentage



Percentage Change

The change in the value of an item can be expressed as a percentage increase or decrease in the original value.

An **increase** of 5% in the salary of a man who earns \$1600 per month means that for every \$100 of the **original** salary, there is an increase of \$5, i.e. each \$100 in the original salary becomes \$105 in the **new** salary.

$$\therefore \quad \frac{\text{New salary}}{\text{Original salary}} = \frac{105}{100}$$

$$\begin{aligned} \text{New salary} &= \frac{105}{100} \times \text{Original salary} \\ &= \frac{105}{100} \times \$1600 \\ &= \$1680 \end{aligned}$$

ATTENTION

We can also say that the new salary is 105% of the original salary.

$$\begin{aligned} \text{Increase in salary} &= \$1680 - \$1600 \\ &= \$80 \end{aligned}$$

Alternatively,

$$\begin{aligned} \text{Increase in salary} &= \frac{5}{100} \times \$1600 \\ &= \$80 \end{aligned}$$

On the other hand, a **decrease** of 5% in his salary means that for every \$100 of the **original** salary, there is a decrease of \$5, i.e. each \$100 in the original salary becomes \$95 in the **new** salary.

$$\therefore \quad \frac{\text{New salary}}{\text{Original salary}} = \frac{95}{100}$$

$$\begin{aligned} \text{New salary} &= \frac{95}{100} \times \text{Original salary} \\ &= \frac{95}{100} \times \$1600 \\ &= \$1520 \end{aligned}$$

ATTENTION

We can also say that the new salary is 95% of the original salary.

$$\begin{aligned} \text{Decrease in salary} &= \$1600 - \$1520 \\ &= \$80 \end{aligned}$$

Alternatively,

$$\begin{aligned} \text{Decrease in salary} &= \frac{5}{100} \times \$1600 \\ &= \$80 \end{aligned}$$

In general,

> New value = final percentage × original value
>
> Increase/Decrease = **percentage increase/decrease** × original value

Worked Example 6

(Problem involving Simple Percentage Changes)

In Singapore, the Good Progress Award (GPA) is given out to the top 10% of students within each level and stream in every school who make significant improvement in their academic performance as compared to the previous year. There was an increase in the value of the award from 2008 to 2009. The table shows the value of this award for each level from 2008 to 2009.

INFORMATION

A student is eligible for the GPA only if he or she does not qualify for the Edusave Scholarship or Edusave Merit Bursary.

Year/Level	Primary 2 – 3	Primary 4 – 6	Secondary 1 – 5	Junior Colleges/ Centralised Institute (JCs/CI)	Institute of Technical Education (ITE)
2008	\$50	\$100	\$150	\$200	\$300
2009	\$100	?	\$200	\$250	\$400

(i) Given that there was an increase of 50% in the value of the award from 2008 to 2009 for a Primary 5 student, calculate the value of the award for a Primary 5 student in 2009.

(ii) Calculate the percentage increase in the value of the award from 2008 to 2009 for

(a) a Primary 2 student,　　**(b)** a Secondary 1 student.

Solution:

(i) The value of the award for a Primary 5 student in 2009 was 150% of that in 2008.

$\therefore$ Value of the award for a Primary 5 student in 2009 = 150% of \$100

$$= \frac{150}{100} \times \$100$$

$$= \$150$$

(ii) (a) Increase = \$100 – \$50 = \$50

Percentage increase in the value of the award from 2008 to 2009 for a Primary 2 student

$$= \frac{\text{Increase}}{\text{Original value}} \times 100\%$$

$$= \frac{\$50}{\$50} \times 100\%$$

$$= 100\%$$

(b) Percentage increase in the value of the award from 2008 to 2009 for a Secondary 1 student

$$= \frac{\$200 - \$150}{\$150} \times 100\%$$

$$= \frac{\$50}{\$150} \times 100\%$$

$$= 33\frac{1}{3}\%$$

PRACTISE NOW 6

SIMILAR QUESTIONS

Exercise 8B Questions 1(a)-(d), 3–6, 10–11, 16

1. In Singapore, the Edusave Merit Bursary (EMB) is given out to the top 25% of students within each level and stream in every school. There was an increase in the value of the award from 2008 to 2009. The table below shows the value of this award for each level in 2008 and 2009.

Year/ Level	Primary 1 – 3	Primary 4 – 6	Secondary 1 – 5	JCs/CI	ITE
2008	\$150	\$200	\$250	\$300	\$400
2009	\$250	\$300	?	\$400	\$500

(a) Given that there was an increase of 40% in the value of the award from 2008 to 2009 for a Secondary 1 student, find the value of the award for a Secondary 1 student in 2009.

(b) Find the percentage increase in the value of the award from 2008 to 2009 for

(i) a Primary 1 student, **(ii)** a Primary 6 student.

2. **(a)** What is the result when 32 is decreased by 25%?

(b) A new car costs \$127 000. After one year, its value is \$119 380. Find the percentage decrease in the value of the car.

INFORMATION

A student is eligible for the EMB only if he or she does not qualify for the Edusave Scholarship and if his or her gross monthly household income is below \$4000.

Worked Example 7

(More Complicated Problem involving Percentage Changes)

The cost of a piece of furniture consists of the cost of wood at \$300, the cost of paint at \$200 and wages at \$200. If the costs of wood and paint are increased by 12% and 7% respectively, while wages are decreased by 10%, calculate the percentage increase or decrease in the cost of the furniture.

Just For Fun

Why is it possible to have an increase of 110% in the cost of an item but not a 110% decrease in its cost?

Solution:

	Original Cost	Percentage Change	New Cost
Wood	\$300	+12%	$\frac{112}{100} \times \$300 = \$336$
Paint	\$200	+7%	$\frac{107}{100} \times \$200 = \$214$
Wages	\$200	–10%	$\frac{90}{100} \times \$200 = \$180$
Furniture	\$700		\$730

$$\text{Percentage increase in the cost of the furniture} = \frac{\$730 - \$700}{\$700} \times 100\%$$

$$= \frac{\$30}{\$700} \times 100\%$$

$$= 4\frac{2}{7}\%$$

ATTENTION

Explain why we cannot do the following: $12\% + 7\% + (-10\%) = 9\%$, to obtain the resulting change in the cost of the furniture.

PRACTISE NOW 7

SIMILAR QUESTIONS

Exercise 8B Question 12

The monthly cost of running a small business consists of retail space rental at \$2400, wages at \$1800 and utilities at \$480. If the retail space rental and wages are decreased by 5% and 6% respectively, while utilities are increased by 7%, find the percentage increase or decrease in the monthly cost of running the business.

Reverse Percentage

Let us now take a look at some problems involving reverse percentage.

Worked Example 8

(Simple Reverse Percentage Problem)

In a student council, 30% of the students wear spectacles. If 48 students wear spectacles, calculate the number of students in the student council.

Solution:

$$\begin{aligned} 30\% \text{ of the student council} &= 48 \\ 1\% \text{ of the student council} &= \frac{48}{30} \\ 100\% \text{ of the student council} &= \frac{48}{30} \times 100 \\ &= 160 \end{aligned}$$

There are 160 students in the student council.

PRACTISE NOW 8

SIMILAR QUESTIONS

Exercise 8B Questions 2(a)–(b), 7

70% of the books on a bookshelf are English books. If there are 35 English books on the bookshelf, find the number of books on the bookshelf.

Worked Example 9

(Reverse Percentage involving an Increase)

After an increase of 5%, Shirley's monthly salary becomes \$2205. Find her original monthly salary.

Solution:

Method 1:

After an increase of 5%, Shirley's monthly salary becomes 105% of her original monthly salary.

$$105\% \text{ of original monthly salary} = \$2205$$

$$1\% \text{ of original monthly salary} = \frac{\$2205}{105}$$

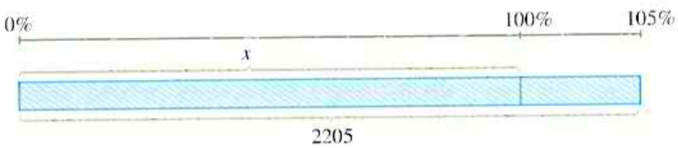
$$100\% \text{ of original monthly salary} = \frac{\$2205}{105} \times 100$$

$$= \$2100$$

Shirley's original monthly salary is \$2100.

Method 2:

Let Shirley's original monthly salary = \$$x$.



Problem Solving Tip

You may draw a model to help you make sense of a problem.

From the model, we form the equation:

$$105\% \times x = 2205$$

$$1.05x = 2205$$

$$x = 2100$$

Shirley's original monthly salary is \$2100.

PRACTISE NOW 9

SIMILAR QUESTIONS

Exercise 8B Questions 2(c), 8, 13

1. If the cost of an article is raised by 9% to \$654, what is the original cost of the article?

2. Every year, the value of an antique vase appreciates by 20% of its value in the previous year. If the value of the vase was \$180 000 in 2012, find its value in 2010.

Worked Example 10

(Reverse Percentage involving a Decrease)

If 6% is deducted from a bill, \$282 remains to be paid. How much is the original bill?

Solution:

Method 1:

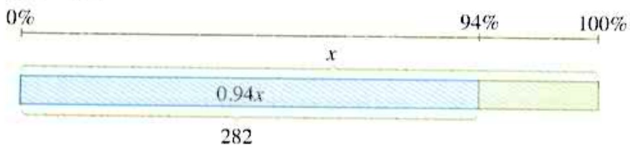
After 6% is deducted from the bill, 94% of the bill remains to be paid.

$$\begin{aligned} 94\% \text{ of the original bill} &= \$282 \\ 1\% \text{ of the original bill} &= \frac{\$282}{94} \\ 100\% \text{ of the original bill} &= \frac{\$282}{94} \times 100 \\ &= \$300 \end{aligned}$$

The original bill is \$300.

Method 2:

Let the original bill = $\$x$.



From the model, we form the equation:

$$\begin{aligned} 94\% \times x &= 282 \\ 0.94x &= 282 \\ x &= 300 \end{aligned}$$

The original bill is \$300.

Internet Resources

Visit http://www.shinglee.com.sg/StudentResources/ to access the AlgeTools™ software. Select the AlgeBar™ application to solve word problems on 'Percentage'. You may draw models to help you formulate the equations.

PRACTISE NOW 10

1. After a pay cut of 3%, Devi's monthly salary becomes \$3346.50. Find her original monthly salary.
2. Every year, the value of a car depreciates by 15% of its value in the previous year. If the value of the car was \$86 700 in 2012, find its value in 2010.

SIMILAR QUESTIONS

Exercise 8B Questions 2(d), 9, 14–15

Thinking Time

1. Ethan conducted a survey on 20% of a group of students and on 80% of another group of students. Is it correct to say that $\frac{20\% + 80\%}{2}$, i.e. 50% of the total number of students in the two groups had done the survey? Explain your answer.
2. Mr Lee was earning a monthly salary of $\$x$ in 2010. In 2011, his salary was increased by 10%. However, in 2012, due to the financial situation of his company, his salary was decreased by 10%. Is it correct to say that his monthly salary in 2012 was $\$x$? Explain your answer.

Exercise 8B

BASIC LEVEL

1. Find the value of each of the following.
   (a) Increase 60 by 35%
   (b) Increase 28 by 125%
   (c) Decrease 120 by 45%
   (d) Decrease 216 by $37\frac{1}{2}\%$

2. (a) 20% of a number is 17. Find the number.
   (b) 175% of a number is 49. Find the number.
   (c) The result of a number, when increased by 15%, is 161. Find the number.
   (d) The result of a number, when decreased by 20%, is 192. Find the number.

3. An elastic band which is 72 cm long, is stretched to 90 cm. Find the percentage increase in its length.

4. In Singapore, the Edusave Scholarship is given out to the top 10% of students within each level and stream in every school. There was an increase in the value of the award from 2008 to 2009. The table shows the value of this award for each level in 2008 and 2009.

	Level/Year	2008	2009
Top 5%	Primary 1 – 6	\$300	\$400
	Secondary 1 – 5	\$500	?
Next 5%	Primary 1 – 6	\$250	\$350
	Secondary 1 – 5	\$300	\$400

   (i) Given that there was an increase of 30% in the value of the award from 2008 to 2009 for a Secondary 1 student in the top 5%, find the value of the award for a Secondary 1 student in the top 5% in 2009.
   (ii) Find the percentage increase in the value of the award from 2008 to 2009 for a Primary 4 student in the next 5%.

5. The price of a desktop computer decreases from \$1360 to \$1020. Find the percentage decrease in its price.

6. A car was bought in 2009 for \$120 000. In 2010, its value decreased by 20%. In 2011, its value decreased by 10% of its value in 2010. Find the value of the car at the end of 2011.

7. 45% of the students who take part in a creative writing competition are boys. If 135 boys take part in the competition, find the total number of students who take part in the competition.

8. A house costs 36% more today than when it was built. If the cost of the house today is \$333 200, find its cost when it was built.

9. If 10% is deducted from a bill, \$58.50 remains to be paid. How much is the original bill?

INTERMEDIATE LEVEL

10. The number 2400 is first increased by 30%. The value obtained is next decreased by 20%. Find the final number.

11. In 2011, a train carried 8% more passengers than in 2010. In 2012, it carried 8% more passengers than in 2011. Find the percentage increase in the number of train passengers from 2010 to 2012.

12. The production cost of a printer consists of the cost of raw materials at \$100, the cost of overheads at \$80 and wages at \$120. If the costs of raw materials and overheads are increased by 11% and 20% respectively, while wages are decreased by 15%, find the percentage increase or decrease in the production cost of the printer.

13. Every year, the value of a condominium in Singapore appreciates by 15% of its value in the previous year. If the value of the condominium was \$899 300 in 2012, find its value in 2010.

14. Every year, the value of a surveying machine depreciates by 25% of its value in the previous year. If the value of the machine was \$11 250 in 2012, find its value in 2010.

15. The value of an investment portfolio decreased by 8% in 2010. In 2011, its value increased by 5% of its value in 2010. Given that the value of the portfolio at the end of 2011 was \$61 824, find its original value.

ADVANCED LEVEL

16. Amirah is 8% taller than Huixian and Priya is 10% shorter than Huixian. Express the height of Amirah as a percentage of that of Priya.

1. We use % to represent '**percent**'. Thus x percent is defined as x parts per hundred, i.e. $x\% = \frac{x}{100}$.
2. To express one quantity, a, as a **percentage** of another quantity, b, we write a as a fraction of b before converting the fraction $\frac{a}{b}$ into a percentage. Both a and b must be of the *same* unit.
3. New value = final percentage × original value
   Increase/Decrease = **percentage increase/decrease** × original value

Review Exercise 8

1. Express 1 m as a percentage of 56 mm.

2. Michael receives a weekly pocket money of $28. If he decides to save 20% of it, find his
   (i) savings in a year,
   (ii) spending in a year.
   Assume that there are 52 weeks in a year.

3. Given that a is 30% of b, find the value of $\frac{a}{4b}$.

4. Huixian scores 68 out of 80 for her Science test, Priya scores 86 out of 120 for her Science test and Rui Feng scores 120 out of 150 for his Science test. Who performs the best in his/her Science test?

5. A vendor sells apples, pears and oranges. He has 120 oranges. Given that he has 20% more apples than oranges and 40% fewer oranges than pears, find the total number of fruits he has.

6. Kate reads 60 pages of a book on the first day. This is 20% more than the number of pages she reads on the second day. Given that she reads $\frac{1}{6}$ of the book on the second day, find the number of pages in the book.

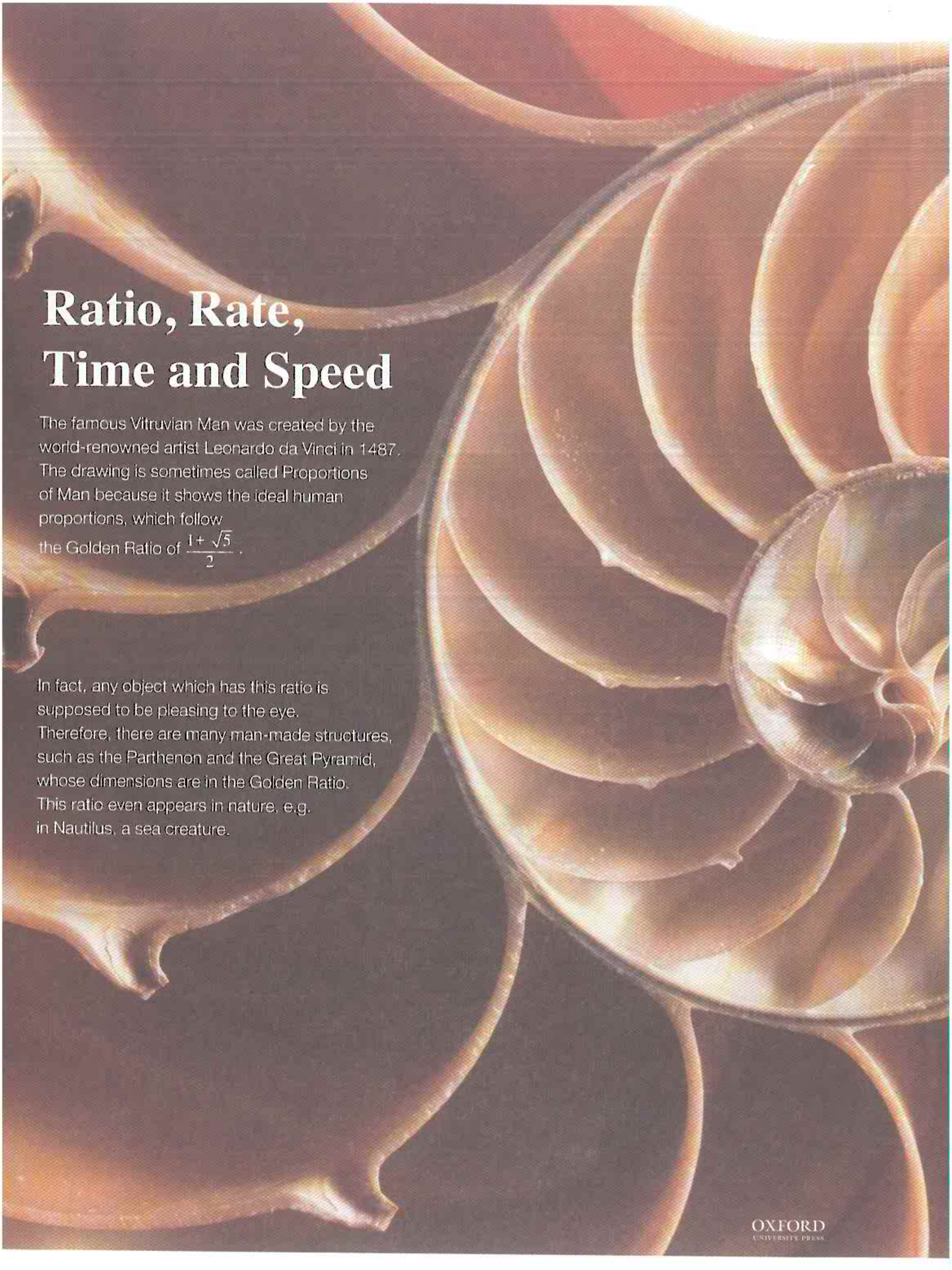
7. A village loses 14% of its goats in a flood and 6% of the remainder die from diseases. Given that the number of goats left is 8084, find the original number of goats in the village.

8. Mr Neo's salary was decreased by 15% when his company was not doing well. Now, his company's financial situation has improved and his boss wants to restore his original salary. By what percentage must his reduced salary be increased?

Challenge Yourself

1. Amirah has 2 bottles of jellybeans, A and B. Bottle A has 400 jellybeans while Bottle B has 300 jellybeans. 75% of the jellybeans in Bottle A are red while the rest are yellow; 50% of the jellybeans in Bottle B are red while the rest are yellow. If Amirah moves some jellybeans from Bottle A to Bottle B such that 80% of the jellybeans in Bottle A are now red and 40% of those in Bottle B are yellow, find the number of jellybeans Amirah moves from Bottle A to Bottle B.

2. Cup A is 40% filled with water. Cup B, which is identical to Cup A, is completely filled with a mixed solution containing 70% water and 30% hydrochloric acid. 60% of the content in Cup B is then poured into Cup A. After mixing, 60% of the mixed solution in Cup A is poured into Cup B. Find the percentage of water in Cup A now.



Ratio, Rate, Time and Speed

The famous Vitruvian Man was created by the world-renowned artist Leonardo da Vinci in 1487. The drawing is sometimes called Proportions of Man because it shows the ideal human proportions, which follow the Golden Ratio of $\frac{1+\sqrt{5}}{2}$.

In fact, any object which has this ratio is supposed to be pleasing to the eye. Therefore, there are many man-made structures, such as the Parthenon and the Great Pyramid, whose dimensions are in the Golden Ratio. This ratio even appears in nature, e.g. in Nautilus, a sea creature.

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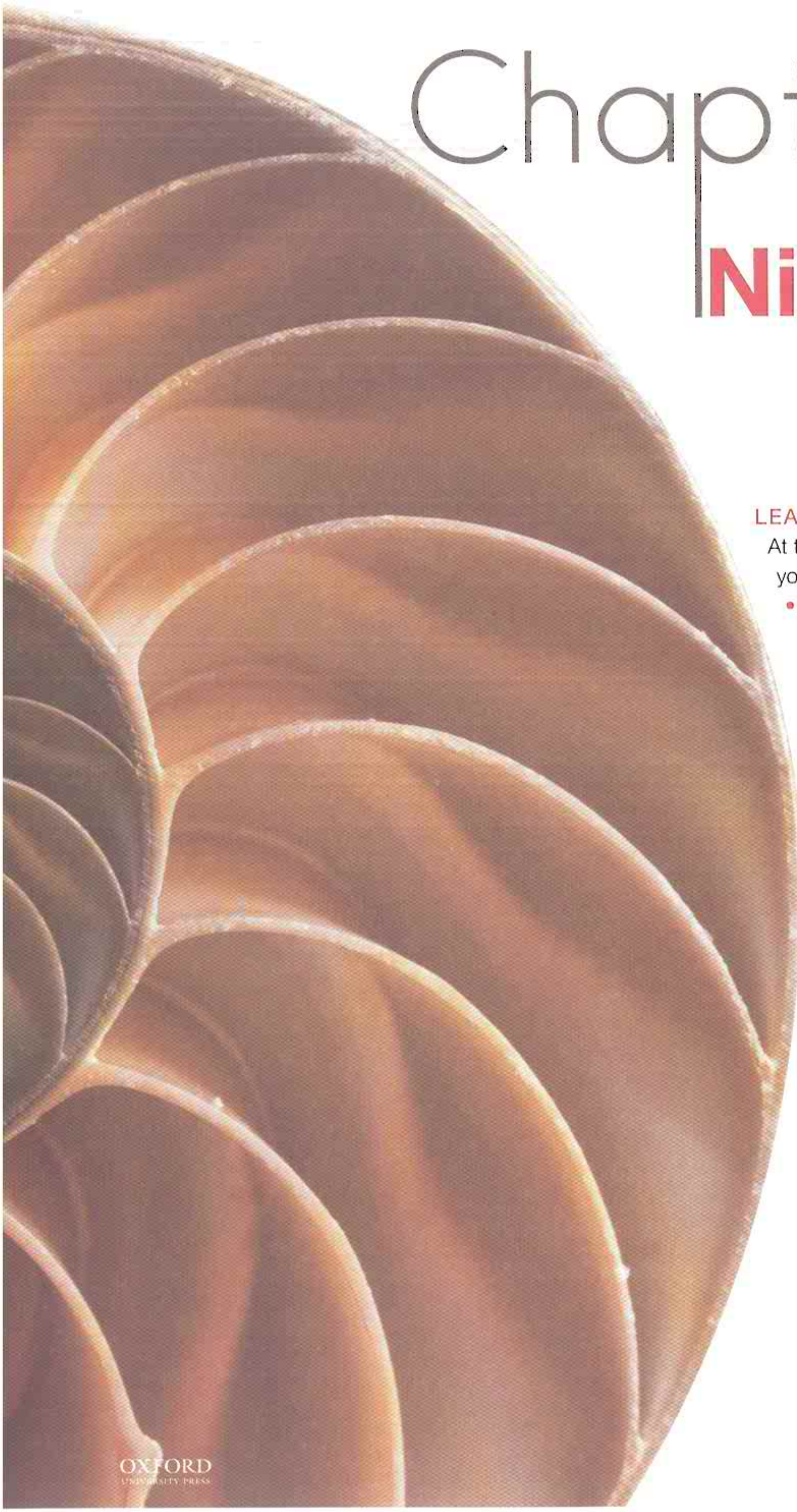


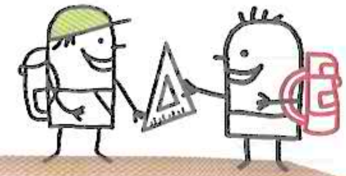
Chapter Nine

LEARNING OBJECTIVES

At the end of this chapter, you should be able to:

- find ratios involving rational numbers,
- find ratios involving three quantities,
- distinguish between constant and average rates,
- denote time in 12-hour clock and 24-hour clock formats,
- discuss real-life examples of rates such as speed and rate of rotation,
- solve problems involving ratio, rate and speed.

9.1 Ratio



Recap (Concept of Ratio)

In primary school, we have learnt how to solve problems involving ratios. We shall have a quick revision. A **ratio** is used to compare two or more quantities of the *same kind* which are measured in the same unit.

> The **ratio** of a is to b, where a and b represent two quantities of the *same kind*, and $b \neq 0$, is written as $a : b$.

ATTENTION

A ratio has *no* units.

Worked Example 1

(Problem involving Ratios)

There are 17 boys and 19 girls in a class. Find the ratio of
(i) the number of boys to the number of girls,
(ii) the number of girls to the number of boys.

Solution:

(i) Ratio of the number of boys to the number of girls = 17 : 19
(ii) Ratio of the number of girls to the number of boys = 19 : 17

ATTENTION

The order in which a ratio is expressed is important.

PRACTISE NOW 1

There are 33 lemons and 20 pears in a basket. Find the ratio of
(i) the number of lemons to the number of pears,
(ii) the number of pears to the total number of fruits in the basket.

SIMILAR QUESTIONS

Exercise 9A Question 5

Equivalent Ratios

$$\frac{6}{12} \overset{\times 3}{=} \frac{2}{4} \overset{\div 2}{=} \frac{1}{2}$$

$$\times 3 \qquad \div 2$$

We have learnt in primary school that $\frac{1}{2}$, $\frac{2}{4}$ and $\frac{6}{12}$ are equivalent fractions.

These fractions can be expressed as ratios:

$$\overset{\times 3 \quad \div 2}{6 : 12 = 2 : 4 = 1 : 2}_{\times 3 \quad \div 2}$$

1 : 2, 2 : 4 and 6 : 12 are known as **equivalent ratios**. The simplest form is 1 : 2.

Is 23 : 46 and $\frac{1}{6} : \frac{1}{3}$ equivalent to 1 : 2?

$23 : 46 = \frac{23}{23} : \frac{46}{23}$ (divide both parts by 23)

$= 1 : 2$

$\frac{1}{6} : \frac{1}{3} = \frac{1}{6} \times 6 : \frac{1}{3} \times 6$ (multiply both parts by the lowest common multiple (LCM) of 3 and 6, i.e. 6)

$= 1 : 2$

$\therefore$ 23 : 46 and $\frac{1}{6} : \frac{1}{3}$ are equivalent to 1 : 2.

In general,

- a ratio is said to be in its simplest form $a : b$ when a and b are integers with no common factors (other than 1),
- similar to equivalent fractions, we can obtain **equivalent ratios** by multiplying or dividing both parts by the same constant.

  $x : y = hx : hy = \frac{x}{k} : \frac{y}{k}$, where h and k are not equal to 0

  $x : y$, $hx : hy$ and $\frac{x}{k} : \frac{y}{k}$ are equivalent ratios.

INFORMATION

You can use the $\boxed{a\,^b/_c}$ key on your calculator to help you express a fraction in its lowest term, e.g. to express $\frac{6}{12}$ in its lowest term, press $\boxed{6}\boxed{a\,^b/_c}\boxed{1}\boxed{2}\boxed{=}$ to obtain $\frac{1}{2}$.

Similarly, to simplify a ratio, you can also use the $\boxed{a\,^b/_c}$ key, e.g. to simplify 6 : 12, you can press the same sequence of calculator keys and write the answer as 1 : 2.

Worked Example 2

(Simplifying Ratios)

Simplify each of the following.

(a) 600 g : 1.6 kg **(b)** $\frac{2}{3} : \frac{5}{6}$ **(c)** 0.12 : 0.56

Solution:

(a) 600 g : 1.6 kg = 600 g : 1600 g (convert to the same unit)

= 3 : 8 (divide both parts by 200)

Alternatively,

$\frac{600\text{ g}}{1.6\text{ kg}} = \frac{600\text{ g}}{1600\text{ g}}$ (convert to the same unit)

$= \frac{3}{8}$ (divide the numerator and the denominator by 200 respectively)

$\therefore$ 600 g : 1.6 kg = 3 : 8

(b) $\frac{2}{3} : \frac{5}{6} = \frac{2}{3} \times 6 : \frac{5}{6} \times 6$ (multiply both parts by 6)

$= 4 : 5$

(c) $0.12 : 0.56 = 0.12 \times 100 : 0.56 \times 100$ (multiply both parts by 100)

$= 12 : 56$

$= 3 : 14$ (divide both parts by 4)

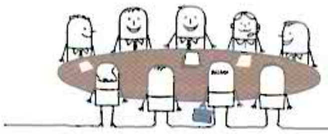
PRACTISE NOW 2

SIMILAR QUESTIONS

Exercise 9A Questions 1(a)–(c), 9(a)–(b)

Simplify each of the following.

(a) 240 g : 1.8 kg **(b)** $\frac{3}{5} : \frac{8}{9}$ **(c)** 0.36 : 1.2



Class Discussion

Making Sense of the Relationship between Ratios and Fractions

There are 40 green balls and 60 red balls in a bag.

Let A and B represent the number of green balls and red balls respectively.

1. Find the ratio of A to B.

$A : B = 40 : 60$

$= $ ______ : ______

We can conclude that:

The ratio of A to B is ______ : ______.

The following statement is equivalent to the above statement.

A is ______ (fraction) of B, i.e. $\frac{A}{B} =$ ______ (fraction).

2. Find the ratio of B to A.

$B : A = 60 : 40$

$= $ ______ : ______

We can conclude that:

The ratio of B to A is ______ : ______.

The following statement is equivalent to the above statement.

B is ______ (fraction) of A, i.e. $\frac{B}{A} =$ ______ (fraction).

3. Draw a model to illustrate the relationship between A and B.

4. Work in pairs.

Come up with other scenarios involving two quantities similar to that given above. Challenge your classmate to write equivalent statements involving ratios and fractions to compare the two quantities. Draw models to illustrate the relationships.

> In general, using ratios to compare two quantities of the same unit is equivalent to using fractions to compare the two quantities, e.g. $a : b = 5 : 7$ is equivalent to $\frac{a}{b} = \frac{5}{7}$.

Worked Example 3

(Expressing Ratios as Fractions)

Given that $4x : 9 = 7 : 3$, calculate the value of x.

Solution:

$4x : 9 = 7 : 3$

$\frac{4x}{9} = \frac{7}{3}$ (express ratios as fractions)

$4x = 21$

$x = 5\frac{1}{4}$

PRACTISE NOW 3

SIMILAR QUESTIONS

Exercise 9A Questions 2(a)–(b), 3

Given that $3a : 7 = 8 : 5$, find the value of a.

Worked Example 4

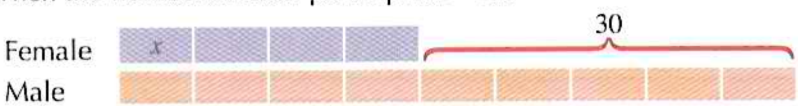
(Problem involving Ratios of Two Quantities)

The ratio of the number of female participants to the number of male participants at a party is 4 : 9. If there are 30 more male participants than female participants, calculate the total number of people who attended the party.

Solution:

Let the number of female participants = $4x$.

Then the number of male participants = $9x$.



From the model, we form the equation:

$$\begin{aligned} 9x - 4x &= 30 \\ 5x &= 30 \\ x &= 6 \end{aligned}$$

Total number of people who attended the party $= (4 + 9) \times 6$

$= 13 \times 6$

$= 78$

Problem Solving Tip

You may draw a model to help you make sense of a problem.

Internet Resources

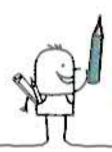
Visit http://www.shinglee.com.sg/StudentResources/ to access the AlgeTools™ software. Select the AlgeBar™ application to solve word problems on 'Ratio'. You may draw models to help you formulate the equations.

PRACTISE NOW 4

SIMILAR QUESTIONS

Exercise 9A Questions 7, 12, 14

1. The ratio of the number of fiction books to the number of non-fiction books in a library is 5 : 2. If there are 1421 fiction and non-fiction books altogether, how many more fiction than non-fiction books are there in the library?
2. Kate and Nora each have a sum of money. The ratio of the amount of money Kate has to that of Nora is 3 : 5. After Nora gives \$150 to Kate, the ratio of the amount of money Kate has to that of Nora becomes 7 : 9. Find the sum of money Kate had initially.



Journal Writing

1. An example of a use of ratio in everyday life can be found in technology such as the aspect ratios of televisions. The two most common sizes of televisions are standard and widescreen, where the aspect ratios are 4 : 3 and 16 : 9 respectively. Find out more about aspect ratios and record your findings.
2. Search on the Internet to find out other uses of ratios in everyday life and give a description for each example.

Investigation

Golden Ratio

1. Fig. 9.1(a) shows the famous *Vitruvian Man* drawn by the world-renowned artist Leonardo da Vinci in 1487. Measure the lengths of AB and BC and find the ratios $\frac{AC}{AB}$ and $\frac{AB}{BC}$ correct to 2 significant figures. What do you notice?

(a) Vitruvian Man

(b) Parthenon in Athens

Fig. 9.1

2. Fig. 9.1(b) shows the Parthenon in Athens, Greece. It was built in 438 BC but has survived to this day.

   Measure the length XY and breadth YZ of the rectangle, and find the ratio $\frac{XY}{YZ}$ correct to 2 significant figures.

3. Use a calculator to find the value of $\frac{1+\sqrt{5}}{2}$ correct to 2 significant figures.

4. What do you notice about the values obtained in the previous three questions?

5. $\frac{1+\sqrt{5}}{2}$ is called the **Golden Ratio** and is denoted by the symbol Φ (pronounced as 'phi'). It is an irrational number and its value, truncated to 50 decimal places, is

   1.61803 39887 49894 84820 45868 34365 63811 77203 09179 80576 ...

   It is called the Golden Ratio because any object which has this ratio is supposed to be the most pleasing to the eye. For example, most people like the Parthenon in Athens as the dimensions of the rectangle are in the golden ratio and so it is called the 'Golden Rectangle'.

6. The Golden Ratio Φ has some interesting properties.

   (a) Use a calculator to find the values of Φ^2 and $\Phi + 1$. What do you notice?

   (b) Use a calculator to find the value of $\frac{1}{\Phi}$. What do you think it is equal to?

   $$\frac{1}{\Phi} = \Phi - \_\_\_\_\_$$

Performance Task

Search on the Internet to find out what other man-made structures (e.g. the Great Pyramid) or natural occurrences (e.g. Nautilus, a sea creature, as shown in Fig. 9.2) have in common with the Golden Ratio. Present your findings on an A4-sized poster. Remember to include some photos.

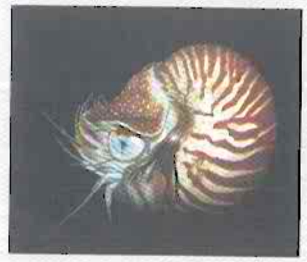


Fig. 9.2

Ratios involving Three Quantities

Ratios can also be used to make comparisons among three or more quantities.

For example, if $x = 18$, $y = 27$ and $z = 54$, then

$$x : y : z = 18 : 27 : 54 = 2 : 3 : 6.$$

From the above, we can deduce that

$$x : y = 2 : 3,$$
$$y : z = 3 : 6 = 1 : 2,$$
$$x : z = 2 : 6 = 1 : 3.$$

Note: The ratio of three quantities can be simplified by multiplying or dividing each term by the same constant but it cannot be written as a fraction.

Worked Example 5

(Ratio involving Three Quantities)

If $x : y = 11 : 8$ and $y : z = 6 : 7$, calculate

(i) $x : y : z$, **(ii)** $x : z$.

Solution:

(i) $x : y = 11 : 8 \xrightarrow{\times 3} = 33 : 24$

$y : z = 6 : 7 \xrightarrow{\times 4} = 24 : 28$

$\therefore x : y : z = 33 : 24 : 28$

(ii) From **(i)**, $x : z = 33 : 28$.

Problem Solving Tip

y is the common part in the ratios $x : y$ and $y : z$. To calculate $x : y : z$, we find the equivalent ratios of $x : y$ and $y : z$ such that y has the same number of units in both ratios.

ATTENTION

The LCM of 8 and 6 is 24.

PRACTISE NOW 5

If $x : y = 5 : 6$ and $y : z = 4 : 9$, find

(i) $x : y : z$, **(ii)** $x : z$.

SIMILAR QUESTIONS

Exercise 9A Questions 1(d)–(f), 4, 6, 9(c)–(e), 10(a)–(b), 11, 16

Worked Example 6

(Problem involving Ratios of Three Quantities)

A sum of money is divided among Devi, Lixin and Shirley in the ratio 9 : 8 : 7. After Devi gives \$25 each to Lixin and Shirley, the ratio becomes 16 : 17 : 15. Calculate the amount of money Devi had initially.

Solution:

Let the amount of money Devi had initially be $\$9x$.

Then the amount of money Lixin and Shirley had initially is $\$8x$ and $\$7x$ respectively.

	Devi	Lixin	Shirley
Before	$\$9x$	$\$8x$	$\$7x$
After	$\$(9x - 50)$	$\$(8x + 25)$	$\$(7x + 25)$

$$\therefore \frac{9x - 50}{8x + 25} = \frac{16}{17}$$

$$17(9x - 50) = 16(8x + 25)$$

$$153x - 850 = 128x + 400$$

$$153x - 128x = 400 + 850$$

$$25x = 1250$$

$$x = 50$$

$\therefore$ Amount of money Devi had initially $= 9 \times \$50$

$= \$450$

Problem Solving Tip

You can also find the value of x by solving $\frac{9x - 50}{7x + 25} = \frac{16}{15}$ or $\frac{8x + 25}{7x + 25} = \frac{17}{15}$.

PRACTISE NOW 6

A sum of money is divided among Khairul, Michael and Ethan in the ratio 6 : 4 : 5. After Khairul gives \$30 to Michael and \$15 to Ethan, the ratio becomes 7 : 6 : 7. Find the amount of money Khairul had initially.

SIMILAR QUESTIONS

Exercise 9A Questions 8, 13, 15

Exercise 9A

BASIC LEVEL

1. Simplify each of the following ratios.

   (a) 1.5 kg : 350 g (b) $\frac{15}{24} : \frac{9}{7}$

   (c) 0.45 : 0.85 (d) 580 ml : 1.12 l : 104 ml

   (e) $\frac{2}{3} : \frac{3}{2} : \frac{5}{8}$ (f) 0.33 : 0.63 : 1.8

2. (a) Find the value of a if $a : 400 = 6 : 25$.

   (b) Given that $5b : 8 = 2 : 5$, find the value of b.

3. Given that $\frac{2x}{5} = \frac{3y}{8}$, find the ratio of $x : y$.

4. Given that $a : b : c = 75 : 120 : 132$,

   (i) simplify $a : b : c$,

   (ii) find $b : a$,

   (iii) find $b : c$.

5. There are 14 boys and 25 girls in a school badminton team. Find the ratio of

   (i) the number of boys to the number of girls,

   (ii) the number of girls to the total number of players in the team.

6. A total of 3600 athletes participated in the Singapore 2010 Youth Olympic Games. There were 1200 media representatives who reported on the Games, 20 000 volunteers who helped out during the Games and 370 000 spectators who attended the Games. Find the ratio of

   (i) the number of athletes to the number of volunteers,

   (ii) the number of media representatives to the number of athletes to the number of spectators.

7. A certain amount of money is shared between Rui Feng and Vishal in the ratio 5 : 9. If Rui Feng gets $44 less than Vishal, find the total amount of money that is shared between the two boys.

8. Amirah, Huixian and Priya make a total of 1530 toys in the ratio 12 : 16 : 17. Find

   (i) the number of toys Huixian makes,

   (ii) the amount of money Priya earns if she is paid $1.65 for each toy.

INTERMEDIATE LEVEL

9. Simplify each of the following.

   (a) $4\frac{1}{5}$ kg : 630 g (b) $0.75 : 3\frac{5}{16}$

   (c) 0.6 kg : $\frac{3}{4}$ kg : 400 g (d) $\frac{1}{3} : 2.5 : 3\frac{3}{4}$

   (e) $1.2 : 3\frac{3}{10} : 5.5$

10. (a) Find the value of m if $2\frac{1}{4} : 6 = m : 1\frac{1}{5}$.

(b) Given that $x : 3 : \frac{9}{2} = \frac{15}{4} : 4\frac{1}{2} : y$,

find the value of x and of y.

11. If $p : q = \frac{3}{4} : 2$ and $p : r = \frac{1}{3} : \frac{1}{2}$, find

(i) $p : q : r$,

(ii) $q : r$.

12. In a school of 1200 students, the ratio of the number of teachers to students is 1 : 15. After some teachers join the school, the ratio of the number of teachers to students becomes 3 : 40. Find

(i) the initial number of teachers in the school,

(ii) the number of teachers who join the school.

13. Ethan, Farhan and Michael invested \$427 000, \$671 000 and \$305 000 in a property respectively and they agreed to share the profit in the ratio of their investments. After a few years, they sold the property for \$1 897 500. Find the amount of profit each of them received.

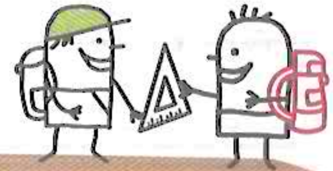
ADVANCED LEVEL

14. Find the number that must be added to 3 and 8 so that the ratio of the first number to the second number becomes 2 : 3.

15. A sum of money is divided among Ethan, Jun Wei and Raj in the ratio 5 : 6 : 9. After Ethan gives \$50 to his mother, the ratio becomes 3 : 4 : 6. Find the amount of money Ethan has after giving \$50 to his mother.

16. Given that $x : y = 3 : 4$ and $y : z = 5 : 8$, find the value of $\frac{2y}{3x - y + 2z}$.

9.2 Rate



Concept of Rate

We have learnt that a ratio compares two or more quantities of the same kind. We shall now learn how to compare two or more quantities of *different kinds*.

Worked Example 7

(Problem involving Rates)

Shop *A* sells eggs at $1.50 per half dozen whereas Shop *B* sells eggs of the same size and quality at $2.40 per dozen. Which shop should we buy the eggs from?

Solution:

To find the answer to the problem, we have to find each of their prices for an equal number of eggs, such as one egg.

Price in Shop *A*: $\frac{\$1.50}{6} = \$0.25$ per egg

Price in Shop *B*: $\frac{\$2.40}{12} = \$0.20$ per egg

Thus we should buy the eggs from Shop *B*.

Is there a simpler method of obtaining the answer to this problem?

PRACTISE NOW 7

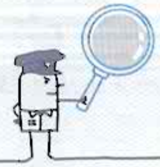
Amirah can type 720 words in 16 minutes, Lixin can type 828 words in 18 minutes and Shirley can type 798 words in 19 minutes. Who is the fastest typist?

SIMILAR QUESTIONS

Exercise 9B Questions 1(a)–(d), 2

In Worked Example 7, '$0.25 per egg' and '$0.20 per egg' are known as **rates**. We use 'per' to mean 'for each'.

Average Rates and Constant Rates



Investigation

Average Pulse Rate

1. Take your pulse count for 1 minute. Record your reading in Table 9.1.
2. Repeat Step 1 twice.

	First reading	Second reading	Third reading
Pulse rate (per minute)			

Table 9.1

3. Are the three pulse rates in Table 9.1 the same? Explain your answer.
4. Find your average pulse rate per minute.

From the investigation, we notice that a pulse rate may not be constant (uniform) throughout. Another real-life example of rate which is not constant is the rate of growth of bacteria.

On the other hand, some of the rates we encounter in daily lives are **constant**. For example, if the hourly wage for working in a cafe is \$6, we will earn \$12 if we work for 2 hours, \$18 if we work for 3 hours, and so on.

Worked Example 8

(Rates in Everyday Life)

(a) Khairul pays \$6 for parking his car in a shopping centre for 2.5 hours. Calculate

(i) the parking charges per minute,

(ii) the amount that he will have to pay if he parks his car in the shopping centre for $1\frac{3}{4}$ hours.

(b) On another occasion, Khairul drives from Singapore to Malaysia. His car requires 18 litres of petrol to travel a distance of 243 km.

(i) How far can his car travel on 44 litres of petrol?

(ii) Given that the cost of petrol is \$1.87 per litre, how much does he have to pay for the petrol to travel a distance of 576 km?

Solution:

(a) (i) 2.5 hours = 2.5 × 60 minutes
= 150 minutes

Parking charges per minute = $\frac{\$6}{150}$
= $0.04

(ii) $1\frac{3}{4}$ hours = $1\frac{3}{4} \times 60$ minutes
= 105 minutes

Amount that he will have to pay = $0.04 × 105
= $4.20

(b) (i) Distance travelled on 1 litre of petrol = $\frac{243}{18}$
= 13.5 km

Distance travelled on 44 litres of petrol = 13.5 × 44
= 594 km

(ii) Amount of petrol required to travel a distance of 576 km = $\frac{576}{13.5}$
= $42\frac{2}{3}$ litres

Amount that he will have to pay = $42\frac{2}{3} \times \$1.87$
= $79.79 (to the nearest cent)

PRACTISE NOW 8

SIMILAR QUESTIONS

Exercise 9B Questions 3–10

1. **(a)** A school bus company charges $2.70 per kilometre to ferry 36 children for an outing. How much does each child have to pay if the distance travelled for the trip is 32.5 km?
   (b) A car requires 25 litres of petrol to travel a distance of 265 km. Find
   (i) the distance that the car can travel on 58 litres of petrol,
   (ii) the amount that the car owner has to pay to travel a distance of 1007 km if a litre of petrol costs $1.95.
2. In a competition, 5 people can finish 20 steamed buns in 3 minutes 20 seconds. Assuming that everyone consumes steamed buns at the same rate and that the rate of consumption remains constant throughout the competition, find the number of steamed buns 10 people can finish in 5 minutes.

Thinking Time

1. In Worked Example 8, the parking charges per minute are $0.04 while the rate of petrol consumption is 13.5 km per litre. Which of the above is an average rate and which is a constant rate? Explain your answer.
2. Give 3 other examples of average rates and of constant rates that can be found in everyday life.

Exercise 9B

BASIC LEVEL

1. (a) A typist types 1800 words in 1 hour. Find the number of words that she can type per minute.
   (b) If \$120.99 is charged for 654 units of electricity used, find the cost of one unit of electricity.
   (c) A man pays a total of \$4800 in flat rental for 3 months. Find his monthly rental rate.
   (d) If the mass of a metal bar which is 3.25 m long is 15 kg, find its mass per metre.

2. Ethan can blow 15 balloons in 20 minutes, Jun Wei can blow 18 balloons in 25 minutes and Vishal can blow 16 balloons in 21 minutes. Assuming that all balloons are blown to the same size, who can blow balloons at the fastest rate?

3. For each ornament that a worker makes, he is paid \$1.15. He makes 4 ornaments every 15 minutes. Find the amount earned by the worker if he works for 3 hours.

4. Michael has to pay \$39 for 650 minutes of outgoing calls made using his handphone. Find
   (i) the amount he is charged for each minute of outgoing calls,
   (ii) the amount he has to pay if he makes 460 minutes of outgoing calls.

5. A car requires 22 litres of petrol to travel a distance of 259.6 km. Find
   (i) the distance that the car can travel on 63 litres of petrol,
   (ii) the amount that the car owner has to pay to travel a distance of 2013.2 km if a litre of petrol costs \$1.99.

6. 200 g of fertiliser is required for a plot of land that has an area of 8 m^2. Find
   (i) the amount of fertiliser needed for a plot of land that has an area of 14 m^2,
   (ii) the area of land that can be fertilised by 450 g of fertiliser.

INTERMEDIATE LEVEL

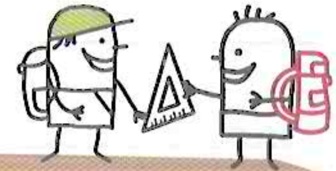
7. A piece of metal is heated to 428 °C before it is left to cool. The temperature of the metal falls at a rate of 23 °C per minute for the first 3 minutes, at a rate of 15 °C per minute for the next 15 minutes and then at a rate of 8 °C per minute until it reaches room temperature of 25 °C. Find
   (i) the temperature of the metal after 9 minutes,
   (ii) the total time needed for the metal to reach a temperature of 25 °C from 428 °C.

8. A cook uses fifteen 2-litre bottles of cooking oil in 4 weeks. If he decides to buy 5-litre tins of cooking oil instead, how many tins of cooking oil will he use over a 10-week period if the rate at which he uses it remains unchanged?

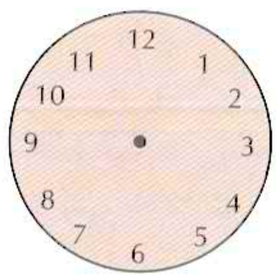
9. 224 hours are required to complete a project. 4 men are employed for this project.
   (i) The hourly rate of each man is \$7.50. If the 4 men do not work overtime, find the total amount to be paid to the men.
   (ii) Their normal working hours are from 9 a.m. to 6 p.m., with a one-hour lunch break. Given that their overtime rate is 1.5 times their hourly rate, find the total amount to be paid to the 4 men if the project is to be completed in 4 days.

10. 10 chefs can prepare a meal for 536 people in 8 hours. Assuming that the chefs cook at the same rate and that the rate at which they cook remains constant throughout the preparation, find the number of people 22 chefs can prepare a meal for in 5 hours.

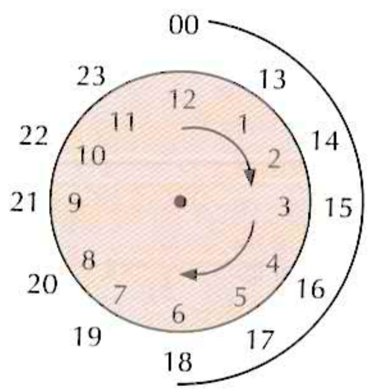
9.3 Time



To record the time of the day, we can either use the 12-hour clock or the 24-hour clock. In the 12-hour clock, morning (from midnight to just before noon) is denoted by a.m. while afternoon, evening and night are denoted by p.m. In the 24-hour clock, four digits are used to indicate time. The first two digits denote hours and the last two denote minutes.



12-hour clock



24-hour clock

INFORMATION

a.m. stands for "ante meridiem' (Latin word) meaning "before midday" and p.m. stands for "post meridiem" meaning "after midday".

Table 9.2 shows some examples of representing time in 12-hour and 24-hour clocks respectively.

Time	12-hour clock	24-hour clock
2 o'clock early morning	2.00 a.m.	02 00
5 to 11 in the morning	10.55 a.m.	10 55
Noon	12.00 p.m.	12 00
Half past 12 early afternoon	12.30 p.m.	12 30
Quarter to 3 in the afternoon	2.45 p.m.	14 45
5 past 8 in the evening	8.05 p.m.	20 05
One minute to midnight	11.59 p.m.	23 59
Midnight	12.00 a.m.	00 00
One minute past midnight	12.01 a.m.	00 01

Table 9.2

Worked Example 9

(Problem involving Time)

A car leaves Town A at 21 15 on Wednesday and arrives at Town B $5\frac{1}{2}$ hours later. At what time and day does the car arrive at Town B?

Solution:

$5\frac{1}{2}$ h = 5 h 30 min

$$21\ 15 \xrightarrow{+\,5\text{ h}} \underset{(02\ 15)}{26\ 15} \xrightarrow{+\,30\text{ min}} 02\ 45$$

∴ The car arrives at Town B at 02 45 or 2.45 a.m. on Thursday.

PRACTISE NOW 9

SIMILAR QUESTIONS

Exercise 9C Question 5

A ship left Port X at 22 45 on Friday and arrived at Port Y $7\frac{1}{4}$ hours later. At what time and day did the ship arrive at Port Y?

Worked Example 10

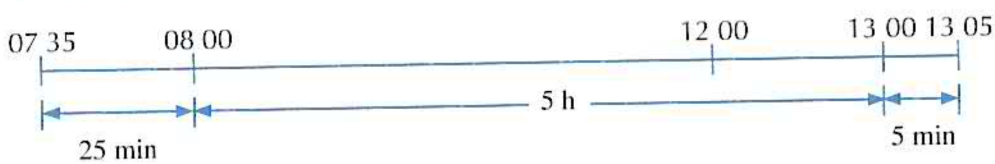
(Problem involving Time)

A train left Singapore at 07 35 and arrived at Seremban at 13 05 on the same day. How long was the train journey?

Problem Solving Tip

1 hour is converted to 60 minutes.

Solution:



25 min + 5 min = 30 min

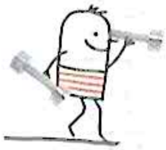
∴ The train journey was 5 h 30 min long.

PRACTISE NOW 10

SIMILAR QUESTIONS

Exercise 9C Question 4

A bus left City P at 10 45 and arrived at City Q at 23 11 on the same day. How long was the bus journey?



Exercise 9C

BASIC LEVEL

1. Convert the following times to 24-hour clock notation.

(a) 8.00 a.m.
(b) 9.42 p.m.
(c) midnight
(d) 2.42 a.m.

2. Convert the following times to 12-hour clock notation.

(a) 03 30
(b) 23 12
(c) 19 15
(d) 00 00

3. Copy and complete the following table:

	Departure Time	Journey Time	Arrival Time
(a)	02 40	55 minutes	
(b)	22 35	8 hours	
(c)	15 45		17 50
(d)	09 48		22 16
(e)	20 35 (Tuesday)		07 15 (Wednesday)
(f)		$1\frac{1}{4}$ hour	23 50

INTERMEDIATE LEVEL

4. A train left a station at 8.35 a.m. and arrived at its destination at 3.12 p.m. How long did the journey take?

5. An overnight train left at 21 55 on a journey that took 9 h 18 min. Given that the train departed on a Monday, find the day and time at which it arrived at its destination.

6. A car arrived at a town at 15 06 after travelling for $4\frac{1}{4}$ hours. Find the time the car started the journey.

7. According to a timetable, a coach was due to leave a station at 22 55 and arrive at its destination at 06 05 the next day. Find

(i) the time taken for the journey,
(ii) the time when the coach reached its destination given that it reached 35 minutes before schedule.

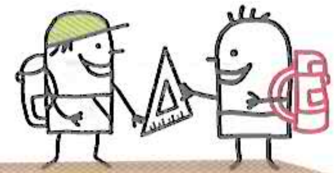
8. The schedule of the arrival and departure times of a long-distance express overnight coach is shown below.

Destination	Arrival	Departure
A	–	21 30
B	22 15	22 30
C	02 25	02 30
D	03 50	04 20
E	07 50	08 00
F	09 20	09 30
G	10 45	–

Find the time taken for the coach to travel from

(a) *A* to *C*
(b) *B* to *E*
(c) *C* to *F*
(d) *D* to *G*
(e) *A* to *G*

9.4 Speed



In this section, we will explore speed, which is a special type of rate.

Recap (Concept of Speed)

In primary school, we have learnt that the **speed** of an object is defined as the distance travelled by the object per unit time:

$$\boxed{\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}}$$

The speed of an object indicates how fast it is moving. Speed can be expressed in different units, such as m/s, km/h, m/min and cm/s.

Worked Example 11

(Problem involving Speed)

The distance from the southern tip of Singapore to Batam is 20 km. If a ferry starts on its journey from Singapore at 10.50 a.m. and reaches Batam at 11.30 a.m., calculate the speed of the ferry in

(i) km/h, **(ii)** m/s.

A car leaves Singapore for Kuala Lumpur at the same time as a bus that leaves Kuala Lumpur for Singapore. They travel along the same road, the car at 90 km/h and the bus at 45 km/h. Which vehicle is further away from Singapore when they meet?

Solution:

$$10.50 \text{ a.m.} \xrightarrow{40 \text{ minutes}} 11.30 \text{ a.m.}$$

(i) 40 minutes $= \frac{2}{3}$ hour

$$\text{Speed of ferry} = \frac{\text{Distance travelled}}{\text{Time taken}} = \frac{20}{\frac{2}{3}} = 20 \div \frac{2}{3} = \cancel{20}^{10} \times \frac{3}{\cancel{2}_1}$$

$$= 30 \text{ km/h}$$

(ii) 20 km = 20 × 1000 = 20 000 m

40 minutes = 40 × 60 = 2400 s

$$\text{Speed of ferry} = \frac{20\,000}{2400} = 8\frac{1}{3} \text{ m/s}$$

PRACTISE NOW 11

SIMILAR QUESTIONS

Exercise 9D Questions 1–2, 11

1. A train travels 16.8 km in 25 minutes. Find the speed of the train in
   (i) km/h, **(ii)** m/s.
2. A car travels at a speed of 55 km/h. Find the distance travelled by the car in 12 minutes 30 seconds, giving your answer in metres.
3. A car and a bus are travelling towards each other. They are 510 km apart at 13 20 and they pass each other at 16 20. If the car is travelling at a speed of 90 km/h, find the speed of the bus.

Conversion of Units

An alternative method for Worked Example 11(ii) is to convert the speed of the ferry directly from 30 km/h to m/s.

$$30 \text{ km/h} = \frac{30 \text{ km}}{1 \text{ h}}$$
$$= \frac{30 \times 1000 \text{ m}}{3600 \text{ s}} \quad \text{(convert 30 km into m and 1 h into s)}$$
$$= \frac{30\,000 \text{ m}}{3600 \text{ s}}$$
$$= \frac{25}{3} \text{ m/s}$$
$$= 8\frac{1}{3} \text{ m/s}$$

ATTENTION

- 1 km = 1000 m
- 1 h = 60 min
  = 60 × 60 s
  = 3600 s

Worked Example 12

(Conversion of Units)

In Singapore, the speed limits for cars depend on the type of roads. The highest speed limit for cars is 90 km/h. Express this speed in

(i) m/s, **(ii)** cm/min.

Internet Resources

Search on the Internet for the number of demerit points that a car owner will be given if he or she is caught speeding on the expressway at

(a) 100 km/h,
(b) 120 km/h,
(c) 160 km/h.

Solution:

(i) Highest speed limit for cars = 90 km/h

$$= \frac{90 \text{ km}}{1 \text{ h}}$$
$$= \frac{90\,000 \text{ m}}{3600 \text{ s}} \quad \text{(convert 90 km into m and 1 h into s)}$$
$$= 25 \text{ m/s}$$

(ii) Highest speed limit for cars = 90 km/h

$$= \frac{90 \text{ km}}{1 \text{ h}}$$
$$= \frac{9\,000\,000 \text{ cm}}{60 \text{ min}} \quad \text{(convert 90 km into cm and 1 h into min)}$$
$$= 150\,000 \text{ cm/min}$$

PRACTISE NOW 12

SIMILAR QUESTIONS

Exercise 9D Questions 3–5

1. A train travels at a speed of 48.6 km/h. Express this speed in
   (i) m/s, **(ii)** cm/min.
2. The cheetah is the fastest land animal, capable of attaining a speed of 110 km/h. In August 2009, the world record for the men's 100 m sprint was set at 9.58 seconds. How many times is a cheetah as fast as the fastest human sprinter?

Constant Speeds and Average Speeds

If the speed of an object does not change throughout its journey, the object is said to be travelling at a **constant speed**.

However, in real-life situations, it is unlikely for an object to travel at the same speed throughout its journey. Consider the speed of the ferry in Worked Example 11. The **average speed** of the ferry is 30 km/h. This means that on average, the ferry travels 30 km every hour. Why is it not possible for the ferry to maintain a speed of 30 km/h from Singapore to Batam?

The average speed of an object is defined as the total distance travelled by the object per unit time:

$$\boxed{\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}}$$

INFORMATION

If a cyclist is equipped with a speedometer, which gives his speed at a particular instant, the reading of the speedometer will change from time to time.

Worked Example 13

(Problem involving Average Speed)

Priya sets off from her home on a 70 km journey to her friend's house. She travels the first 40 km of her journey at an average speed of 40 km/h and the remaining at an average speed of 60 km/h. Calculate the average speed for her entire journey.

Solution:

$$\text{Time taken for first part of journey} = \frac{40}{40} = 1 \text{ hour}$$

$$\text{Time taken for second part of journey} = \frac{70-40}{60} = \frac{1}{2} \text{ hour}$$

$$\text{Total time taken} = 1 + \frac{1}{2} = 1\frac{1}{2} \text{ hour}$$

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{70}{1\frac{1}{2}} = 46\frac{2}{3} \text{ km/h}$$

PRACTISE NOW 13

In a triathlon, Farhan swims a distance of 1.5 km at an average speed of 2.5 km/h, cycles 40 km in $1\frac{1}{2}$ hours and runs at an average speed of 9 km/h for $1\frac{1}{9}$ hours. Find his average speed for the entire competition.

SIMILAR QUESTIONS

Exercise 9D Questions 6–9

Thinking Time

In Worked Example 13, Huixian did this calculation:

$$\text{Average speed} = \frac{40 + 60}{2} = 50 \text{ km/h}.$$

How would you explain to Huixian why she is wrong?

Worked Example 14

(Average Speed Problem involving Algebra)

A car travels at an average speed of 60 km/h from Town A to Town B. If the car travels at an average speed of 72 km/h instead, it would reach Town B 15 minutes earlier. Calculate the distance between Town A and Town B.

Solution:

Let the distance between Town A and Town $B = x$ km.

Then the time taken for the car to travel from Town A to Town B at an average speed of 60 km/h $= \frac{x}{60}$ hour,

the time taken for the car to travel from Town A to Town B at an average speed of 72 km/h $= \frac{x}{72}$ hour.

$$\therefore \frac{x}{60} - \frac{x}{72} = \frac{15}{60}$$

$$\frac{x}{360} = \frac{1}{4}$$

$$x = 90$$

Distance between Town A and Town B = 90 km

PRACTISE NOW 14

SIMILAR QUESTIONS

Exercise 9D Question 12

A car leaves Town A for Town B, which are 550 km apart, at an average speed of 72 km/h. At the same time, a truck leaves Town B for Town A and travels along the same road as the car at an average speed of 38 km/h. Find the time taken for the two vehicles to meet.

Performance Task

1. Complete Table 9.3 by answering each of the following questions.

   (a) Walk one round around the field in your school at your normal pace. Use a stopwatch to record the time taken. Find out the perimeter of your school field and thus find your average walking speed.

   (b) A 13-year-old boy wants to score an A for his 2.4 km run in his National Physical Fitness Award Test. To do so, he needs to complete the run in less than 11 minutes 31 seconds. Find the minimum average speed at which he needs to run.

   (c) Find out the average speed of a bicycle and of a sports car.

   (d) In Singapore, it is easy to travel from one place to another with the Mass Rapid Transit (MRT) trains. What is the average speed of an MRT train?

   (e) The aeroplane is one of the greatest inventions in the history of mankind as it allows for the ease of transportation of people over long distances. Find out the average speed of an aeroplane.

   (f) The spaceship 'Discovery' made several trips to the International Space Station in recent years. What is the average speed of the spaceship during one of its missions?

		Average speed (km/h)
(a)	Walking	
(b)	Running	
(c)	Bicycle	
(d)	Sports car	
(e)	MRT train	
(f)	Aeroplane	
(g)	Spaceship	

Table 9.3

2. Devi and Priya each travel the same distance to school. If Priya takes 5 times as long as Devi to get to school, using Table 9.3, determine the mode of transport each of them takes to get to school.
3. Based on Table 9.3, how many times is a spaceship as fast as an aeroplane?
4. What are other examples of speeds which can be encountered in real life?
5. Present your findings to the class.

Worked Example 15

(Rate of Revolution)

The Singapore Flyer has a diameter of 150 m. If each capsule of the Singapore Flyer travels at an average speed of 0.262 m/s, calculate the number of revolutions made by a capsule per hour, giving your answer correct to the nearest whole number. (Take π to be 3.142.)

Solution:

Radius of the Singapore Flyer $= \frac{150}{2}$

$= 75$ m

Circumference of the Singapore Flyer $= 2 \times \pi \times 75$

$= 2 \times 3.142 \times 75$

$= 471.3$ m

Distance travelled by a capsule in 1 hour $= 0.262 \times 3600$

$= 943.2$ m

Number of revolutions made by a capsule per hour $= \frac{943.2}{471.3}$

$= 2.00$ (to the nearest whole number)

RECALL

Circumference of a circle $= 2\pi r$

PRACTISE NOW 15

The diameter of a wheel of a car is 0.75 m. If the car travels at an average speed of 14 m/s, find the number of revolutions made by the wheel per minute, giving your answer correct to the nearest whole number. (Take π to be 3.142.)

SIMILAR QUESTIONS

Exercise 9D Question 10

Exercise 9D

BASIC LEVEL

1. A particle travels 24.6 km in 30 minutes. Find the speed of the particle in
   (i) km/h, **(ii)** m/s.

2. A high-speed train travels at a speed of 200 km/h. If the train sets off from Station A at 12 24 and reaches Station B at 14 12, find the distance between the two stations, giving your answer in metres.

3. Express each of the following in km/h.
   (a) 8.4 km/min **(b)** 315 m/s
   (c) 242 m/min **(d)** 125 cm/s

4. Express each of the following in m/s.
   (a) 65 cm/s **(b)** 367 km/h
   (c) 1000 cm/min **(d)** 86 km/min

5. A bullet train travels at a speed of 365 km/h. In August 2001, Singapore's national record for the men's 100 m sprint was set at 10.37 seconds. How many times is a bullet train as fast as the fastest Singaporean sprinter?

6. A car travels the first 19 km of its journey at an average speed of 57 km/h and the remaining 55 km at an average speed of 110 km/h. Find the average speed of the car for its entire journey.

INTERMEDIATE LEVEL

7. Two points, X and Y, are 120 m apart. M is the midpoint of X and Y. An object travels from X to M in 12 seconds and then from M to Y at an average speed of 15 m/s. Find
   (i) the time taken for the object to travel from M to Y,
   (ii) the average speed of the object for its entire journey from X to Y.

8. Two points, L and N, are 160 m apart. M lies on the straight line joining L and N. An object travels from L to M at an average speed of 10 m/s in 6 seconds and then from M to N at an average speed of 25 m/s. Find the average speed of the object for its entire journey from L to N.

9. A car travels the first 50 km of its journey at an average speed of 25 m/s and the next 120 km at an average speed of 80 km/h. The car completes the last part of its journey at an average speed of 90 km/h in 35 minutes. Find the average speed for its entire journey, giving your answer in km/h.

10. The diameter of a wheel of a car is 60 cm. If the car travels at an average speed of 13.2 m/s, find the number of revolutions made by the wheel per hour, giving your answer correct to the nearest whole number. (Take π to be 3.142.)

ADVANCED LEVEL

11. A passenger train travels at a speed of 72 km/h. A man on the passenger train observes a goods train travelling at a speed of 54 km/h in the opposite direction. If the goods train passes him in 8 seconds, find the length of the goods train.

12. Nora leaves Town A and walks towards Town B at a speed of 100 m/min. At the same time, Kate and Lixin walk from Town B towards Town A at a speed of 80 m/min and 75 m/min respectively. If Nora meets Lixin 6 minutes after passing Kate, find the distance between Town A and Town B.

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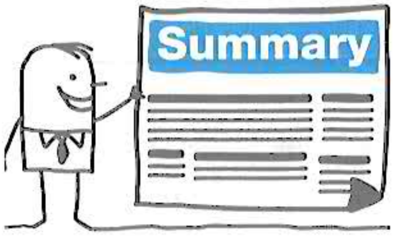
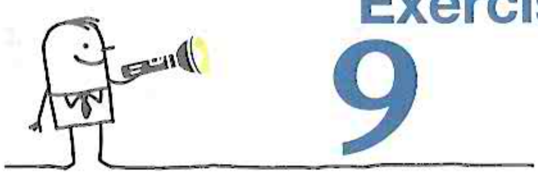


Summary

1. The **ratio** of a is to b, where a and b represent two quantities of the *same kind*, and $b \neq 0$, is written as $a : b$. A ratio has *no* units.
2. **Rate** is a comparison of two quantities of *different kinds*.
3. An object is said to be travelling at a **constant speed** when its speed does not change throughout the journey.
4. The formula for the **average speed** of an object is:

$$\boxed{\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}}$$



Review Exercise 9

1. If $a : b = \frac{1}{2} : \frac{1}{3}$ and $b : c = 3 : 4$, find $a : c$.

2. Three types of coffee beans, A, B and C, are blended in the ratio 3 : 5 : 7 to make a bag of coffee powder.
   (i) Given that the bag contains 45 kg of coffee powder, find the mass of each type of coffee beans in the mixture.
   (ii) If A costs \$7 per kg, B costs \$10 per kg and C costs \$13 per kg, find the cost per kg of the mixture.

3. A box contains 36 books and some toys. The ratio of the number of books to the number of toys is 4 : 5. After some toys are given away, the ratio of the number of books to the number of toys becomes 12 : 11. Find
   (i) the initial number of toys in the box,
   (ii) the number of toys that are given away.

4. The total cost of placing an advertisement in a newspaper comprises of a fixed cost of \$3.50 and a variable cost that depends on the number of words. Each word costs 25 cents.
   (i) Find the total cost of placing an advertisement containing 22 words.
   (ii) If Michael does not want to spend more than \$15 on an advertisement, what is the greatest number of words he can use?

5. A car took 2 hours and 15 minutes to travel 198 km. If it arrived at its destination at 12 06, find
   (i) the time it started its journey,
   (ii) the average speed of the car, giving your answer in km/h.

6. A lorry leaves a factory on a journey of 195 km at 08 45, travelling at an average speed of 52 km/h.
   (i) Find the time at which the lorry arrives at its destination.
   (ii) On the return journey, the lorry leaves at 14 55 and arrives at the factory at 18 15.
   Calculate the time taken and the average speed of the lorry on the return journey.

7. In a triathlon, an athlete swims 750 m in 15 minutes, cycles at an average speed of 40 km/h for 30 minutes and runs 5 km at an average speed of 3 m/s. Find his average speed for the entire competition, giving your answer in km/h.

8. Two points, A and B, are x m apart. C lies on the straight line joining A and B such that the ratio of the length of AC to that of CB is 2 : 3. An object travels from A to C in half a minute and then from C to B at an average speed of 30 m/s. Find an expression for
   (i) the time taken for the object to travel from C to B,
   (ii) the average speed of the object for its entire journey from A to B.

9. Farhan cycles the first part of a 150-km journey at an average speed of 35 km/h and walks the remaining distance at an average speed of 5 km/h. If he takes 4.5 hours for his entire journey, find the distance that he cycles.

10. The diameter of a wheel of a car is 48 cm. If the car travels at an average speed of 3.5 km/h, find the number of revolutions made by the wheel per minute, giving your answer correct to the nearest whole number. (Take π to be 3.142.)

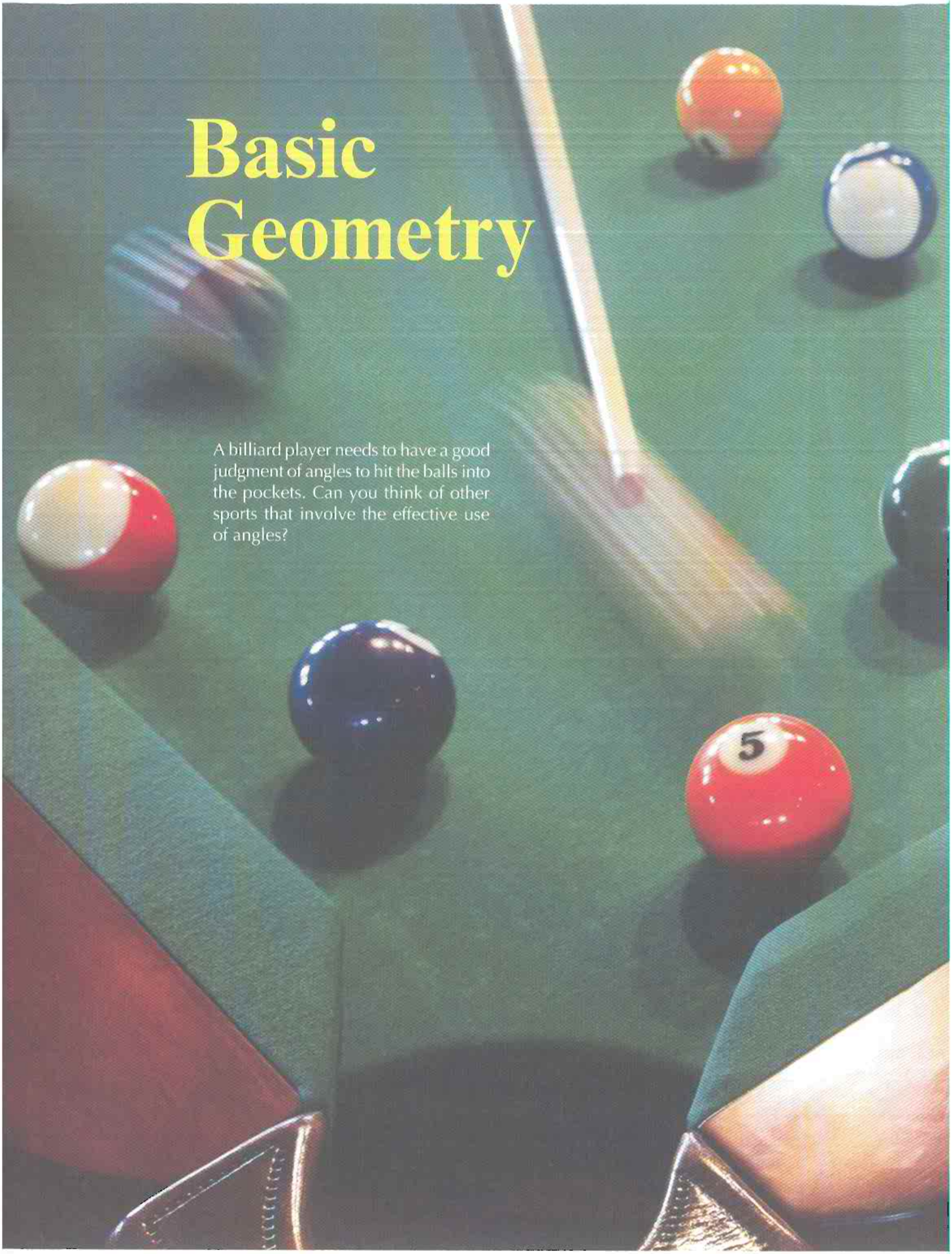
Challenge Yourself

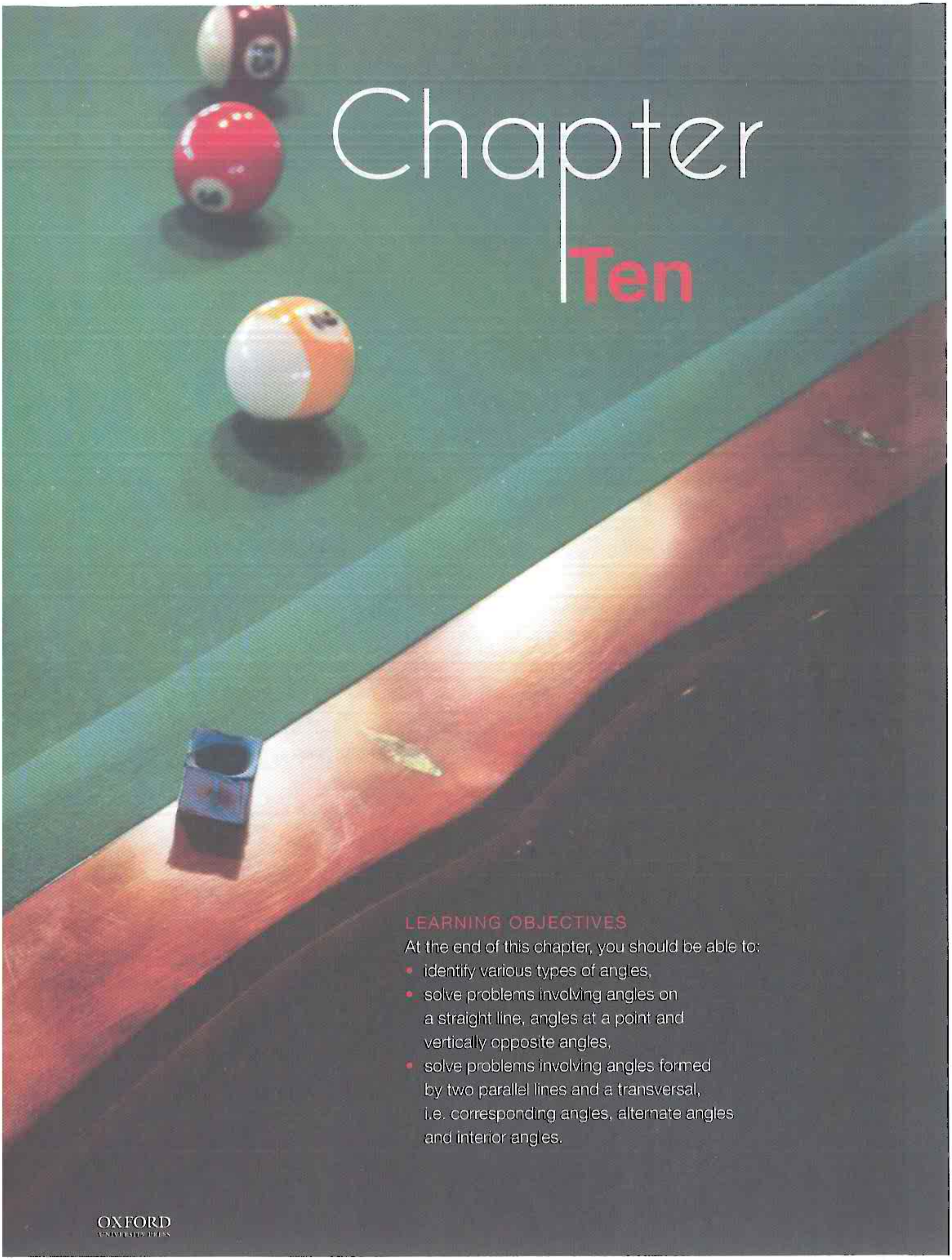
1. Kate likes to walk down moving escalators and she takes one step at a time. When she walks down at a rate of 2 steps per second, she reaches the bottom from the top after 18 steps. When she is exhausted, she will walk down at a rate of 1 step per second and she will reach the bottom from the top after 12 steps. Find the time taken for her to reach the bottom from the top if she just stands on the escalator.

2. In a 100-metre race, Vishal beats Jun Wei by 10 m. The two boys plan to have another race, where Vishal will start 10 m behind the starting point. Both boys run at the same rate as before. Will it be a draw? Or who will win the race and by how many metres?

Basic Geometry

A billiard player needs to have a good judgment of angles to hit the balls into the pockets. Can you think of other sports that involve the effective use of angles?





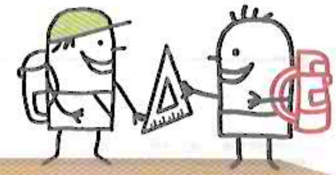
Chapter Ten

LEARNING OBJECTIVES

At the end of this chapter, you should be able to:

- identify various types of angles,
- solve problems involving angles on a straight line, angles at a point and vertically opposite angles,
- solve problems involving angles formed by two parallel lines and a transversal, i.e. corresponding angles, alternate angles and interior angles.

10.1 Points, Lines and Planes



In **geometry**, we study shapes, relative positions of figures and properties of space. We will be taking a close look at geometry in this chapter, as well as in the next few chapters.

INFORMATION

One of the most important geometry books ever written is 'The Elements' by a Greek mathematician, Euclid of Alexandria.

Points

The most basic geometric figure is a **point**. All other geometric figures are made up of a collection of points. A point has a position but it has neither size nor shape. We use a dot or a cross to mark the position of a point. We normally use capital letters to name points, for example, point A and point B, as shown in Fig. 10.1.

ATTENTION

Since a dot has a size, if we want to mark the position of a point more precisely, a cross is preferred.

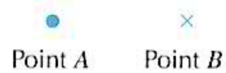


Fig. 10.1

Lines

When we join two points A and B together, a straight **line segment** AB is formed. A and B are called the **endpoints**.

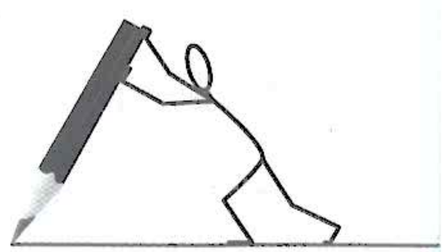
Fig. 10.2

If we extend the line segment AB in Fig. 10.2 in both directions indefinitely, we get a **line**.

Fig. 10.3

Sometimes, an arrowhead is drawn at each end of the line in Fig. 10.3 to indicate that the line continues indefinitely.

Fig. 10.4



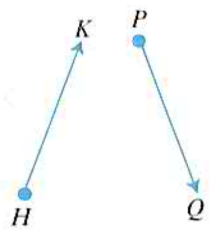


Fig. 10.5

Fig. 10.5 shows parts of lines which extend indefinitely in only one direction, i.e. they have only one endpoint. *HK* and *PQ* are known as **rays**. For each of the rays, the arrowhead indicates the direction in which the ray extends.

Intersecting Lines

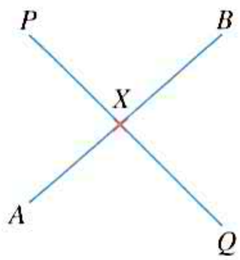


Fig. 10.6

Fig. 10.6 shows two lines *AB* and *PQ* that have a common point *X*. We say that *AB* and *PQ* *intersect* and *X* is called the **point of intersection**.

INFORMATION

Points *A*, *X* and *B* are **collinear**, i.e. they lie on the same line. However, points *P*, *X* and *B* are not collinear.

Planes

A **plane** is a flat surface in which any two points are joined by a line that lies entirely on the surface. The floor of a classroom is an example of a **horizontal plane** and the wall of a classroom is an example of a **vertical plane**.

A surface which is not flat is called a **curved surface** and it does not form part of a plane. For example, the surface of a basketball is a curved surface.

Internet Resources

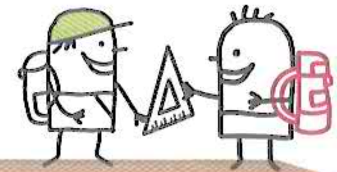
'A plane has two dimensions.' Search on the Internet to find out the meaning of 'dimension' in such a context, and the dimensions of a point and a line.

Thinking Time

Determine whether each of the following statements is true or false.
If it is false, give the correct statement.

(a) There are only two points lying on a line segment.
(b) There is only one line which can pass through any three distinct points.
(c) There are at least two different lines that pass through any two distinct points.
(d) Any two lines will intersect at one point.
(e) If two points lie on a plane, then the line containing the points lies on the same plane.

10.2 Angles



When two rays OA and OB share a common point O, an **angle** is formed. O is known as the **vertex** of the angle, and OA and OB are the **sides** (or arms) of the angle.

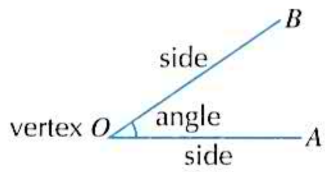


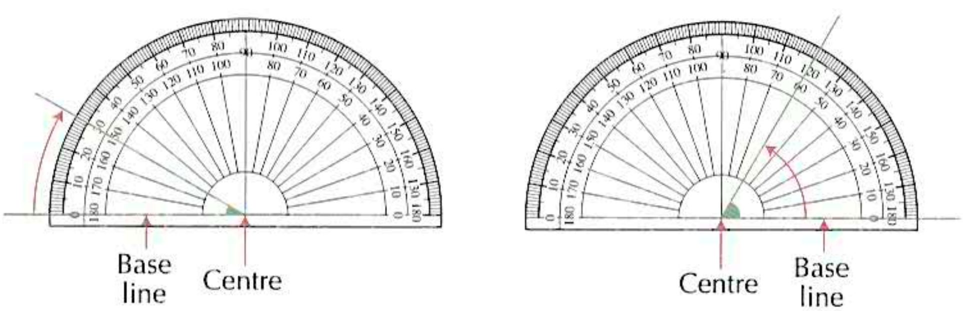
Fig. 10.7

The angle is called angle AOB or angle BOA and is written as $\angle AOB$ or $\angle BOA$. Another way of writing this angle is $A\hat{O}B$ or $B\hat{O}A$. When it is clear which angle we are referring to, we may also call it angle O and write it as $\angle O$ or $\hat{O}$.

Recap (Angle Measurement)

The standard unit for measuring angles is the **degree** (°). It is defined as $\frac{1}{360}$ of a complete revolution. Thus, one complete revolution about a point is 360°.

Angles are measured using a **protractor**.



(a) Outer Scale **(b)** Inner Scale

Fig. 10.8

Step 1: Place a protractor such that its centre is at the vertex of the angle and its base line is along one side of the angle.

Step 2a: Read off the angle from the *outer* scale in Fig. 10.8(a). The angle is _____.

Step 2b: Read off the angle from the *inner* scale in Fig. 10.8(b). The angle is _____.

A magnifying glass can enlarge an object three times its original size. How many degrees will an angle of 2° appear to a man using the magnifying glass?

SIMILAR QUESTIONS

Exercise 10A Questions 1(a)–(d)

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Types of Angles

The different types of angles are given in Table 10.1.

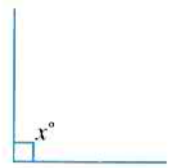
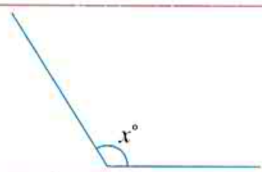
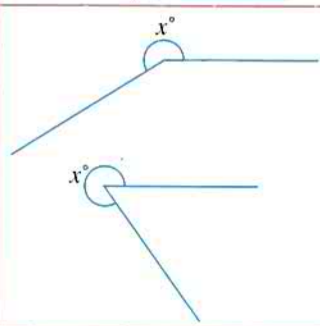
Name	Definition	Illustration	Real-life example
Acute angle	$0^\circ < x^\circ < 90^\circ$, i.e. x° is between 0° and 90°		
Right angle	$x^\circ = 90^\circ$, i.e. x° is equal to 90°		
Obtuse angle	$90^\circ < x^\circ < 180^\circ$, i.e. x° is between 90° and 180°		
Reflex angle	$180^\circ < x^\circ < 360^\circ$, i.e. x° is between 180° and 360°		

Table 10.1

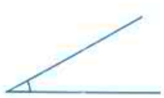
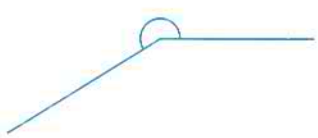
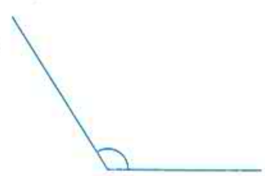
Note: The names of the angles from the smallest to the largest size (excluding right angle and straight angle) are in alphabetical order: **a**cute angle, **o**btuse angle, **r**eflex angle.

INFORMATION

A **straight angle** is equal to 180°.

PRACTISE NOW

Classify each of the following as an acute, obtuse or reflex angle.

(a)  **(b)**  **(c)** 

(d) 176° **(e)** 326° **(f)** 48°

SIMILAR QUESTIONS

Exercise 10A Questions 2(a)–(f)

Perpendicular Lines

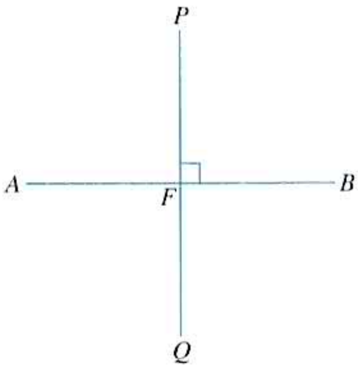
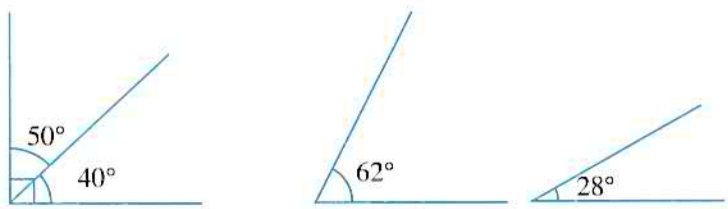


Fig. 10.9

Fig. 10.9 shows two lines AB and PQ intersecting at *right angles*. We say that AB and PQ are **perpendicular** to each other, i.e. $AB \perp PQ$. F is known as the **foot of the perpendicular** from P to AB.

Complementary Angles

Two angles are known as **complementary angles** if they add up to 90°. Fig. 10.10 shows two examples of complementary angles.



(a) 40° and 50°

(b) 62° and 28°

Fig. 10.10

Supplementary Angles

Two angles are known as **supplementary angles** if they add up to 180°. Fig. 10.11 shows two examples of supplementary angles.

(a) **(b)**

Fig. 10.11

ATTENTION

'Complementary angles' and 'supplementary angles' only apply to a pair of angles and not three or more angles.

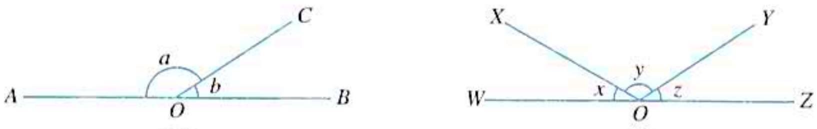
SIMILAR QUESTIONS

Exercise 10A Questions 3(a)–(d), 4(a)–(d)

Recap (Angles on a Straight Line)

Adjacent angles on a straight line are angles which

- share a common vertex,
- have a common side,
- lie on opposite sides of the common side.



(a) (b)

Fig. 10.12

In Fig. 10.12(a), AOB is a straight line. $A\hat{O}C$ and $B\hat{O}C$ are adjacent angles.

$$\angle a + \angle b = \underline{\qquad}$$

In Fig. 10.12(b), WOZ is a straight line. $W\hat{O}X$, $X\hat{O}Y$ and $Y\hat{O}Z$ are adjacent angles.

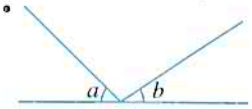
$$\angle x + \angle y + \angle z = \underline{\qquad}$$

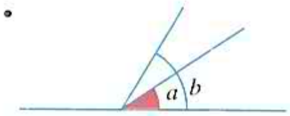
In general, we have:

The **sum of the adjacent angles on a straight line** is 180°.   (Abbreviation: adj. ∠s on a str. line)

ATTENTION

The following shows some examples of angles on a straight line, which are not adjacent.

- 

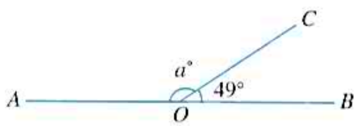
- 

Worked Example 1

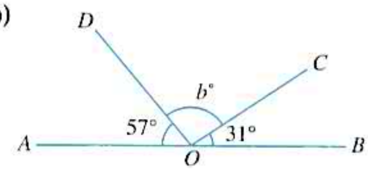
(Angles on a Straight Line)

Given that AOB is a straight line, calculate the value of the unknown in each of the following.

(a)



(b)



Solution:

(a) $a° + 49° = 180°$ (adj. ∠s on a str. line)

$a° = 180° - 49°$

$= 131°$

$\therefore a = 131$

(b) $57° + b° + 31° = 180°$ (adj. ∠s on a str. line)

$b° = 180° - 57° - 31°$

$= 92°$

$\therefore b = 92$

ATTENTION

- Indicate the angle property which you have used in your working.
- The unknown marked angles are $a°$ and $b°$, which consist of a letter and the degree symbol each, thus a and b represent values that have no units.

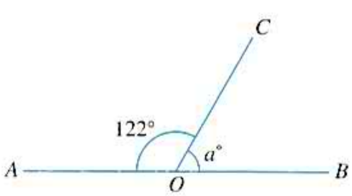
PRACTISE NOW 1

SIMILAR QUESTIONS

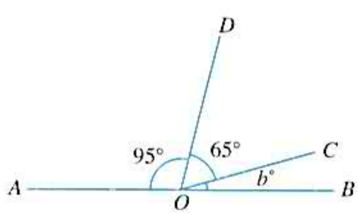
Exercise 10A Questions 5(a)–(d), 6(a)–(b), 10(a)–(b)

1. Given that AOB is a straight line, find the value of the unknown in each of the following figures.

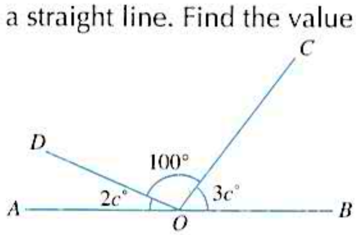
(a)



(b)



2. In the figure, AOB is a straight line. Find the value of c.



Recap (Angles at a Point)

In Fig. 10.13, *BOD* is a straight line.

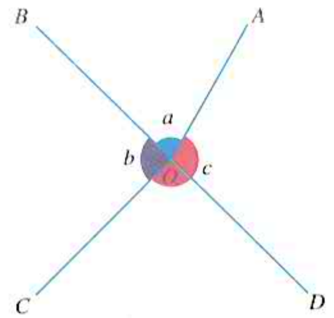


Fig. 10.13

$$\begin{aligned}\angle a + \angle b + \angle c &= \angle a + \angle b + A\hat{O}D + C\hat{O}D\\ &= \angle a + A\hat{O}D + \angle b + C\hat{O}D\\ &= 180^\circ + 180^\circ \text{ (adj. } \angle\text{s on a str. line)}\\ &= 360^\circ\end{aligned}$$

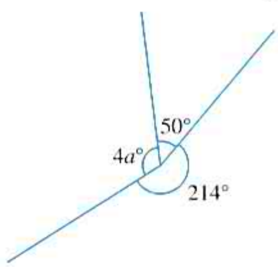
Hence, we have:

> The **sum of angles at a point** is 360°.
> (Abbreviation: ∠s at a point)

Worked Example 2

(Angles at a Point)

Calculate the value of a in the figure.



Solution:

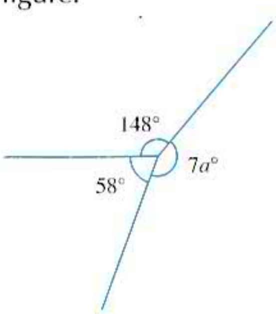
$$\begin{aligned}4a^\circ + 50^\circ + 214^\circ &= 360^\circ \text{ (}\angle\text{s at a point)}\\ 4a^\circ &= 360^\circ - 50^\circ - 214^\circ\\ &= 96^\circ\\ a^\circ &= 24^\circ\\ \therefore a &= 24\end{aligned}$$

PRACTISE NOW 2

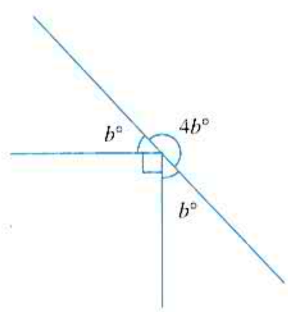
SIMILAR QUESTIONS

Exercise 10A Questions 7(a)–(d), 11

1. Find the value of a in the figure.



2. Find the value of b in the figure.



Problem Solving Tip

You may use 'angles at a point' or 'adjacent angles on a straight line' to solve Question 2.

Recap (Vertically Opposite Angles)

In Fig. 10.14, two straight lines AB and CD intersect at the point O. $A\hat{O}C$ and $B\hat{O}D$ are called **vertically opposite angles**. $B\hat{O}C$ and $A\hat{O}D$ are also vertically opposite angles.

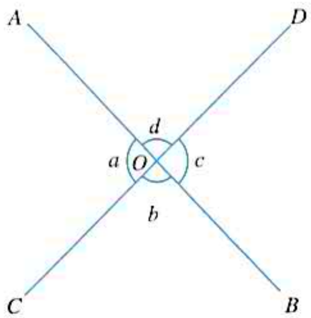


Fig. 10.14

To find the relationship between vertically opposite angles, fill in the blanks below.

From Fig. 10.14,

$\angle a + \angle b = 180^\circ$ (adj. $\angle$s on a str. line)

$\angle b + \angle c =$ ________________

$\therefore \angle a + \angle b = \angle b + \angle c$

$\therefore \angle a =$ __________

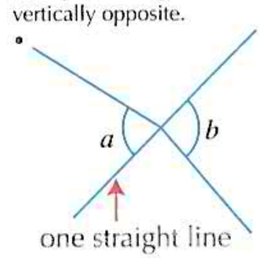
Similarly, $\angle b =$ __________.

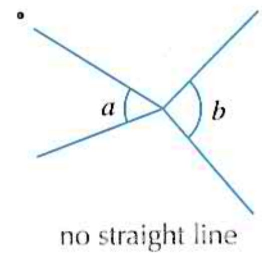
Hence, we have:

Vertically opposite angles are equal.
(Abbreviation: vert. opp. $\angle$s)

ATTENTION

The following shows some examples of angles that are not vertically opposite.

- 

- 

Worked Example 3

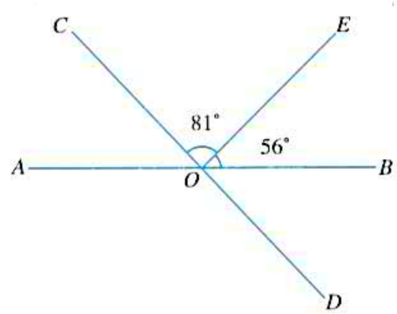
(Vertically Opposite Angles)

In the figure, AOB and COD are straight lines.

If $B\hat{O}E = 56^\circ$ and $C\hat{O}E = 81^\circ$, calculate

(i) $A\hat{O}D$,

(ii) $A\hat{O}C$.



Solution:

(i) $A\hat{O}D = 56^\circ + 81^\circ$ (vert. opp. $\angle$s)

$= 137^\circ$

(ii) $A\hat{O}C = 180^\circ - 137^\circ$ (adj. $\angle$s on a str. line)

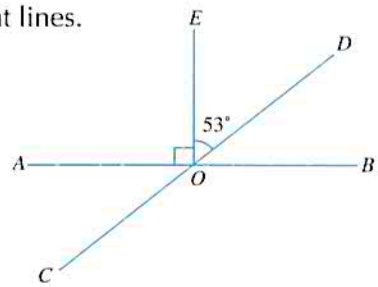
$= 43^\circ$

PRACTISE NOW 3

SIMILAR QUESTIONS

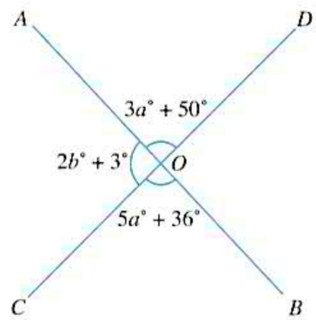
Exercise 10A Question 8

In the figure, AOB and COD are straight lines.
If $A\hat{O}E = 90^\circ$ and $D\hat{O}E = 53^\circ$, find
(i) $A\hat{O}C$,
(ii) $B\hat{O}D$.



Worked Example 4

(Vertically Opposite Angles)
In the figure, AOB and COD are straight lines.
Calculate the value of a and of b.



Solution:

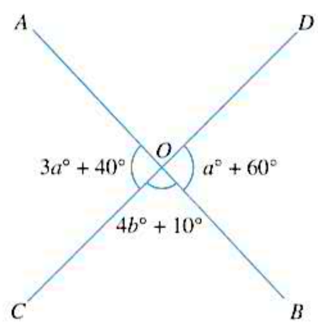
$3a^\circ + 50^\circ = 5a^\circ + 36^\circ$ (vert. opp. ∠s)
$5a^\circ - 3a^\circ = 50^\circ - 36^\circ$
$2a^\circ = 14^\circ$
$a^\circ = 7^\circ$
$\therefore a = 7$
$5a^\circ + 36^\circ + 2b^\circ + 3^\circ = 180^\circ$ (adj. ∠s on a str. line)
$5(7^\circ) + 36^\circ + 2b^\circ + 3^\circ = 180^\circ$ (substitute in $a = 7$)
$2b^\circ = 180^\circ - 35^\circ - 36^\circ - 3^\circ$
$= 106^\circ$
$b^\circ = 53^\circ$
$\therefore b = 53$

Problem Solving Tip

You can also find the value of b by considering the line COD.

PRACTISE NOW 4

In the figure, AOB and COD are straight lines. Find the value of a and of b.



SIMILAR QUESTIONS

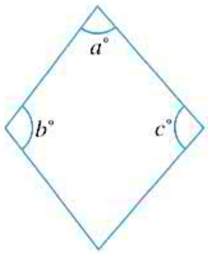
Exercise 10A Questions 9(a)–(b), 12(a)–(d), 13

Exercise 10A

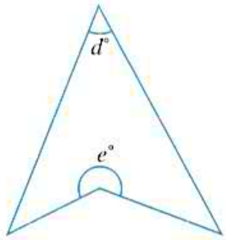
BASIC LEVEL

1. Using a protractor, measure and write down the value of each of the following unknowns.

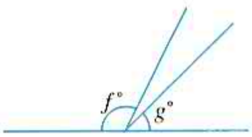
(a)



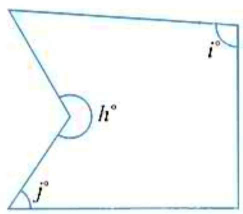
(b)



(c)

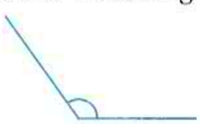


(d)

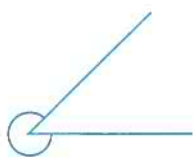


2. Classify each of the following as an acute, obtuse or reflex angle.

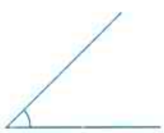
(a)



(b)



(c)



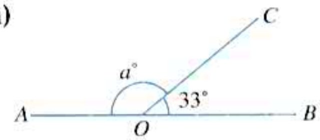
(d) 181°
(e) 89°
(f) 124°

3. Find the complementary angle of each of the following angles.
(a) 18° (b) 46° (c) 53° (d) 64°

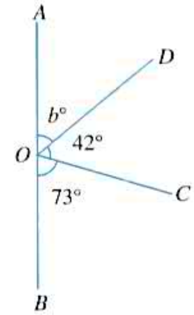
4. Find the supplementary angle of each of the following angles.
(a) 36° (b) 12° (c) 102° (d) 171°

5. Given that AOB is a straight line, find the value of the unknown in each of the following figures.

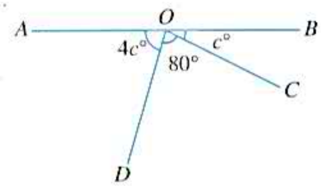
(a)



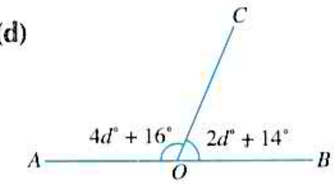
(b)



(c)



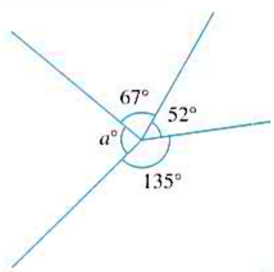
(d)



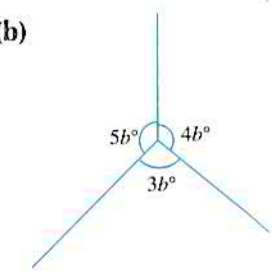
6. $x°$, $y°$ and $z°$ are three angles lying on a straight line.
(a) If $y° = 45°$ and $z° = 86°$, find the value of x.
(b) If $x° = 2y°$ and $z° = 3y°$, find the value of y.

7. For each of the following figures, find the value of the unknown.

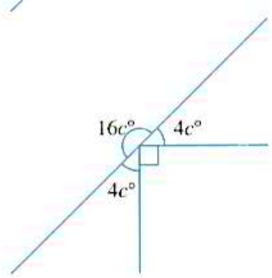
(a)



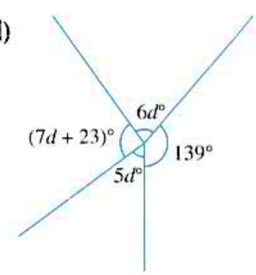
(b)



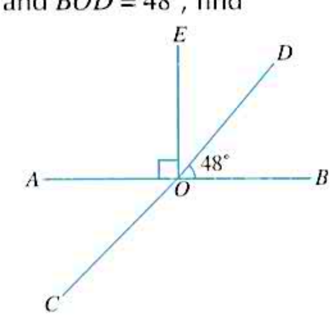
(c)



(d)

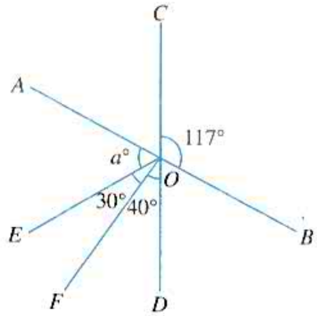


8. In the figure, AOB and COD are straight lines. If $A\hat{O}E = 90°$ and $B\hat{O}D = 48°$, find
(i) $A\hat{O}C$,
(ii) $D\hat{O}E$.

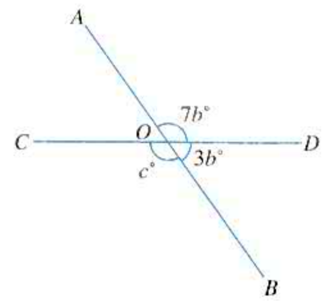


9. Given that AOB and COD are straight lines, find the value(s) of the unknown(s) in each of the following figures.

(a)



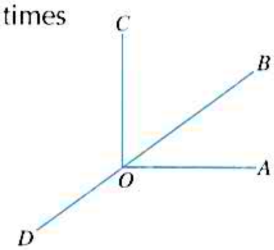
(b)



INTERMEDIATE LEVEL

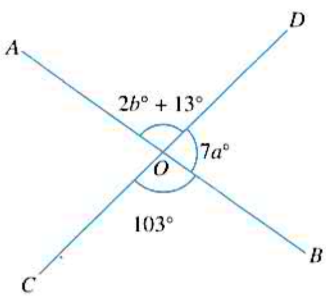
10. $x°$, $y°$ and $z°$ are three angles lying on a straight line.
   (a) If $y° = x° + z°$, find the value of y.
   (b) If $x° = y° = z°$, find the value of z.

11. In the figure, DOB is a straight line. If $B\hat{O}C$ is twice of $A\hat{O}B$, $C\hat{O}D$ is four times of $A\hat{O}B$ and $D\hat{O}A$ is five times of $A\hat{O}B$, find all the four angles.

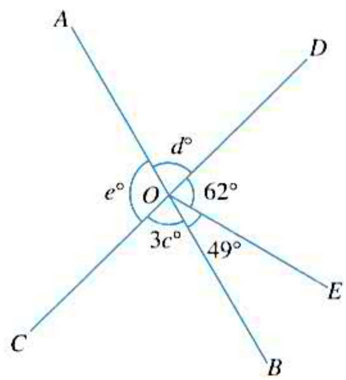


12. Given that AOB and COD are straight lines, find the values of the unknowns in each of the following figures.

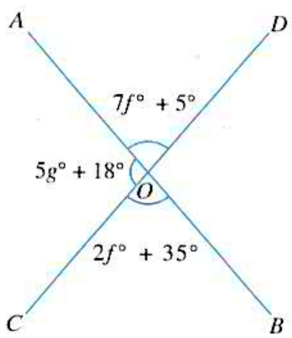
(a)



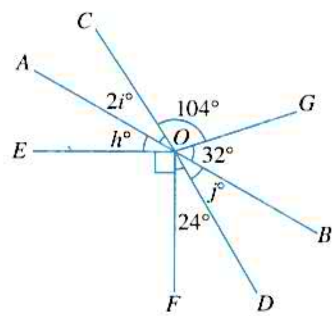
(b)



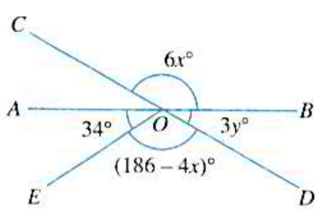
(c)



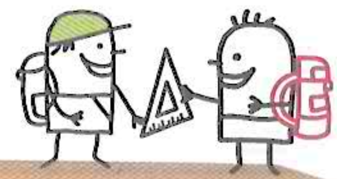
(d)



13. In the figure, AOB and COD are straight lines.
   (i) Find the value of x and of y.
   (ii) Find obtuse $A\hat{O}D$ and reflex $C\hat{O}E$.



10.3 Angles Formed by Two Parallel Lines and a Transversal



Parallel Lines

When two lines lying on the same plane do not intersect, they are known as **parallel lines**. Fig. 10.15 shows a pair of railway tracks which are parallel.

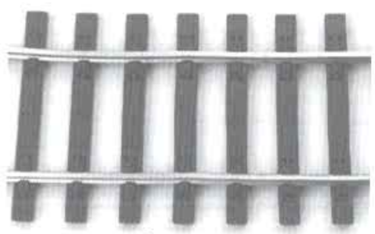


Fig. 10.15

Parallel lines are represented by either single or double arrowheads pointing in the same direction as shown in Fig. 10.16.

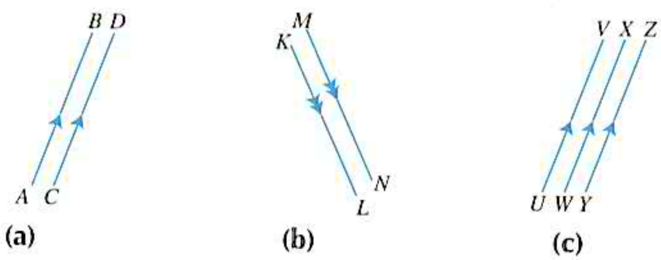


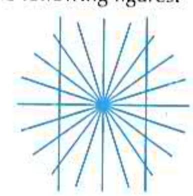
Fig. 10.16

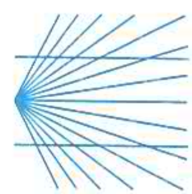
We use the symbol '//' to denote 'is parallel to', i.e. in Fig. 10.16(a), $AB \parallel CD$ means that AB is parallel to CD.

INFORMATION

Parallel lines can be constructed by using a ruler and a set square.

Identify the parallel lines in each of the following figures.





Corresponding Angles, Alternate Angles and Interior Angles

Fig. 10.17 shows a line PQ which cuts two parallel lines AB and CD. PQ is known as a **transversal**.

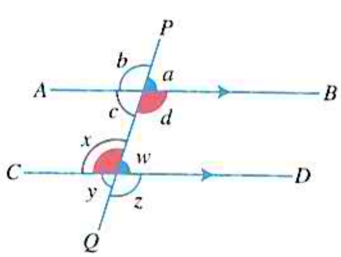


Fig. 10.17

- $\angle a$ and $\angle w$ are called **corresponding angles**. Name another pair of corresponding angles.
- $\angle d$ and $\angle x$ are called **alternate angles**. Name another pair of alternate angles.
- $\angle c$ and $\angle x$ are called **interior angles**. Name another pair of interior angles.

INFORMATION

In general, a line that cuts any two lines is called a transversal.

Investigation

Corresponding Angles, Alternate Angles and Interior Angles

In this investigation, we shall explore three angle properties which exist when a pair of parallel lines is cut by a transversal.

Go to http://www.shinglee.com.sg/StudentResources/ and open the geometry software template 'Parallel Lines'.

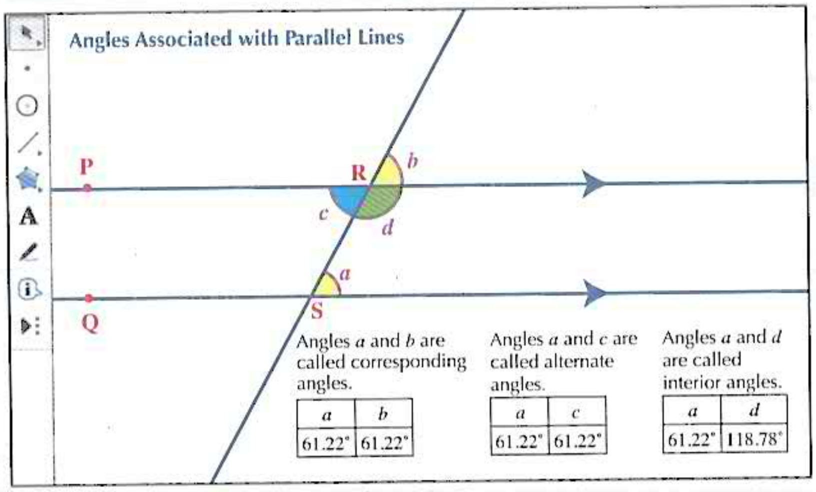


Fig. 10.18

ATTENTION

To change the sizes of the angles, click and drag the points P, Q, R and S to move the parallel lines and the transversal.

1. $\angle a$ and $\angle b$ are corresponding angles. Change the sizes of $\angle a$ and $\angle b$. What is the relationship between $\angle a$ and $\angle b$?
2. $\angle a$ and $\angle c$ are alternate angles. Change the sizes of $\angle a$ and $\angle c$. What is the relationship between $\angle a$ and $\angle c$?
3. $\angle a$ and $\angle d$ are interior angles. Change the sizes of $\angle a$ and $\angle d$. What is the relationship between $\angle a$ and $\angle d$?
   Hint: Consider their sum.

Summarise the 3 angle properties that you have learnt from the above:

(a) $\angle a = \angle$_____ (corr. $\angle$s) **(b)** $\angle a = \angle$_____ (alt. $\angle$s) **(c)** $\angle a + \angle d =$ _____ (int. $\angle$s)

Go to page 2 by clicking on the tab at the bottom left corner.

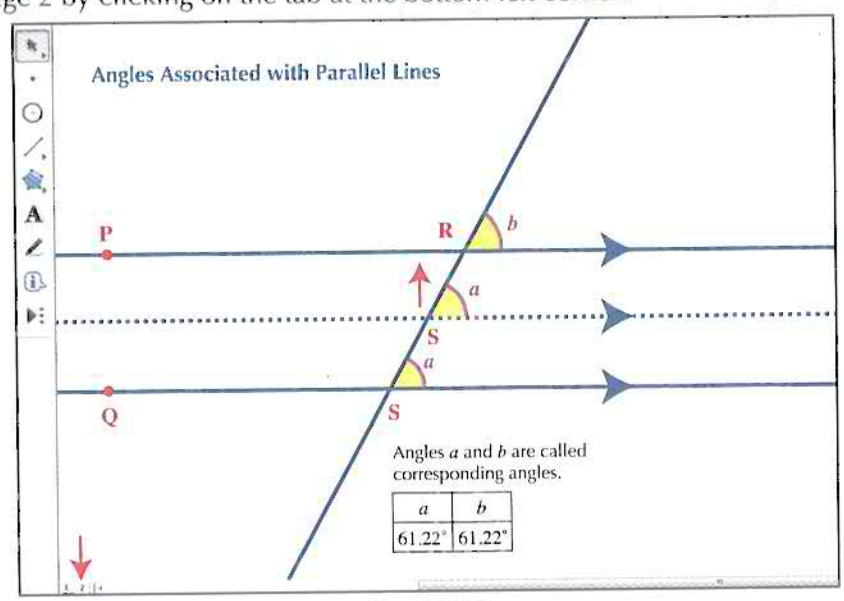


Fig. 10.19

4. $\angle a$ and $\angle b$ are corresponding angles. Click and drag the point S upwards towards the point R. This will shift $\angle a$ towards $\angle b$. As long as the dotted line remains parallel to the two parallel lines, the size of $\angle a$ will remain the same. Do you understand why $\angle b$ is equal to $\angle a$ now?

Go back to page 1 of the template.

5. $\angle a$ and $\angle c$ are alternate angles. How do you prove that they are equal?
   Hint: Use corresponding angles and vertically opposite angles.
6. $\angle a$ and $\angle d$ are interior angles. Use two methods to prove that the sum of their angles is 180°.
   Hint: Use corresponding angles (or alternate angles) and adjacent angles on a straight line.

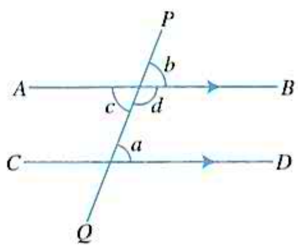


Fig. 10.20

From the investigation, we observe that when two parallel lines, AB and CD, are cut by a transversal PQ, then

- corresponding angles are equal, e.g. $\angle a = \angle b$ (corr. $\angle$s, $AB \parallel CD$);
- alternate angles are equal, e.g. $\angle a = \angle c$ (alt. $\angle$s, $AB \parallel CD$);
- interior angles are supplementary, e.g. $\angle a + \angle d = 180°$ (int. $\angle$s, $AB \parallel CD$).

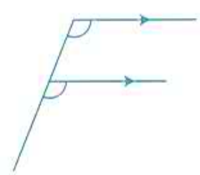
The converse for each of the above is also true, i.e. when two lines AB and CD are cut by a transversal PQ, and

- if $\angle a = \angle b$, then $AB \parallel CD$;
- if $\angle a = \angle c$, then $AB \parallel CD$;
- if $\angle a + \angle d = 180°$, then $AB \parallel CD$.

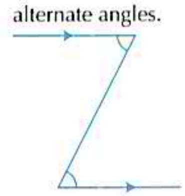
INFORMATION

You may identify the 3 angle properties by looking at the letters F, Z and C.

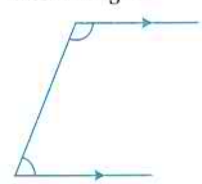
- The letter F shows corresponding angles.



- The letter Z shows alternate angles.



- The letter C shows interior angles.

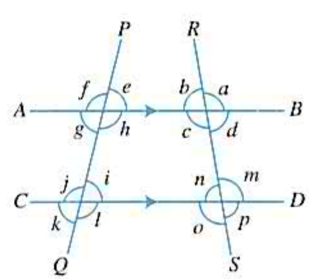


PRACTISE NOW

In the figure, $AB \parallel CD$.

(a) List down
   (i) one pair of equal corresponding angles,
   (ii) one pair of equal alternate angles,
   (iii) one pair of interior angles which are supplementary.

(b) Is $\angle c = \angle g$? Explain your answer.



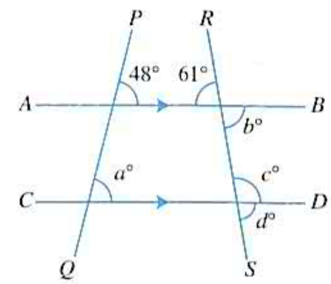
SIMILAR QUESTIONS

Exercise 10B Question 1

Worked Example 5

(Corresponding Angles and Interior Angles)

In the figure, $AB \parallel CD$. Calculate the values of a, b, c and d.



Solution:

$a° = 48°$ (corr. $\angle$s, $AB \parallel CD$)

$\therefore a = 48$

$b° = 61°$ (vert. opp. $\angle$s)

$\therefore b = 61$

$c° + b° = 180°$ (int. $\angle$s, $AB \parallel CD$)

$c° + 61° = 180°$

$c° = 180° - 61°$

$= 119°$

$\therefore c = 119$

$d° = b°$

$= 61°$ (corr. $\angle$s, $AB \parallel CD$)

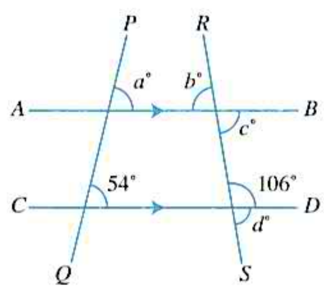
$\therefore d = 61$

PRACTISE NOW 5

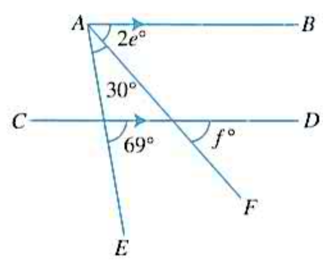
SIMILAR QUESTIONS

Exercise 10B Questions 2(a), 3(a)–(c), 4(a), 5(a)–(b)

1. In the figure, $AB \parallel CD$. Find the values of a, b, c and d.



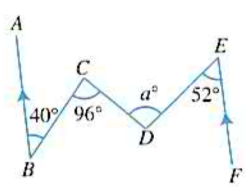
2. In the figure, $AB \parallel CD$. Find the value of e and of f.



Worked Example 6

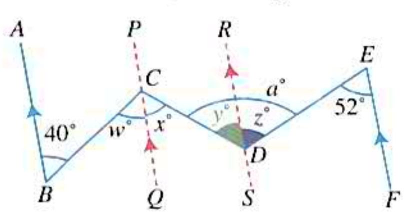
(Alternate Angles)

In the figure, $BA \parallel FE$. Calculate the value of a.



Solution:

Draw a line QP through C, and a line SR through D, that are parallel to BA and FE.



$w° = 40°$ (alt. $\angle$s, $BA \parallel QP$)

$x° = 96° - 40°$

$= 56°$

$y° = x° = 56°$ (alt. $\angle$s, $QP \parallel SR$)

$z° = 52°$ (alt. $\angle$s, $SR \parallel FE$)

$a° = y° + z°$

$= 56° + 52°$

$= 108°$

$\therefore a = 108$

Problem Solving Tip

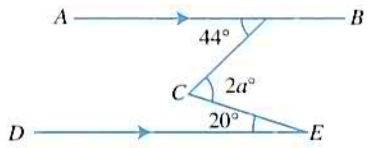
Draw additional lines if necessary to make use of the properties of the angles formed by parallel lines and transversals to solve problems.

PRACTISE NOW 6

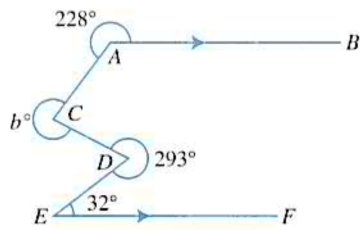
SIMILAR QUESTIONS

Exercise 10B Questions 2(b), 3(d), 4(b), 5(c)

1. In the figure, $AB \parallel DE$. Find the value of a.



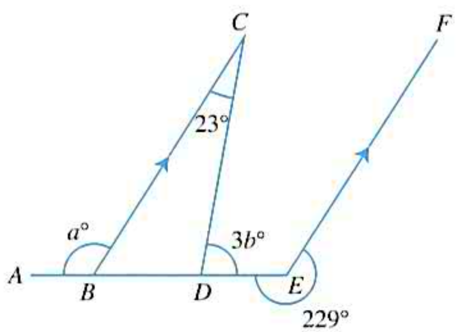
2. In the figure, $AB \parallel EF$. Find the value of b.



Worked Example 7

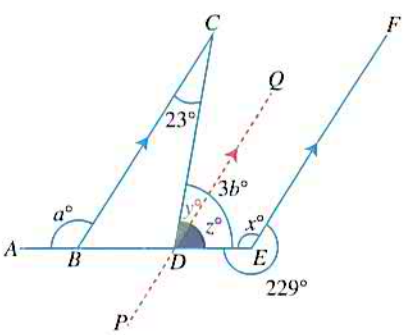
(Corresponding Angles, Alternate Angles and Interior Angles)

In the figure, $ABDE$ is a straight line and $BC \parallel EF$. Calculate the value of a and of b.



Solution:

Draw a line PQ through D that is parallel to BC and EF.



$x° = 360° - 229°$ (∠s at a point)

$= 131°$

$a° = 131°$ (corr. ∠s, $BC \parallel EF$)

$\therefore a = 131$

$y° = 23°$ (alt. ∠s, $BC \parallel PQ$)

$z° = 180° - 131°$ (int. ∠s, $PQ \parallel EF$)

$= 49°$

$3b° = y° + z°$

$= 23° + 49°$

$= 72°$

$b° = \dfrac{72°}{3}$

$= 24°$

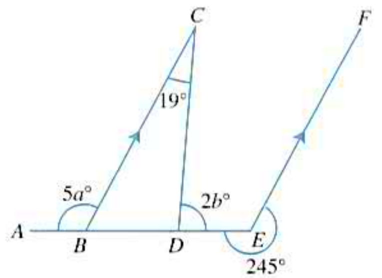
$\therefore b = 24$

PRACTISE NOW 7

SIMILAR QUESTIONS

Exercise 10B Questions 2(c)–(d), 6–10, 12

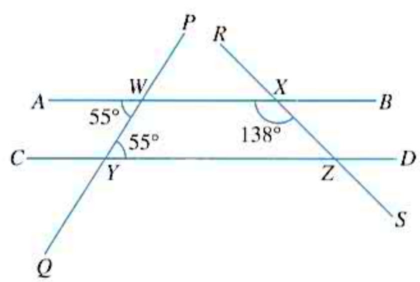
In the figure, $ABDE$ is a straight line and $BC \parallel EF$. Find the value of a and of b.



Worked Example 8

(Application of Converse Statements)

In the figure, the lines AB and CD are cut by the transversals PQ and RS. If $A\hat{W}Q = D\hat{Y}P = 55°$ and $A\hat{X}S = 138°$, calculate $C\hat{Z}S$.



Solution:

Since $A\hat{W}Q = D\hat{Y}P$ (= 55°), then $AB \parallel CD$ (converse of alt. ∠s).

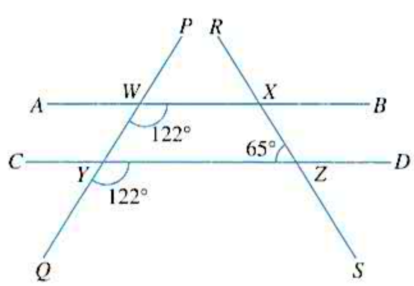
∴ $C\hat{Z}S = A\hat{X}S = 138°$ (corr. ∠s, $AB \parallel CD$)

PRACTISE NOW 8

SIMILAR QUESTIONS

Exercise 10B Question 11

In the figure, the lines AB and CD are cut by the transversals PQ and RS. If $B\hat{W}Q = D\hat{Y}Q = 122°$ and $C\hat{Z}R = 65°$, find $B\hat{X}S$.



Exercise 10B

BASIC LEVEL

1. In the figure, $AB \parallel CD$.

(a) List down

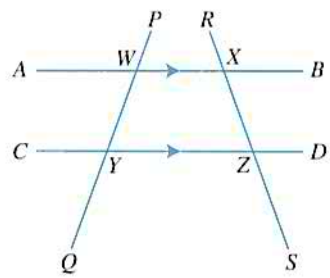
(i) two pairs of equal corresponding angles,

(ii) two pairs of equal alternate angles,

(iii) two pairs of interior angles which are supplementary.

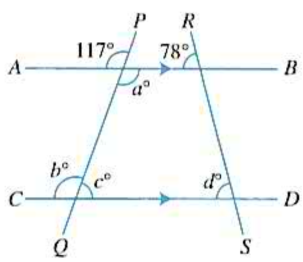
(b) Is $B\hat{W}Q = A\hat{X}R$? Explain your answer.

(c) Is the sum of $D\hat{Y}P$ and $C\hat{Z}R$ equal to 180°? Explain your answer.

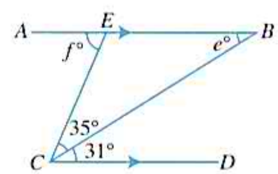


2. Given that $AB \parallel CD$, find the values of the unknowns in each of the following figures.

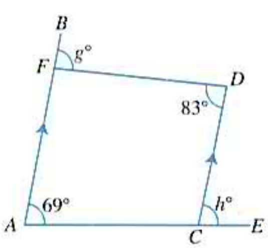
(a)



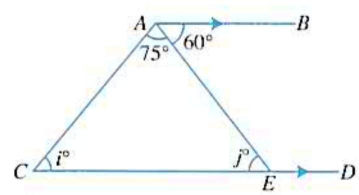
(b)



(c)

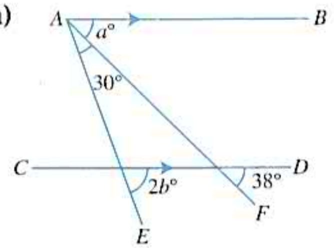


(d)

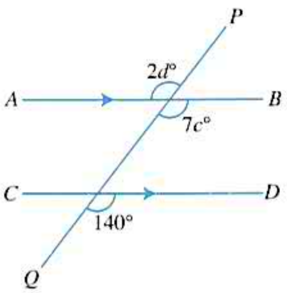


3. In each of the following figures, $AB \parallel CD$. Find the value(s) of the unknown(s).

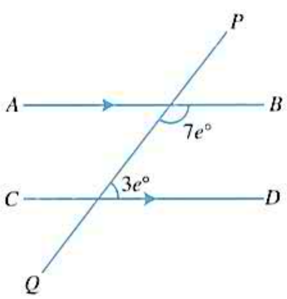
(a)



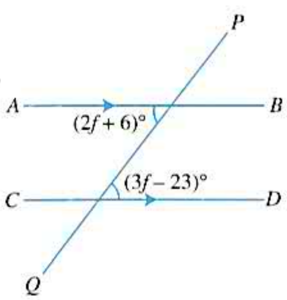
(b)



(c)



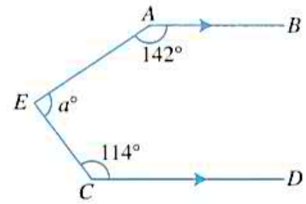
(d)



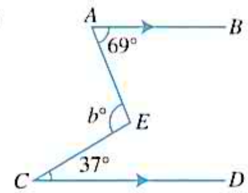
INTERMEDIATE LEVEL

4. Given that $AB \parallel CD$, find the value of the unknown in each of the following figures.

(a)

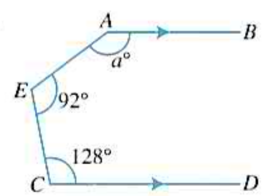


(b)

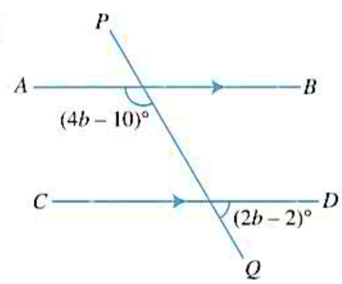


5. In each of the following figures, $AB \parallel CD$. Find the value of the unknown.

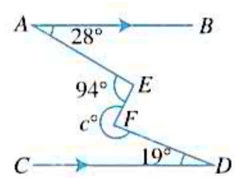
(a)



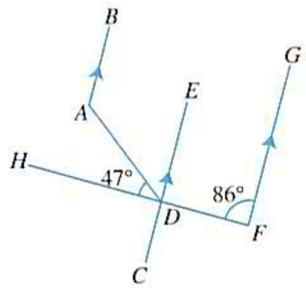
(b)



(c)

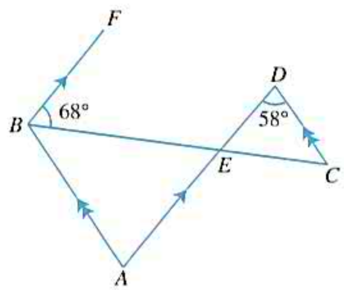


6. In the figure, HDF is a straight line, $AB \parallel CE \parallel FG$, $A\hat{D}H = 47°$ and $D\hat{F}G = 86°$.



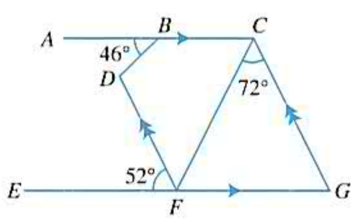
Find
(i) $C\hat{D}F$,
(ii) $B\hat{A}D$.

7. In the figure, $AB \parallel CD$, $BF \parallel AD$, $E\hat{B}F = 68°$ and $C\hat{D}E = 58°$.



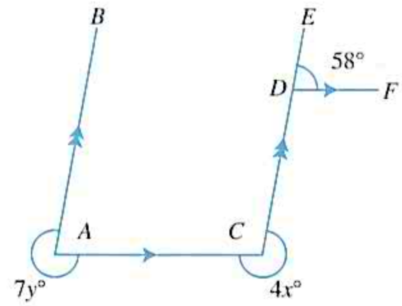
Find
(i) $A\hat{E}B$,
(ii) $A\hat{B}E$.

8. In the figure, $AC \parallel EG$, $FD \parallel GC$, $A\hat{B}D = 46°$, $D\hat{F}E = 52°$ and $F\hat{C}G = 72°$.

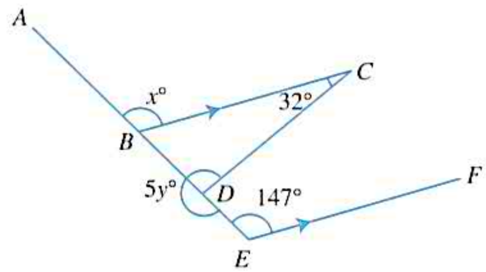


Find
(i) $C\hat{G}F$,
(ii) $B\hat{C}F$,
(iii) reflex $B\hat{D}F$.

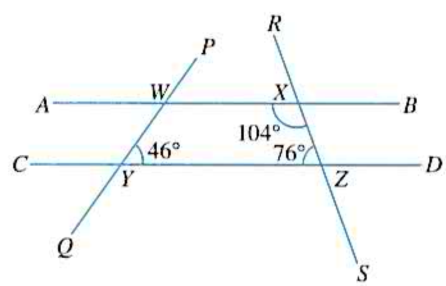
9. In the figure, $AB \parallel CE$ and $DF \parallel AC$. Find the value of x and of y.



10. In the figure, $ABDE$ is a straight line and $BC \parallel EF$. Find the value of x and of y.

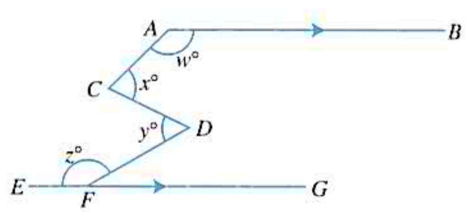


11. In the figure, the lines AB and CD are cut by the transversals PQ and RS. If $D\hat{Y}P = 46°$, $A\hat{X}S = 104°$ and $C\hat{Z}R = 76°$, find $B\hat{W}P$.

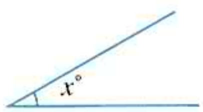
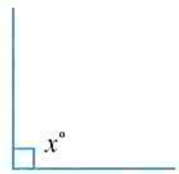
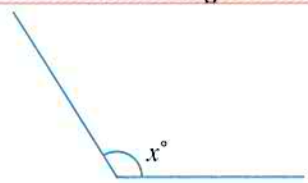


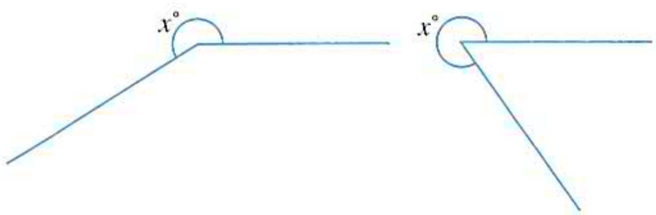
ADVANCED LEVEL

12. In the figure, $AB \parallel EG$, $B\hat{A}C = w°$, $A\hat{C}D = x°$, $C\hat{D}F = y°$ and $D\hat{F}E = z°$. Form an equation connecting w, x, y and z.



1.

Acute angle	Right angle	Obtuse angle
		
$0° < x° < 90°$	$x° = 90°$	$90° < x° < 180°$

Reflex angle

$180° < x° < 360°$

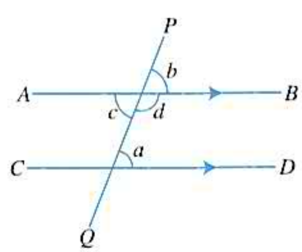
2. Two angles, $\angle a$ and $\angle b$, are known as **complementary angles** if their sum is 90°, i.e. $\angle a + \angle b = 90°$.
   Two angles, $\angle a$ and $\angle b$, are known as **supplementary angles** if their sum is 180°, i.e. $\angle a + \angle b = 180°$.

3. The **sum of the adjacent angles on a straight line** is 180° (adj. $\angle$s on a str. line).

4. The **sum of angles at a point** is 360° ($\angle$s at a point).

5. **Vertically opposite angles** are equal (vert. opp. $\angle$s).

6. When two parallel lines, AB and CD, are cut by a transversal PQ, then

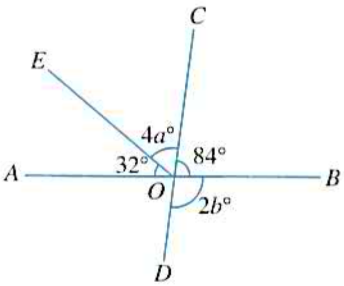
   - **corresponding angles** are equal, e.g. $\angle a = \angle b$ (corr. $\angle$s, AB // CD);
   - **alternate angles** are equal, e.g. $\angle a = \angle c$ (alt. $\angle$s, AB // CD);
   - **interior angles** are supplementary, e.g. $\angle a + \angle d = 180°$ (int. $\angle$s, AB // CD).

   The converse for each of the above is also true.

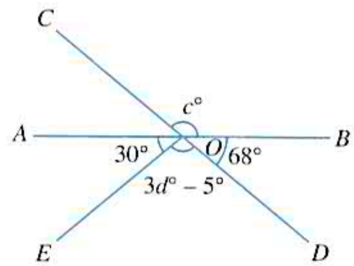
Review Exercise 10

1. Given that AOB and COD are straight lines, find the values of the unknowns in each of the following figures.

(a)

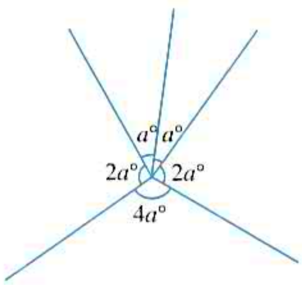


(b)

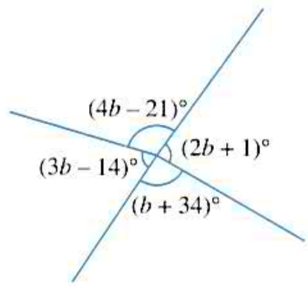


2. For each of the following figures, find the value of the unknown.

(a)

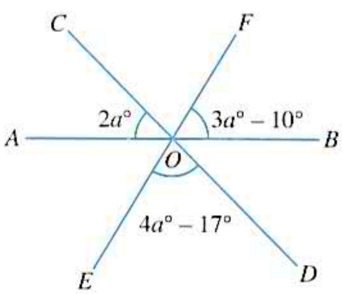


(b)

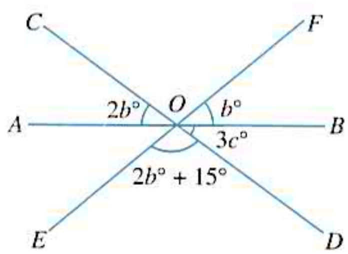


3. Given that AOB, COD and EOF are straight lines, find the value(s) of the unknown(s) in each of the following figures.

(a)

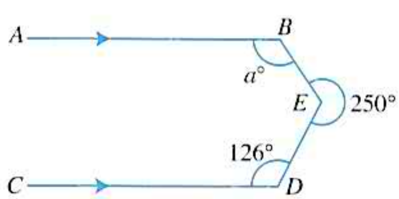


(b)

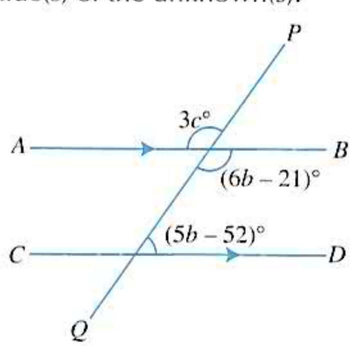


4. In each of the following figures, AB // CD. Find the value(s) of the unknown(s).

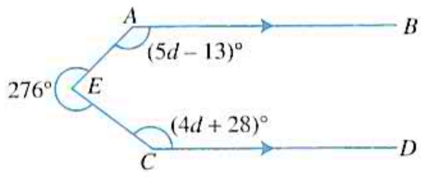
(a)



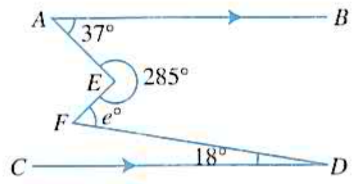
(b)



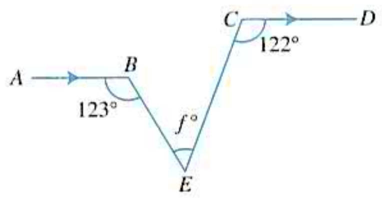
(c)



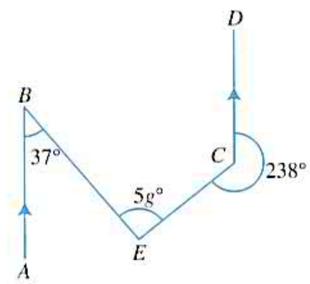
(d)



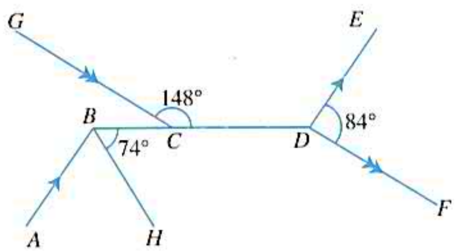
(e)



(f)

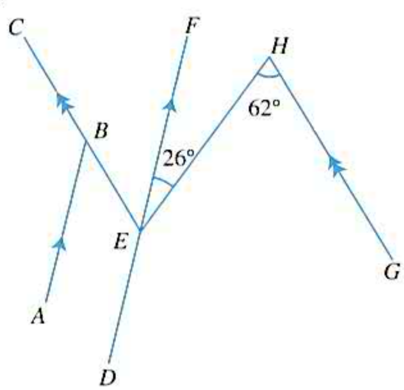


5. In the figure, $AB \parallel DE$, $GC \parallel DF$, $C\hat{B}H = 74°$, $D\hat{C}G = 148°$ and $E\hat{D}F = 84°$.



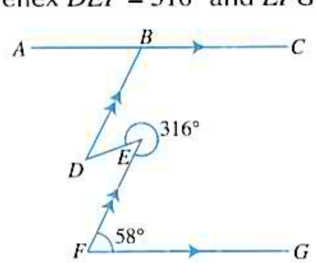
Find
(i) $C\hat{D}E$,
(ii) $A\hat{B}H$.

6. In the figure, $AB \parallel DF$, $EC \parallel GH$, $F\hat{E}H = 26°$ and $E\hat{H}G = 62°$.

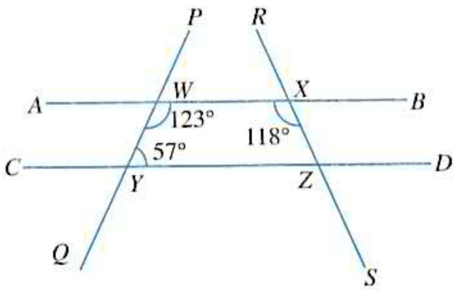


Find
(i) $D\hat{E}H$,
(ii) $A\hat{B}C$.

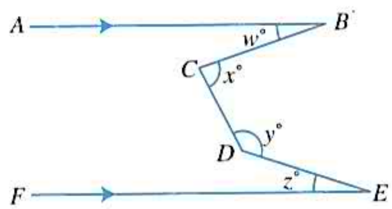
7. In the figure, $AC \parallel FG$, $DB \parallel FE$, reflex $D\hat{E}F = 316°$ and $E\hat{F}G = 58°$.
Find
(i) $B\hat{D}E$,
(ii) $A\hat{B}D$.



8. In the figure, the lines AB and CD are cut by the transversals PQ and RS. If $B\hat{W}Q = 123^\circ$, $D\hat{Y}P = 57^\circ$ and $A\hat{X}S = 118^\circ$, find $D\hat{Z}R$.

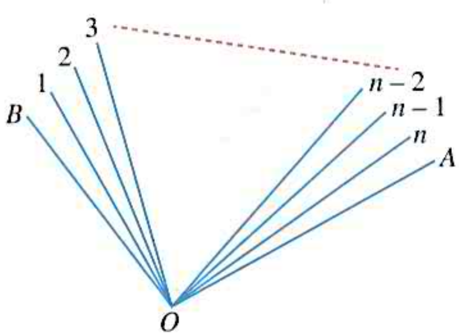


9. In the figure, $AB \parallel FE$, $A\hat{B}C = w^\circ$, $B\hat{C}D = x^\circ$, $C\hat{D}E = y^\circ$ and $D\hat{E}F = z^\circ$. Form an equation connecting w, x, y and z.



Challenge Yourself

1. In the figure, two rays OA and OB share a common vertex O.



Find an expression, in terms of n, for the number of different angles in the figure if there are n rays between OA and OB.

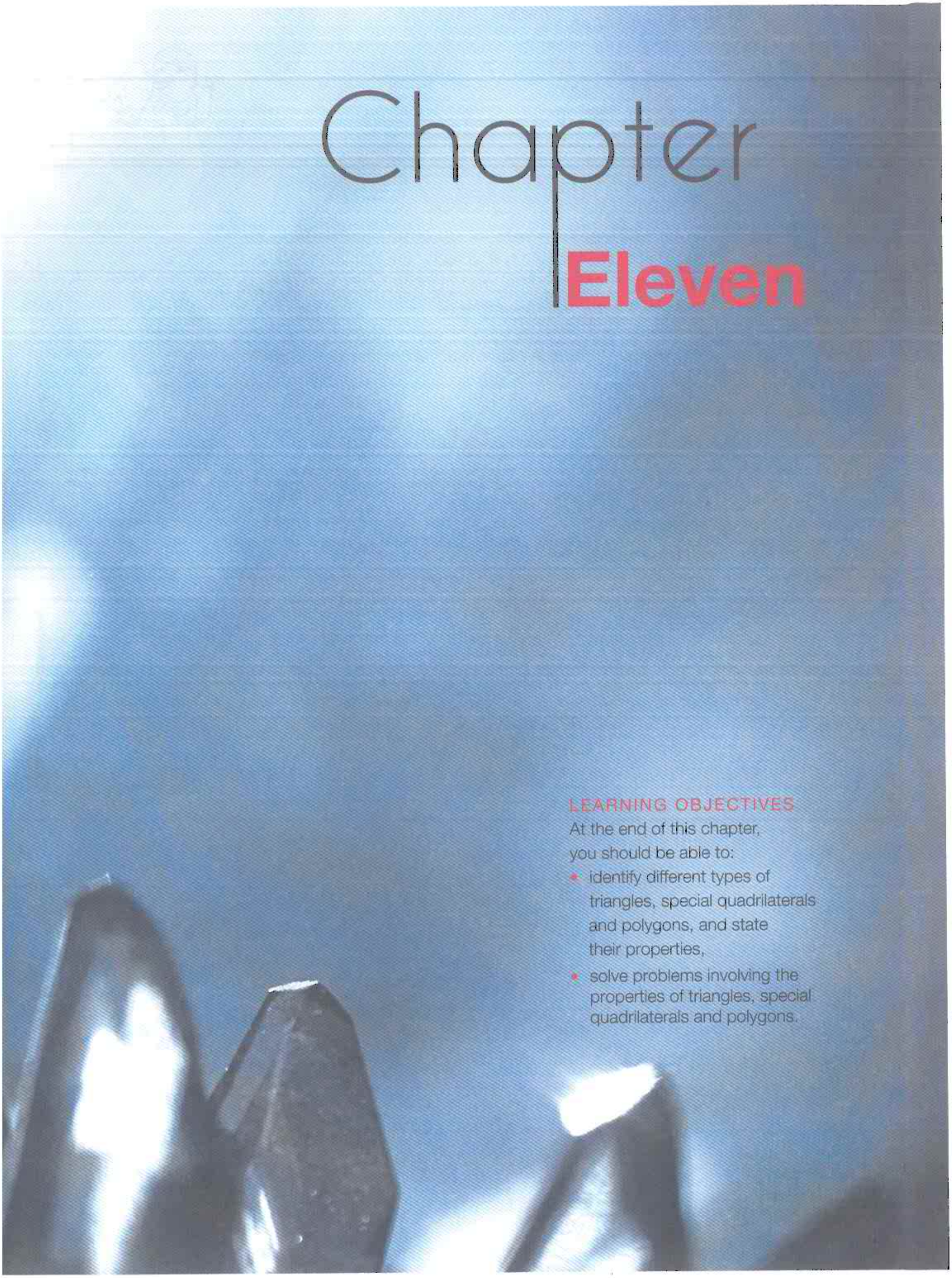
2. Find the larger angle between the minute hand and the hour hand of a 12-hour clock at 7.20 p.m.

3. When the minute hand of a 12-hour clock is at a right angle to the hour hand of the clock, a bell will sound once. Find the number of times the bell will sound from 9 a.m. on a particular day to 9 p.m. the next day.

Triangles, Quadrilaterals and Polygons

Quasicrystals are structures which can be found in many metallic alloys. Polygonal quasicrystals can have 8, 10 or 12 planes of symmetry. They are known as octagonal, decagonal and dodecagonal quasicrystals respectively.

Chapter Eleven

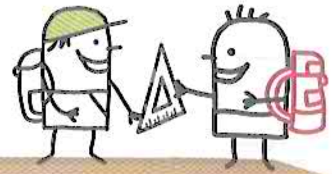


LEARNING OBJECTIVES

At the end of this chapter, you should be able to:

- identify different types of triangles, special quadrilaterals and polygons, and state their properties,
- solve problems involving the properties of triangles, special quadrilaterals and polygons.

11.1 Triangles



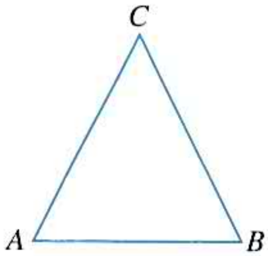


Fig. 11.1

Fig. 11.1 shows a **triangle** ABC ($\triangle ABC$) that has three sides AB, BC and AC. The points A, B and C are called the **vertices** (singular: vertex) of the triangle. $B\hat{A}C$, $A\hat{B}C$ and $A\hat{C}B$ are known as the **interior angles** of $\triangle ABC$.

In primary school, we have learnt about triangles and some of their properties. In this section, we shall learn more about triangles.

Classification of Triangles

Triangles can be classified according to

- the number of equal sides they have,

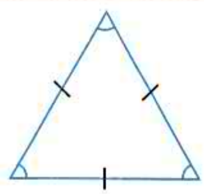
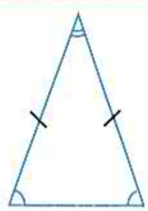
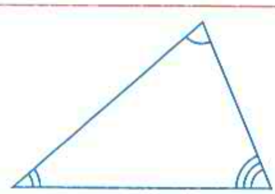
Name	Definition	Figure	Properties
Equilateral triangle	A triangle with 3 equal sides		All the angles in an equilateral triangle are equal, i.e. 60°. (Abbreviation: $\angle$s of equilateral $\triangle$)
Isosceles triangle	A triangle with at least 2 equal sides		The base angles of an isosceles triangle are equal. (Abbreviation: base $\angle$s of isos. $\triangle$)
Scalene triangle	A triangle with no equal sides		All the angles in a scalene triangle are different.

Table 11.1

INFORMATION

Euclid first defined an isosceles triangle to have exactly 2 equal sides. However, nowadays, an isosceles triangle is defined to have at least 2 equal sides. Hence, an equilateral triangle is a special type of an isosceles triangle.

• the types of angles they have.

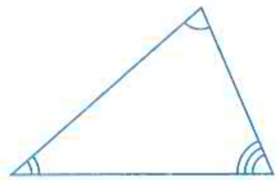
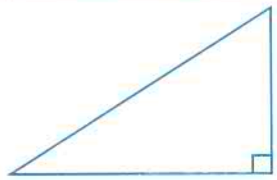
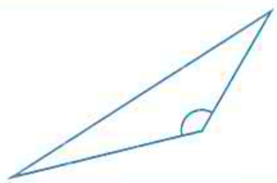
Name	Definition	Figure
Acute-angled triangle	A triangle with 3 acute angles	
Right-angled triangle	A triangle with 1 right angle	
Obtuse-angled triangle	A triangle with 1 obtuse angle	

Table 11.2

Thinking Time

How are the 6 types of triangles in Tables 11.1 and 11.2 related to one another? For example, can an obtuse-angled triangle be an equilateral triangle?

The relationships among them are illustrated in Fig. 11.2.

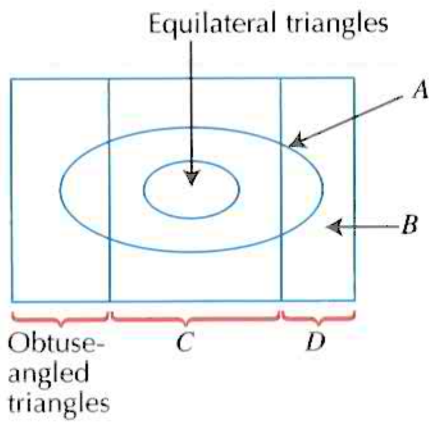


Fig. 11.2

What types of triangles do A, B, C and D represent?

INFORMATION

Fig. 11.2 is a Venn diagram which will be covered in the chapter on Sets in Upper Secondary Mathematics.

Recap (Angle Sum of a Triangle)

We have learnt in primary school that:

> The sum of interior angles of a triangle is 180°.
> (Abbreviation: ∠ sum of △)

A proof of the above result is given as follows:

Consider $\triangle ABC$ in Fig. 11.3.

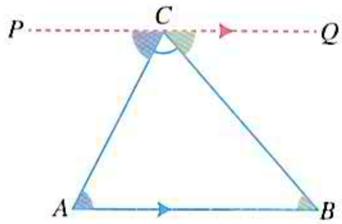


Fig. 11.3

Draw a line PQ that is parallel to AB and passes through C.

$B\hat{A}C = A\hat{C}P$ (alt. ∠s, PQ // AB)

$A\hat{B}C = B\hat{C}Q$ (alt. ∠s, PQ // AB)

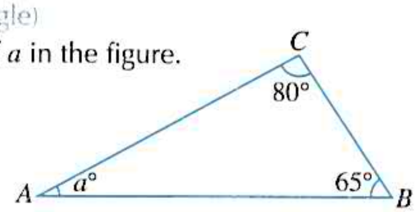
We also have $A\hat{C}P + A\hat{C}B + B\hat{C}Q = 180°$ (adj. ∠s on a str. line).

$\therefore B\hat{A}C + A\hat{C}B + A\hat{B}C = 180°$ (∠ sum of △)

Worked Example 1

(Angle Sum of a Triangle)

Calculate the value of a in the figure.

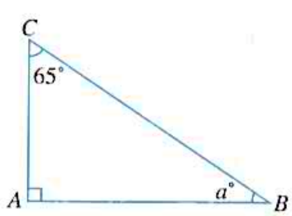


Solution:

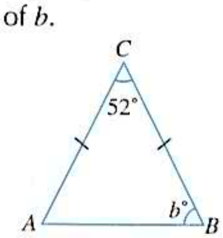
$$\begin{aligned} a° + 65° + 80° &= 180° \text{ (∠ sum of △)} \\ a° &= 180° - 65° - 80° \\ &= 35° \\ \therefore a &= 35 \end{aligned}$$

PRACTISE NOW 1

1. Find the value of a in the figure.



2. In the figure, $AC = BC$. Find the value of b.



SIMILAR QUESTIONS

Exercise 11A Questions 1(a)–(d), 2(a)–(d), 3(a)–(f), 5, 9(b), (d), 10, 12–15

RECALL

The base angles of an isosceles triangle are equal. (Abbreviation: base ∠s of isos. △)

Basic Properties of a Triangle

Investigation

Basic Properties of a Triangle

In this investigation, we shall explore two basic properties of a triangle.

Go to http://www.shinglee.com.sg/StudentResources/ and open the geometry software template 'Basic Properties of Triangle'.

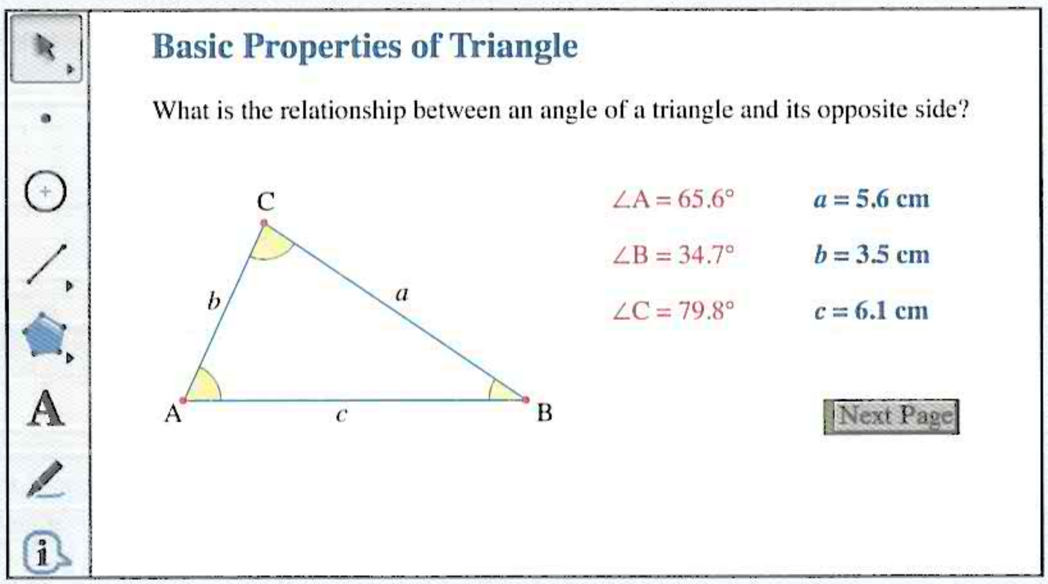


Fig. 11.4

1. The template in Fig. 11.4 shows a triangle with 3 angles and their opposite sides, e.g. the side a is opposite $\angle A$. Name the side opposite $\angle B$ and the side opposite $\angle C$.
2. State the largest and the smallest angle. Compare the lengths of the sides opposite these angles. What do you observe?
3. Click and move a point A, B or C to change the size of the triangle. What can you conclude about the relationship between an angle of a triangle and the length of its opposite side?
4. Use a calculator to add the lengths of the two shorter sides of a triangle and compare it with the length of the longest side. What do you notice?
5. Repeat Step 3. Is your observation in Step 4 still applicable? Explain your answer.

Click '2' in the template to proceed to the next page as shown in Fig. 11.5.

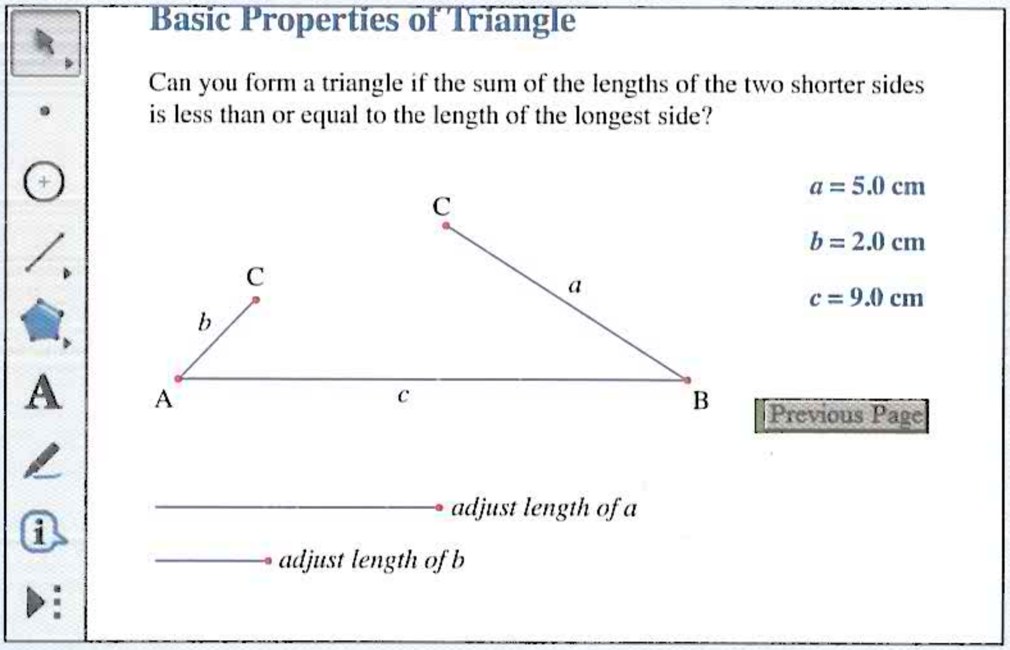
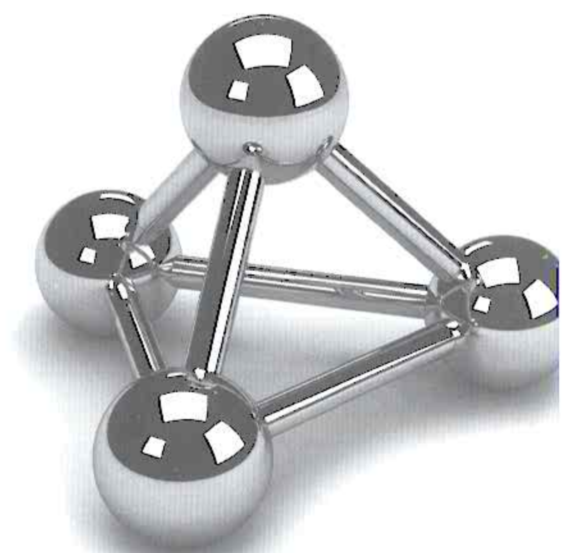


Fig. 11.5

6. The template in Fig. 11.5 shows three line segments with lengths $a = 5$ cm, $b = 2$ cm and $c = 9$ cm. Notice that the sum of the lengths of the two shorter line segments, i.e. $a + b$, is shorter than the length of the longest line segment. Click and move the two points labelled C to see if it is possible to form a triangle.
7. Adjust the lengths a and b so that $a = 3$ cm and $b = 4$ cm. Adjust the length c by clicking and moving either point A or B such that $c = 7$ cm. What do you notice about $a + b$ and c? Try to form a triangle if possible.
8. Change the lengths of the three line segments. Are you able to form a triangle? What can you conclude about the relationship between the sum of the lengths of any two line segments and the length of the third line segment?

From the investigation, two basic properties of a triangle are:

- The largest angle of a triangle is opposite the longest side, and the smallest angle is opposite the ________ side.
- The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.



Exterior Angles of a Triangle

Fig. 11.6 shows $\triangle ABC$ with AB produced to P, BC produced to Q and CA produced to R. $\angle a$, $\angle b$ and $\angle c$ are the interior angles of $\triangle ABC$. $\angle p$, $\angle q$ and $\angle r$ are the **exterior angles** of $\triangle ABC$.

INFORMATION

There are two ways in which the exterior angles of a triangle may be drawn.

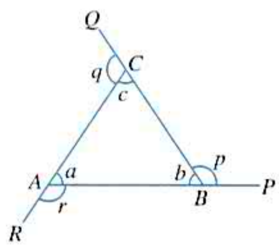


Fig. 11.6

In particular, $\angle a$ and $\angle b$ are called the **interior opposite angles** with reference to $\angle q$.

Similarly, $\angle a$ and $\angle c$ are called the interior opposite angles with reference to $\angle p$.

Which are the interior opposite angles with reference to $\angle r$?

Consider $\triangle ABC$ in Fig. 11.7.

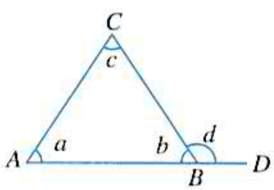


Fig. 11.7

$\angle b + \angle d = 180^\circ$ (adj. $\angle$s on a str. line)

We also have $\angle a + \angle b + \angle c = 180^\circ$ ($\angle$ sum of $\triangle$).

$\therefore \angle b + \angle d = \angle a + \angle b + \angle c$

$\therefore \angle d = \angle a + \angle c$

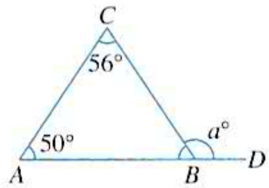
Hence, we conclude that:

> An exterior angle of a triangle is equal to the sum of its interior opposite angles.
> (Abbreviation: ext. $\angle$ of $\triangle$)

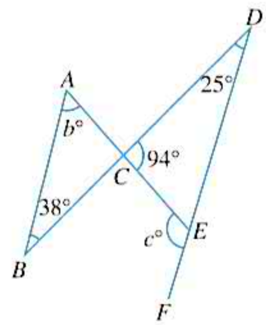
Worked Example 2

(Exterior Angles of Triangles)

(a) In the figure, ABD is a straight line.
Calculate the value of a.



(b) In the figure, ACE, BCD and DEF are straight lines.
Calculate the value of b and of c.



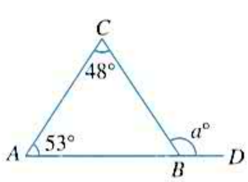
Solution:

(a) $a° = 56° + 50°$ (ext. $\angle$ of $\triangle$)
$= 106°$
$\therefore a = 106$

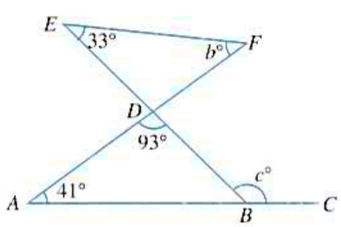
(b) $A\hat{C}B = 94°$ (vert. opp. $\angle$s)
$b° + 38° + 94° = 180°$ ($\angle$ sum of $\triangle ABC$)
$b° = 180° - 38° - 94°$
$= 48°$
$\therefore b = 48$
$c° = 25° + 94°$
$= 119°$ (ext. $\angle$ of $\triangle CDE$)
$\therefore c = 119$

PRACTISE NOW 2

(a) In the figure, ABD is a straight line. Find the value of a.



(b) In the figure, ABC, ADF and BDE are straight lines. Find the value of b and of c.



SIMILAR QUESTIONS

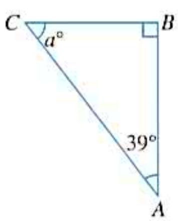
Exercise 11A Questions 4(a)–(c), 6, 7(a)–(b), 8(a)–(b), 9(a), (c), 11

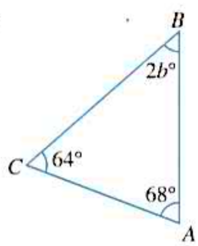
Exercise 11A

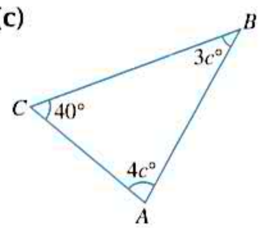
BASIC LEVEL

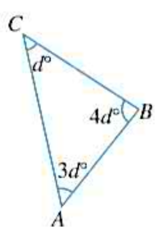
1. For each of the following, given the sizes of $\angle A$ and $\angle B$, sketch $\triangle ABC$. Find $\angle C$ and hence classify each triangle according to the number of equal sides it has, and the types of angles it has, e.g. equilateral triangle and acute-angled triangle.
   (a) $\angle A = 20^\circ$, $\angle B = 60^\circ$
   (b) $\angle A = 70^\circ$, $\angle B = 40^\circ$
   (c) $\angle A = 60^\circ$, $\angle B = 60^\circ$
   (d) $\angle A = 42^\circ$, $\angle B = 48^\circ$

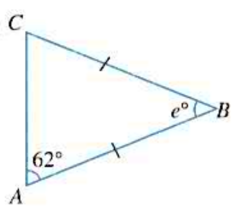
2. For each of the following, the given angle is the base angle of an isosceles triangle. Find the third angle of the triangle.
   (a) 40° (b) 87°
   (c) 15° (d) 79°

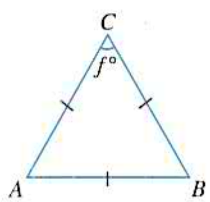
3. Find the value of the unknown in each of the following figures.
   (a)


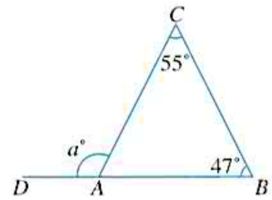
   (b)


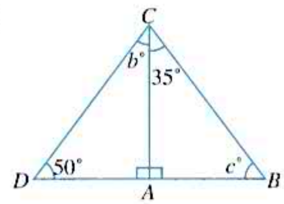
   (c)


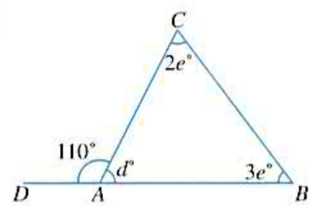
   (d)


   (e)


   (f)


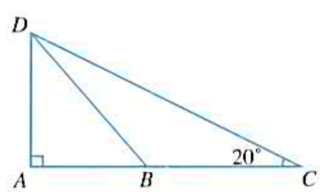
4. Given that DAB is a straight line, find the value(s) of the unknown(s) in each of the following figures.
   (a)


   (b)


   (c)


5. If the sizes of the angles of a triangle are $3x^\circ$, $4x^\circ$ and $5x^\circ$, find the smallest angle of the triangle.

6. In the figure, ABC is a straight line.

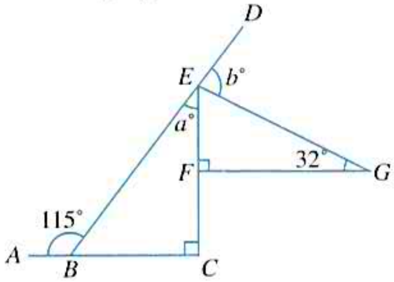


Given that $A\hat{D}B = B\hat{D}C$, find
(i) $B\hat{D}C$,
(ii) $C\hat{B}D$.

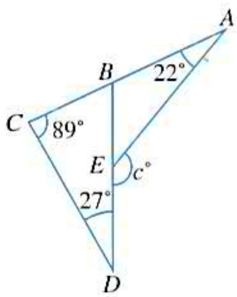
INTERMEDIATE LEVEL

7. Given that ABC and BED are straight lines, find the value(s) of the unknown(s) in each of the following figures.

(a)

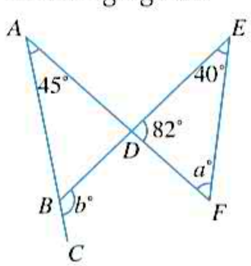


(b)

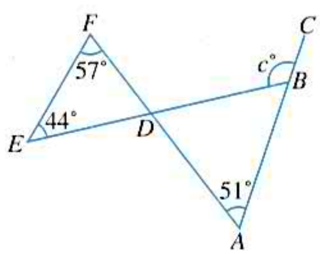


8. Given that ABC, ADF and BDE are straight lines, find the value(s) of the unknown(s) in each of the following figures.

(a)

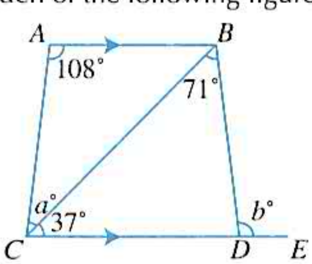


(b)

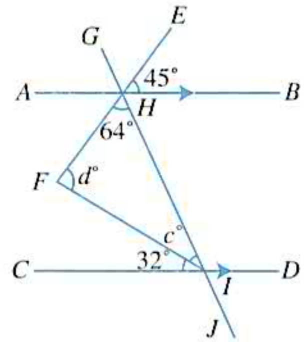


9. Given that $AB \parallel CD$, find the values of the unknowns in each of the following figures.

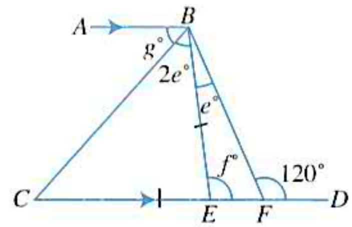
(a)



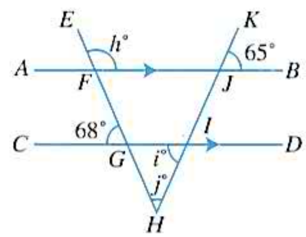
(b)



(c)



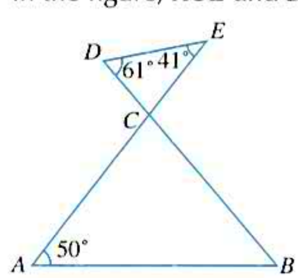
(d)



10. If the sizes of the angles of a triangle are $(x-35)°$, $(x-25)°$ and $\left(\frac{1}{2}x-10\right)^{\circ}$, find the value of x.

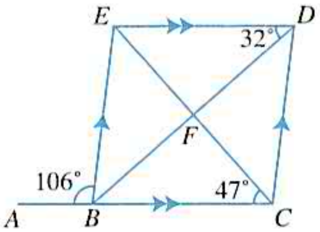
11. In $\triangle ABC$, $B\hat{A}C = 50°$ and $B\hat{C}A = 26°$.
   (i) Find $A\hat{B}C$.
   (ii) If AB is produced to D, find $C\hat{B}D$.

12. In the figure, ACE and BCD are straight lines.



Find
(i) $A\hat{C}B$,
(ii) $A\hat{B}C$.

13. In the figure, $AC \parallel ED$ and $BE \parallel CD$.



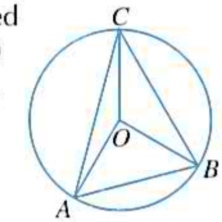
Find

(i) $D\hat{F}E$,

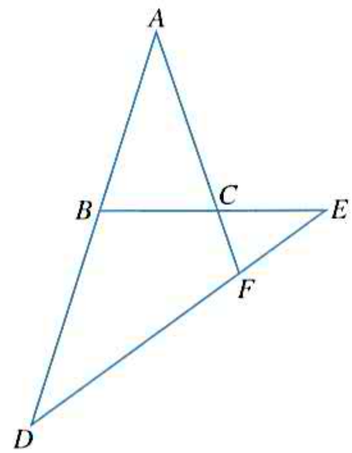
(ii) $B\hat{D}C$.

ADVANCED LEVEL

14. The figure shows $\triangle ABC$ inscribed in a circle with centre O. If $C\hat{B}O$ is twice of $C\hat{A}O$ and $B\hat{A}O$ is one and a half times of $C\hat{B}O$, find $C\hat{A}O$.



15. In the figure, each side of $\triangle ABC$ is produced. If $AB = AC$, $BD = BE$ and $AF = DF$, find $A\hat{B}C$.



11.2 Quadrilaterals

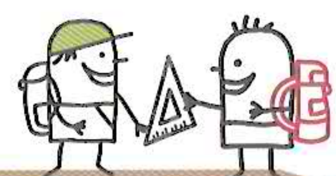


Fig. 11.8 shows a **quadrilateral** $ABCD$. A quadrilateral is a closed plane figure that has four sides, four vertices and four interior angles. The line segment BD that joins the vertices, B and D, is a **diagonal** of the quadrilateral $ABCD$.

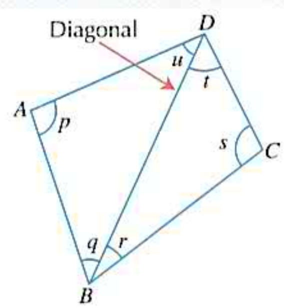


Fig. 11.8

ATTENTION

A quadrilateral is named by taking the vertices either in a clockwise or anticlockwise order. Hence, $ABCD$, $BCDA$, $CDAB$ and $DABC$ are correct ways of naming the quadrilateral but $ABDC$ and $CDBA$ are not.

Now let us find the sum of interior angles of a quadrilateral.

$\angle p + \angle q + \angle u = 180°$ ($\angle$ sum of $\triangle ABD$)

$\angle r + \angle s + \angle t = 180°$ ($\angle$ sum of $\triangle BCD$)

Sum of interior angles of the quadrilateral $ABCD$

$= \angle p + \angle q + \angle r + \angle s + \angle t + \angle u$

$= \angle p + \angle q + \angle u + \angle r + \angle s + \angle t$

$= 180° + 180°$

$= 360°$

The sum of interior angles of a quadrilateral is 360°.

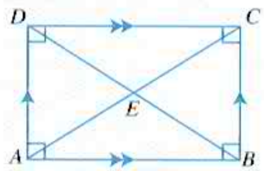
Properties of Special Quadrilaterals

Investigation

Properties of Special Quadrilaterals

In this investigation, we shall explore the properties of special quadrilaterals.

Go to http://www.shinglee.com.sg/StudentResources/ and open the geometry software template 'Special Quadrilaterals'.



Rectangle

Fig. 11.9

1. A rectangle $ABCD$ is shown. Measure the lengths of AB, BC, DC and AD. What do you notice?
2. Measure $B\hat{A}D$, $A\hat{B}C$, $B\hat{C}D$ and $A\hat{D}C$. What do you notice?
3. Measure the lengths of AE, BE, CE and DE. What do you notice?
4. Find the sum of the lengths AE and CE and the sum of the lengths BE and DE. What do you notice?
5. Click and drag the points A, B and D to form rectangles of different dimensions and repeat Steps 1 – 4. What conclusions can you draw?

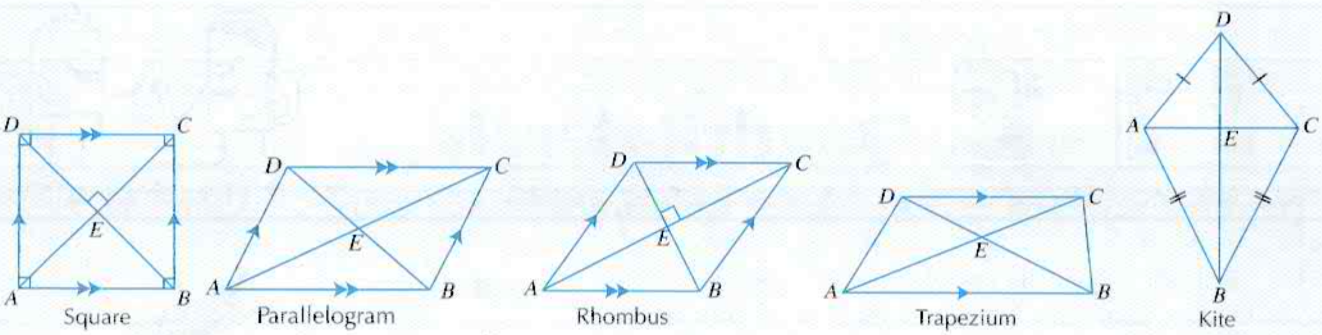


Fig. 11.10

6. Click on the different tabs to view other special quadrilaterals, namely a square, a parallelogram, a rhombus, a trapezium and a kite. Consider each of the quadrilaterals and answer the following questions.
   (a) What can you say about the lengths of its sides?
   (b) What can you say about its interior angles?
   (c) Do the diagonals have the same length?
   (d) Do the diagonals bisect each other? For example, in the parallelogram $ABCD$, is $AE = EC$ and $BE = ED$?
   (e) Are the diagonals perpendicular to each other?
   (f) Do the diagonals bisect the interior angles? For example, in the rhombus $ABCD$, is $B\hat{A}C = C\hat{A}D$ and $A\hat{B}D = C\hat{B}D$?

Table 11.3 gives a summary of the properties of the special quadrilaterals which have been covered in the investigation.

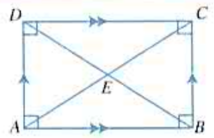
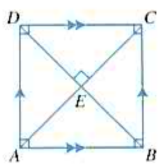
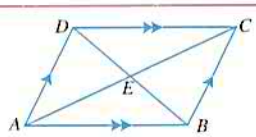
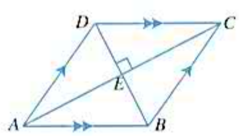
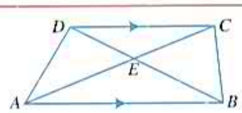
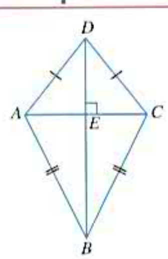
Quadrilateral	Parallel sides	Equal sides	Interior angles	Diagonals
 **Rectangle**	There are two pairs of parallel sides.	Opposite sides are equal in length.	All four angles are right angles.	• The two diagonals are equal in length. • Diagonals **bisect** each other, i.e. $AE = EC$ and $BE = ED$.
 **Square**	There are two pairs of parallel sides.	All four sides are of the same length.	All four angles are right angles.	• The two diagonals are equal in length. • Diagonals bisect each other at right angles, i.e. $A\hat{E}B = B\hat{E}C = A\hat{E}D = C\hat{E}D = 90^\circ$. • Diagonals bisect the interior angles, e.g. $B\hat{A}C = C\hat{A}D = 45^\circ$ and $A\hat{B}D = C\hat{B}D = 45^\circ$.
 **Parallelogram**	There are two pairs of parallel sides.	Opposite sides are equal in length.	Opposite angles are equal, i.e. $A\hat{D}C = A\hat{B}C$ and $B\hat{A}D = B\hat{C}D$.	Diagonals bisect each other, i.e. $AE = EC$ and $BE = ED$.
 **Rhombus**	There are two pairs of parallel sides.	All four sides are of the same length.	Opposite angles are equal. i.e. $A\hat{D}C = A\hat{B}C$ and $B\hat{A}D = B\hat{C}D$.	• Diagonals bisect each other at right angles, i.e. $A\hat{E}B = B\hat{E}C = A\hat{E}D = C\hat{E}D = 90^\circ$. • Diagonals bisect the interior angles, e.g. $B\hat{A}C = C\hat{A}D$ and $A\hat{B}D = C\hat{B}D$.
 **Trapezium**	There is at least one pair of parallel sides.			
 **Kite**	The opposite sides may be parallel.	There are two pairs of equal adjacent sides.		• Diagonals cut each other at right angles, i.e. $A\hat{E}B = B\hat{E}C = A\hat{E}D = C\hat{E}D = 90^\circ$. • One diagonal bisects the interior angles, i.e. $A\hat{D}B = C\hat{D}B$ and $A\hat{B}D = C\hat{B}D$.

Table 11.3

Thinking Time

How are the 6 types of special quadrilaterals in Table 11.3 related to one another? For example,

(a) is a square a special type of rectangle?
(b) is a rectangle a special type of parallelogram?
(c) is a parallelogram a special type of trapezium?
(d) is a square a special type of rhombus?
(e) is a rhombus a special type of kite?

The relationships among them are illustrated in Fig. 11.11, where for instance, '$C \rightarrow A$' means C is a special type of A.

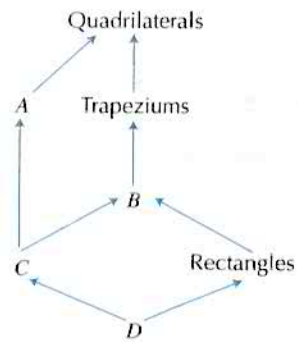


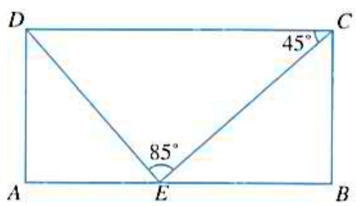
Fig. 11.11

What types of quadrilaterals do A, B, C and D represent?

Worked Example 3

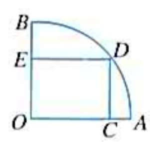
(Angles in a Rectangle)

The figure shows a rectangle $ABCD$. E lies on AB such that $C\hat{E}D = 85°$ and $D\hat{C}E = 45°$.



Calculate
(i) $A\hat{D}E$,
(ii) $A\hat{E}D$.

OAB is a quadrant of a circle with centre O and $OCDE$ is a rectangle. If $OE = 3$ cm and $EB = 2$ cm, find the length of EC.



Solution:

(i) $C\hat{D}E + 85° + 45° = 180°$ ($\angle$ sum of $\triangle CDE$)

$$C\hat{D}E = 180° - 85° - 45°$$
$$= 50°$$

$A\hat{D}E + 50° = 90°$ ($A\hat{D}C$ is a right angle.)

$$A\hat{D}E = 90° - 50°$$
$$= 40°$$

(ii) $B\hat{A}D = 90°$

$A\hat{E}D + 40° + 90° = 180°$ ($\angle$ sum of $\triangle ADE$)

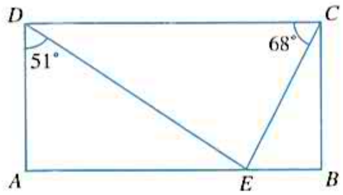
$$A\hat{E}D = 180° - 40° - 90°$$
$$= 50°$$

PRACTISE NOW 3

SIMILAR QUESTIONS

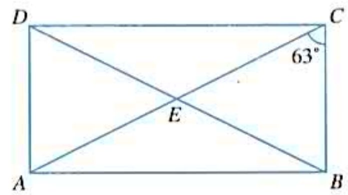
Exercise 11B Questions 1(a)–(b), 4(a)–(b), 6, 12

1. The figure shows a rectangle $ABCD$. E lies on AB such that $A\hat{D}E = 51°$ and $D\hat{C}E = 68°$.



Find
(i) $A\hat{E}D$,
(ii) $C\hat{E}D$.

2. The figure shows a rectangle $ABCD$ where the diagonals AC and BD intersect at E.



Given that $A\hat{C}B = 63°$, find
(i) $B\hat{E}C$,
(ii) $C\hat{D}E$.

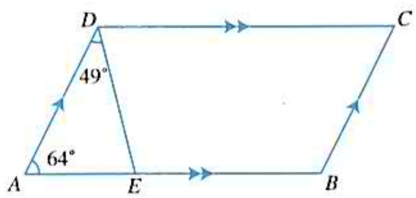
Use four identical 3-by-2 rectangles to form two squares.

Worked Example 4

(Angles in a Parallelogram)

The figure shows a parallelogram $ABCD$ where $B\hat{A}D = 64°$. E lies on AB such that $A\hat{D}E = 49°$.

Calculate
(i) $A\hat{B}C$,
(ii) $C\hat{D}E$.



Solution:

(i) $A\hat{B}C + 64° = 180°$ (int. ∠s, AD // BC)
$A\hat{B}C = 180° - 64°$
$= 116°$

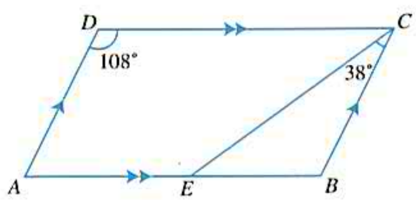
(ii) $A\hat{D}C = 116°$ (opp. ∠s of //gram)
$C\hat{D}E + 49° = 116°$
$C\hat{D}E = 116° - 49°$
$= 67°$

PRACTISE NOW 4

SIMILAR QUESTIONS

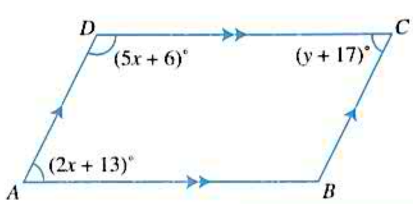
Exercise 11B Questions 2(a)–(b), 7, 13

1. The figure shows a parallelogram $ABCD$ where $A\hat{D}C = 108°$. E lies on AB such that $B\hat{C}E = 38°$.



(i) Given that $A\hat{B}C = 9x°$, find the value of x.
(ii) Find $D\hat{C}E$.

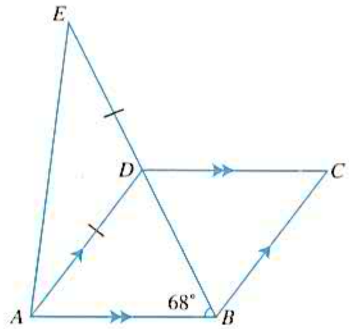
2. The figure shows a parallelogram $ABCD$. Find the value of x and of y.



Worked Example 5

(Angles in a Rhombus)

The figure shows a rhombus $ABCD$. The diagonal BD is produced to E such that $AD = DE$.



If $A\hat{B}E = 68°$, calculate

(i) $B\hat{C}D$,

(ii) $D\hat{A}E$.

Solution:

(i) $C\hat{B}D = 68°$ (diagonals bisect interior angles of a rhombus)

$B\hat{C}D + 68° + 68° = 180°$ (int. $\angle$s, $AB // DC$)

$B\hat{C}D = 180° - 68° - 68°$

$= 44°$

(ii) $A\hat{D}B = 68°$ (base $\angle$s of isos. $\triangle ABD$)

$D\hat{A}E + A\hat{E}D = 68°$ (ext. $\angle$ of $\triangle$)

$D\hat{A}E = \dfrac{68°}{2}$ (base $\angle$s of isos. $\triangle ADE$)

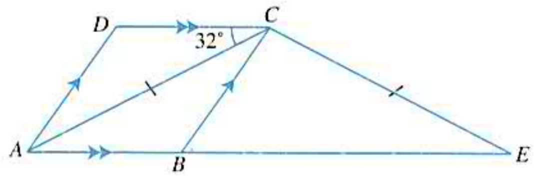
$= 34°$

PRACTISE NOW 5

SIMILAR QUESTIONS

Exercise 11B Questions 5(a)–(c), 8–9, 14

1. The figure shows a rhombus $ABCD$ where $A\hat{C}D = 32°$. AB is produced to E such that $AC = CE$.

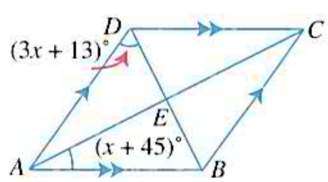


Find

(i) $A\hat{B}C$,

(ii) $B\hat{C}E$.

2. The figure shows a rhombus $ABCD$ where the diagonals AC and BD intersect at E. Find the value of x.

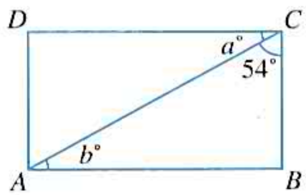


Exercise 11B

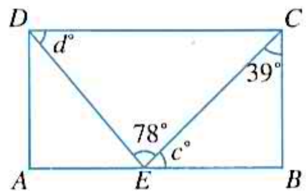
BASIC LEVEL

1. Find the values of the unknowns in each of the following rectangles.

(a)

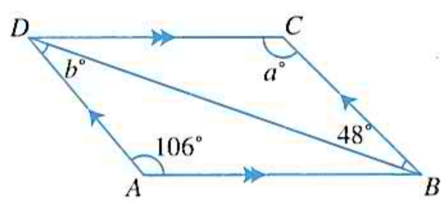


(b)

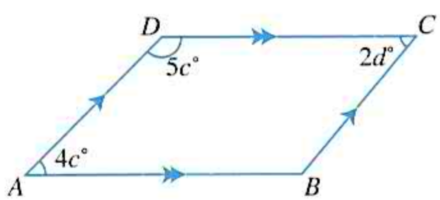


2. Find the values of the unknowns in each of the following parallelograms.

(a)

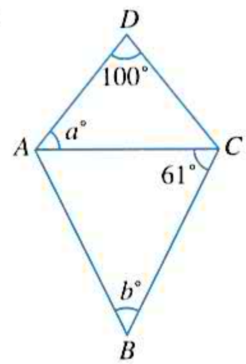


(b)

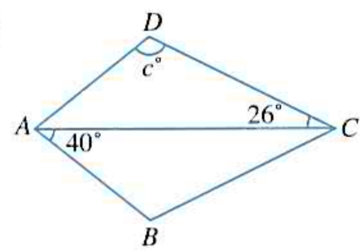


3. Find the value(s) of the unknown(s) in each of the following kites.

(a)



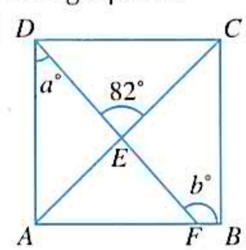
(b)



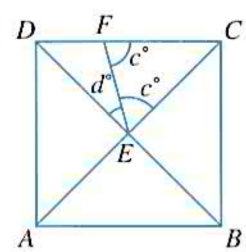
INTERMEDIATE LEVEL

4. Find the values of the unknowns in each of the following squares.

(a)

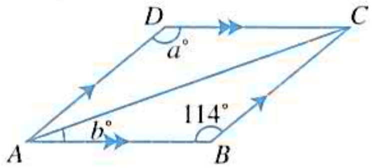


(b)

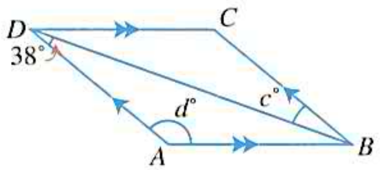


5. Find the values of the unknowns in each of the following rhombuses.

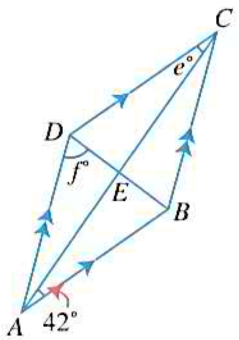
(a)



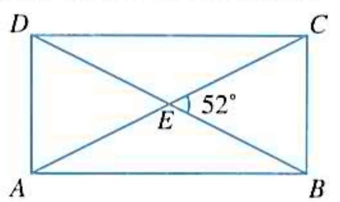
(b)



(c)



6. The figure shows a rectangle $ABCD$ where the diagonals AC and BD intersect at E.

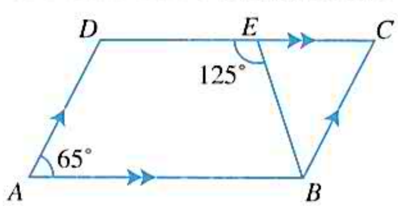


Given that $B\hat{E}C = 52°$, find

(i) $A\hat{D}B$,

(ii) $A\hat{C}D$.

7. The figure shows a parallelogram $ABCD$ where $B\hat{A}D = 65°$. E lies on DC such that $B\hat{E}D = 125°$.

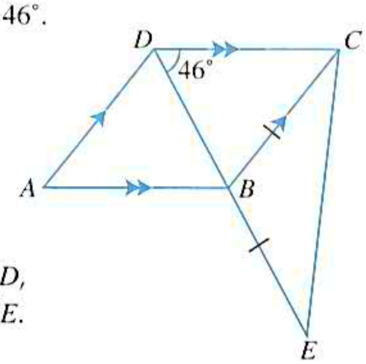


Find

(i) $A\hat{D}E$,

(ii) $C\hat{B}E$.

8. The figure shows a rhombus $ABCD$. The diagonal DB is produced to E such that $BC = BE$ and $C\hat{D}E = 46°$.

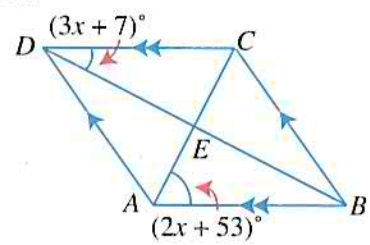


Find

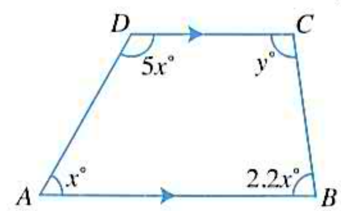
(i) $B\hat{A}D$,

(ii) $B\hat{C}E$.

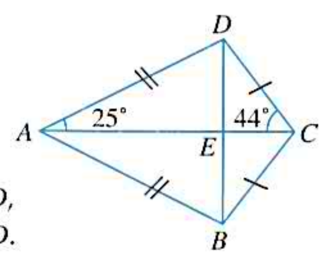
9. The figure shows a rhombus $ABCD$ where the diagonals AC and BD intersect at E. Find the value of x.



10. The figure shows a trapezium $ABCD$ where $AB \parallel DC$. Find the value of x and of y.



11. The figure shows a kite $ABCD$ where $AB = AD$, $BC = CD$ and the diagonals AC and BD intersect at E.



Find

(i) $A\hat{B}D$,

(ii) $C\hat{B}D$.

12. In a rectangle $ABCD$, E is the midpoint of AB and $C\hat{E}D = 118°$. Find

(i) $A\hat{D}E$,

(ii) $D\hat{C}E$.

13. In a parallelogram $PQRS$, $Q\hat{P}R = 42°$ and $Q\hat{R}S = 70°$. Find

(i) $P\hat{Q}R$,

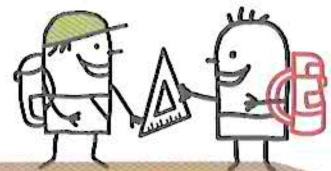
(ii) $P\hat{R}Q$.

14. In a rhombus $WXYZ$, $W\hat{X}Y = 108°$. Find
   (i) $X\hat{Z}Y$,
   (ii) $X\hat{Y}Z$,
   (iii) $X\hat{W}Y$.

15. In a trapezium $ABCD$, $AB // DC$, $AB = AD$, $B\hat{C}D = 52°$ and $A\hat{D}C = 62°$. Find
   (i) $A\hat{B}D$,
   (ii) $C\hat{B}D$.

16. In a kite $PQRS$, $PQ = QR$, $PS = RS$, $Q\hat{P}R = 42°$ and $P\hat{S}R = 64°$. Find
   (i) $P\hat{R}S$,
   (ii) $P\hat{Q}R$.

11.3 Polygons



We have learnt about triangles and quadrilaterals in the previous sections. Triangles and quadrilaterals are examples of polygons.

A **polygon** is a *closed* plane figure with three or more *straight* line segments as its sides. Fig. 11.12 shows some other examples of polygons.

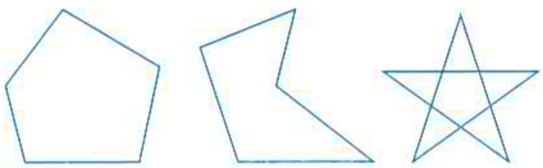


Fig. 11.12

The shapes shown in Fig. 11.13 are *not* polygons. Why?

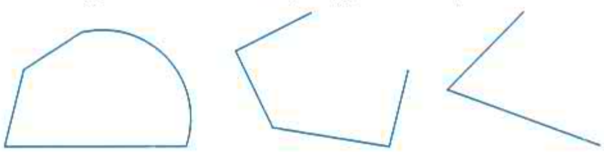


Fig. 11.13

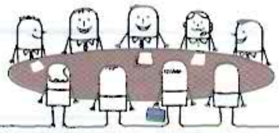
The first two polygons in Fig. 11.12 are called **simple polygons** because their boundaries do not cross themselves, i.e. the line segments do not intersect one another, unlike the third polygon.

The first polygon in Fig. 11.12 is called a **convex polygon** because *all* of its interior angles are less than 180°.

Indicate the interior angle that is greater than 180° on the second polygon in Fig. 11.12. A *simple* polygon in which at least one of its interior angles is more than 180° is called a **concave polygon** (notice that the second polygon in Fig. 11.12 *caves in* at that vertex whose interior angle is more than 180°).

The third polygon in Fig. 11.12 is neither convex nor concave because it is not a simple polygon.

In this section, we will study only convex polygons.



Class Discussion

Naming of Polygons

Polygons are named after the number of sides that they have. Search on the Internet for a video titled 'Polygon Song'. Listen to the song and work in pairs to write down the names of the following polygons.

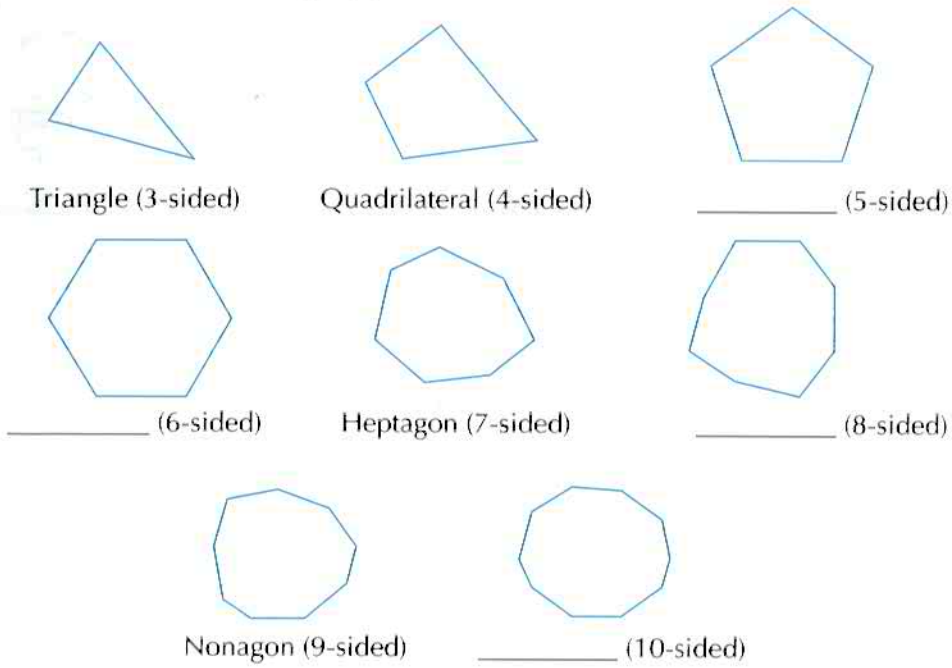


Fig. 11.14

Internet Resources

The names of the polygons in Fig. 11.14 contain prefixes which are determined by their number of sides. Search the Internet to find out other uses of these prefixes.

For polygons with more than 10 sides, some of them also have special names, but they are not easy to remember. To make naming of polygons easier, we call a polygon with n sides an n-sided polygon or an ***n*-gon**. For example, a polygon with 12 sides is known as a 12-sided polygon or a 12-gon.

Regular Polygons

A **regular polygon** is a polygon with *all sides equal* and *all angles equal*. Which polygons in Fig. 11.14 are regular polygons?

Thinking Time

What is the name of a regular triangle and of a regular quadrilateral?

Investigation

Properties of a Regular Polygon

1. Is it possible for a polygon to have all sides equal without being a regular polygon?
   (a) What is the name of a non-regular quadrilateral with all sides equal?
   (b) Fig. 11.15 shows the pulling of a regular hexagon as indicated by the arrows to form a non-regular hexagon with all sides equal. Draw another non-regular hexagon with all sides equal but of a different shape as the one shown in Fig. 11.15.

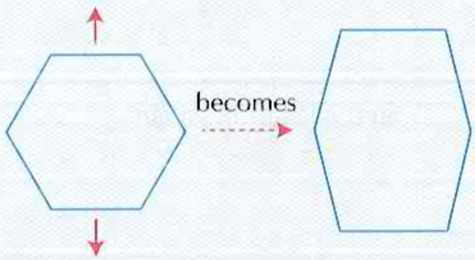
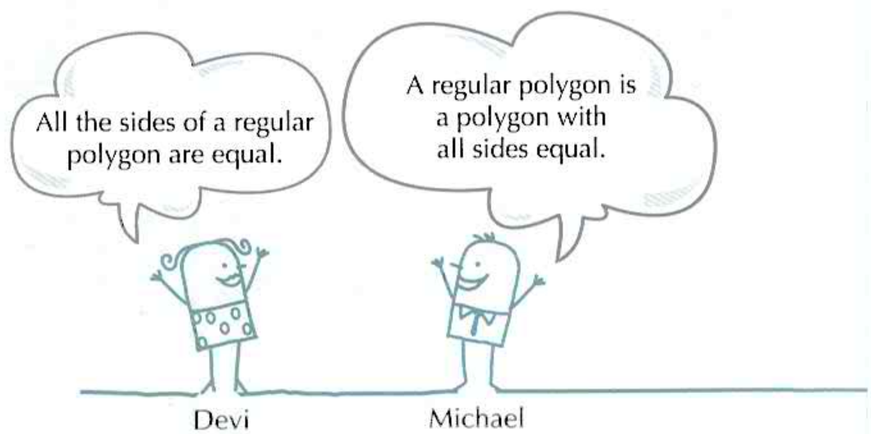


Fig. 11.15

2. Is it possible for a polygon to have all angles equal without being a regular polygon?
   (a) What is the name of a non-regular quadrilateral with all angles equal?
   (b) Draw two different non-regular hexagons with all angles equal.

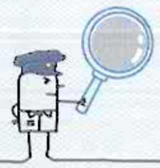
Journal Writing

The teacher says that Devi is correct but Michael is wrong. In your journal, explain the differences between the statements made by Devi and Michael.



Sum of Interior Angles of a Polygon

In the previous sections, we have learnt that the sum of interior angles of a triangle and of a quadrilateral is 180° and 360° respectively. What is the sum of interior angles of other polygons?



Investigation

Sum of Interior Angles of a Polygon

In this investigation, we shall deduce a general expression for the sum of interior angles of an n-sided polygon.

1. Complete Table 11.4.

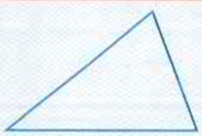
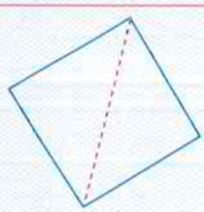
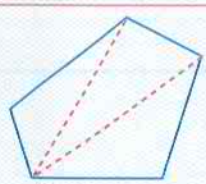
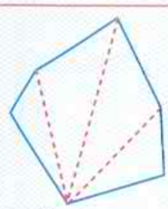
Polygon	Number of sides	Number of Triangle(s) formed	Sum of Interior Angles
 Triangle	3	1	$1 \times 180° = (3-2) \times 180°$
 Quadrilateral	4	2	$2 \times 180° = (4-2) \times 180°$
 Pentagon			
 Hexagon			

Table 11.4

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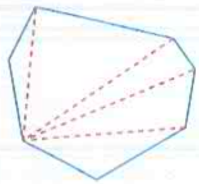
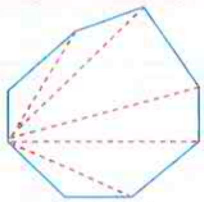
Polygon	Number of sides	Number of Triangle(s) formed	Sum of Interior Angles
 Heptagon			
 Octagon			
n-gon			

Table 11.4

2. From Table 11.4, what can you say about the number of triangles formed by a polygon in relation to the number of sides it has?

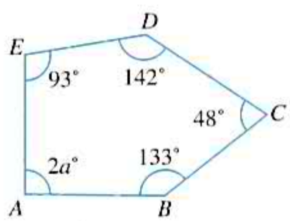
From the investigation, we have deduced that:

The sum of interior angles of an n-sided polygon is $(n - 2) \times 180°$.

Worked Example 6

(Finding the Value of an Unknown in a Pentagon)

Calculate the value of a in the figure.



Solution:

$$
\begin{aligned}
\text{Sum of interior angles of a pentagon} &= (n - 2) \times 180° \\
&= (5 - 2) \times 180° \\
&= 540° \\
2a° + 133° + 48° + 142° + 93° &= 540° \\
2a° &= 540° - 133° - 48° - 142° - 93° \\
&= 124° \\
a° &= 62° \\
\therefore a &= 62
\end{aligned}
$$

INFORMATION

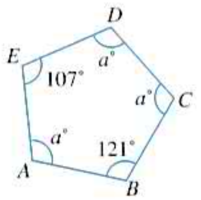
A pentagon has 5 sides, i.e. $n = 5$.

PRACTISE NOW 6

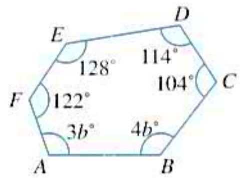
SIMILAR QUESTIONS

Exercise 11C Questions 1(a)–(d), 2(a)–(d), 7

1. Find the value of a in the figure.



2. Find the value of b in the figure.



Worked Example 7

(Interior Angles of a Regular Decagon)

(i) Calculate the sum of interior angles of a regular decagon.

(ii) Hence, calculate the size of each interior angle of a regular decagon.

Solution:

(i) Sum of interior angles of a regular decagon $= (10 - 2) \times 180°$
$= 1440°$

(ii) Size of each interior angle of a regular decagon $= \dfrac{1440°}{10}$
$= 144°$

INFORMATION

A regular decagon has 10 sides.

PRACTISE NOW 7

SIMILAR QUESTIONS

Exercise 11C Questions 3(a)–(b)

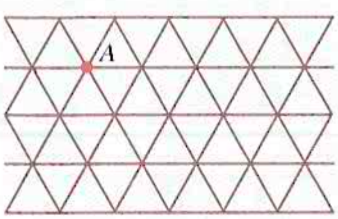
(i) Find the sum of interior angles of a regular polygon with 24 sides.

(ii) Hence, find the size of each interior angle of a regular polygon with 24 sides.

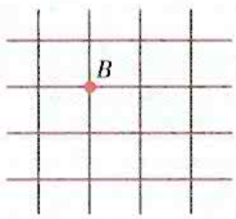
Investigation

Tessellation

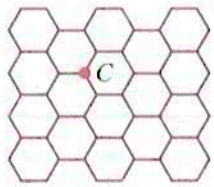
If you look at the floor of houses and shopping centres, you will most probably notice that many of them are tiled. The tiles used are usually in the shape of a square or a rectangle. A pattern formed by fitting together regular figures which completely cover a plane surface is called a **tessellation**. The following diagrams show how planes may be tessellated by equilateral triangles, squares and regular hexagons.



A tessellation formed by equilateral triangles.



A tessellation formed by squares.



A tessellation formed by regular hexagons.

ATTENTION

A tessellation made by repeating a regular polygon is also known as a regular tessellation.

1. What other tessellations are made up of combinations of other regular polygons?
2. Can you design one on your own?
3. What is the sum of the corner angles at each of the points A, B and C?

However, not all regular polygons tessellate. For example, regular pentagons do not tessellate. When they are put together as shown in Fig. 11.16, they leave a gap in between.

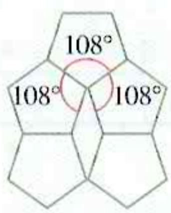


Fig. 11.16

Sometimes a tessellation may be made up of two or more regular polygons. Fig. 11.17 shows a tessellation formed by squares and regular octagons.

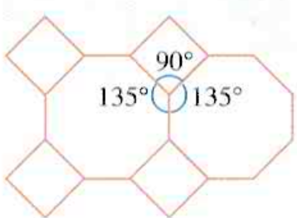
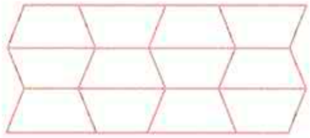


Fig. 11.17

4. Find out whether rhombuses, regular octagons and regular decagons tessellate.

We can also tessellate irregular polygons.

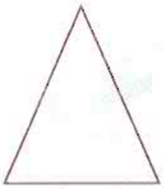
A tessellation formed by parallelograms.



A tessellation formed by isosceles trapeziums (two sides equal).

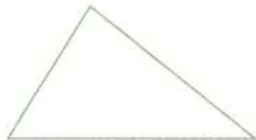
5. Which of the following figures will tessellate?

(a)



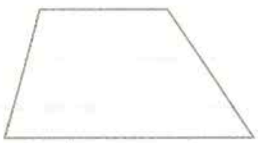
Isosceles triangle

(b)



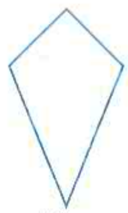
Scalene triangle

(c)



Scalene trapezium (No sides equal)

(d)

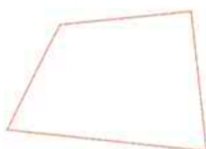


Kite

(e)

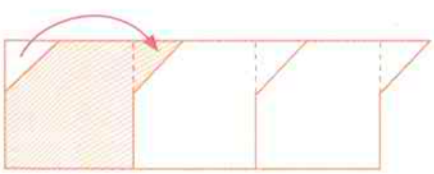
Circle

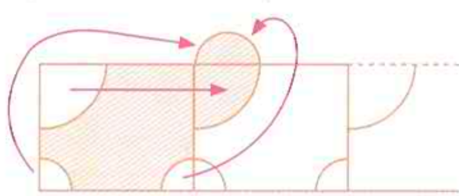
(f)



Quadrilateral

6. We can design figures of different shapes which tessellate. In each of the diagrams below, we start with a simple design. Then, we remove a piece from a corner and add it onto the opposite side and we will have a new figure which tessellates. Create a few new tessellating patterns on your own in this way.





From the investigation, we can conclude that there are only 3 regular tessellation namely those formed by equilateral triangles, squares and regular hexagons.

Sum of Exterior Angles of a Polygon

Fig. 11.18 shows a pentagon *ABCDE* with *AB* produced to *P*, *BC* produced to *Q*, *CD* produced to *R*, *DE* produced to *S* and *EA* produced to *T*. $\angle a$, $\angle b$, $\angle c$, $\angle d$ and $\angle e$ are the interior angles of the pentagon. $\angle p$, $\angle q$, $\angle r$, $\angle s$ and $\angle t$ are the exterior angles of the pentagon.

INFORMATION

There are two ways in which the exterior angles of the pentagon may be drawn.

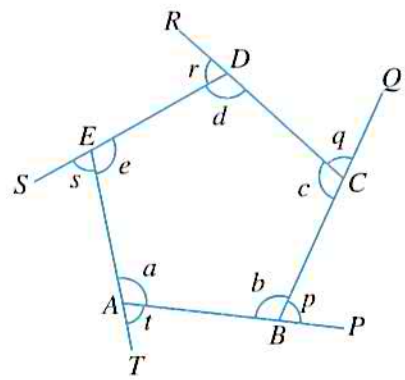


Fig. 11.18

Investigation

Sum of Exterior Angles of a Pentagon

Go to http://www.shinglee.com.sg/StudentResources/ and open the geometry software template 'Exterior Angles of Polygon'.

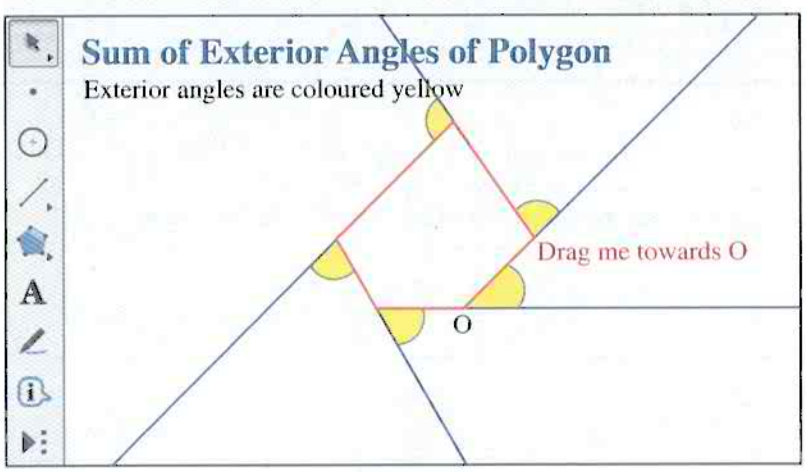


Fig. 11.19

1. The template shows a pentagon with 5 exterior angles. Click on the point 'Drag me towards O' and drag it until it reaches O. Fig. 11.20 shows the figure just before the point reaches O.

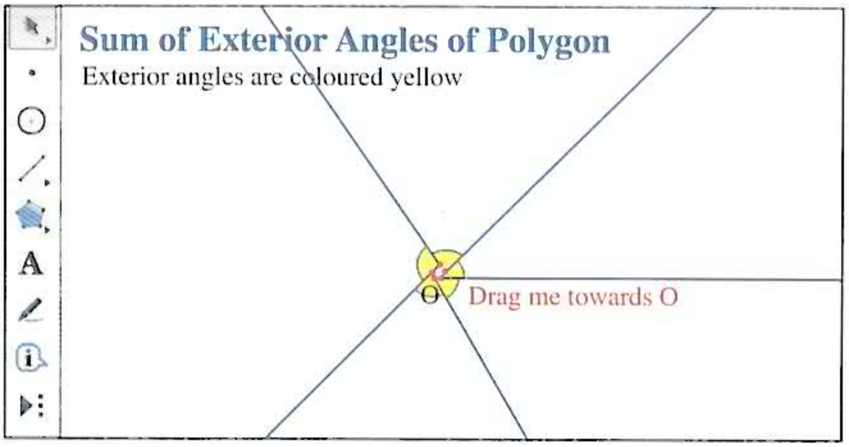


Fig. 11.20

2. What do you think is the sum of the exterior angles of a pentagon? Explain your answer.

From the investigation, we observe that the sum of exterior angles of a pentagon is 360°.
A proof of the above result is given as follows:

Consider the pentagon in Fig. 11.21.

We have $\angle a + \angle p = 180°$, $\angle b + \angle q = 180°$,

$\angle c + \angle r =$ ______, $\angle d + \angle s =$ ______________

and $\angle e + \angle t =$ ______.

$\therefore \angle a + \angle p + \angle b + \angle q + \angle c + \angle r + \angle d + \angle s + \angle e + \angle t =$ ____ $\times 180°$

$(\angle a + \angle b + \angle c + \angle d + \angle e) + (\angle p + \angle q + \angle r + \angle s + \angle t) = 900°$

Since the sum of interior angles of a pentagon $= \angle a + \angle b + \angle c + \angle d + \angle e$

$= (5 - 2) \times 180° = 540°$,

$540° + (\angle p + \angle q + \angle r + \angle s + \angle t) = 900°$.

$\therefore \angle p + \angle q + \angle r + \angle s + \angle t = 900° -$ __________ $=$ __________

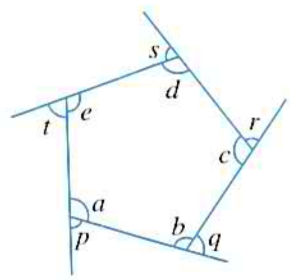


Fig. 11.21

By using this method, we can show that the sum of exterior angles of a hexagon, of a heptagon and of an octagon is also 360°.

In general, we have:

The sum of exterior angles of any polygon is 360°.

Thinking Time

1. **(i)** Is it possible for a regular polygon to have an exterior angle of 70°? Explain your answer.
   (ii) If an exterior angle of a regular polygon is an integer, what are all the possible values of the angle?
2. Is it possible for a regular polygon to be a concave polygon? Explain your answer.

Worked Example 8

(Finding the Number of Sides of a Regular Polygon)

Calculate the number of sides of a regular polygon if

(a) each exterior angle of the polygon is 24°,

(b) each interior angle of the polygon is 162°.

Solution:

(a) The sum of exterior angles of the regular polygon is 360°.

$$\therefore \text{ Number of sides of the polygon} = \frac{360^\circ}{24^\circ} = 15$$

(b) Method 1:

Let the number of sides of the regular polygon = n.

$$\text{Size of each interior angle of the polygon} = \frac{(n-2)\times 180^\circ}{n}$$

$$\begin{aligned}
\therefore \frac{(n-2)\times 180^\circ}{n} &= 162^\circ \\
(n-2)\times 180^\circ &= 162^\circ n \\
180n - 360 &= 162n \\
180n - 162n &= 360 \\
18n &= 360 \\
\therefore n &= 20
\end{aligned}$$

Method 2:

$$\begin{aligned}
\text{Size of each exterior angle of the regular polygon} &= 180^\circ - 162^\circ \\
&= 18^\circ \text{ (adj. } \angle\text{s on a str. line)}
\end{aligned}$$

$$\therefore \text{ Number of sides of the polygon} = \frac{360^\circ}{18^\circ} = 20$$

Problem Solving Tip

Generally, it is easier to use the formula for the sum of exterior angles of a polygon. We can find each exterior angle by using this formula: int. $\angle$ + ext. $\angle$ = 180°.

PRACTISE NOW 8

SIMILAR QUESTIONS

Exercise 11C Questions 4(a)–(b), 5(a)–(d), 6(a)–(d), 8–10

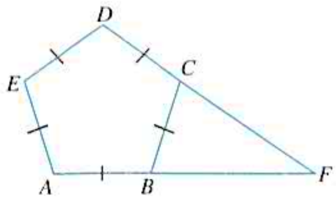
1. Find the number of sides of a regular polygon if
   (a) each exterior angle of the polygon is 40°,
   (b) each interior angle of the polygon is 178°.
2. By finding the size of each exterior angle of a regular decagon, find the size of each interior angle of the decagon.
   Note: Refer to Worked Example 7. This is another method to find the size of each interior angle of a regular polygon.
3. Two of the exterior angles of an n-sided polygon are 25° and 26°, three of its interior angles are 161° each and the remaining interior angles are 159° each. Find the value of n.

Worked Example 9

(Problem involving a Regular Pentagon)

$ABCDE$ is a regular pentagon. If AB and DC are produced to meet at F, find the value of $B\hat{F}C$.

Solution:



Size of each exterior angle of the pentagon $= \frac{360^\circ}{5}$

$= 72^\circ$

$C\hat{B}F = B\hat{C}F = 72^\circ$

$B\hat{F}C + 72^\circ + 72^\circ = 180^\circ$ ($\angle$ sum of $\triangle BCF$)

$B\hat{F}C = 180^\circ - 72^\circ - 72^\circ$

$= 36^\circ$

PRACTISE NOW 9

SIMILAR QUESTIONS

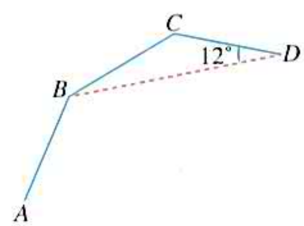
$ABCDEF$ is a regular hexagon. If AB and DC are produced to meet at G, find the value of $B\hat{G}C$.

Exercise 11C Question 11

Worked Example 10

(Problem involving an n-sided Polygon)

AB, BC and CD are adjacent sides of an n-sided regular polygon.



If $B\hat{D}C = 12^\circ$, calculate

(i) the size of an exterior angle of the polygon,

(ii) the value of n,

(iii) $A\hat{B}D$.

Solution:

(i) $C\hat{B}D = 12^\circ$ (base $\angle$s of isos. $\triangle BCD$)

Size of each exterior angle of the regular polygon $= 12^\circ + 12^\circ$ (ext. $\angle$ of $\triangle BCD$)

$= 24^\circ$

(ii) Value of $n = \frac{360^\circ}{24^\circ}$

$= 15$

(iii) $A\hat{B}C + 24^\circ = 180^\circ$ (adj. $\angle$s on a str. line)

$A\hat{B}C = 180^\circ - 24^\circ$

$= 156^\circ$

$A\hat{B}D = A\hat{B}C - C\hat{B}D$

$= 156^\circ - 12^\circ$

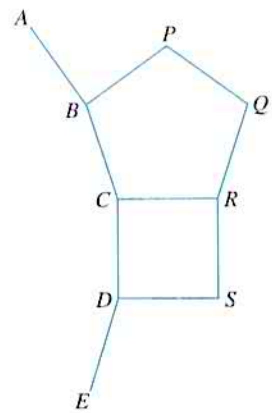
$= 144^\circ$

PRACTISE NOW 10

SIMILAR QUESTIONS

Exercise 11C Questions 12–15

In the figure, $ABCDE$ is part of an n-sided regular polygon, $BPQRC$ is a regular pentagon and $CRSD$ is a square.



Find

(i) $P\hat{B}C$, **(ii)** $Q\hat{C}R$,
(iii) $B\hat{C}D$, **(iv)** $B\hat{D}C$,
(v) the value of n.

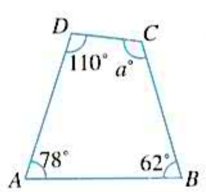
Exercise 11C

BASIC LEVEL

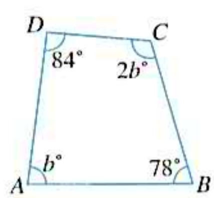
1. Find the sum of the interior angles of each of the following polygons.
(a) 11-gon **(b)** 12-gon
(c) 15-gon **(d)** 20-gon

2. Find the value of the unknown in each of the following figures.

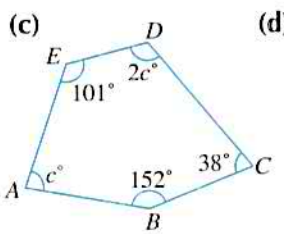
(a)



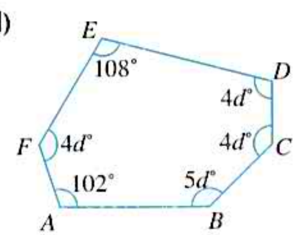
(b)



(c)



(d)



3. **(a)** **(i)** Find the sum of interior angles of a regular hexagon.
(ii) Hence, find the size of each interior angle of a regular hexagon.
(b) **(i)** Find the sum of interior angles of a regular polygon with 18 sides.
(ii) Hence, find the size of each interior angle of a regular polygon with 18 sides.

4. **(a)** By finding the size of each exterior angle of a regular polygon with 24 sides, calculate the size of each interior angle of the polygon.
(b) By finding the size of each exterior angle of a regular polygon with 36 sides, calculate the size of each interior angle of the polygon.

5. Find the number of sides of a regular polygon if each exterior angle of the polygon is
(a) 90°, **(b)** 45°, **(c)** 12°, **(d)** 4°.

6. Find the number of sides of a regular polygon if each interior angle of the polygon is
(a) 140°, **(b)** 162°, **(c)** 172°, **(d)** 175°.

7. If the sizes of the interior angles of a pentagon are $2x°$, $3x°$, $4x°$, $5x°$ and $6x°$, find the largest interior angle of the pentagon.

8. The exterior angles of a triangle are $3y°$, $4y°$ and $5y°$.
(i) Find the value of y.
(ii) Find the smallest interior angle of the triangle.

INTERMEDIATE LEVEL

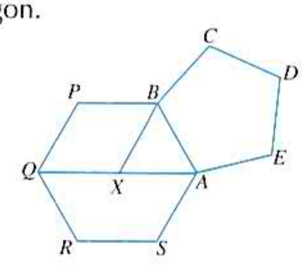
9. Three of the exterior angles of an n-sided polygon are 15°, 25° and 70°, and the remaining exterior angles are 50° each. Find the value of n.

10. Three of the exterior angles of an n-sided polygon are 50° each, two of its interior angles are 127° and 135°, and the remaining interior angles are 173° each. Find the value of n.

11. $ABCDEFG$ is a regular heptagon. If AB and DC are produced to meet at H, find the value of $B\hat{H}C$.

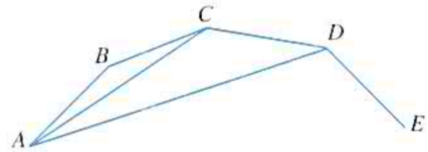
12. The points A, B, C and D are consecutive vertices of a regular polygon with 20 sides. Find
(i) $A\hat{B}C$,
(ii) $A\hat{B}D$.

13. In the figure, $ABCDE$ is a regular pentagon and $ABPQRS$ is a regular hexagon. X is the centre of the hexagon.



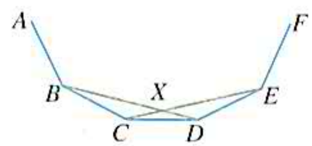
Find
(i) $A\hat{B}P$,
(ii) $P\hat{Q}X$,
(iii) $A\hat{X}B$,
(iv) $A\hat{B}C$,
(v) $A\hat{C}D$,
(vi) $A\hat{S}E$.

14. In the figure, $ABCDE$ is part of an n-sided regular polygon. The ratio of an interior angle to an exterior angle of this polygon is 5 : 1.



Find
(i) the value of n,
(ii) $A\hat{C}D$,
(iii) $A\hat{D}E$.

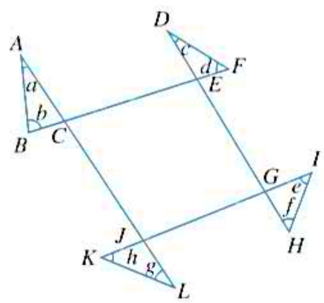
15. In the figure, $ABCDEF$ is part of an n-sided regular polygon. Each exterior angle of this polygon is 36°.



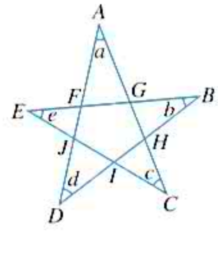
Find
(i) the value of n,
(ii) $B\hat{D}E$,
(iii) $C\hat{X}D$.

ADVANCED LEVEL

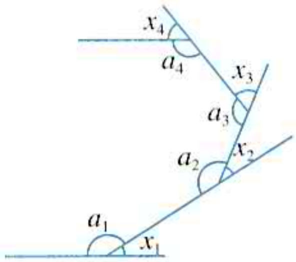
16. In the figure, $ACJL$, $BCEF$, $DEGH$ and $IGJK$ are straight lines.
Find $\angle a + \angle b + \angle c + \angle d + \angle e + \angle f + \angle g + \angle h$.



17. In the figure, $AFJD$, $AGHC$, $BGFE$, $BHID$ and $CIJE$ are straight lines.
Find $\angle a + \angle b + \angle c + \angle d + \angle e$.

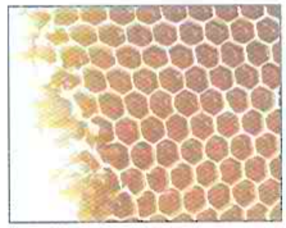


18. The figure shows part of an n-sided polygon where each side is produced. a_1, a_2, a_3, a_4, ..., and a_n are the interior angles of the polygon and x_1, x_2, x_3, x_4, ..., and x_n are its exterior angles.



Show that the sum of exterior angles of the polygon is 360°, i.e. $x_1 + x_2 + x_3 + x_4 + \ldots + x_n = 360°$.

19. The figure shows the internal structure of a beehive made up of regular hexagonal cells that can form tessellations with no overlaps or gaps.



(i) Name another two **regular** polygons that can form tessellations and sketch their tessellations.

(ii) Why are these regular polygons able to form tessellations?
Hint: What do you notice about their interior angles?

(iii) Show that there are no other regular polygons that can form tessellations.

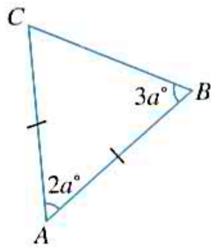
(iv) Suggest a reason why beehives are made up of regular hexagonal cells and not the other two regular polygons that can form tessellations too.

1. The sum of interior angles of a **triangle** is 180° (∠ sum of △).
2. An exterior angle of a triangle is equal to the sum of its interior opposite angles (ext. ∠ of △).
3. A **quadrilateral** is a closed 4-sided plane figure.
   - The sum of the interior angles of a quadrilateral is 360°.
   - The diagonals of a **rectangle** bisect each other and are equal in length.
   - The diagonals of a **square** bisect each other at 90°, are equal in length and they bisect the interior angles.
   - The diagonals of a **parallelogram** bisect each other.
   - The diagonals of a **rhombus** bisect each other at 90° and they bisect the interior angles.
   - The diagonals of a **kite** cut each other at 90° and one of them bisects the interior angles.
4. In a polygon, interior angle + exterior angle = 180°.
5. Sum of interior angles of an n-sided polygon = $(n - 2) \times 180°$
6. A pattern formed by fitting together regular figures which completely cover a plane surface is called a **tessellation**.
7. Sum of exterior angles of an n-sided polygon = 360°

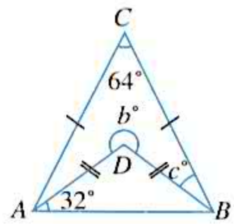
Review Exercise 11

1. Find the value(s) of the unknown(s) in each of the following figures.

(a)

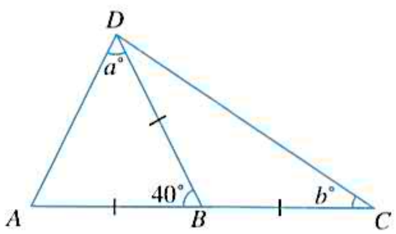


(b)

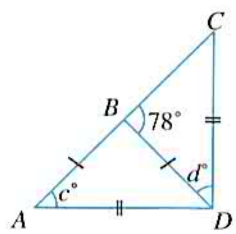


2. Given that *ABC* is a straight line, find the values of the unknowns in each of the following figures.

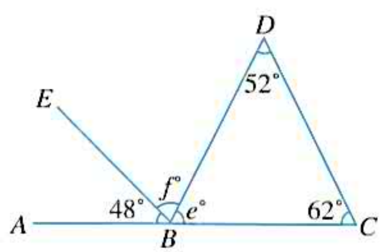
(a)



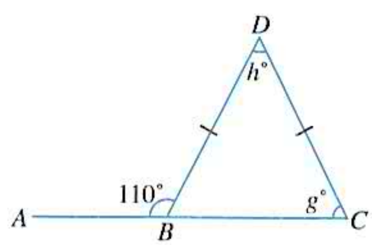
(b)



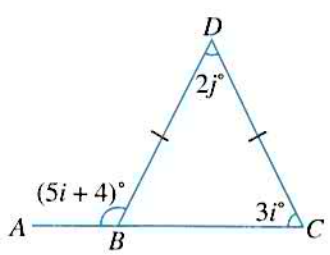
(c)



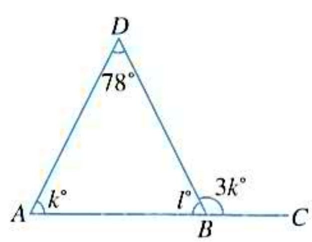
(d)



(e)

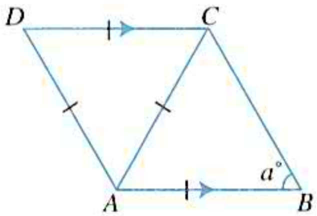


(f)

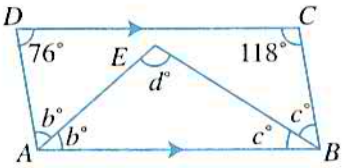


3. Given that $AB \parallel DC$, find the value(s) of the unknown(s) in each of the following figures.

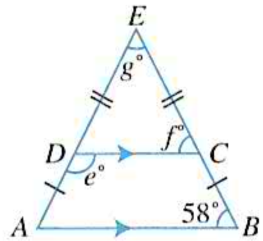
(a)



(b)

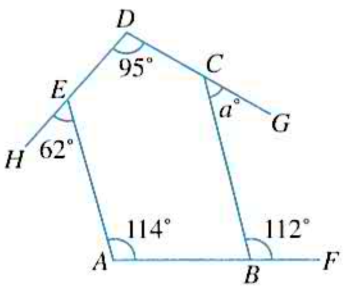


(c)

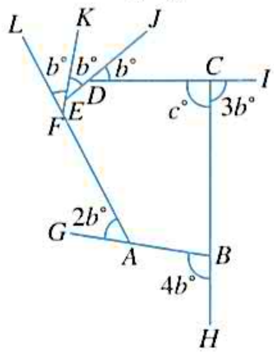


4. Find the value(s) of the unknown(s) in each of the following figures.

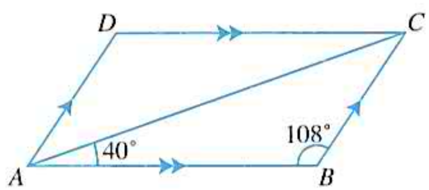
(a)



(b)

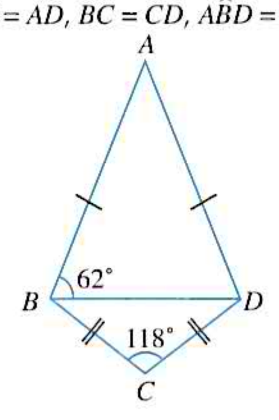


5. The figure shows a parallelogram $ABCD$ where $A\hat{B}C = 108°$ and $B\hat{A}C = 40°$.



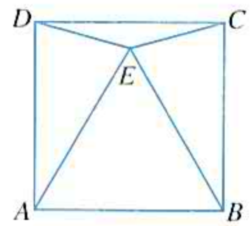
Find
(i) $A\hat{C}D$,
(ii) $C\hat{A}D$.

6. The figure shows a kite $ABCD$ where $AB = AD$, $BC = CD$, $A\hat{B}D = 62°$ and $B\hat{C}D = 118°$.



Find
(i) $B\hat{A}D$,
(ii) $B\hat{D}C$.

7. The figure shows a square $ABCD$ and an equilateral triangle ABE. Find $C\hat{E}D$.



8. The sum of interior angles of a polygon with $(2n - 3)$ sides is equal to 62 right angles. Find the value of n.

9. One of the interior angles of an n-sided polygon is 126° and the remaining interior angles are 162° each. Find the value of n.

10. The ratio of the interior angles of a pentagon is $3 : 4 : 5 : 5 : 7$. Find
    (i) the largest interior angle,
    (ii) the largest exterior angle.

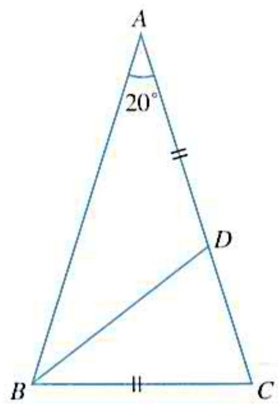
11. Two of the exterior angles of an n-sided polygon are 35° and 72°, and the remaining exterior angles are 23° each. Find the value of n.

12. The ratio of an interior angle to an exterior angle of an n-sided regular polygon is $13 : 2$. Find the value of n.

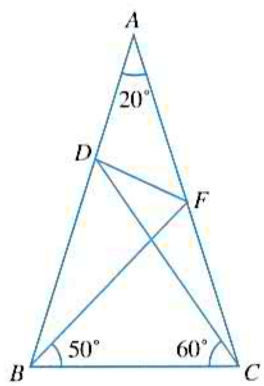
13. If the sum of the interior angles of an n-sided polygon is four times the sum of its exterior angles, find the value of n.

Challenge Yourself

1. In the figure, $\triangle ABC$ is an isosceles triangle in which $AB = AC$ and $B\hat{A}C = 20°$. If $AD = BC$, find $A\hat{D}B$.



2. In the figure, $\triangle ABC$ is an isosceles triangle in which $AB = AC$ and $B\hat{A}C = 20°$. If $C\hat{B}F = 50°$ and $B\hat{C}D = 60°$, find $C\hat{D}F$.

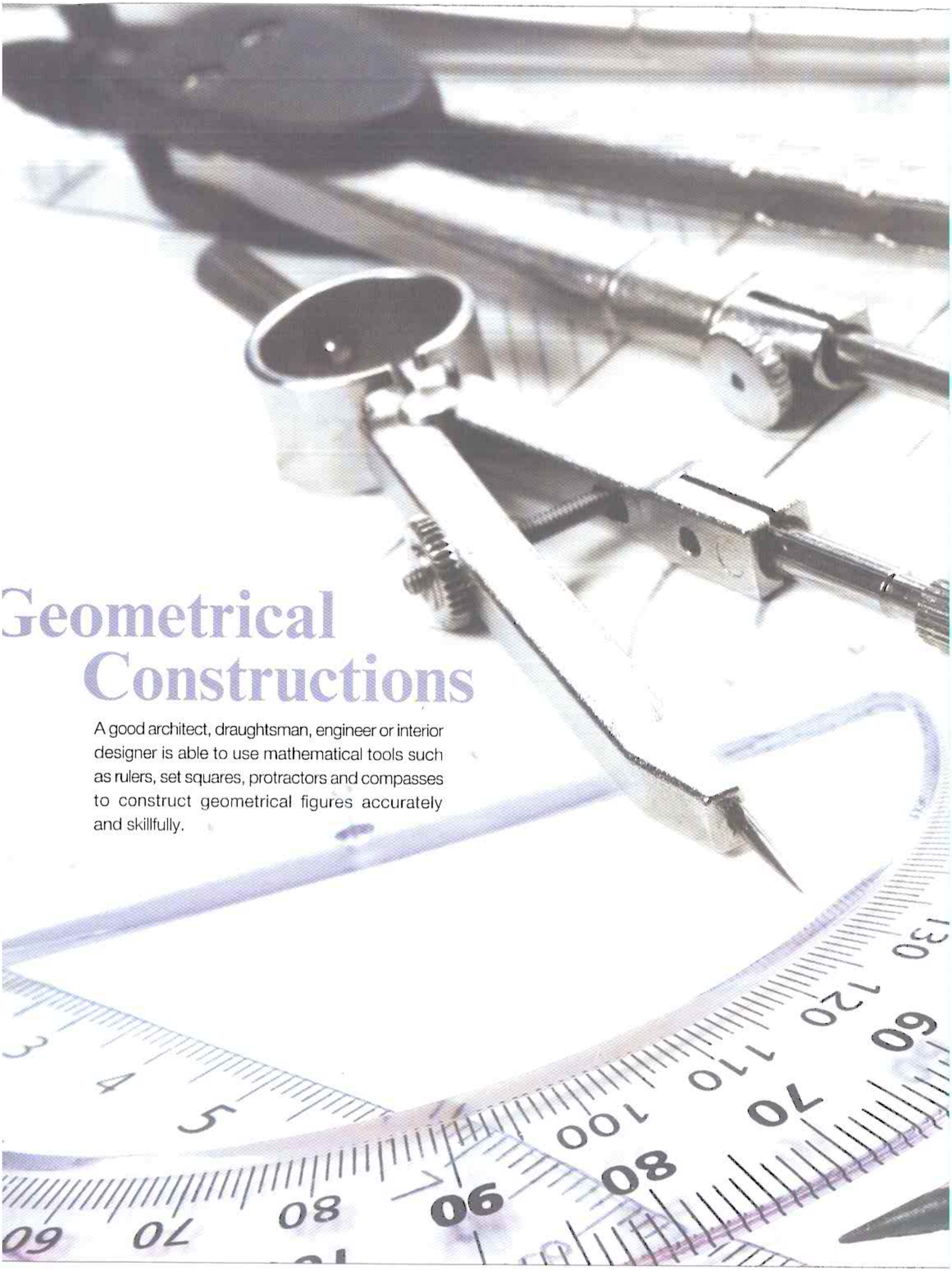


3. The sum of interior angles of a convex n-gon is $(n - 2) \times 180°$. Does this formula apply to the sum of interior angles of a concave n-gon? Explain your answer.

4. **(i)** How do you define an exterior angle of a concave polygon?

   (ii) The sum of exterior angles of any convex n-gon is $360°$. Is the sum of exterior angles of a concave n-gon also $360°$? Explain your answer.

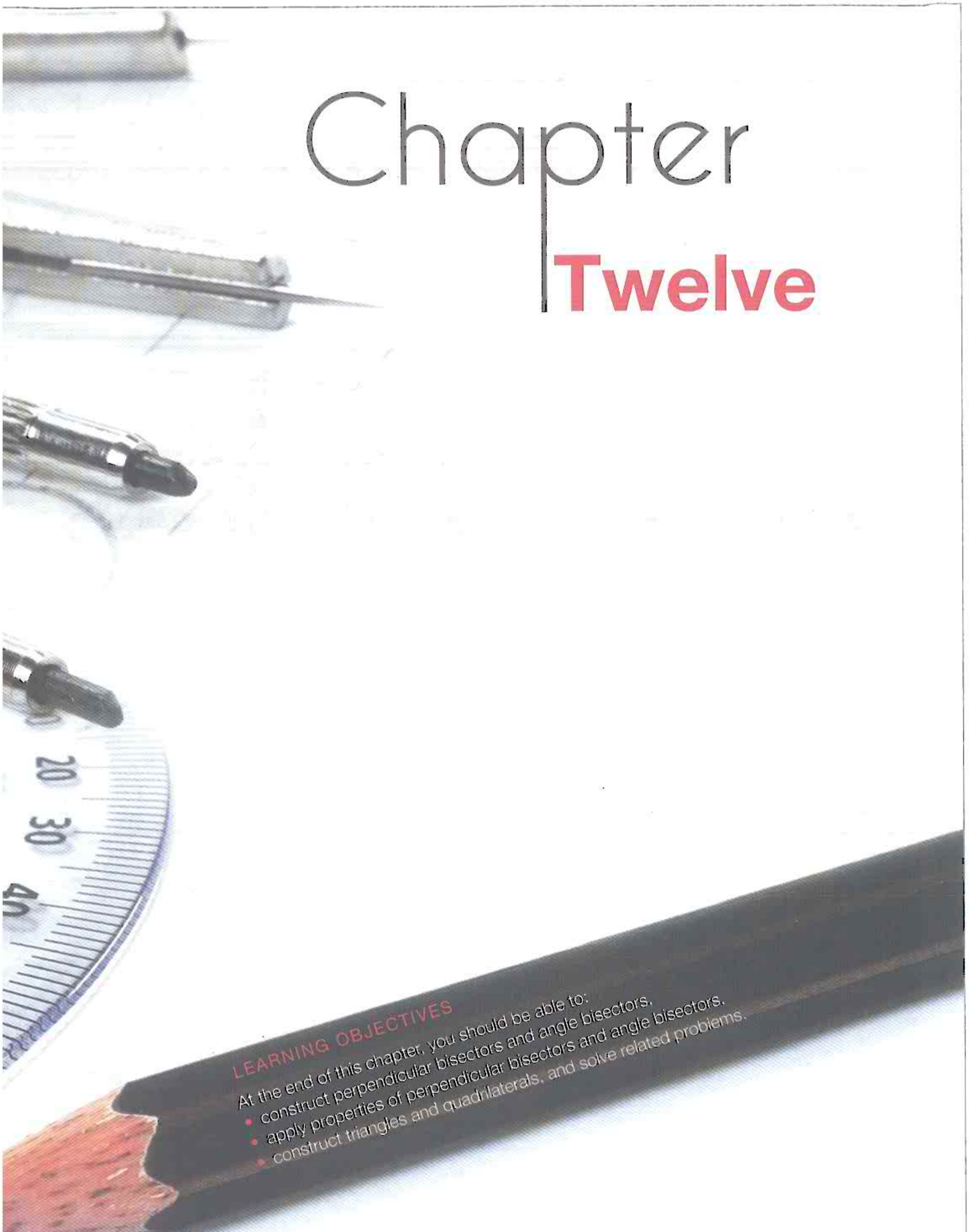
5. Find a formula for the number of diagonals of an n-sided polygon.



Geometrical Constructions

A good architect, draughtsman, engineer or interior designer is able to use mathematical tools such as rulers, set squares, protractors and compasses to construct geometrical figures accurately and skillfully.

Chapter Twelve



LEARNING OBJECTIVES

At the end of this chapter, you should be able to:

- construct perpendicular bisectors and angle bisectors,
- apply properties of perpendicular bisectors and angle bisectors,
- construct triangles and quadrilaterals, and solve related problems.

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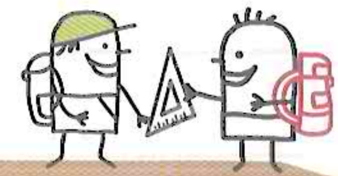
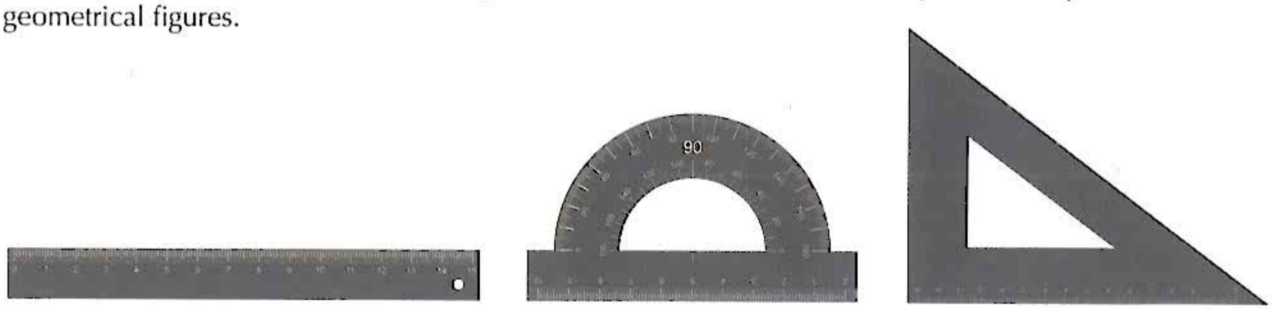
12.1 Introduction to Geometrical Constructions



Recap (Use of Rulers, Protractors and Set Squares)

In primary school, we have learnt how to draw triangles and quadrilaterals using rulers, protractors and set squares (see Fig. 12.1). In this chapter, we shall learn how to use a pair of compasses to construct geometrical figures.



(a) (b) (c)

Fig. 12.1

Use of Compasses

A pair of compasses (see Fig. 12.2) is a mathematical instrument consisting of two moveable arms attached together by a hinge. It is used for drawing a circle or an arc of a circle, and for marking off a length.

INFORMATION

A pair of compasses is different from a compass which is used to tell directions.

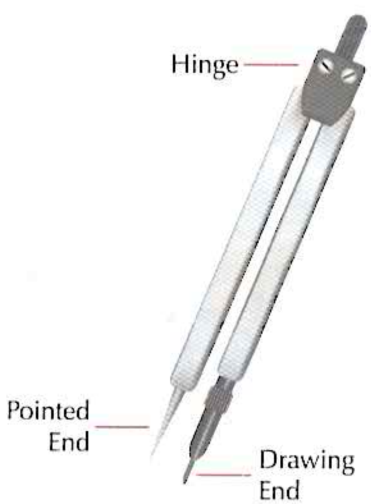


Fig. 12.2

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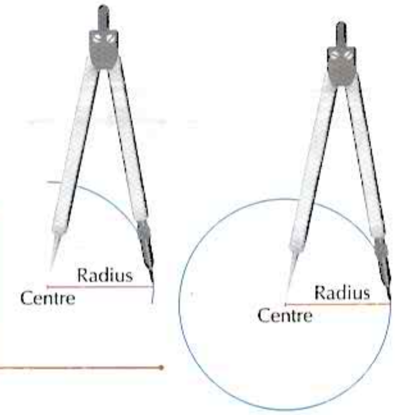
The following show how compasses can be used.

- Drawing a Circle or an Arc of a circle

Step 1: Adjust the arms of the compasses so that the distance between the ends is equal to the radius of the circle.

Step 2: Fix the pointed end at the point which is the centre of the circle and move the other arm to draw the circle or an arc of a circle.



- Marking off a Length

Step 1: Adjust the arms of the compasses until the ends touch points A and B.

Step 2: Mark a point P on another line L.

Step 3: Without adjusting the arms of the compasses, fix the pointed end at P and move the other arm to draw an arc cutting L at Q. Hence, $PQ = AB$.

Useful Tips for Geometrical Constructions

The following tips are useful for the construction of geometrical figures:

- Use a sharp pencil so that points and lines can be drawn finely and clearly.
- When making an intersection with a line or an arc, ensure that the angle of intersection does not differ greatly from 90° if possible (see Fig. 12.3).

(a) Good

(b) Bad

Fig. 12.3

- Exercise caution when drawing a line through a point to ensure accuracy (see Fig. 12.4).

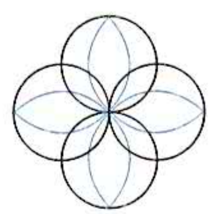
(a) Good

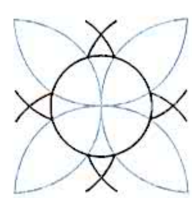
(b) Bad

Fig. 12.4

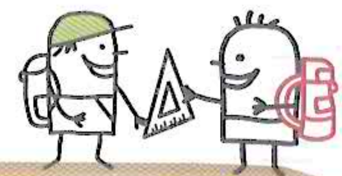
- All construction lines must be clearly shown. Do not erase the construction lines that have been drawn.

Create the following designs using a pair of compasses.





12.2 Perpendicular Bisectors and Angle Bisectors



Perpendicular Bisectors

In Fig. 12.5, M is the midpoint of the line segment AB. As the line XY passes through M, XY is said to *bisect* AB, i.e. XY divides AB into two equal parts. XY is also *perpendicular* to AB. Therefore, XY is known as the **perpendicular bisector** of AB.

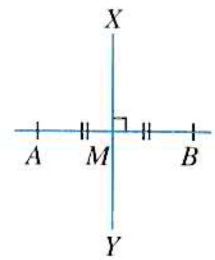


Fig. 12.5

Worked Example 1

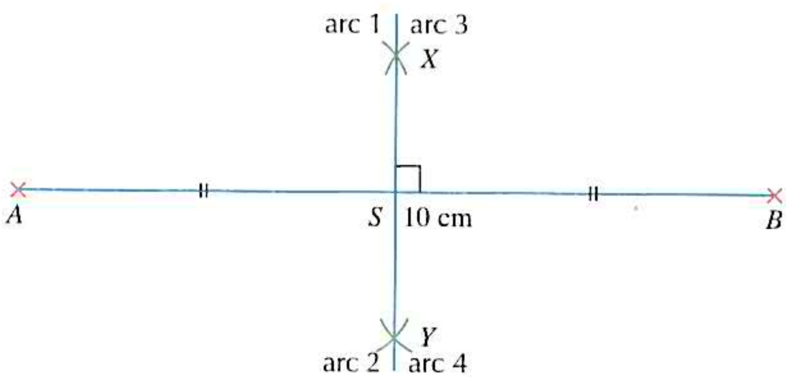
(Construction of a Perpendicular Bisector)

Draw a line segment AB of length 10 cm. Construct the perpendicular bisector of AB.

Solution:

Construction Steps:

1. Using a ruler, draw a line segment AB of length 10 cm.
2. Adjust the arms of the compasses so that the distance between the ends is more than half the length of AB, i.e. more than 5 cm. With A as centre, draw arc 1 above AB and draw arc 2 below AB.
3. Using the same radius as in Step 2, with B as centre, draw arc 3 to cut arc 1 at X and draw arc 4 to cut arc 2 at Y.
4. Join XY to cut AB at S.



Note: XY is the perpendicular bisector of AB, where $AS = SB$ and XY is perpendicular to AB.

INFORMATION

A perpendicular bisector can also be constructed by using a ruler and a protractor.

PRACTISE NOW 1

Draw a line segment AB of length 8 cm. Construct the perpendicular bisector of AB.

SIMILAR QUESTIONS

Exercise 12A Questions 1, 17

Investigation

Property of a Perpendicular Bisector

In this investigation, we shall construct and deduce a property of the perpendicular bisector of a line segment. A suitable interactive geometry software may be used.

1. Draw a line segment AB.
2. Construct the perpendicular bisector of AB.
3. Join A to any point C on the perpendicular bisector. Then join B to C.
4. Measure the length of AC and of BC. What do you notice?
5. Click and drag the point C along the perpendicular bisector. What conclusion can you make?
6. Join A and B to any point D on the left of the perpendicular bisector. Measure the length of AD and of BD. Click and drag the point D such that D is not on the perpendicular bisector. What conclusion can you make?

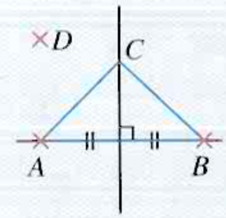


Fig. 12.6

ATTENTION

The perpendicular bisector of AB is the line of symmetry of ΔABC. Therefore, C is equidistant from the two end points of the line segment, i.e. A and B.

From the investigation, we can conclude that:

Any point on the perpendicular bisector of a line segment is *equidistant* from the two end points of the line segment.

Angle Bisectors

In Fig. 12.7, the ray AX divides $B\hat{A}C$ into two *equal* angles, i.e. $B\hat{A}X = C\hat{A}X$. Hence, AX is known as the **angle bisector** of $B\hat{A}C$.

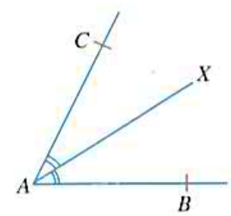


Fig. 12.7

Worked Example 2

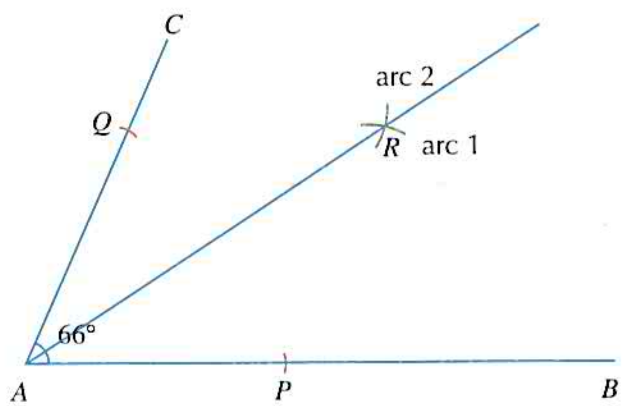
(Construction of an Angle Bisector)

Draw an angle BAC of $66°$. Construct the angle bisector of $B\hat{A}C$.

Solution:

Construction Steps:

1. Using a ruler and a protractor, draw an angle BAC of $66°$.
2. With A as centre and with a suitable fixed radius, draw an arc to cut AB at P and AC at Q.
3. With P as centre and with a suitable radius (may be different from that in Step 2), draw arc 1.



4. Using the same radius as in Step 3, with Q as centre, draw arc 2 to cut arc 1 at R.
5. Join AR.

Note: AR is the angle bisector of $B\hat{A}C$, where $B\hat{A}R = C\hat{A}R$.

INFORMATION

An angle bisector can also be constructed by using a ruler and a protractor.

Internet Resources

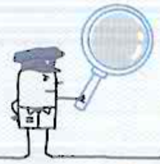
It can be proven that in general, it is not possible to use a pair of compasses and a straightedge (unmarked ruler) to trisect an angle, i.e. divide the angle into three equal parts. However, we can trisect an angle using origami (paper folding). Search on the Internet to find out how this can be done.

PRACTISE NOW 2

SIMILAR QUESTIONS

Draw an angle BAC of 78°. Construct the angle bisector of $B\hat{A}C$.

Exercise 12A Question 2



Investigation

Property of an Angle Bisector

In this investigation, we shall construct and deduce a property of the angle bisector of an angle. A suitable interactive geometry software may be used.

1. Draw an angle BAC.
2. Construct the angle bisector of $B\hat{A}C$.
3. Take any point R on the angle bisector and construct a line passing through R and perpendicular to AB. Let the point of intersection of this line and AB be P. PR is the *perpendicular (or shortest) distance* of R from AB. Measure the length of PR.
4. Similarly, construct a line passing through R and perpendicular to AC. Let the point of intersection of this line and AC be Q. QR is the perpendicular (or shortest) distance of R from AC. Measure the length of QR.
5. What do you notice about the length of PR and of QR?
6. Click and drag the point R along the angle bisector. What conclusion can you make?
7. Take any point S on the left of the angle bisector and construct a line passing through A and S. Measure the perpendicular distance from the line to AB and AC respectively. Click and drag the point S such that S is not on the angle bisector. What conclusion can you make?

RECALL

A line passing through a point and perpendicular to another line can be constructed using a ruler and a set square.

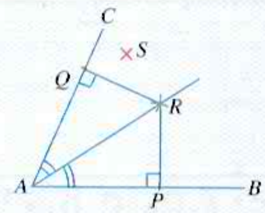


Fig. 12.8

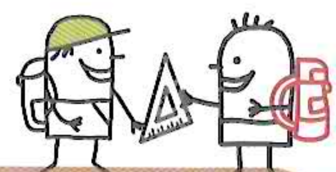
From the investigation, we can conclude that:

Any point on the angle bisector of an angle is *equidistant* from the two sides of the angle.

ATTENTION

AR is the line of symmetry of the quadrilateral $APRQ$. Therefore, R is equidistant from the lines AP and AQ.

12.3 Construction of Triangles



In this section, we shall learn how to construct triangles and solve related problems.

Worked Example 3

(Construction of a Triangle Given 2 Sides and an Included Angle)

Construct ΔABC such that $AB = 10.5$ cm, $BC = 7.5$ cm and $A\hat{B}C = 60°$.

(i) Measure and write down the length of AC.

(ii) Construct the perpendicular bisector of AB such that it cuts AC. Measure and write down the length of BS, such that S is the point where the perpendicular bisector of AB cuts AC.

Solution:

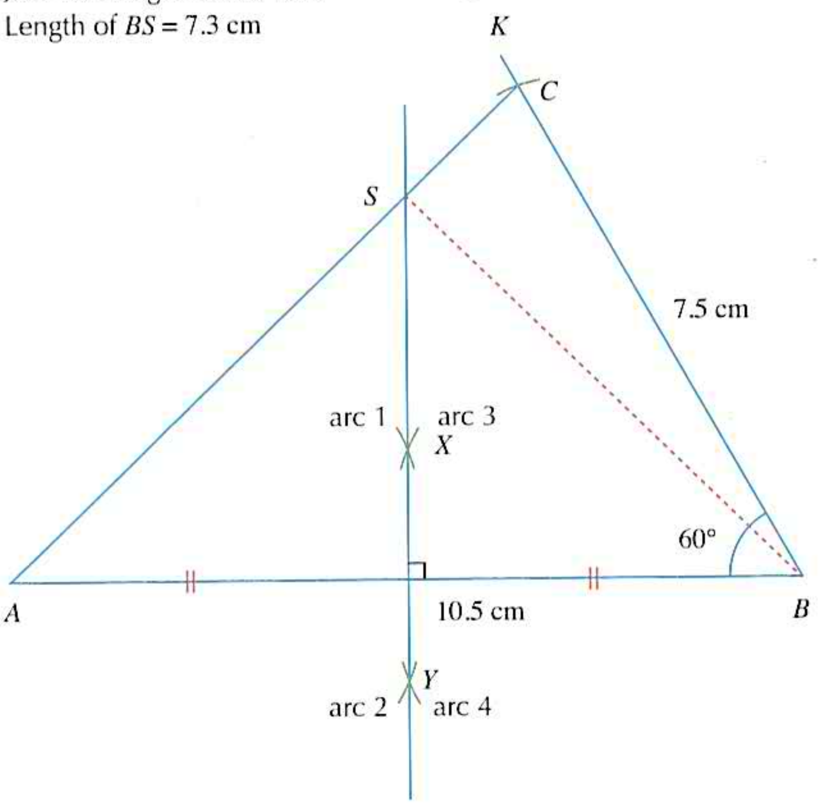
Construction Steps:

1. Using a ruler, draw $AB = 10.5$ cm.
2. Since $\angle B = 60^\circ$, using a protractor at B, mark off an angle of 60° and draw a line BK such that $A\hat{B}K = 60^\circ$.
3. Since C is 7.5 cm away from B, with B as centre and 7.5 cm as radius, draw an arc to cut BK at C.
4. Join AC.

(i) Length of $AC = 9.4$ cm

(ii) Construction Steps:

1. With A as centre and radius more than half the length of AB, draw arc 1 above AB and draw arc 2 below AB.
2. With B as centre and radius the same as in the previous step, draw arc 3 to cut arc 1 at X and draw arc 4 to cut arc 2 at Y.
3. Join XY to obtain the perpendicular bisector of AB.
4. Extend the perpendicular bisector of AB such that it cuts AC. Label the point where the perpendicular bisector of AB cuts AC as S.
   Join BS using a dotted line.
   Length of $BS = 7.3$ cm



ATTENTION

Alternatively, the line BC can be drawn first. However, AB is drawn first as it is the longer line and hence, allows us to ensure that the figure does not exceed the space given.

ATTENTION

Use dotted lines for lines that are part of your working.

PRACTISE NOW 3

Construct ΔABC such that $AB = 7.6$ cm, $BC = 4.8$ cm and $A\hat{B}C = 130^\circ$.

(i) Measure and write down the length of AC.

(ii) Construct the perpendicular bisector of AB such that it cuts AC.
Measure and write down the length of BS, such that S is the point where the perpendicular bisector of AB cuts AC.

SIMILAR QUESTIONS

Exercise 12A Questions 3–4, 8–9, 14–15

Worked Example 4

(Construction of a Triangle and Given 3 Sides)

Construct ΔPQR such that $PQ = 10.5$ cm, $PR = 8.5$ cm and $QR = 9.5$ cm.

(i) Measure and write down the size of the angle facing the longest side.

(ii) Construct the angle bisector of $Q\hat{P}R$ such that it cuts QR. Measure and write down the length of QT, such that T is the point where the angle bisector of $Q\hat{P}R$ cuts QR.

Solution:

Construction Steps:

1. Using a ruler, draw $PQ = 10.5$ cm.
2. Since R is 8.5 cm away from P, with P as centre and 8.5 cm as radius, draw arc 1.
3. Since R is 9.5 cm away from Q, with Q as centre and 9.5 cm as radius, draw arc 2 to cut arc 1 at R.
4. Join PR and QR.

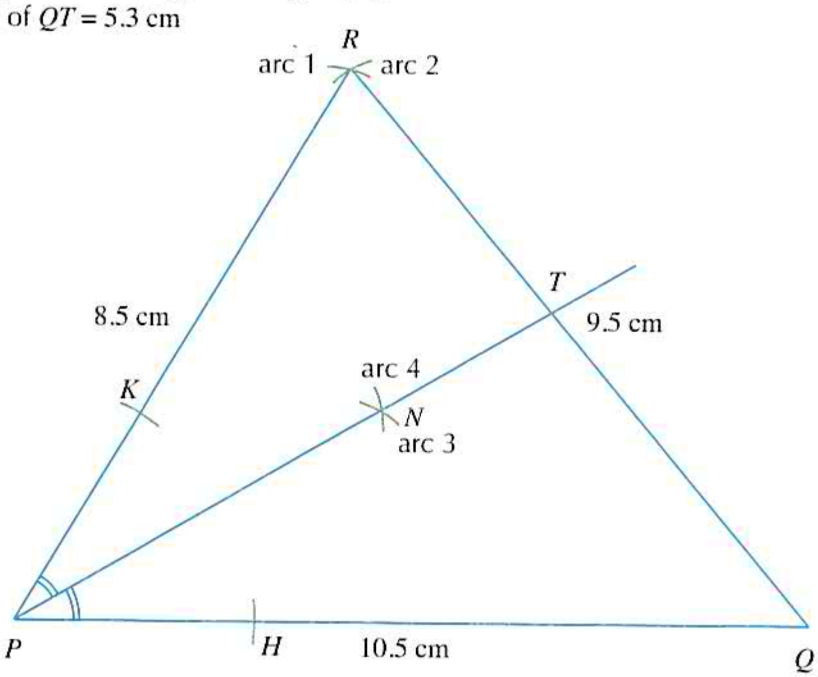
(i) PQ is the longest side.

$\therefore$ Required angle, $P\hat{R}Q = 71°$

(ii) Construction Steps:

1. With P as centre and with a suitable fixed radius, draw an arc to cut PQ at H and PR at K.
2. With H as centre and with a suitable radius (may be different from that in the previous step), draw arc 3. Using the same radius, with K as centre, draw arc 4 to cut arc 3 at N.
3. Join PN to obtain the angle bisector of $Q\hat{P}R$.
4. Extend the angle bisector of $Q\hat{P}R$ such that it cuts QR. Label the point where the angle bisector of $Q\hat{P}R$ cuts QR as T.

Length of $QT = 5.3$ cm



ATTENTION

Alternatively, the lines PR or QR can be drawn first. However, PQ is drawn first as it is the longest line and hence, allows us to ensure that the figure does not exceed the space given.

Construct an angle of 60° by using a ruler and a pair of compasses only.

PRACTISE NOW 4

Construct ΔPQR such that $PQ = 8.4$ cm, $PR = 7.2$ cm and $QR = 9.8$ cm.

(i) Measure and write down the size of the angle facing the longest side.

(ii) Construct the angle bisector of $Q\hat{P}R$ such that it cuts QR.
Measure and write down the length of QT, such that T is the point where the angle bisector of $Q\hat{P}R$ cuts QR.

SIMILAR QUESTIONS

Exercise 12A Questions 5–6, 10–11, 16

Worked Example 5

(Construction of a Triangle Given 1 Side and 2 Angles)

Construct ΔXYZ such that $XY = 9$ cm, $X\hat{Y}Z = 38°$ and $Y\hat{X}Z = 67°$.

(i) Construct the perpendicular bisector of XZ.

(ii) Construct the angle bisector of $X\hat{Z}Y$.

(iii) The two bisectors intersect at U.

Complete the following statement:

The point U is equidistant from the points _____ and _____, and equidistant from the lines _____ and _____.

Solution:

Construction Steps:

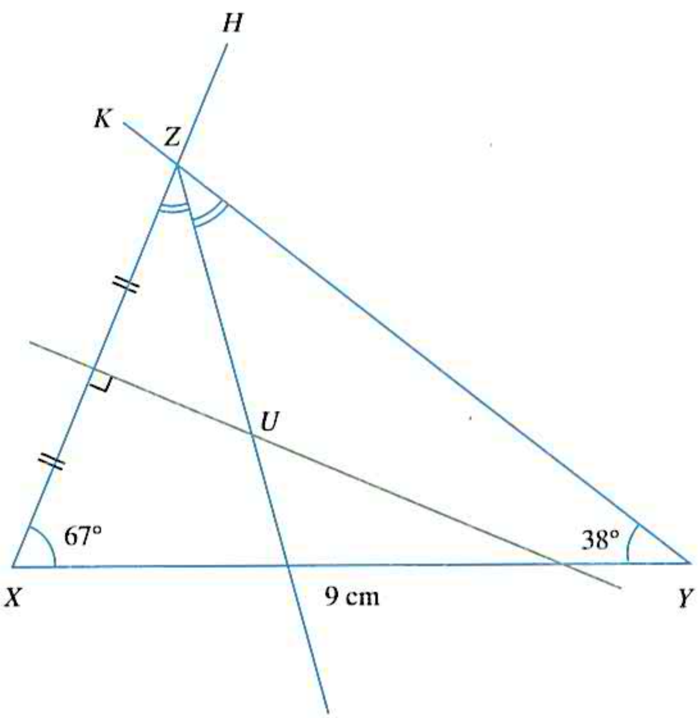
1. Using a ruler, draw $XY = 9$ cm.
2. Since $\angle X = 67°$, using a protractor at X, mark off an angle of $67°$ and draw a line XH such that $Y\hat{X}H = 67°$.
3. Since $\angle Y = 38°$, using a protractor at Y, mark off an angle of $38°$ and draw a line YK such that $X\hat{Y}K = 38°$.
4. Label the intersection of XH and YK as Z.

(i) Construct the perpendicular bisector of XZ.

(ii) Construct the angle bisector of $X\hat{Z}Y$.

(iii) Label the intersection of the two bisectors as U.

The point U is equidistant from the points X and Z, and equidistant from the lines XZ and YZ.



PRACTISE NOW 5

SIMILAR QUESTIONS

Exercise 12A Questions 7, 12–13

Construct ΔXYZ such that $XY = 8$ cm, $X\hat{Y}Z = 56°$ and $Y\hat{X}Z = 48°$.

(i) Construct the perpendicular bisector of YZ.

(ii) Construct the angle bisector of $Y\hat{X}Z$.

(iii) These two bisectors intersect at U.

Complete the following statement:

The point U is equidistant from the points _____ and _____, and equidistant from the lines _____ and _____.

Exercise 12A

BASIC LEVEL

1. Draw a line segment AB of length 9.5 cm. Construct the perpendicular bisector of AB.

2. Draw an angle BAC of 56°. Construct the angle bisector of $B\hat{A}C$.

3. Construct ΔABC such that $AB = 8$ cm, $BC = 6.5$ cm and $A\hat{B}C = 80°$. Measure and write down the length of AC.

4. Construct ΔABC such that $AB = 5$ cm, $BC = 9$ cm and $B\hat{A}C = 90°$. Measure and write down the length of AC.

5. Construct an isosceles triangle PQR such that $PQ = PR = 10$ cm and $QR = 9$ cm. Measure and write down the size of $Q\hat{P}R$.

6. Construct an equilateral triangle with sides 9.5 cm each.

7. Construct ΔXYZ such that $XY = 10.2$ cm, $X\hat{Y}Z = 60°$ and $Y\hat{X}Z = 45°$. Measure and write down the length of XZ.

INTERMEDIATE LEVEL

8. Construct ΔABC such that $AB = 9.8$ cm, $BC = 6.5$ cm and $A\hat{B}C = 88°$.
   (i) Measure and write down the length of AC.
   (ii) Construct the perpendicular bisector of AB such that it cuts AC. Measure and write down the length of BS, such that S is the point where the perpendicular bisector of AB cuts AC.

9. Construct ΔABC such that $AB = 9.4$ cm, $AC = 8.8$ cm and $A\hat{B}C = 60°$.
   (i) Measure and write down the size of the angle facing the shortest side.
   (ii) Construct the angle bisector of $B\hat{A}C$ such that it cuts BC. Measure and write down the length of CS, such that S is the point where the angle bisector of $B\hat{A}C$ cuts BC.

10. Construct ΔPQR such that $PQ = 9.5$ cm, $PR = 8.5$ cm and $QR = 9.8$ cm.
    (i) Measure and write down the size of the angle facing the shortest side.
    (ii) Construct the perpendicular bisector of QR such that it cuts PQ. Measure and write down the length of QT, such that T is the point where the perpendicular bisector of QR cuts PQ.

11. Construct ΔPQR such that $PQ = 8.8$ cm, $PR = 9.2$ cm and $QR = 10.4$ cm.
    (i) Measure and write down the size of the angle facing the longest side.
    (ii) Construct the angle bisector of $P\hat{Q}R$ such that it cuts PR. Measure and write down the length of PT, such that T is the point where the angle bisector of $P\hat{Q}R$ cuts PR.

12. Construct ΔXYZ such that $XY = 8$ cm, $X\hat{Y}Z = 55°$ and $Y\hat{X}Z = 64°$.
    (i) Measure and write down the length of XZ.
    (ii) Construct the perpendicular bisector of YZ such that it cuts XY. Measure and write down the length of UY, such that U is the point where the perpendicular bisector of YZ cuts XY.

13. Construct ΔXYZ such that $XY = 8$ cm, $X\hat{Y}Z = 49°$ and $Y\hat{X}Z = 74°$.
   (i) Construct the perpendicular bisector of XY.
   (ii) Construct the angle bisector of $X\hat{Y}Z$.
   (iii) These two bisectors intersect at U.
   Complete the following statement:
   The point U is equidistant from the points ______ and ______, and equidistant from the lines ______ and ______.

14. Construct ΔABC such that $AB = 10.2$ cm, $AC = 11$ cm and $B\hat{A}C = 62°$.
   (i) Measure and write down the length of BC.
   (ii) Construct a circle of radius 5 cm with its centre at C.
   (iii) Construct the angle bisector of $A\hat{C}B$ such that it cuts the circle at S inside the triangle and AB at T. Measure and write down the length of ST.

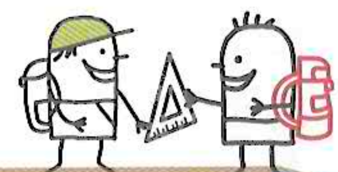
ADVANCED LEVEL

15. Construct ΔABC such that $AB = 8.5$ cm, $AC = 4.6$ cm and $B\hat{A}C = 54°$. The point S is such that it is equidistant from A, B and C. Find and label S.

16. Construct ΔPQR such that $PQ = 8.3$ cm, $PR = 9.2$ cm and $QR = 7.9$ cm. The point T is such that it is equidistant from PQ, PR and QR. Find and label T.

17. Construct a circle of any radius. Label the centre of the circle as O.
   (i) A chord of a circle is a line segment such that its end points lie on the circumference of the circle. A chord that passes through the centre of the circle is known as the diameter of the circle. Draw a chord in your circle that does not pass through O.
   (ii) Construct the perpendicular bisector of the chord which you have drawn in **(i)**. The perpendicular bisector should cut your circle at two points P and Q. What is the name given to PQ?

12.4 Construction of Quadrilaterals

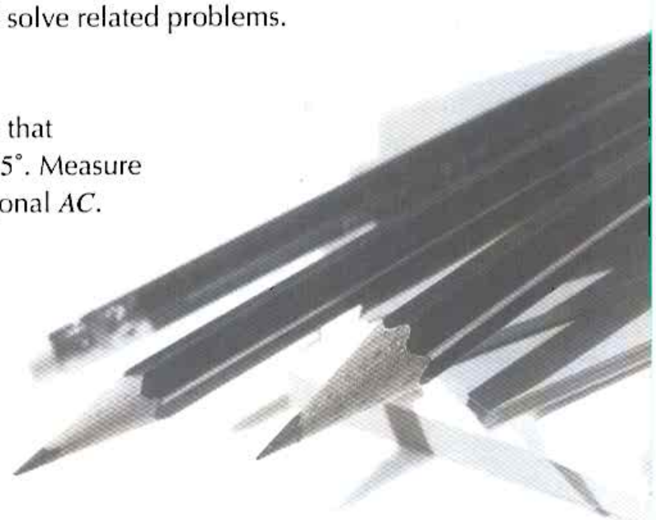


In this section, we shall learn how to construct quadrilaterals and solve related problems.

Worked Example 6

(Construction of a Parallelogram)

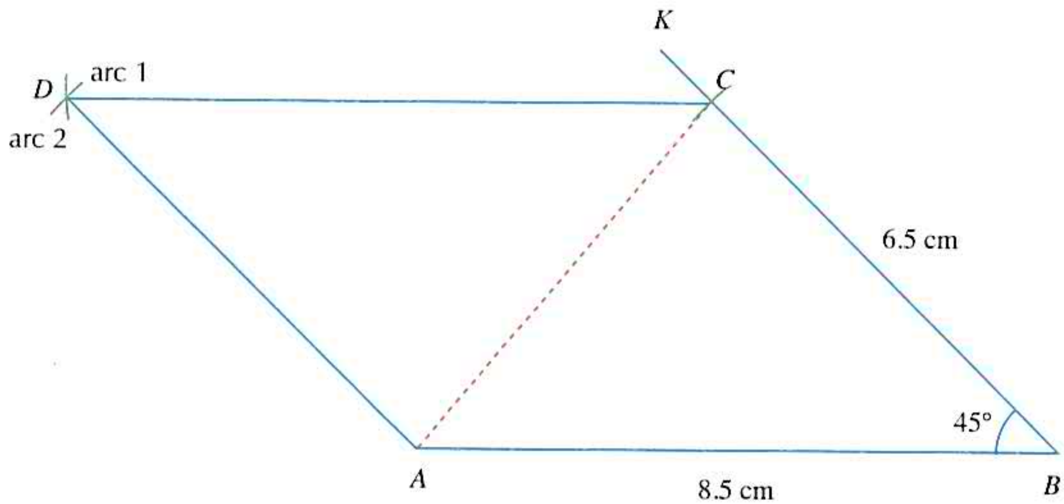
Construct a parallelogram $ABCD$ such that $AB = 8.5$ cm, $BC = 6.5$ cm and $A\hat{B}C = 45°$. Measure and write down the length of the diagonal AC.



Solution:

Construction Steps:

1. Using a ruler, draw $AB = 8.5$ cm.
2. Since $\angle B = 45^\circ$, using a protractor at B, mark off an angle of 45° and draw a line BK such that $A\hat{B}K = 45^\circ$.
3. Since C is 6.5 cm away from B, with B as centre and 6.5 cm as radius, draw an arc to cut BK at C.
4. Since $ABCD$ is a parallelogram, $AD = BC = 6.5$ cm. With A as centre and 6.5 cm as radius, draw arc 1.
5. Similarly, $CD = BA = 8.5$ cm. With C as centre and 8.5 cm as radius, draw arc 2 to cut arc 1 at D.
6. Join AD and CD.
7. Join AC using a dotted line.
   Length of $AC = 6.0$ cm



PRACTISE NOW 6

SIMILAR QUESTIONS

Exercise 12B Questions 1–3, 9

1. Construct a parallelogram $ABCD$ such that $AB = 8.5$ cm, $BC = 5.5$ cm and $A\hat{B}C = 120^\circ$. Measure and write down the length of the diagonal AC.
2. Construct a rectangle $ABCD$ such that $AB = 10.5$ cm and $BC = 6.5$ cm. Measure and write down the length of the diagonal AC.

Worked Example 7

(Construction of a Quadrilateral)

Construct a quadrilateral $PQRS$ such that $PQ = 6$ cm, $QR = 7.5$ cm, $RS = 8.2$ cm, $PS = 5.8$ cm and the diagonal $PR = 9.2$ cm. Measure and write down the size of $Q\hat{R}S$.

Solution:

Construction Steps:

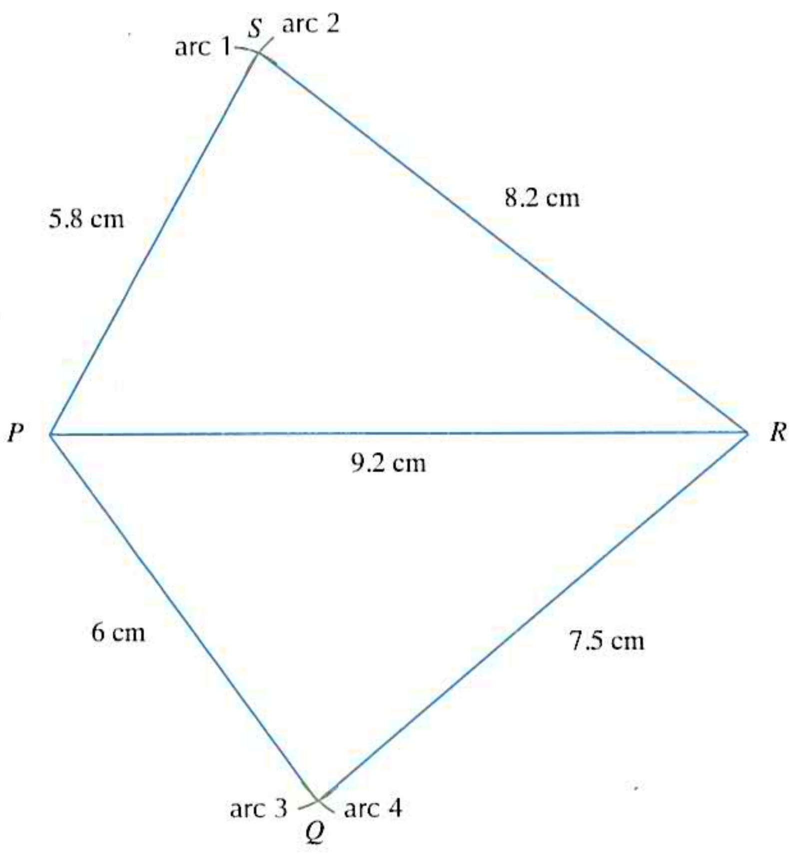
1. Using a ruler, draw $PR = 9.2$ cm.
2. Since S is 5.8 cm away from P, with P as centre and 5.8 cm as radius, draw arc 1.
3. Since S is 8.2 cm away from R, with R as centre and 8.2 cm as radius, draw arc 2 to cut arc 1 at S.
4. Join PS and RS.
5. Since Q is 6 cm away from P, with P as centre and 6 cm as radius, draw arc 3.
6. Since Q is 7.5 cm away from R, with R as centre and 7.5 cm as radius, draw arc 4 to cut arc 3 at Q.
7. Join PQ and QR.
   $Q\hat{R}S = 79°$

Problem Solving Tip

The point S is located by using its distance from P and from R, i.e. PS and RS.

Problem Solving Tip

The point Q is located by using its distance from P and from R, i.e. PQ and QR.



PRACTISE NOW 7

SIMILAR QUESTIONS

Exercise 12B Questions 4–5

1. Construct a quadrilateral $PQRS$ such that $PQ = 4.5$ cm, $QR = 6$ cm, $RS = 9$ cm, $PS = 6$ cm and the diagonal $QS = 9$ cm. Measure and write down the size of $Q\hat{R}S$.
2. Construct a rhombus $PQRS$ such that $PQ = 7.5$ cm and the diagonal $PR = 12$ cm. Measure and write down the size of $Q\hat{R}S$.

Worked Example 8

(Construction of a Quadrilateral)

Construct a quadrilateral $PQRS$ such that $PQ = 6.5$ cm, $QR = 4.8$ cm, $RS = 8.5$ cm, $P\hat{Q}R = 75^\circ$ and $Q\hat{R}S = 98^\circ$.

(i) Measure and write down the length of PS.

(ii) Measure and write down the size of $P\hat{S}R$.

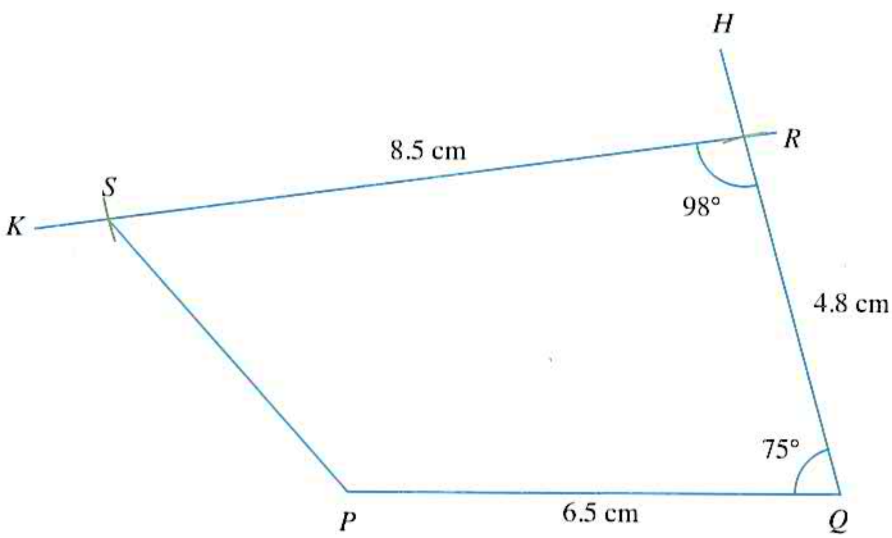
Solution:

Construction Steps:

1. Using a ruler, draw $PQ = 6.5$ cm.
2. Since $\angle Q = 75^\circ$, using a protractor at Q, mark off an angle of 75° and draw a line QH such that $P\hat{Q}H = 75^\circ$.
3. Since R is 4.8 cm away from Q, with Q as centre and 4.8 cm as radius, draw an arc to cut QH at R.
4. Since $\angle R = 98^\circ$, using a protractor at R, mark off an angle of 98° and draw a line RK such that $Q\hat{R}K = 98^\circ$.
5. Since S is 8.5 cm away from R, with R as centre and 8.5 cm as radius, draw an arc to cut RK at S.
6. Join PS.

(i) Length of $PS = 4.8$ cm

(ii) $P\hat{S}R = 56^\circ$



Internet Resources

Lorenzo Maschroni (1750 – 1800), an Italian Mathematician, had proven that any geometrical construction that can be done with a pair of compasses and a straightedge, i.e. Euclidean construction, can also be done with only a pair of compasses. This suggests that a pair of compasses is more fundamental than a straightedge although Maschroni construction is more difficult and tedious than Euclidean construction. Search on the Internet to find out more about Maschroni construction.

PRACTISE NOW 8

Construct a quadrilateral $PQRS$ such that $PQ = 5.6$ cm, $QR = 6.2$ cm, $RS = 9.2$ cm, $P\hat{Q}R = 80^\circ$ and $Q\hat{R}S = 95^\circ$.

(i) Measure and write down the length of PS.

(ii) Measure and write down the size of $P\hat{S}R$.

SIMILAR QUESTIONS

Exercise 12B Questions 6–8, 10, 12–15

Exercise 12B

BASIC LEVEL

1. Construct a parallelogram $ABCD$ such that $AB = 10$ cm, $BC = 12$ cm and $A\hat{B}C = 80°$. Measure and write down the length of the diagonal BD.

2. Construct a rectangle of sides 96 mm and 84 mm. Measure and write down the length of each of the two diagonals.

3. Construct a rhombus $ABCD$ such that $AB = 6$ cm and $A\hat{B}C = 115°$. Measure and write down the length of each of the two diagonals.

4. Construct a quadrilateral $PQRS$ such that $PQ = PS = PR = 9$ cm, $QR = 12$ cm and $RS = 7.5$ cm. Measure and write down the size of $Q\hat{P}S$.

5. Construct a rhombus $PQRS$ such that $PQ = 60$ mm and the diagonal $PR = 9$ mm. Measure and write down the size of $Q\hat{P}S$.

6. Construct a quadrilateral $PQRS$ such that $PQ = 5.3$ cm, $QR = 6.3$ cm, $RS = 6.7$ cm, $P\hat{Q}R = 75°$ and $Q\hat{R}S = 60°$.
   (i) Measure and write down the length of PR.
   (ii) Measure and write down the size of $R\hat{P}S$.

7. Construct a trapezium $WXYZ$ such that WZ is parallel to XY, $WX = 4.5$ cm, $XY = 8$ cm, $WZ = 6$ cm and $W\hat{X}Y = 60°$. Measure and write down the length of YZ and of WY.

INTERMEDIATE LEVEL

8. Construct a trapezium $WXYZ$ such that $WX = 56$ mm, $XY = 112$ mm, $W\hat{X}Y = 80°$ and $X\hat{Y}Z = 70°$. Measure and write down the length of WY and of XZ.

9. Construct a parallelogram $ABCD$ such that $AB = 9$ cm, $BC = 6$ cm and $A\hat{B}C = 115°$.
   (i) Measure and write down the length of the diagonal BD.
   (ii) Construct the perpendicular bisector of AB such that it cuts CD. Measure and write down the length of AT, such that T is the point where the perpendicular bisector of AB cuts CD.

10. Construct a quadrilateral $PQRS$ such that $PQ = 4$ cm, $QR = RS = 4.8$ cm, $PS = 3.6$ cm and $Q\hat{P}S = 90°$.
    (i) Measure and write down the length of QS.
    (ii) Construct the perpendicular from S to QR. Measure and write down the length of SU, such that U is the point where the perpendicular from S cuts QR.

11. A square $PQRS$ is such that the diagonal $PR = 10$ cm.
    (i) Draw PR.
    (ii) By constructing the perpendicular bisector of PR, locate the points Q and S and join the points to form the square $PQRS$.
    (iii) Measure and write down the length of PQ.

12. Construct a quadrilateral $PQRS$ such that $PQ = 10$ cm, $QR = 6$ cm, $PS = 3.5$ cm, $P\hat{Q}R = 45°$ and $Q\hat{P}S = 60°$.
   (i) Measure and write down the size of $Q\hat{R}S$.
   (ii) Construct the perpendicular bisector of QS such that it cuts PQ. Measure and write down the length of PT, such that T is the point where the perpendicular bisector of QS cuts PQ.

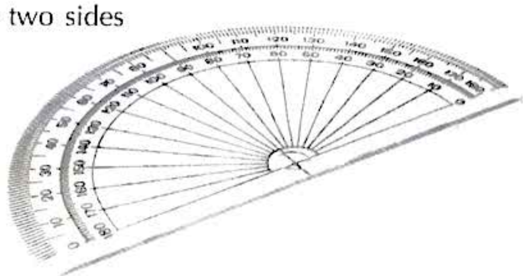
13. Construct a quadrilateral $PQRS$ such that $PQ = 11$ cm, $QR = 3.1$ cm, $PS = 7$ cm, $P\hat{Q}R = 90°$ and $Q\hat{P}S = 50°$.
   (i) Measure and write down the size of $Q\hat{R}S$.
   (ii) Construct a line parallel to PR that passes through S to meet QR produced at U. Measure and write down the length of RU.

14. Construct a trapezium $WXYZ$ such that WX is parallel to ZY, $WX = 5$ cm, $YZ = 9.8$ cm, $WZ = 6$ cm and $X\hat{W}Z = 120°$.
   (i) Measure and write down the length of WY.
   (ii) Construct the perpendicular bisector of WY such that it cuts WX produced at S and YZ at T. Measure and write down the length of ST.
   (iii) Construct the angle bisector of $W\hat{Z}Y$ such that it cuts WY at U. Measure and write down the size of $W\hat{U}X$.

ADVANCED LEVEL

15. Construct a quadrilateral $PQRS$ such that $PQ = 6.5$ cm, $QR = 4.6$ cm, $PS = 5.8$ cm, $P\hat{Q}R = 120°$ and $Q\hat{P}S = 105°$. The point T is such that it is equidistant from P and Q, and equidistant from PQ and PS. Find and label T.

1. If the line XY is the **perpendicular bisector** of a line segment AB, then
   - $XY \perp AB$,
   - XY passes through the midpoint of AB.
2. Any point on the perpendicular bisector of a line segment is *equidistant* from the two end points of the line segment.
3. If the ray AX is the **angle bisector** of $B\hat{A}C$, then $B\hat{A}X = C\hat{A}X$.
4. Any point on the angle bisector of an angle is *equidistant* from the two sides of the angle.



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Review Exercise 12

1. Construct ΔABC such that $AB = 4.5$ cm, $BC = 6$ cm and $A\hat{B}C = 60^\circ$.
   (i) Measure and write down the length of AC.
   (ii) Construct the angle bisector of $B\hat{A}C$ such that it cuts BC. Measure and write down the length of CS, such that S is the point where the angle bisector of $B\hat{A}C$ cuts BC.

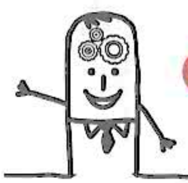
2. Construct ΔPQR such that $PQ = 12$ cm, $PR = 10.2$ cm and $QR = 8.8$ cm.
   (i) Measure and write down the size of the angle facing the shortest side.
   (ii) Construct the perpendicular bisector of QR such that it cuts PQ. Measure and write down the length of RT, such that T is the point where the perpendicular bisector of QR cuts PQ.

3. Construct ΔXYZ such that $YZ = 8$ cm and $X\hat{Y}Z = X\hat{Z}Y = 60^\circ$.
   (i) Construct the perpendicular bisector of YZ, which passes through X, and let U be the point where it cuts YZ.
   (ii) Find and label V on XU produced such that $XU = UV$.
   (iii) Join YV and ZV to obtain a quadrilateral. State the name of this quadrilateral.

4. Construct a parallelogram $ABCD$ such that $AB = 8$ cm, $BC = 5.5$ cm and $A\hat{B}C = 120^\circ$.
   (i) Measure and write down the length of the diagonal BD.
   (ii) Construct the perpendicular bisector of BD such that it cuts AB and CD. Measure and write down the length of ST, such that S and T are the points where the perpendicular bisector of BD cuts AB and CD respectively.

5. Construct a quadrilateral $PQRS$ such that $PQ = 8$ cm, $QR = 2$ cm, $PS = 6$ cm, $P\hat{Q}R = 90^\circ$ and $Q\hat{P}S = 60^\circ$.
   (i) Measure and write down the size of $Q\hat{R}S$.
   (ii) Construct a line parallel to PR that passes through S to meet QR produced at U. Measure and write down the length of QU.

6. Construct a circle with diameter $AC = 10$ cm.
   (i) Find and label a point B on the circumference of the circle such that $AB = BC$.
   (ii) Find and label the point D on the circumference of the circle such that it lies on the side of AC opposite to B and $C\hat{A}D = 45^\circ$.
   (iii) Join the points to form a quadrilateral $ABCD$. State the name of this quadrilateral.
   (iv) Construct the angle bisector of $B\hat{A}C$ such that it cuts the circle at A and at another point S. Measure and write down the length of DS.



Challenge Yourself

1. Construct ΔPQR such that $PQ = 8$ cm, $PR = 5$ cm and $QR = 6$ cm. Construct a circle which will pass through P, Q and R. What is the special name given to this circle?

2. Construct ΔPQR such that $PQ = 7$ cm, $PR = 6$ cm and $QR = 8$ cm. Construct a circle which will touch the sides of PQ, PR and QR. What is the special name given to this circle?

C1 Revision Exercise

1. 35% of students who take part in a dance competition are girls. If 140 girls take part in the competition, find the total number of students who take part in the competition.

2. The number 5600 is first decreased by 15%. The value obtained is next increased by 10%. Find the final number.

3. The length and breadth of a hall are 28 m and 21 m respectively. If the ratio of its length to its height is 7 : 6, find the ratio of its breadth to its height.

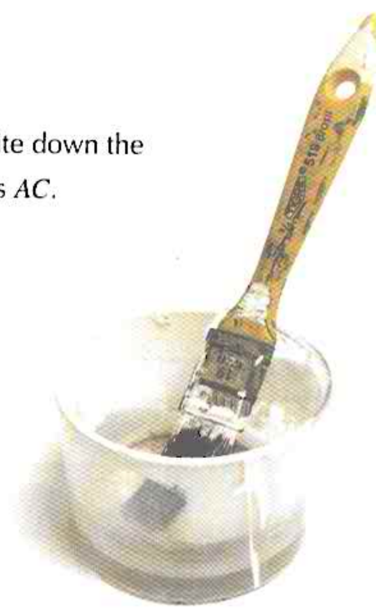
4. Priya can type 575 words in 25 minutes. She starts typing a report at 1035 hours and finishes at 1128 hours. Assuming she types non-stop at her usual rate and does not make any mistakes, find the number of words in the report.

5. A train leaves Town A at 0845 hours and arrives at Town B at 1510 hours.
   (i) How long does the train take to travel from Town A to Town B?
   (ii) If the average speed of the train is 108 km/h, find the distance between Town A and Town B.

6. In a parallelogram $ABCD$, $B\hat{A}C = 56°$ and $B\hat{C}D = 70°$. Find
   (i) $A\hat{B}C$,
   (ii) $A\hat{C}B$.

7. The interior angles of a hexagon are $(2x + 17)°$, $(3x - 25)°$, $(2x + 49)°$, $(x + 40)°$, $(4x - 17)°$ and $(3x - 4)°$. Find
   (i) the value of x,
   (ii) the smallest interior angle,
   (iii) the smallest exterior angle.

8. **(i)** Construct ΔABC such that $AB = 6$ cm, $BC = 7$ cm and $AC = 6.5$ cm.
   (ii) Construct the angle bisector of $A\hat{B}C$ such that it cuts AC. Measure and write down the length of BX, such that X is the point where the angle bisector of $A\hat{B}C$ cuts AC.



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C2 Revision Exercise

1. After a pay raise, Kate's monthly salary increased from \$3500 to \$3780. Find the percentage increase in her pay raise.

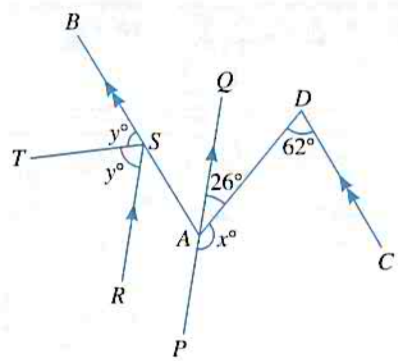
2. In a class of 40 students, 25% of them were made to stay back for detention. Find the number of students who did not have to stay back for detention.

3. An alloy consists of three metals, X, Y and Z. If $X : Y = 8 : 15$ and $Y : Z = 21 : 32$, find $X : Z$.

4. If light can travel 31 times around the world in 4 seconds, find the number of times it can circle the world in 10 seconds.

5. At 0845 hours, a lorry leaves Town A for Town B at an average speed of 52 km/h. It arrives at Town B at 1230 hours. On the return journey, the lorry leaves Town B at 1455 hours and arrives at Town A at 1815 hours. Find the average speed of the lorry on the return journey.

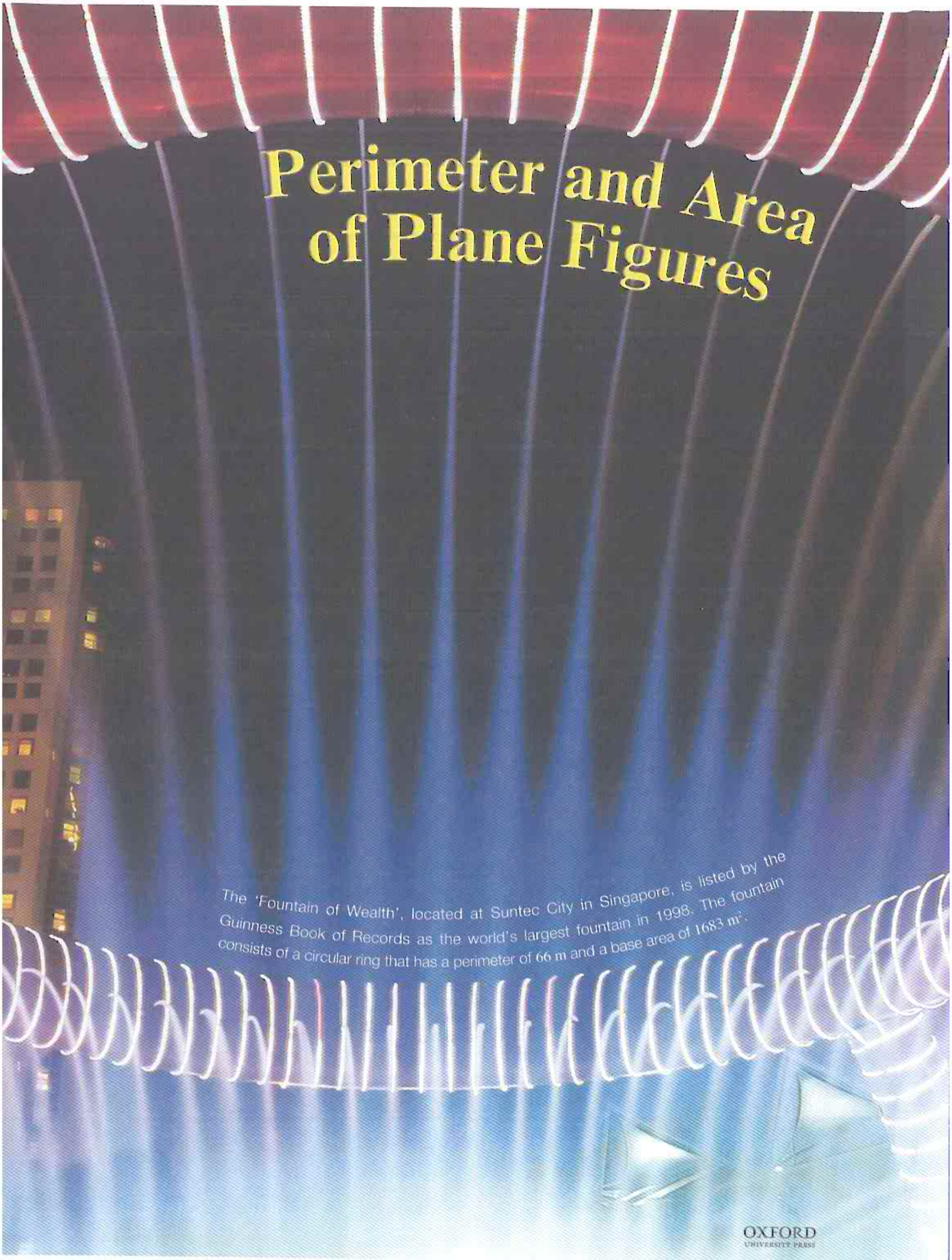
6. In the figure, PAQ and ASB are straight lines. If $PQ \parallel RS$, $AB \parallel CD$, $A\hat{D}C = 62°$ and $D\hat{A}Q = 26°$, find the value of x and of y.



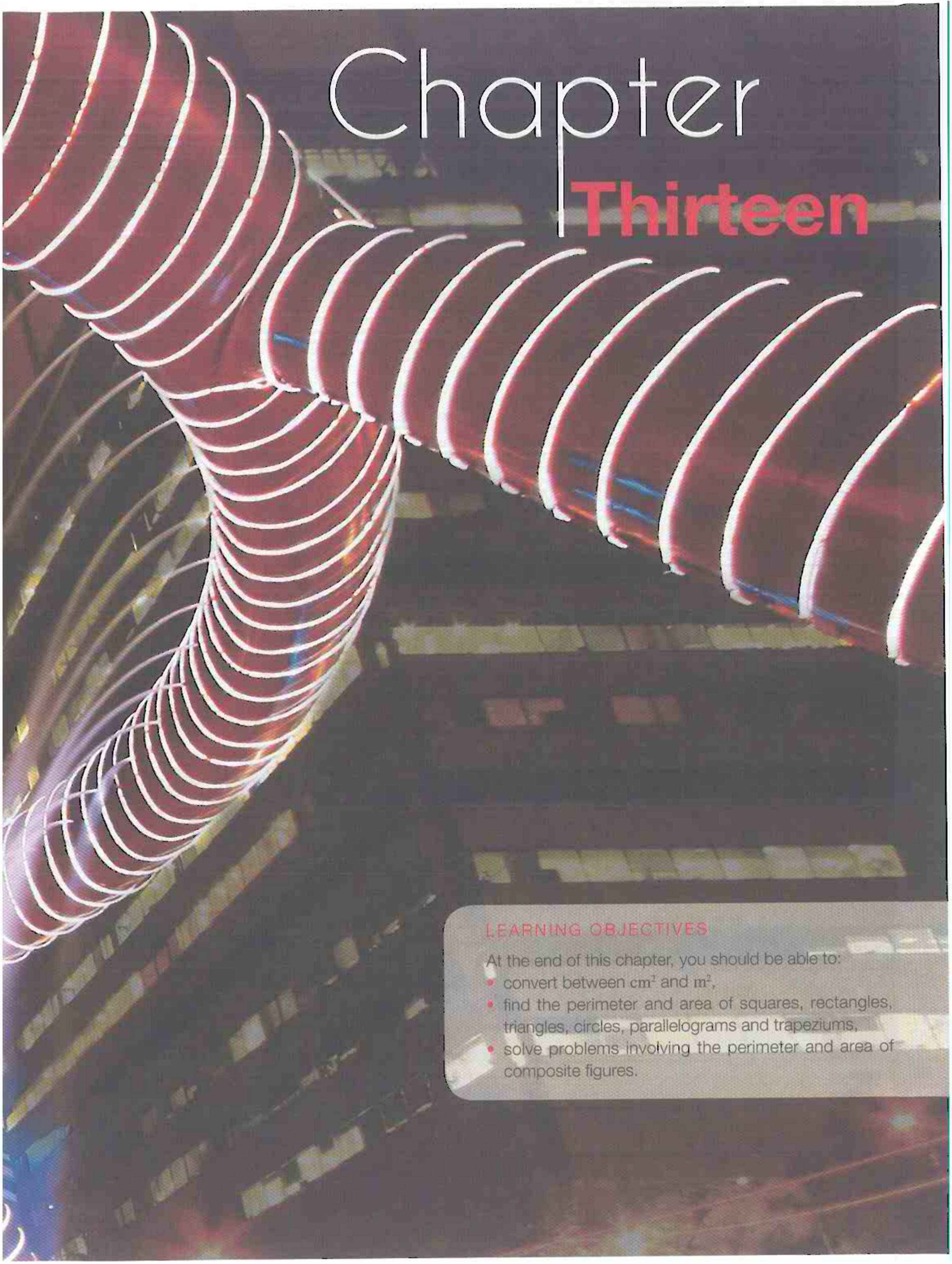
7. One of the interior angles of an n-sided polygon is 95° and the remaining interior angles are 169° each. Find the value of n.

8. Construct a quadrilateral $ABCD$ such that $AB = 7.6$ cm, $AD = 5.3$ cm, $A\hat{B}C = 105°$, $B\hat{A}D = 110°$ and $A\hat{D}C = 82°$. Measure and write down the length of BC and of CD.

Perimeter and Area of Plane Figures



The 'Fountain of Wealth', located at Suntec City in Singapore, is listed by the Guinness Book of Records as the world's largest fountain in 1998. The fountain consists of a circular ring that has a perimeter of 66 m and a base area of 1683 m^2.



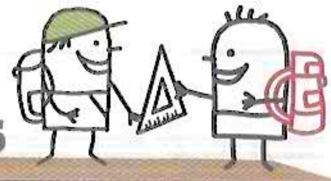
Chapter Thirteen

LEARNING OBJECTIVES

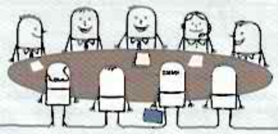
At the end of this chapter, you should be able to:

- convert between cm^2 and m^2,
- find the perimeter and area of squares, rectangles, triangles, circles, parallelograms and trapeziums,
- solve problems involving the perimeter and area of composite figures.

13.1 Conversion of Units



The floor area of a classroom is measured using square metres (m^2). Other units used to measure area of plane figures include square centimetres (cm^2), square millimetres (mm^2), hectares (ha) and square kilometres (km^2).



Class Discussion

International System of Units

1. The International System of Units (SI units) is made up of seven base units which are used in measurements. For example, length is a basic physical quantity and its base unit is the metre (m). Find out the other six basic physical quantities and their base units. Discuss with your classmates why scientists developed this system.
2. The British use feet, inches, yards and miles to measure length, and acres to measure area. Find out more about these units of measurements and how they are related to the SI units.

Sometimes, we need to **convert** from one unit of area to another. For example, it is more common to say that the land area of Singapore is 712.4 km^2 instead of 712 400 000 m^2. Similarly, we do not usually say that the area of the cross section of a piece of wire is 0.000 000 000 002 5 km^2; instead, we say that its cross-sectional area is 0.25 mm^2. In Worked Example 1, we will focus on the conversion between cm^2 and m^2.

Worked Example 1

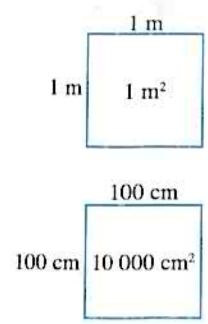
(Conversion between cm^2 and m^2)

Express

(a) 5 m^2 in cm^2, **(b)** 975 cm^2 in m^2.

ATTENTION

The area of a square of sides 1 m is 1 m^2, which is equal to 10 000 cm^2.



Solution:

(a)

$$\begin{aligned} 1 \text{ m} &= 100 \text{ cm} \\ (1 \text{ m})^2 &= (100 \text{ cm})^2 \\ &= 100 \text{ cm} \times 100 \text{ cm} \\ 1 \text{ m}^2 &= 10\,000 \text{ cm}^2 \\ 5 \text{ m}^2 &= 5 \times 10\,000 \text{ cm}^2 \\ &= 50\,000 \text{ cm}^2 \end{aligned}$$

(b)

$$\begin{aligned} 100 \text{ cm} &= 1 \text{ m} \\ 1 \text{ cm} &= \frac{1}{100} \text{ m} \\ &= 0.01 \text{ m} \\ (1 \text{ cm})^2 &= (0.01 \text{ m})^2 \\ &= 0.01 \text{ m} \times 0.01 \text{ m} \\ 1 \text{ cm}^2 &= 0.0001 \text{ m}^2 \\ 975 \text{ cm}^2 &= 975 \times 0.0001 \text{ m}^2 \\ &= 0.0975 \text{ m}^2 \end{aligned}$$

PRACTISE NOW 1

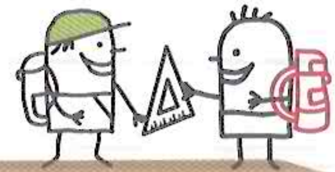
Express

(a) 16 m^2 in cm^2, **(b)** 357 cm^2 in m^2.

SIMILAR QUESTIONS

Exercise 13A Questions 1(a)–(d)

13.2 Perimeter and Area of Basic Plane Figures



Recap (Base and Height of a Triangle)

In primary school, we have learnt that we can use any side of a triangle as its **base**, and that the **height** of a triangle *with reference to the base* is the *perpendicular distance* from the base to the opposite vertex (see Fig. 13.1).

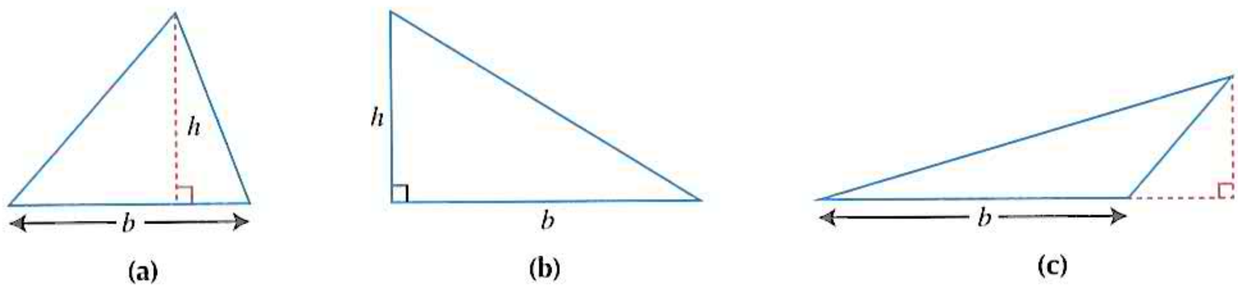
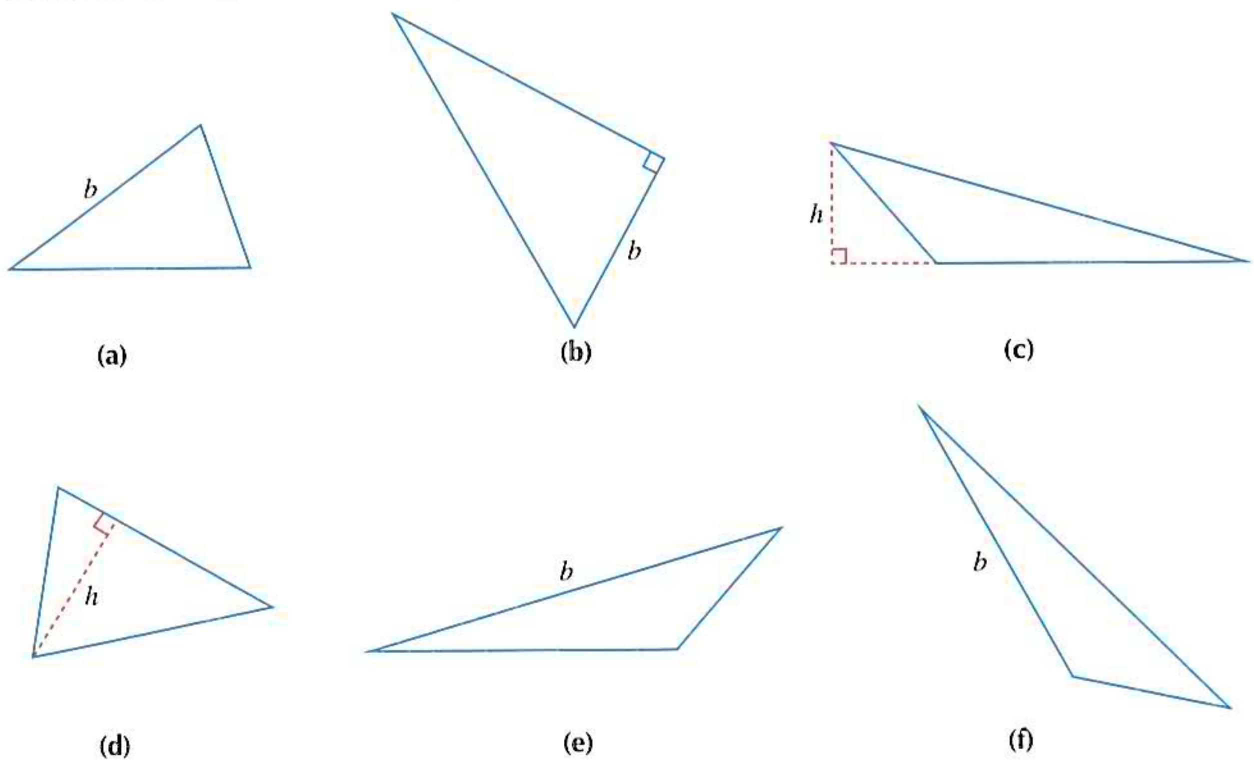


Fig. 13.1

PRACTISE NOW

For each of the following triangles, label the height (or base) with reference to the given base (or height). You need to indicate the right angle where necessary.



Perimeter and Area of Basic Plane Figures

In primary school, we have learnt how to find the perimeter and area of a square, a rectangle, a triangle and a circle. Some important formulae are provided in Table 13.1. Complete the table.

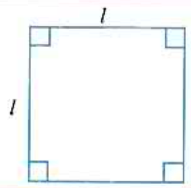
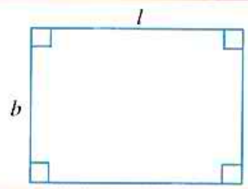
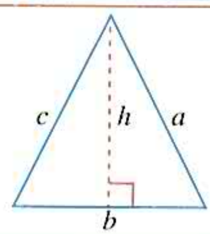
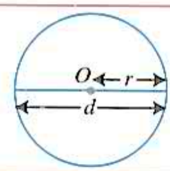
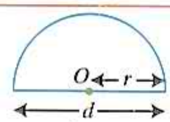
Name	Figure	Perimeter	Area
Square			l^2
Rectangle		$2(l + b)$	lb
Triangle		$a + b + c$	
Circle		$2\pi r$ or πd	πr^2
Semicircle			

Table 13.1

Worked Example 2

(Problem involving Perimeter and Area of a Rectangle)

The length of a rectangular field is 4 m longer than its breadth.

(a) If the perimeter of the field is 44 m, calculate

(i) the breadth,

(ii) the area,

of the field.

(b) The field is surrounded by a cement path of width 2.5 m. Calculate the area of the path.

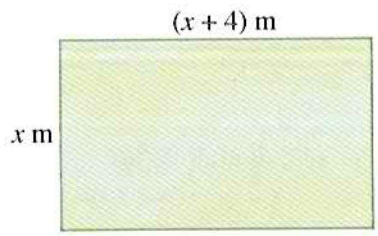
Solution:

(a) (i) Let the breadth of the rectangular field = x m.

Then the length of the field = $(x + 4)$ m.

$$\begin{aligned} \therefore 2[(x+4)+x] &= 44 \\ 2(2x+4) &= 44 \\ 4x+8 &= 44 \\ 4x &= 36 \\ x &= 9 \end{aligned}$$

$\therefore$ Breadth of the field = 9 m



(ii) Area of the field = $(9 + 4) \times 9$

$= 117$ m^2

(b) Total length = $13 + 2.5 + 2.5$

$= 18$ m

Total breadth = $9 + 2.5 + 2.5$

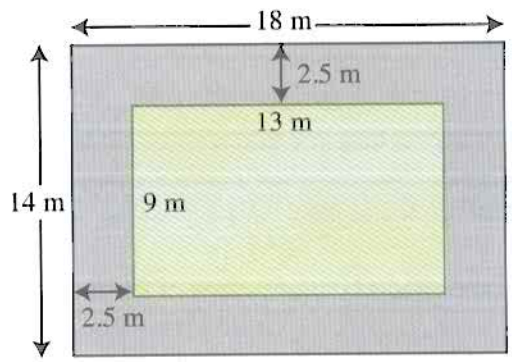
$= 14$ m

Total area of the field and the cement path = 18×14

$= 252$ m^2

Area of the path = $252 - 117$

$= 135$ m^2

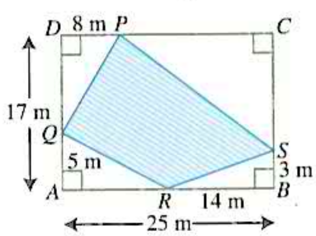


PRACTISE NOW 2

SIMILAR QUESTIONS

Exercise 13A Questions 2–3, 8–11

1. The perimeter of a square field is 64 m. If the field is surrounded by a running path of width 3.5 m, find the area of the path.
2. In the figure, AB = 25 m, AD = 17 m, DP = 8 m, AQ = 5 m, BR = 14 m and BS = 3 m. Find the area of the shaded region.



Worked Example 3

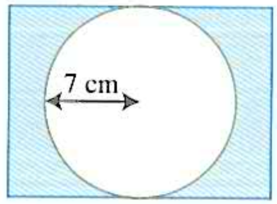
(Problem involving Circumference and Area of a Circle)

The figure shows a circle of radius 7 cm, touching two sides of a rectangle. The length of the rectangle is 9 cm longer than its width. Calculate

(i) the circumference of the circle,

(ii) the area of the circle,

(iii) the area of the shaded region.



Solution:

(i) Circumference of the circle $= 2\pi r$

$= 2\pi(7)$

$= 44.0$ cm (to 3 s.f.)

(ii) Area of the circle $= \pi r^2$

$= \pi(7)^2$

$= 49\pi$

$= 154$ cm^2 (to 3 s.f.)

(iii) Width of the rectangle = diameter of circle $= 7 \times 2$

$= 14$ cm

Length of the rectangle $= 14 + 9$

$= 23$ cm

Area of the shaded region = area of the rectangle − area of the circle

$= 23 \times 14 - 49\pi$

$= 168$ cm^2 (to 3 s.f.)

ATTENTION

If the question does not specify the value of π, we use the value of π stored in the calculator.

Problem Solving Tip

For accuracy, we should use the value of π stored in the calculator in the intermediate steps.

PRACTISE NOW 3

SIMILAR QUESTIONS

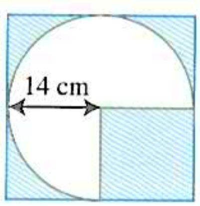
Exercise 13A Questions 4(a)–(d), 5, 6(a)–(c), 7, 12–17

The figure shows a circle of radius 14 cm with one quadrant removed, touching the sides of a square. Find

(i) the perimeter of the unshaded region,

(ii) the area of the unshaded region,

(iii) the area of the shaded region.



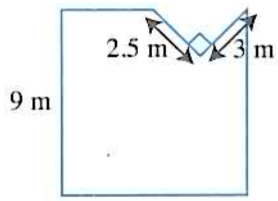
Exercise 13A

BASIC LEVEL

1. Express
   (a) 40 m^2 in cm^2, (b) 16 cm^2 in m^2,
   (c) 0.03 m^2 in cm^2, (d) 28 000 cm^2 in m^2.

2. The area of a rectangle is 259 cm^2 and its length is 18.5 cm. Find
   (i) its breadth,
   (ii) its perimeter.

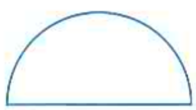
3. In the figure, a triangle is removed from a square of sides 9 m. Find the area of the figure.



4. Complete the table for each circle.

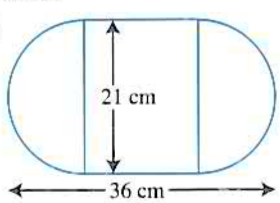
	Diameter	Radius	Circumference	Area
(a)		10 cm		
(b)	3.6 m			
(c)			176 mm	
(d)				616 cm^2

5. A piece of wire 144 cm long is bent to form a semicircle as shown in the figure. Find the diameter of the semicircle, giving your answer in metres. (Take π to be $\frac{22}{7}$.)

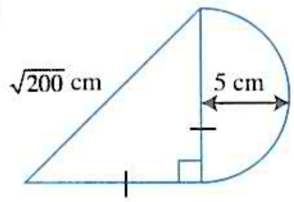


6. The circular portions of the following figures are semicircles. For each of the following figures, find
   (i) its perimeter,
   (ii) its area.

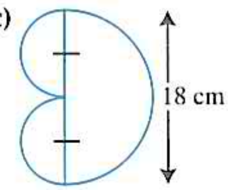
(a)



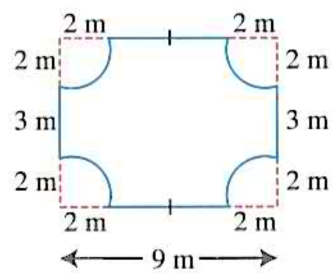
(b)



(c)

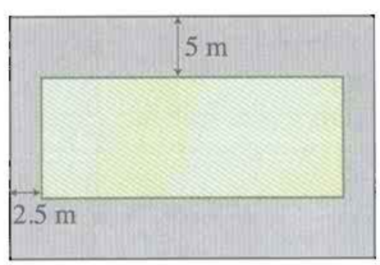


7. In the figure, 4 quadrants, each of radius 2 m, are removed from a rectangle. Find
   (i) the perimeter,
   (ii) the area,
   of the figure.

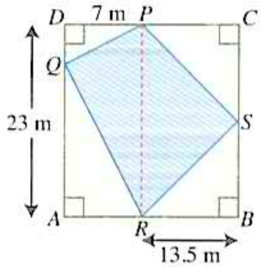


INTERMEDIATE LEVEL

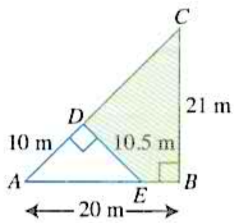
8. The perimeter of a rectangular field is 70 m and its length is 15 m longer than its breadth. The field is surrounded by a concrete path as shown in the figure. Find the area of the path.



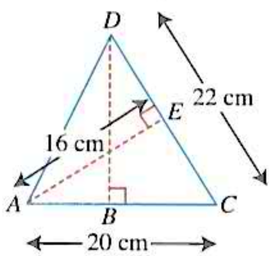
9. In the figure, $AD = 23$ m, $DP = 7$ m and $BR = 13.5$ m. If P is directly above R, find the area of the shaded region.



10. In the figure, $AB = 20$ m, $BC = 21$ m, $AD = 10$ m and $DE = 10.5$ m. Find the area of the shaded region.

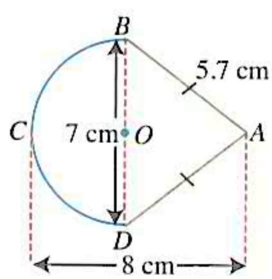


11. In the figure, $AC = 20$ cm, $CD = 22$ cm and $AE = 16$ cm. Find the length of BD.

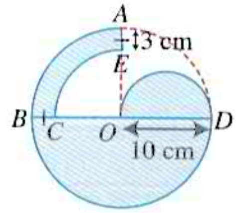


12. (i) Find the area of the surface of a circular pond that has a diameter of 12 m.
(ii) The pond is surrounded by a path of width 2 m. If it costs \$55 per m^2 to pave the path with tiles, find the cost incurred.

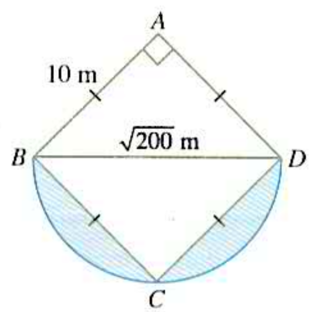
13. A figure is made up of a triangle ABD and a semicircle. BOD is the diameter of the semicircle with centre O. If $AB = AD = 5.7$ cm, $BD = 7$ cm and $AOC = 8$ cm, find
(i) the perimeter,
(ii) the area,
of the figure.



14. BOD is the diameter of the semicircle with centre O. OD is the diameter of the smaller semicircle. AB and CE are arcs of two quadrants with different radii. If $OD = 10$ cm and $AE = BC = 3$ cm, find
(i) the perimeter,
(ii) the area,
of the shaded region.

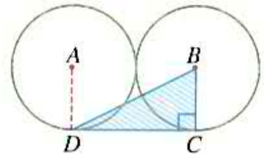


15. In the figure, $ABCD$ is a square of sides 10 m and BCD is a semicircle of diameter $\sqrt{200}$ m. Find
(i) the perimeter,
(ii) the area,
of the shaded region.

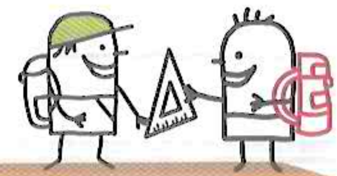


ADVANCED LEVEL

16. In the figure, two identical circles, with centres at A and B respectively, have an area of 0.785 cm^2 each. If A is directly above D, find the area of the shaded region.



17. A goat, tethered by a rope 1.5 m long, is able to eat a square metre of grass in 14 minutes. Find the time it needs to eat all the grass within its reach.

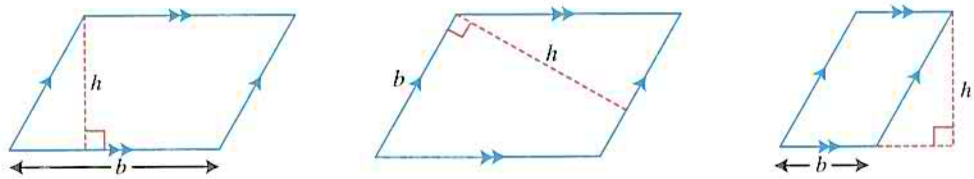


13.3 Perimeter and Area of Parallelograms

We have learnt what a parallelogram is in primary school and its properties in Chapter 11. In this section, we will learn how to find the perimeter and the area of a parallelogram.

Base and Height of a Parallelogram

We can use any side of a parallelogram as its **base**. The **height** of the parallelogram *with reference to the base* is the *perpendicular distance* from the base to the opposite side (see Fig. 13.2).

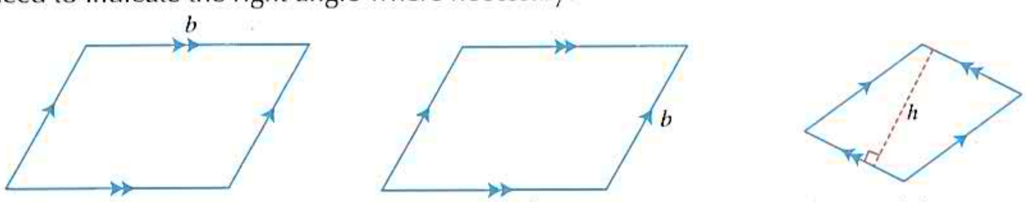


(a) (b) (c)

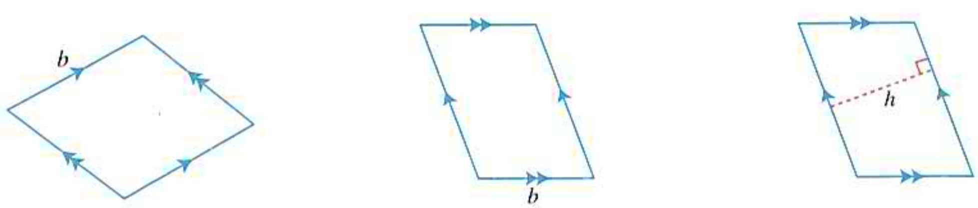
Fig. 13.2

PRACTISE NOW

For each of the following parallelograms, label the height (or base) with reference to the given base (or height). You need to indicate the right angle where necessary.



(a) (b) (c)



(d) (e) (f)

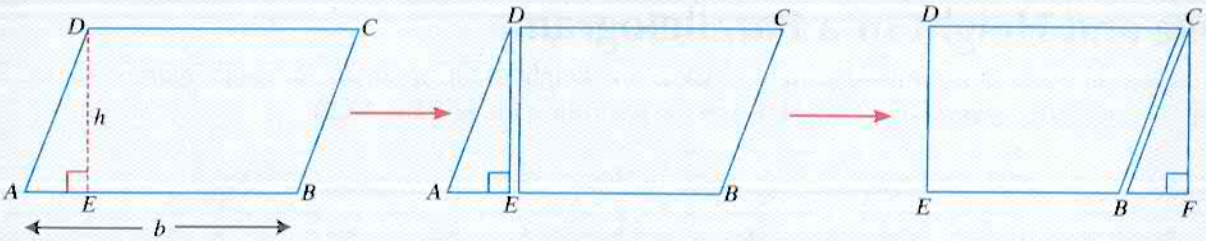
Area of a Parallelogram

Investigation

Formula for Area of a Parallelogram

In this investigation, we will make use of the formula for the area of a rectangle to find a formula for the area of a parallelogram.

Fig. 13.3(a) shows a parallelogram $ABCD$ with base $AB = b$ and height $DE = h$.



(a) (b) (c)

Fig. 13.3

1. If we remove the right-angled triangle ADE from the parallelogram $ABCD$ in Fig. 13.3(b) and place it as shown in Fig. 13.3(c), what is the shape of the new quadrilateral $CDEF$?
2. Find the length of CF and of EF in terms of b and h.
3. Hence, find a formula for the area of the parallelogram $ABCD$ in terms of b and h.
4. Think of another method to find a formula for the area of a parallelogram.

   Hint: Divide the parallelogram $ABCD$ in another way and use the formula for the area of a triangle.

Thinking Time

Fig. 13.4 shows a parallelogram that is slanted so far to one side until we cannot draw the height inside the parallelogram and cut it like in the above investigation.

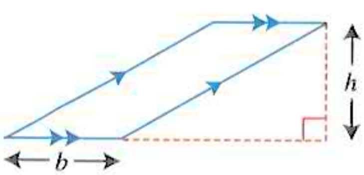


Fig. 13.4

Does the formula which you have found in the investigation for the area of a parallelogram still work for this oblique parallelogram? You may go to http://www.shinglee.com.sg/StudentResources/ and make use of the geometry software template 'Area of Parallelogram' to help you.

From the investigation and thinking time, we can conclude that:

$$\boxed{\text{Area of a parallelogram} = \text{base} \times \text{height} = bh}$$

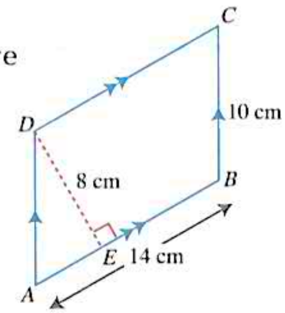
Worked Example 4

(Finding Area and Perimeter of a Parallelogram)

The figure shows a parallelogram $ABCD$ where $AB = 14$ cm and $BC = 10$ cm. If $DE = 8$ cm, calculate

(i) the area, **(ii)** the perimeter,

of the parallelogram.



Solution:

(i) Area of the parallelogram = base × height

$= 14 \times 8$

$= 112 \text{ cm}^2$

(ii) Since opposite sides of a parallelogram are equal in length, $AB = DC$ and $BC = AD$.

Perimeter of the parallelogram $= 14 \times 2 + 10 \times 2$

$= 48$ cm

PRACTISE NOW 4

SIMILAR QUESTIONS

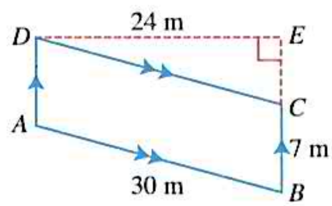
Exercise 13B Questions 1(a), 3, 9(a)

The figure shows a parallelogram $ABCD$ where $AB = 30$ m and $BC = 7$ m. If $DE = 24$ m, find

(i) the area,

(ii) the perimeter,

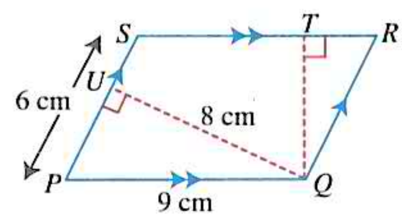
of the parallelogram.



Worked Example 5

(Finding an Unknown in a Parallelogram)

The figure shows a parallelogram $PQRS$ where $PQ = 9$ cm and $PS = 6$ cm. QU is perpendicular to PS and QT is perpendicular to SR. If $QU = 8$ cm, calculate the length of QT.



Solution:

Area of the parallelogram = base × height = $PQ \times QT = PS \times QU$

$9 \times QT$

$= 6 \times 8$

$QT = 5\frac{1}{3}$

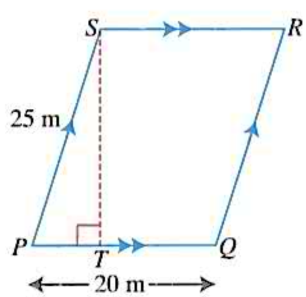
Length of $QT = 5\frac{1}{3}$ cm

PRACTISE NOW 5

SIMILAR QUESTIONS

Exercise 13B Questions 1(b)–(c), 4

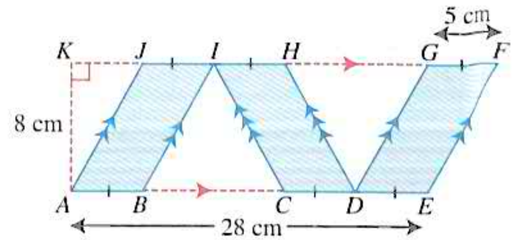
The figure shows a parallelogram $PQRS$ where $PQ = 20$ m and $PS = 25$ m. If the area of the parallelogram is 480 m², find the length of ST.



Worked Example 6

(Finding Total Area of Parallelograms)

In the figure, $AE = 28$ cm, $AK = 8$ cm and $AB = CD = DE = FG = HI = IJ = 5$ cm. Calculate the total area of the shaded regions.



Solution:

The shaded regions are made up of 3 parallelograms. Each parallelogram has a base of 5 cm and a height of 8 cm.

Total area of the shaded regions $= 3 \times (\text{base} \times \text{height})$

$= 3 \times (5 \times 8)$

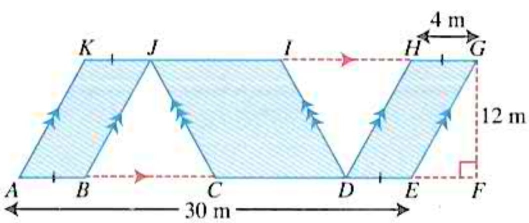
$= 120 \text{ cm}^2$

PRACTISE NOW 6

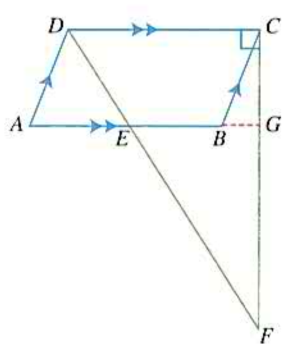
SIMILAR QUESTIONS

Exercise 13B Questions 8, 12

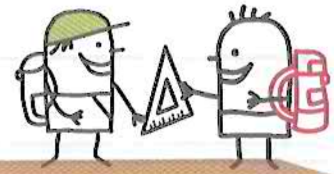
1. In the figure, $AE = 30$ m, $FG = 12$ m and $AB = DE = GH = JK = \frac{1}{2}CD = \frac{1}{2}IJ = 4$ m. Find the total area of the shaded regions.



2. In the figure, $ABCD$ is a parallelogram, CDF is a right-angled triangle, and CGF and DEF are straight lines. If the area of ΔCDF is 60 cm² and $2CG = GF$, find the area of the parallelogram $ABCD$.



13.4 Perimeter and Area of Trapeziums



We have learnt what a trapezium is in primary school, and its properties in Chapter 11. In this section, we will learn how to find the perimeter and the area of a trapezium.

Height of a Trapezium

Unlike a parallelogram, the **height** of a trapezium must be *with reference to the two parallel sides*, i.e. the height of a trapezium is the *perpendicular distance* between the two parallel sides (see Fig. 13.5).

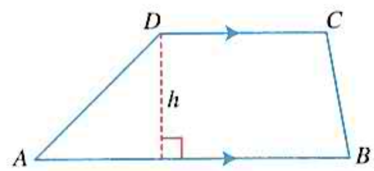
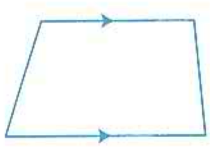


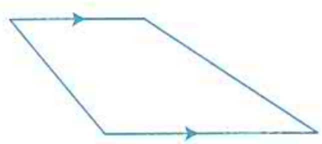
Fig. 13.5

PRACTISE NOW

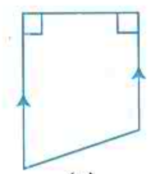
Draw and label the height for each of the following trapeziums.



(a)



(b)



(c)

Area of a Trapezium

Investigation

Formula for Area of a Trapezium

In this investigation, we will make use of the formula for the area of a parallelogram to find a formula for the area of a trapezium.

Fig. 13.6(a) shows two identical trapeziums $ABCD$ and $EFGH$ with $AB = GH = b$, $CD = EF = a$ and height h.

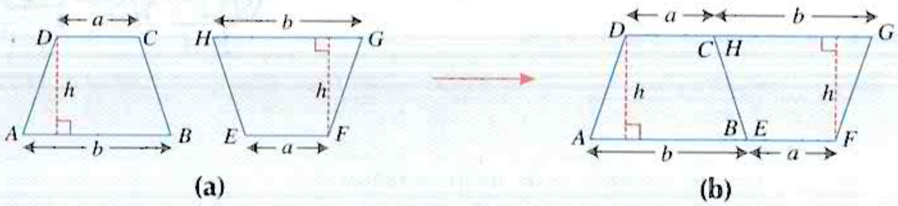


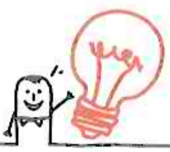
Fig. 13.6

1. If we arrange the two trapeziums as shown in Fig. 13.6(b), what is the shape of the new quadrilateral $AFGD$?
2. Find the length of AF in terms of a and b.
3. Hence, find a formula for the area of the trapezium $ABCD$ in terms of a, b and h.
4. There are at least 12 methods to find a formula for the area of a trapezium. Think of two more methods to find a formula for the area of a trapezium.
   Hint: Divide the trapezium $ABCD$ and use the formula for the area of a triangle.

From the investigation, we can conclude that:

Area of a trapezium = $\frac{1}{2}$ × (sum of lengths of parallel sides) × height = $\frac{1}{2}(a+b)h$,

where a and b are the lengths of the parallel sides.



Thinking Time

How are the formulae for the area of a trapezium, a parallelogram and a triangle related to one another?

1. **(i)** If the parallel sides of a trapezium are equal in length (i.e. $a = b$), what is the shape of the new figure?
   (ii) If we substitute $a = b$ into the formula for the area of a trapezium, what do we get after simplification?
2. **(i)** If we reduce the length of one of the parallel sides of a trapezium until it becomes a point (i.e. $a = 0$), what is the shape of the new figure?
   (ii) If we substitute $a = 0$ into the formula for the area of a trapezium, what do we get after simplification?

Worked Example 7

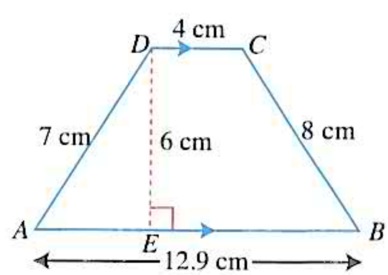
(Finding Area and Perimeter of a Trapezium)

The figure shows a trapezium $ABCD$ where $AB = 12.9$ cm, $BC = 8$ cm, $CD = 4$ cm and $AD = 7$ cm. If $DE = 6$ cm, calculate

(i) the area,

(ii) the perimeter,

of the trapezium.



Solution:

(i) Area of the trapezium $= \frac{1}{2} \times$ (sum of lengths of parallel sides) $\times$ height

$$= \frac{1}{2} \times (12.9 + 4) \times 6$$

$$= 50.7 \text{ cm}^2$$

(ii) Perimeter of the trapezium $= 12.9 + 8 + 4 + 7$

$$= 31.9 \text{ cm}$$

PRACTISE NOW 7

SIMILAR QUESTIONS

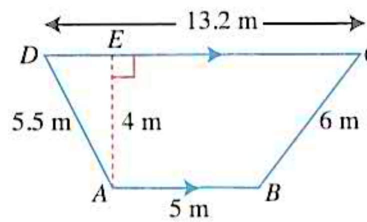
Exercise 13B Questions 2(a), 5, 11

The figure shows a trapezium $ABCD$ where $AB = 5$ m, $BC = 6$ m, $CD = 13.2$ m and $AD = 5.5$ m. If $AE = 4$ m, find

(i) the area,

(ii) the perimeter,

of the trapezium.

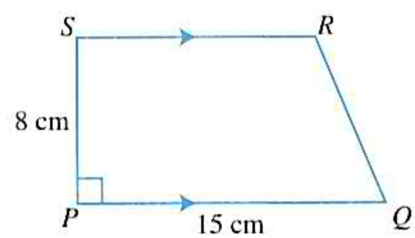


Worked Example 8

(Finding Unknowns in a Trapezium)

The figure shows a trapezium $PQRS$ where $PQ = 15$ cm and $PS = 8$ cm. If the area and the perimeter of the trapezium are 104 cm² and 42.9 cm respectively, calculate the length of

(i) RS, **(ii)** QR.



Solution:

(i) The height of the trapezium is given by the length of $PS = 8$ cm.

Area of the trapezium $= \frac{1}{2} \times$ (sum of lengths of parallel sides) $\times$ height $= 104 \text{ cm}^2$

$$\frac{1}{2} \times (15 + RS) \times 8 = 104$$

$$4 \times (15 + RS) = 104$$

$$15 + RS = 26$$

$$RS = 11$$

Length of $RS = 11$ cm

(ii) Perimeter of the trapezium $= PQ + QR + RS + PS = 42.9$ cm

$$15 + QR + 11 + 8 = 42.9$$

$$34 + QR = 42.9$$

$$QR = 8.9$$

Length of $QR = 8.9$ cm

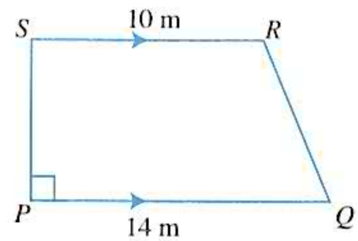
PRACTISE NOW 8

SIMILAR QUESTIONS

Exercise 13B Questions 2(b)–(c), 6

The figure shows a trapezium $PQRS$ where $PQ = 14$ m and $RS = 10$ m. If the area and the perimeter of the trapezium are 72 m^2 and 37.2 m respectively, find the length of

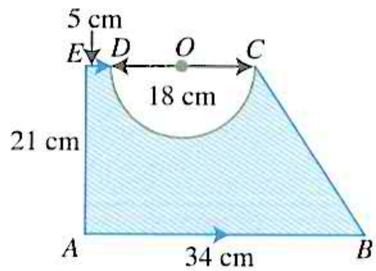
(i) PS, **(ii)** QR.



Worked Example 9

(Problem involving Area of a Trapezium)

In the figure, a semicircle is removed from a trapezium $ABCE$. COD is the diameter of the semicircle with centre O. If $AB = 34$ cm, $DE = 5$ cm, $AE = 21$ cm and $CD = 18$ cm, calculate the area of the figure.



Solution:

Area of the figure = area of the trapezium – area of the semicircle

$$= \frac{1}{2} \times \text{(sum of lengths of parallel sides)} \times \text{height} - \frac{1}{2}\pi r^2$$

$$= \frac{1}{2} \times [34 + (18 + 5)] \times 21 - \frac{1}{2}\pi \left(\frac{18}{2}\right)^2$$

$$= \frac{1}{2} \times 57 \times 21 - \frac{1}{2}\pi(9)^2$$

$$= 598.5 - 40.5\pi$$

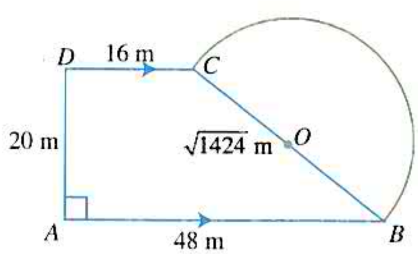
$$= 471 \text{ cm}^2 \text{ (to 3 s.f.)}$$

PRACTISE NOW 9

SIMILAR QUESTIONS

Exercise 13B Questions 7, 9(b), 10

A figure is made up of a trapezium $ABCD$ and a semicircle. BOC is the diameter of the semicircle with centre O. If $AB = 48$ m, $CD = 16$ m, $AD = 20$ m and $BC = \sqrt{1424}$ m, find the area of the figure.



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Exercise 13B

BASIC LEVEL

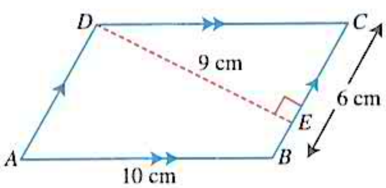
1. Complete the table for each parallelogram.

	Base	Height	Area
(a)	12 cm	7 cm	
(b)		6 m	42 m^2
(c)	7.8 mm		42.9 mm^2

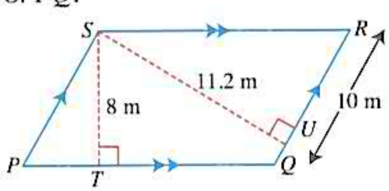
2. Complete the table for each trapezium.

	Parallel side 1	Parallel side 2	Height	Area
(a)	7 cm	11 cm	6 cm	
(b)	8 m	10 m		126 m^2
(c)	5 mm		8 mm	72 mm^2

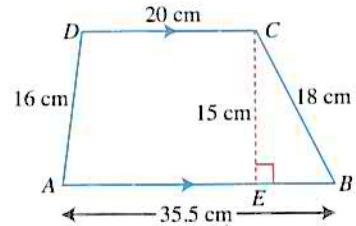
3. The figure shows a parallelogram $ABCD$ where $AB = 10$ cm and $BC = 6$ cm. If $DE = 9$ cm, find
   (i) the area,
   (ii) the perimeter,
   of the parallelogram.



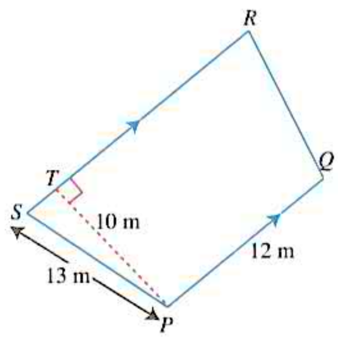
4. The figure shows a parallelogram $PQRS$ where $QR = 10$ m. If $ST = 8$ m and $SU = 11.2$ m, find the length of PQ.



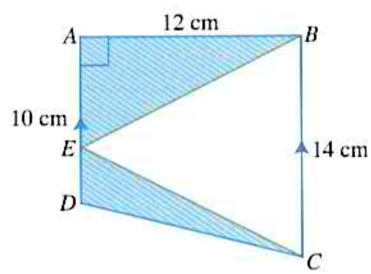
5. The figure shows a trapezium $ABCD$ where $AB = 35.5$ cm, $BC = 18$ cm, $CD = 20$ cm and $AD = 16$ cm. If $CE = 15$ cm, find
   (i) the area,
   (ii) the perimeter,
   of the trapezium.



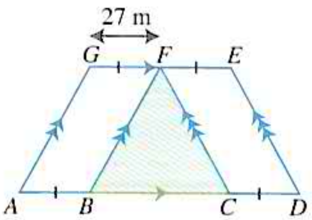
6. The figure shows a trapezium $PQRS$ where $PQ = 12$ m and $PS = 13$ m. If $PT = 10$ m, and the area and the perimeter of the trapezium are 150 m^2 and 54.7 m respectively, find the length of
   (i) RS,
   (ii) QR.



7. The figure shows a trapezium $ABCD$ where AB is perpendicular to AD, $AB = 12$ cm, $BC = 14$ cm and $AD = 10$ cm. Find the area of the shaded regions.

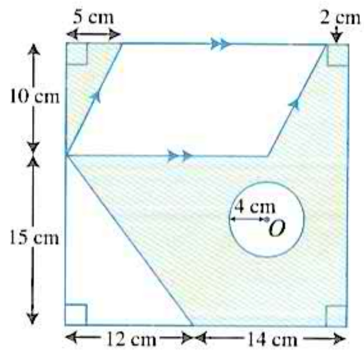


8. In the figure, $ABFG$ and $CDEF$ are two parallelograms such that the sum of their areas is 702 m^2. If $AB = CD = EF = FG = \frac{1}{2}BC$, find the area of the shaded region.

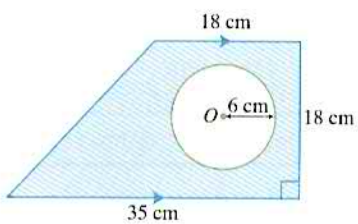


9. For each of the following figures, find the (total) area of the shaded region(s), where O is the centre of the circle.

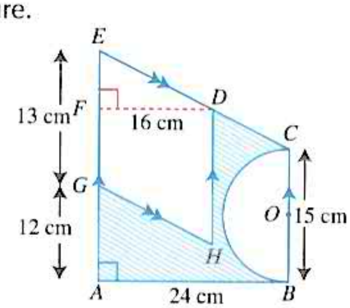
(a)



(b)

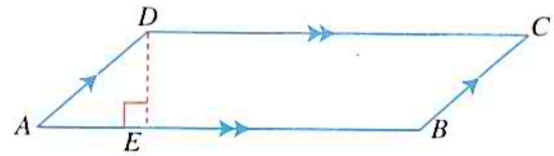


10. In the figure, a parallelogram $GHDE$ and a semicircle are removed from a trapezium $ABCE$. BOC is the diameter of the semicircle with centre O. If $AB = 24$ cm, $BC = 15$ cm, $EG = 13$ cm, $AG = 12$ cm and $DF = 16$ cm, find the area of the figure.



ADVANCED LEVEL

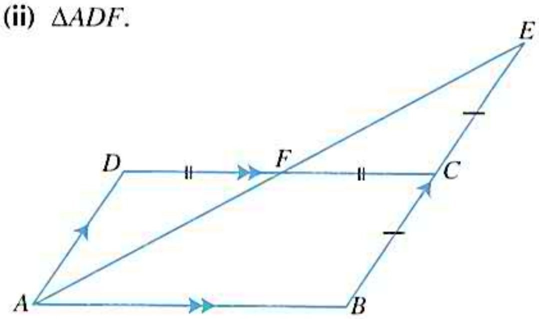
11. In the figure, $ABCD$ is a parallelogram and AED is a right-angled triangle. If the area of ΔAED is 25 cm^2, and the lengths of AE and EB are in the ratio 1 : 3, find the area of the trapezium $BCDE$.



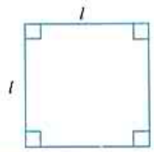
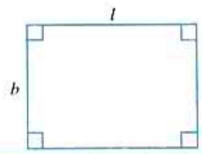
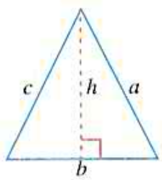
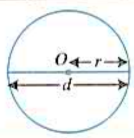
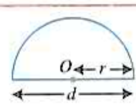
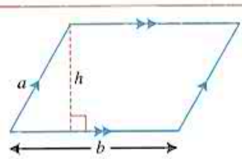
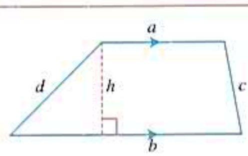
12. In the figure, $ABCD$ is a parallelogram, and AFE and BCE are straight lines. If the area of the parallelogram is 80 cm^2, $BC = CE$ and $DF = FC$, find the area of

(i) ΔABE,

(ii) ΔADF.



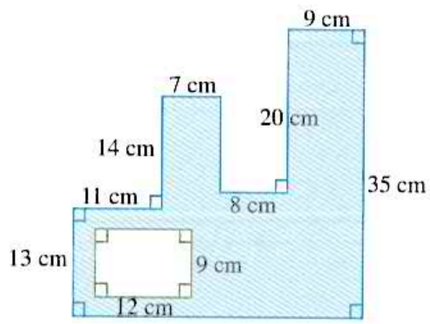
Perimeter and Area of Plane Figures

Name	Figure	Perimeter	Area
Square		$4l$	l^2
Rectangle		$2(l + b)$	lb
Triangle		$a + b + c$	$\frac{1}{2}bh$
Circle		$2\pi r$ or πd	πr^2
Semicircle		$\pi r + 2r$ or $\frac{1}{2}\pi d + d$	$\frac{1}{2}\pi r^2$
Parallelogram		$2(a + b)$	bh
Trapezium		$a + b + c + d$	$\frac{1}{2}(a+b)h$

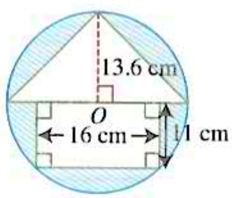
Review Exercise 13

1. For each of the following figures, find the (total) area of the shaded region(s). In **(b)**, O is the centre of the circle.

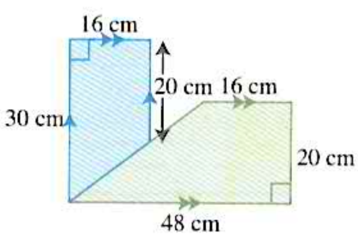
(a)



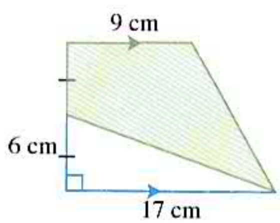
(b)



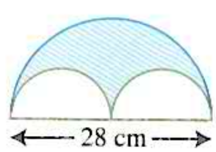
(c)



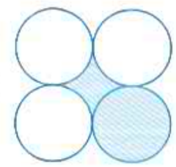
(d)



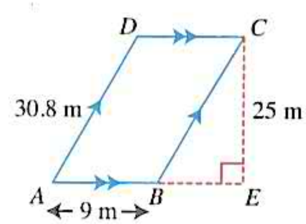
2. The figure shows three semicircles. Find
   (i) the perimeter,
   (ii) the area,
   of the shaded region.



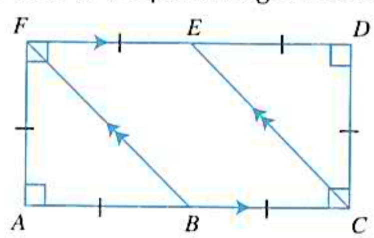
3. The figure shows four circles of equal radius of 12 cm touching one another. Find the area of the shaded region.



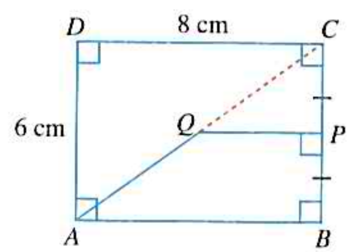
4. The figure shows a parallelogram $ABCD$ where $AB = 9$ m and $AD = 30.8$ m. If $CE = 25$ m, find
   (i) the area,
   (ii) the perimeter,
   of the parallelogram.



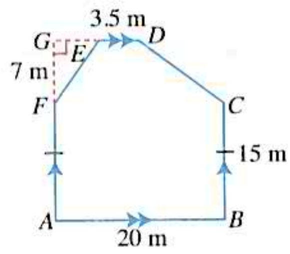
5. The figure shows a rectangle $ACDF$ that has an area of 24 cm². If $AB = BC = CD = DE = EF = AF$, find the area of the parallelogram $BCEF$.



6. The figure shows a rectangle $ABCD$ of length 8 cm and of breadth 6 cm. AQC is a diagonal of the rectangle. If $BP = PC$, find the area of the trapezium $ABPQ$.



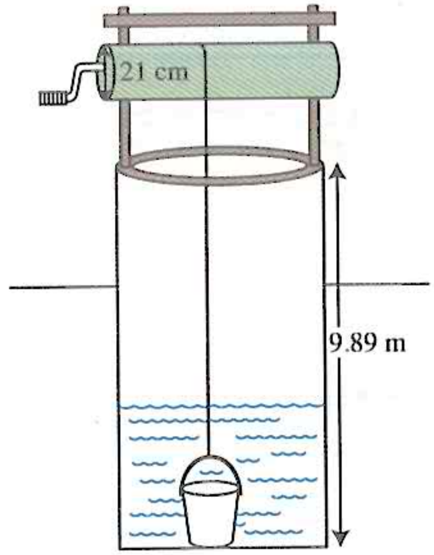
7. In the figure, $AB = 20$ m, $BC = AF = 15$ m, $DE = 3.5$ m and $FG = 7$ m. Find the area of the figure, giving your answer in hectares.
(1 hectare = 10 000 m^2)

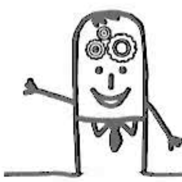


8. The area of a trapezium is 36 cm^2 and the perpendicular distance between its parallel sides is 6 cm.
   (i) If the lengths of these parallel sides are x cm and y cm respectively, find the value of $x + y$.
   (ii) If the value of x is twice that of y, find the value of x and of y.

9. Find the difference between the perimeter of a square of area 1 m^2 and the circumference of a circle that has the same area as the square.

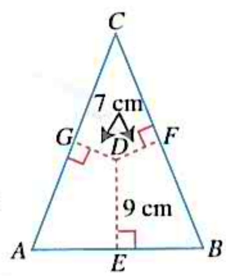
10. A rope which is wound round a drum of diameter 21 cm is attached to a bucket that is resting at the bottom of a well of depth 9.89 m. Find the number of complete turns of the handle required to bring up the bucket such that the bottom of the bucket is suspended just above the mouth of the well.



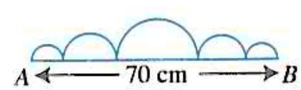


Challenge Yourself

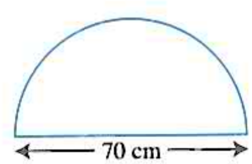
1. In the figure, ABC is an isosceles triangle where the lengths of AB, BC and AC are in the ratio $1 : 2 : 2$. If $DE = 9$ cm and $DF = DG = 7$ cm, find the possible heights of ΔABC.



2. (i) The figure shows five semicircles of different diameters. If $AB = 70$ cm, find the perimeter of the figure.

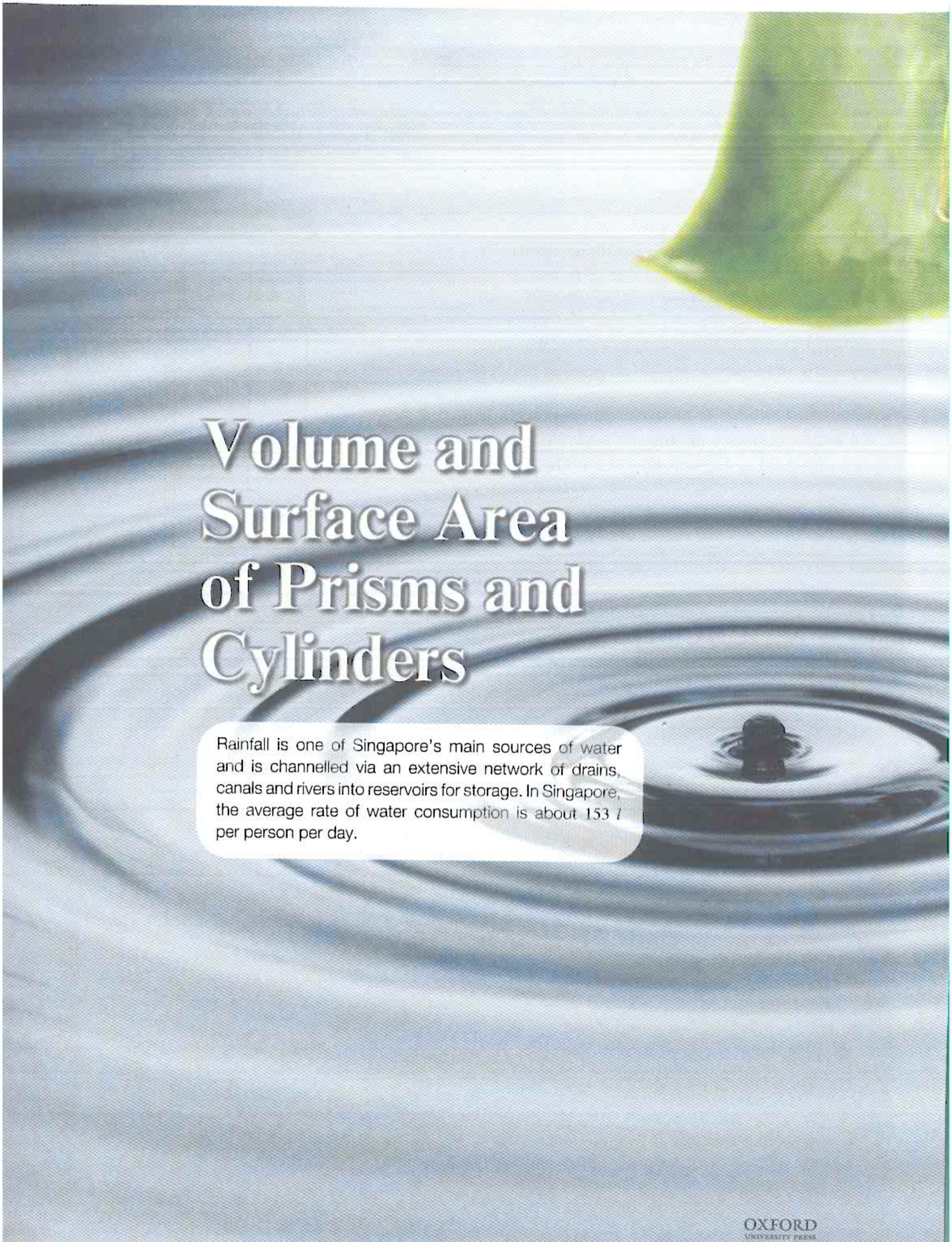


   (ii) The figure shows a semicircle with a diameter of 70 cm. Find the perimeter of the figure.



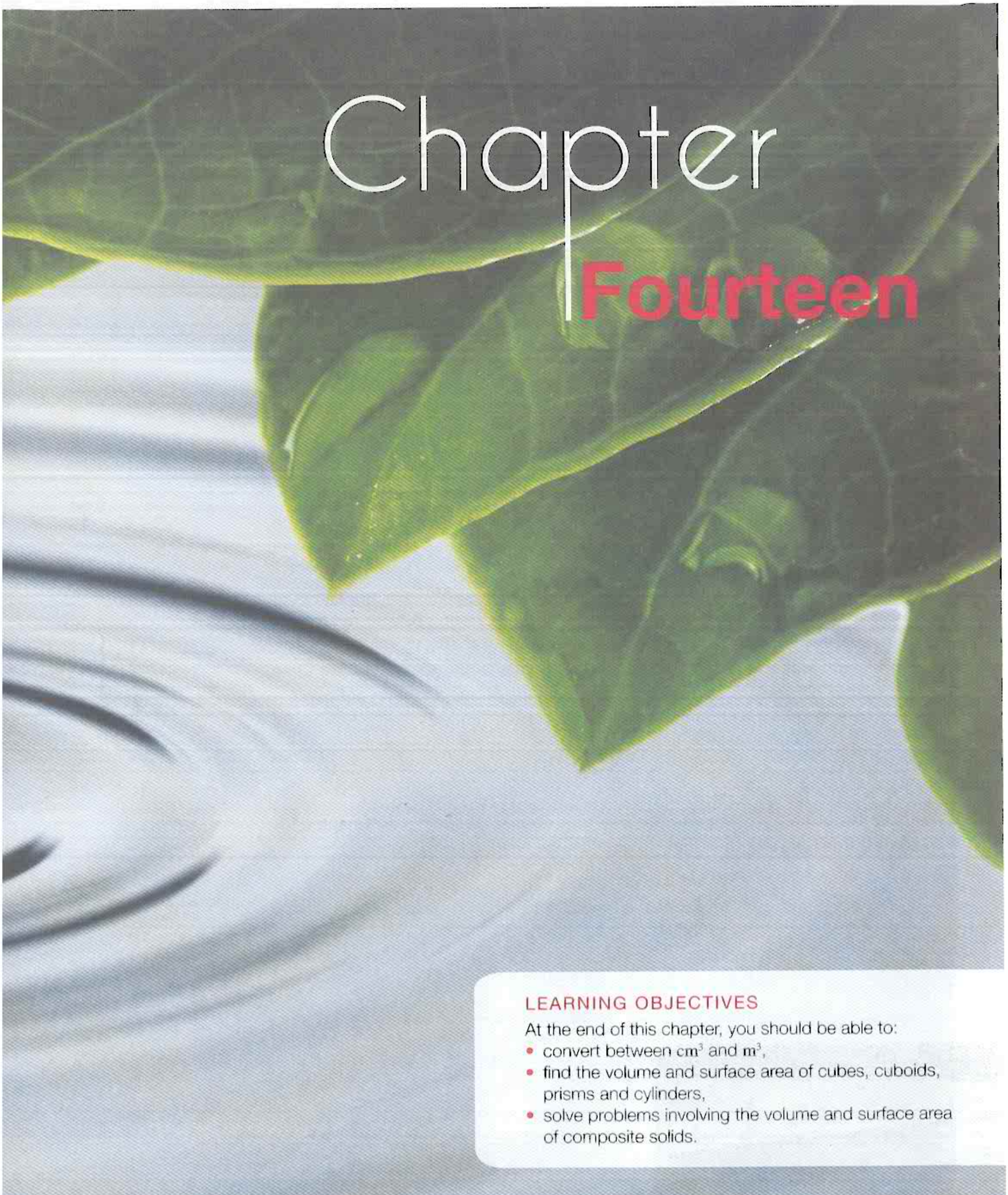
   (iii) From your answers in (i) and (ii), what can you conclude?

3. A trapezium $ABCD$ is such that $AD \parallel BC$. The diagonals AC and BD of the trapezium intersect at E. If the lengths of BD and BE are in the ratio $5 : 4$ and the area of ΔABE is 20 cm^2, find the area of the trapezium.



Volume and Surface Area of Prisms and Cylinders

Rainfall is one of Singapore's main sources of water and is channelled via an extensive network of drains, canals and rivers into reservoirs for storage. In Singapore, the average rate of water consumption is about $153\ l$ per person per day.



Chapter Fourteen

LEARNING OBJECTIVES

At the end of this chapter, you should be able to:

- convert between cm^3 and m^3,
- find the volume and surface area of cubes, cuboids, prisms and cylinders,
- solve problems involving the volume and surface area of composite solids.

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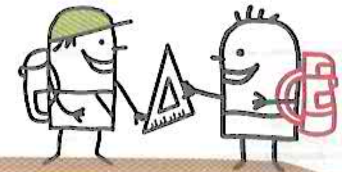
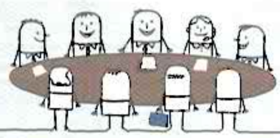
14.1 Conversion of Units



The volume of a solid or a fluid is measured using units such as millilitres (ml), litres (l), cubic centimetres (cm^3) and cubic metres (m^3).

Class Discussion



Measurements in Daily Lives

1. (i) In everyday life, we often encounter units of measurement of volume. The amount of water used in our homes is measured in m^3. Search on the Internet to find out Singaporeans' average water consumption for various domestic activities, e.g. washing the dishes and taking a shower, and determine the activity which requires the greatest amount of water on average.

(ii) By checking your utilities bill, find out the amount of water used in your home last month. Suggest some measures that can be taken to reduce the average water consumption in your household.

2. (i) Mineral water is usually sold in 500 ml bottles while soft drinks are often sold in 330 ml cans. What is the volume, in ml, of one teaspoon of liquid?

(ii) Many health practitioners recommend that we drink at least 8 glasses of water daily. Approximately how many litres does this correspond to?

We have learnt how to convert from one unit of area to another in Chapter 13. To convert from one unit of volume to another, we need to know that:

$$\boxed{\begin{array}{l} 1 \text{ m}l = 1 \text{ cm}^3 \\ 1\ l = 1000 \text{ m}l = 1000 \text{ cm}^3 \\ 1000\ l = 1 \text{ m}^3 \end{array}}$$

ATTENTION

$$\begin{aligned} 1000\ l &= 1\,000\,000 \text{ m}l \\ &= 1\,000\,000 \text{ cm}^3 \\ &= \frac{1\,000\,000}{(100)^3} \text{ m}^3 \\ &= \frac{1\,000\,000}{1\,000\,000} \text{ m}^3 \\ &= 1 \text{ m}^3 \end{aligned}$$

Worked Example 1

(Conversion between Different Units)

Express

(a) 1 m^3 in cm^3,

(b) 1 cm^3 in m^3,

(c) 5 m^3

  (i) in cm^3,

  (ii) in millilitres,

(d) $80\ 000 \text{ cm}^3$

  (i) in m^3,

  (ii) in litres.

Solution:

(a) $1 \text{ m} = 100 \text{ cm}$

$(1 \text{ m})^3 = (100 \text{ cm})^3$

$= 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm}$

$1 \text{ m}^3 = 1\ 000\ 000 \text{ cm}^3$

(b) From **(a)**,

$1 \text{ m}^3 = 1\ 000\ 000 \text{ cm}^3$

i.e. $1\ 000\ 000 \text{ cm}^3 = 1 \text{ m}^3$

$\therefore 1 \text{ cm}^3 = \dfrac{1}{1\ 000\ 000} \text{ m}^3$

$= 0.000\ 001 \text{ m}^3$

(c) **(i)** $1 \text{ m}^3 = 1\ 000\ 000 \text{ cm}^3$

$5 \text{ m}^3 = 5 \times 1\ 000\ 000 \text{ cm}^3$

$= 5\ 000\ 000 \text{ cm}^3$

(ii) $1 \text{ cm}^3 = 1 \text{ m}l$

$5\ 000\ 000 \text{ cm}^3 = 5\ 000\ 000 \text{ m}l$

$\therefore 5 \text{ m}^3 = 5\ 000\ 000 \text{ m}l$

(d) **(i)** $1\ 000\ 000 \text{ cm}^3 = 1 \text{ m}^3$

$80\ 000 \text{ cm}^3 = \dfrac{80\ 000}{1\ 000\ 000} \text{ m}^3$

$= 0.08 \text{ m}^3$

(ii) $1 \text{ cm}^3 = 1 \text{ m}l$

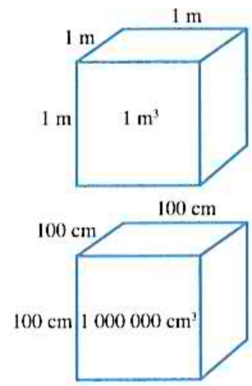
$80\ 000 \text{ cm}^3 = 80\ 000 \text{ m}l$

$= \dfrac{80\ 000}{1000}\ l$

$= 80\ l$

ATTENTION

The volume of a cube of sides 1 m is 1 m^3, which is equal to $1\ 000\ 000 \text{ cm}^3$.



PRACTISE NOW 1

Express

(a) 10 m^3

  (i) in cm^3,

  (ii) in millilitres,

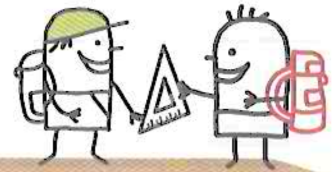
(b) $165\ 000 \text{ cm}^3$

  (i) in m^3,

  (ii) in litres.

SIMILAR QUESTIONS

Exercise 14A Questions 1(a)–(b), 2(a)–(b)



14.2 Nets

Investigation

Cubes, Cuboids, Prisms and Cylinders

For **Part I** and **Part II** of this investigation, paper boxes in the shape of a cube and a cuboid are required.

Part I: Draw a cube and a cuboid.

1. Place the paper boxes on a table.
2. Observe the boxes by looking at each box from different angles, e.g. from the left, right, front, back, etc.
3. Draw each box by copying its shape. A drawing of a cube and of a cuboid is given in Table 14.1.

Part II: Draw a net of a cube and of a cuboid.

1. Consider the box in the shape of a cube. Cut along the edges of the box such that all the surfaces can be laid flat in one piece.
2. In Table 14.1, draw the net of the solid that you have cut out.
3. Compare the net you have drawn with that of your classmates. Are the nets identical? Can a solid have different nets?

Repeat Steps 1 to 3 for the box in the shape of a cuboid.

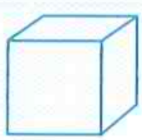
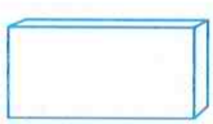
Name	Figure	Net
Cube		
Cuboid		

Table 14.1

Part III: Form a solid when the net is given.

1. Copy each of the following nets onto a sheet of paper and cut them out.
2. Try to fold them into the corresponding solids.
3. In Table 14.2, draw the solids formed from the nets.

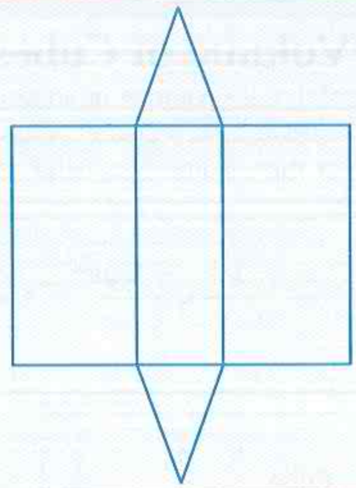
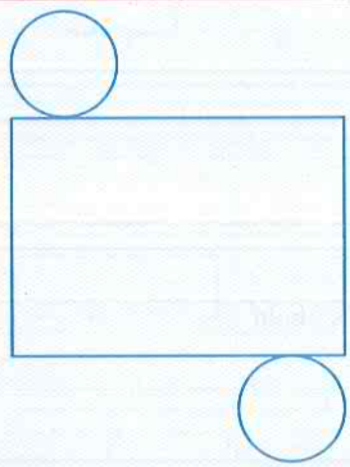
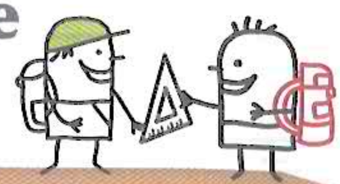
Name	Figure	Net
Triangular Prism		
Cylinder		

Table 14.2

Part IV: Draw a triangular prism and a cylinder.

1. Place the triangular prism and the cylinder from Part III on a table.
2. Observe the solids by looking at each solid from different angles, e.g. from the left, right, front, back, etc.
3. Draw each solid from a different angle as the one you have drawn in Table 14.2.

14.3 Volume and Surface Area of Cubes and Cuboids



Recap (Volume of Cubes and Cuboids)

We have learnt that the **volume** of an object is the amount of space it occupies. The object that occupies *more space* is said to have a *greater volume*.

The formula for the volume of a **cube** and of a **cuboid** is given in Table 14.3. A net of each of them is also provided.

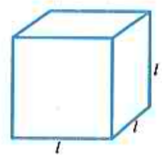
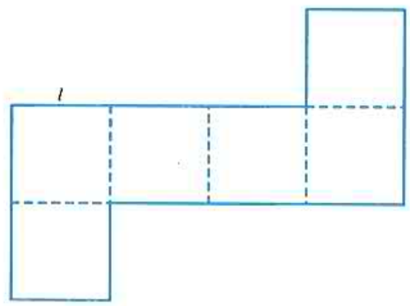
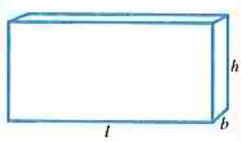
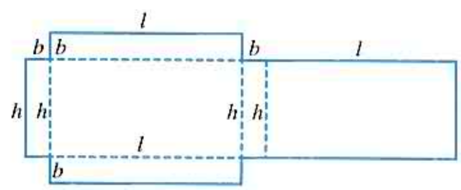
Name	Figure	Volume	Net
Cube		l^3	
Cuboid		lbh	

Table 14.3

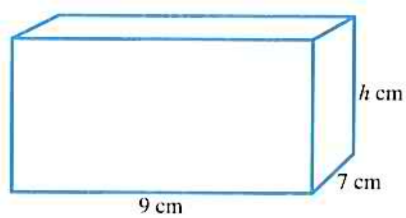
Worked Example 2

(Volume of a Cuboid)

A cuboid, with dimensions 9 cm by 7 cm by h cm, has a volume of 378 cm^3.

(i) Calculate the height, h, of the cuboid.

(ii) The cuboid is melted to form smaller cuboids with dimensions 2 cm by 3 cm by 3 cm. How many smaller cuboids can be obtained?



Solution:

(i) Volume of the cuboid $= 9 \times 7 \times h = 378$ cm^3

$$\therefore h = \frac{378}{9 \times 7} = 6$$

(ii) Volume of each small cuboid $= 2 \times 3 \times 3 = 18$ cm^3

Number of small cuboids that can be obtained $= \frac{378}{18} = 21$

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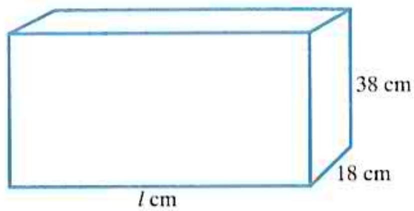
PRACTISE NOW 2

SIMILAR QUESTIONS

Exercise 14A Questions 5–7

1. A cuboid, with dimensions l cm by 18 cm by 38 cm, has a volume of 35 568 cm^3.
   (i) Find the length, l, of the cuboid.
   (ii) The cuboid is melted to form cubes of length 2 cm. How many cubes can be obtained?



2. An open rectangular tank, with dimensions 55 cm by 35 cm by 36 cm, is initially half-filled with water. Find the depth of water in the tank after 7700 cm^3 of water is added to it.

Worked Example 3

(Problem involving the Volume of a Cuboid)

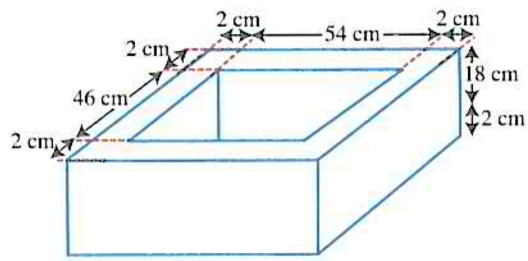
Calculate the volume of wood used in making an open rectangular box 2 cm thick, given that its internal dimensions are 54 cm by 46 cm by 18 cm.

Solution:

External volume $= (54 + 2 + 2) \times (46 + 2 + 2) \times (18 + 2)$
$= 58 \times 50 \times 20$
$= 58\ 000$ cm^3

Internal volume $= 54 \times 46 \times 18$
$= 44\ 712$ cm^3

$\therefore$ Volume of wood used $= 58\ 000 - 44\ 712$
$= 13\ 288$ cm^3

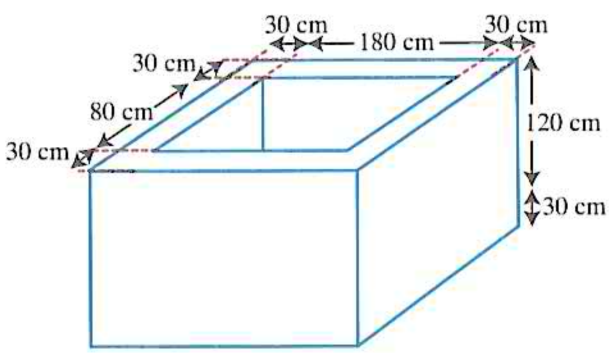


PRACTISE NOW 3

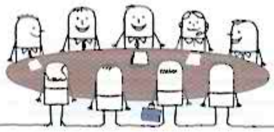
SIMILAR QUESTIONS

Exercise 14A Questions 8–9, 15–16

The internal dimensions of an open, concrete rectangular tank are 180 cm by 80 cm by 120 cm. If the concrete has a thickness of 30 cm, find the volume of concrete used.



Surface Area of Cubes and Cuboids



Class Discussion

Surface Area of Cubes and Cuboids

1. Refer to the nets of a cube and a cuboid in Table 14.3 and fill in the blanks below.
   A cube has ___ surfaces. Each surface is in the shape of a _______________.
   The area of each face is _______________.
   ∴ The total surface area of a cube is _______________.
   A cuboid has ___ surfaces. Each surface is in the shape of a _______________.
   ∴ The total surface area of a cuboid is _______________.
2. What is the relationship between the area of each face of the net and the total surface area of the object?
3. Verify your answers for Questions 1 and 2 with your classmate.

INFORMATION

The word 'surface' is pronounced as sur-fis, not sur-face.

From the class discussion, we can conclude that:

> The **total surface area** of an object is equal to the *area of all the faces* of the net. In particular, we have:
> - Total surface area of a cube = $6l^2$
> - Total surface area of a cuboid = $2(lb + lh + bh)$

Worked Example 4

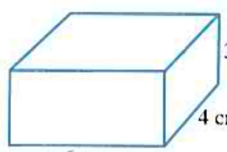
(Surface Area of a Cuboid)

A cuboid is 6 cm long, 4 cm wide and 3 cm high. Calculate
(i) its volume, **(ii)** its total surface area.

Solution:

(i) Volume of the cuboid $= 6 \times 4 \times 3$
$= 72 \text{ cm}^3$

(ii) Surface area of the cuboid $= 2(6 \times 4 + 6 \times 3 + 4 \times 3)$
$= 108 \text{ cm}^2$



PRACTISE NOW 4

SIMILAR QUESTIONS

Exercise 14A Questions 3(a)–(f), 4(a)–(d), 10–14, 17–18

1. A cuboid is 8 cm long, 5 cm wide and 10 cm high. Find
   (i) its volume, **(ii)** its total surface area.
2. An open rectangular tank of length 16 cm and breadth 9 cm contains water to a height of 8 cm. Find
   (i) the volume of water in the tank, giving your answer in litres,
   (ii) the surface area of the tank that is in contact with the water.
3. A metal cube has a volume of 27 cm^3. It is to be painted on all its faces. Find the total area of the faces that will be coated with paint.

Exercise 14A

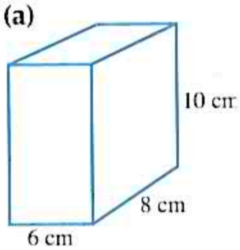
BASIC LEVEL

1. (a) Express each of the following in cm^3.
   (i) $4\ m^3$ (ii) $0.5\ m^3$
   (b) Express each of the following in m^3.
   (i) $250\ 000\ cm^3$ (ii) $67\ 800\ cm^3$

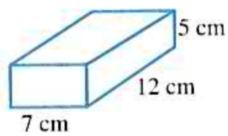
2. Express
   (a) $0.84\ m^3$
   (i) in cm^3, (ii) in millilitres,
   (b) $2560\ cm^3$
   (i) in m^3, (ii) in litres.

3. For each of the following cuboids, find
   (i) its volume, (ii) its total surface area.

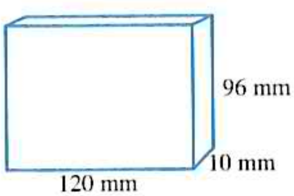
(a)



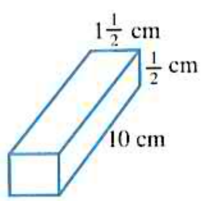
(b)



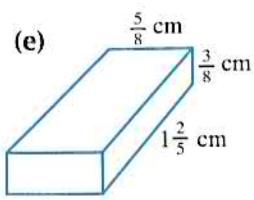
(c)



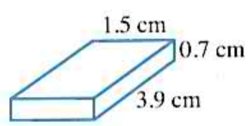
(d)



(e)



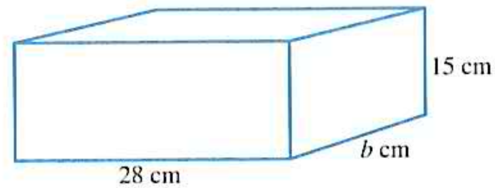
(f)



4. Complete the table for each cuboid.

	Length	Breadth	Height	Volume	Total surface area
(a)	24 mm	18 mm	5 mm		
(b)	5 cm	3 cm		$120\ cm^3$	
(c)		6 cm	3.5 cm	$52.5\ cm^3$	
(d)	12 m		6 m	$576\ m^3$	

5. A cuboid, with dimensions 28 cm by b cm by 15 cm, has a volume of $6720\ cm^3$.



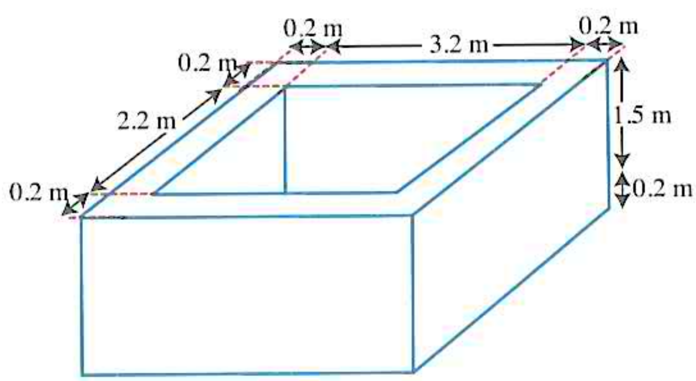
   (i) Find the breadth, b, of the cuboid.
   (ii) The cuboid is melted to form smaller cubes of length 4 cm. How many cubes can be obtained?

INTERMEDIATE LEVEL

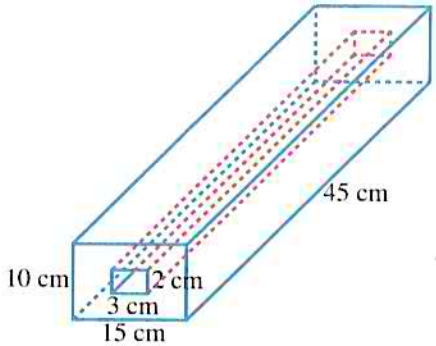
6. A rectangular block of metal is 0.24 m long, 0.19 m wide and 0.15 m high. If the metal block is melted to form a cube, find the length of each side of the cube.

7. An open rectangular tank that is 4 m long, 2 m wide and 4.8 m high, is initially three-quarters filled with water. Find the depth of water in the tank after 4000 litres of water are added to it.

8. The internal dimensions of an open, wooden rectangular box are 3.2 m by 2.2 m by 1.5 m. If the wood has a thickness of 0.2 m, find the volume of wood used.



9. Find the volume of the hollow glass structure.



10. An open rectangular tank of length 0.2 m and breadth 0.15 m contains water to a height of 0.16 m. Find
    (i) the volume of water in the tank, giving your answer in litres,
    (ii) the surface area of the tank that is in contact with the water.

11. A fish tank measuring 80 cm by 40 cm contains water to a height of 35 cm. Find
    (i) the volume of water in the tank, giving your answer in litres,
    (ii) the surface area of the tank that is in contact with the water, giving your answer in m^2.

12. A metal cube has a volume of 64 cm^3. It is to be painted on all its faces. Find the total area of the faces that will be coated with paint.

13. The total surface area of a cube is 433.5 cm^2. Find its volume.

14. 2.85 million cubic metres of earth were required to fill the disused Sin Seng quarry at Rifle Range Road.
    (i) If each truck could carry a maximum load of 6.25 m^3 of earth per trip, how many trips were required to fill the entire quarry?
    (ii) The cost of transporting each truckload of earth was \$55. How much did it cost to fill the quarry?
    (iii) Given that the site of the quarry has a land area of approximately 3 hectares, find the cost to fill 1 m^2 of the land. (1 hectare = 10 000 m^2)

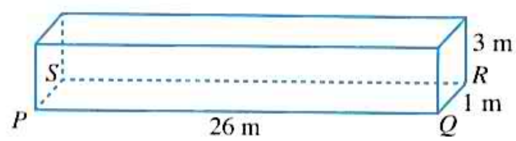
ADVANCED LEVEL

15. A trough, in the form of an open rectangular box, is 1.85 m long, 45 cm wide and 28 cm deep externally. If the trough is made of wood 2.5 cm thick, find the volume of wood used to make this trough, giving your answer in m^3.

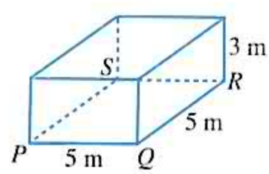
16. The cross section of a drain is a rectangle 30 cm wide. If water 3.5 cm deep flows through the drain at a rate of 22 cm/s, how many litres of water will flow through in one minute?

17. A cuboid of length 12 cm and breadth 9 cm has a total surface area of 426 cm^2.
    (i) Find the height of the cuboid.
    (ii) Hence, find its volume.

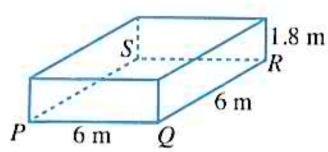
18. Three rooms, each in the shape of a cuboid, where $PQRS$ is the floor, are as shown.
    Cuboid A: PQ = 26 m; QR = 1 m; height = 3 m
    Cuboid B: PQ = QR = 5 m; height = 3 m
    Cuboid C: PQ = 6 m; QR = 6 m; height = 1.8 m



Room A



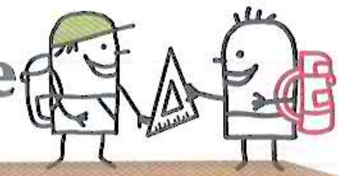
Room B



Room C

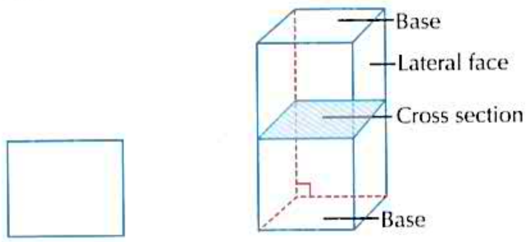
(i) Find the floor area and the volume of each room.
(ii) Which room feels the most spacious? Does a larger floor area or a greater volume necessarily make a room feel more spacious? Explain your answer.

14.4 Volume and Surface Area of Prisms



Introduction to Prisms

Consider a rectangular piece of cardboard in Fig. 14.1(**a**). A large number of identical rectangular pieces of cardboard are stacked up to form a cuboid as shown in Fig. 14.1(**b**).



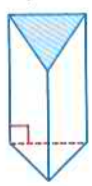
(a) **(b)**

Fig. 14.1

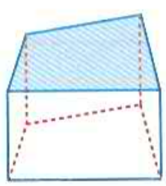
The top and bottom pieces of cardboard are the **bases** of the cuboid. The bases are *parallel* to each other and are *identical* rectangles. Any horizontal cross section of the cuboid is parallel to them and is also a rectangle that is identical to them. We say that a cuboid has a **uniform cross section**. The faces, other than the bases, are the **lateral faces** of the cuboid. A cuboid is an example of a **prism**. We can conclude that:

A prism has a uniform polygonal cross section.

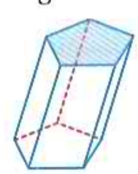
Fig. 14.2 shows some examples of prisms with different bases. A base of each prism is shaded. A prism is named after its polygonal base. Can you name the last two prisms? Write your answers in the spaces provided in Fig. 14.2.



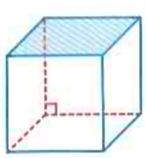
(a) Triangular Prism



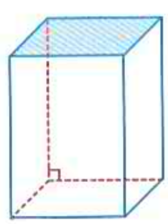
(b) Quadrilateral Prism



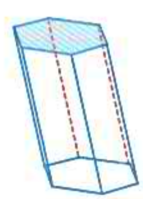
(c) Pentagonal Prism



(d) Cube or Square Prism



(e) Cuboid or ________ Prism

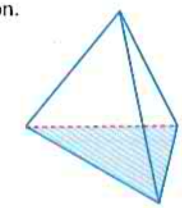


(f) ________ Prism

Fig. 14.2

INFORMATION

The figure shows a counterexample of a prism. It does not have a uniform cross section.



INFORMATION

Some real-life examples of prisms are as follows:

Name the prisms. Can you think of other real-life examples of prisms?

Right Prisms

For the triangular prism in Fig. 14.2**(a)**, do you notice that all the *lateral faces* are *perpendicular* to the *bases*? Such a prism is called a **right prism**. Non-right prisms are called **oblique prisms**. The pentagonal prism in Fig. 14.2**(c)** is an example of an oblique prism. Can you identify another right prism and another oblique prism from Fig. 14.2?

Thinking Time

1. **(i)** What is the shape of all the lateral faces of a right prism?
   (ii) What is the shape of all the lateral faces of an oblique prism?
2. Draw a square prism that is not a cube.
3. Consider building structures and various items which you come across in your daily lives. How many of these are right prisms? Are you able to make sketches of them? Can you think of any reason why they are shaped as prisms?

In this section, we will study only right prisms. Therefore, the term 'prism' is used to refer to a right prism.

Volume of a Prism

Recall that the volume of a cuboid = length × breadth × height
= base area × height.

Now, we will learn how to find the volume of a prism.

Consider a cuboid with dimensions 4 m by 3 m by 2 m as shown in Fig.14.3.
Take the shaded area as the cross section of the cuboid.
The area of the cross section of the cuboid is _______.
Hence, the volume of the cuboid is _______ × _______ = _______.

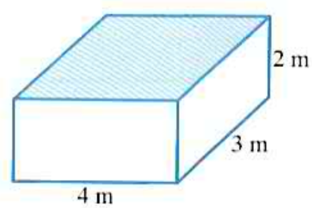


Fig. 14.3

If we are to cut the cuboid vertically as shown in Fig. 14.4, we will obtain two equal *triangular prisms*.

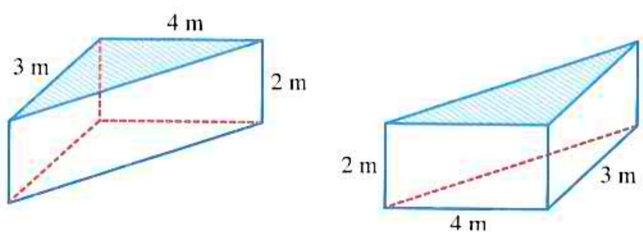


Fig. 14.4

Take the shaded area as the cross section of each triangular prism. The area of the cross section of the triangular prism is _______________.
Area of cross section × height = _______ × 2 m
= _______

Can you find a relationship between the volume of a prism and the area of its cross section?

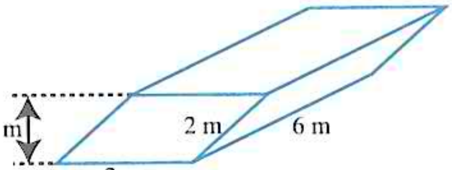
To conclude, we have:

Volume of a prism = area of cross section × distance between cross–sectional bases = base area × height

Worked Example 5

(Volume of a Prism)

Calculate the volume of the prism.



Solution:

Base area = area of parallelogram
$= 2 \times 1$
$= 2\ \text{m}^2$

Volume of the prism = base area × height
$= 2 \times 6$
$= 12\ \text{m}^3$

Problem Solving Tip

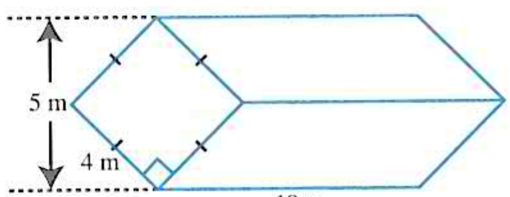
Do not confuse the base and the height of a parallelogram with the base and the height of a prism.

PRACTISE NOW 5

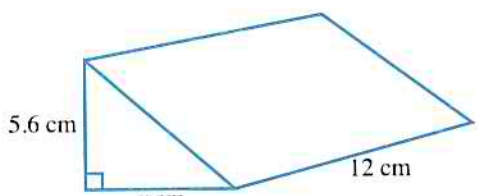
SIMILAR QUESTIONS

Exercise 14B Questions 1(a)–(f), 2(a)–(d), 3

1. Find the volume of the prism.



2. The volume of the prism is 151.2 cm³. Find the value of x.



Surface Area of a Prism

In Section 14.3, we have learnt that the total surface area of a cuboid is equal to the area of all the faces of the net. We shall now extend this concept to find the total surface area of a prism.

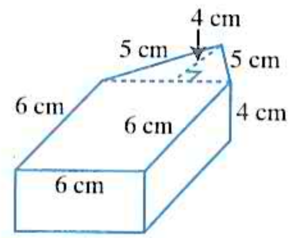


Fig. 14.5

Let us consider a pentagonal prism.

A net of the prism in Fig. 14.5 is shown in Fig. 14.6. The red dotted lines indicate the folds.

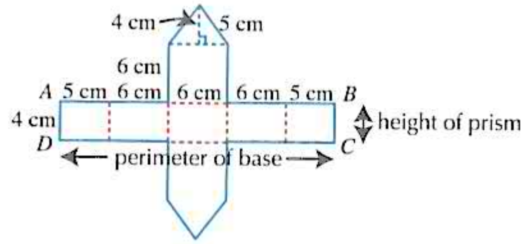


Fig. 14.6

From Fig. 14.6,

Total surface area of prism = area of rectangle $ABCD$ + 2 × area of pentagonal base

$= AB \times BC + 2 \times \quad 6 \times 6 + \frac{1}{2} \times 6 \times 4$

= perimeter of base × height of prism + 2 × (36 + 12)

$= (5 + 6 + 6 + 6 + 5) \times 4 + 2 \times 48$

$= 28 \times 4 + 96$

$= 112 + 96$

$= 208\text{ cm}^2$

To conclude, we have:

> Total surface area of a prism = total area of the lateral faces + 2 × base area
> = perimeter of the base × height + 2 × base area

Worked Example 6

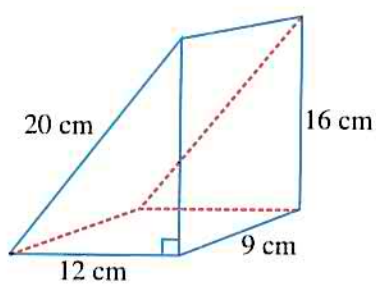
(Surface Area of a Prism)

Calculate

(i) the volume,

(ii) the total surface area,

of the prism.



Solution:

(i) Volume of the prism = base area × height

$= \quad \frac{1}{2} \times 12 \times 16 \quad \times 9$ (The base is a right-angled triangle.)

$= 96 \times 9$

$= 864\text{ cm}^3$

(ii) Total surface area of the prism = perimeter of the base × height + 2 × base area

$= (12 + 16 + 20) \times 9 + 2 \times 96$

$= 48 \times 9 + 192$

$= 432 + 192$

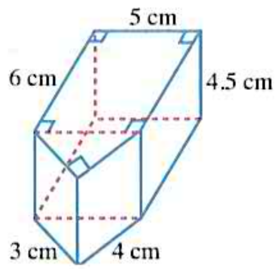
$= 624\text{ cm}^2$

PRACTISE NOW 6

SIMILAR QUESTIONS

Exercise 14B Questions 4(a)–(b), 5

Calculate
(i) the volume,
(i) the total surface area,
of the prism.

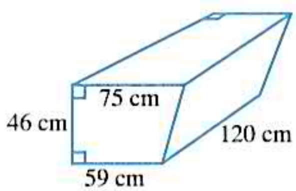


Exercise 14B

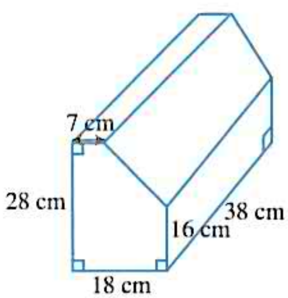
BASIC LEVEL

1. By first identifying the base, find the volume of each of the following prisms.

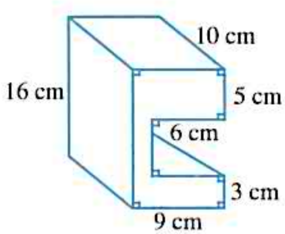
(a)



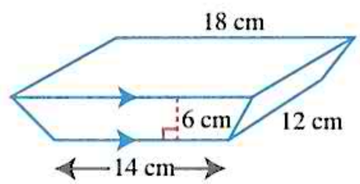
(b)



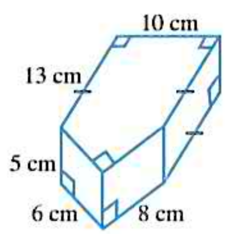
(c)



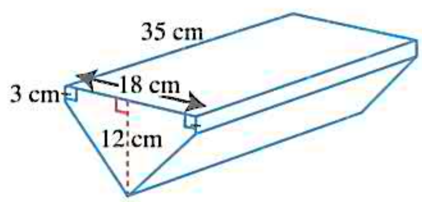
(d)



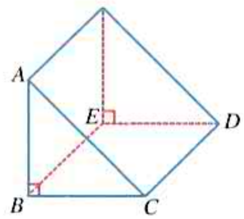
(e)



(f)



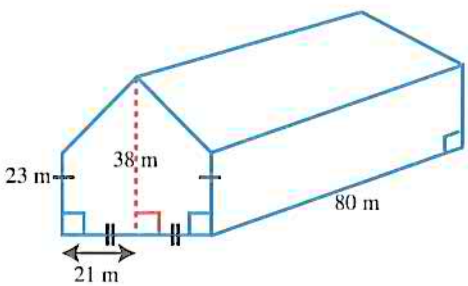
2. The figure shows a prism standing on a horizontal, rectangular base $BCDE$. ΔABC is a vertical cross section of the prism.



Complete the table.

	AB	BC	CD	Area of ΔABC	Volume of prism
(a)	3 cm	4 cm	7 cm		
(b)	9 cm		11 cm	63 cm^2	
(c)		15 cm	300 cm		72 000 cm^3
(d)	24.6 cm	7.8 cm			38 376 cm^3

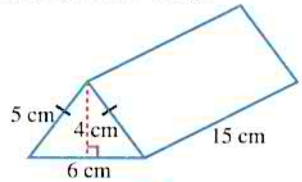
3. The figure shows an empty hall. Without taking into consideration the thickness of the walls and the roof, find the air space in the hall.



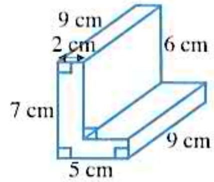
INTERMEDIATE LEVEL

4. For each of the following prisms, find
   (i) its volume,
   (ii) its total surface area.

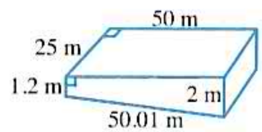
   (a)



   (b)

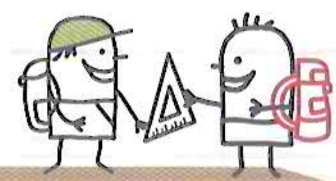


5. A swimming pool is 50 m long and 25 m wide. It is 1.2 m deep at the shallow end and 2 m deep at the other end.



Find
(i) the volume of water in the pool when it is full,
(ii) the area of the pool which is in contact with the water.

14.5 Volume and Surface Area of Cylinders



Cylinders

Consider a 50-cent coin as shown in Fig. 14.7**(a)**. A large number of 50-cent coins have been stacked up vertically to form a circular tower in Fig. 14.7**(b)**.

(a) **(b)**

Fig. 14.7

Fig. 14.8

Every 50-cent coin in the circular tower is *parallel* and *identical* to one another. What is the shape of the cross section?

This object is known as a **cylinder** (see Fig. 14.8). A cylinder has a *uniform* circular cross section.

Formally, the cylinder shown in Fig. 14.8 is known as a right circular cylinder. In this section, we will study only right circular cylinders. Therefore, the term 'cylinder' is used to refer to a right circular cylinder.

Thinking Time

Consider building structures and various items which you come across in your daily lives.

How many of these are cylinders? Are you able to make sketches of them?

Can you think of any reason why they are shaped as cylinders?

Volume of a Cylinder

In Section 14.4, we have learnt:

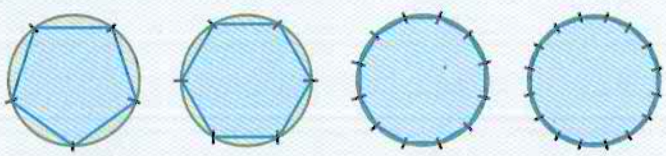
Volume of a prism = area of cross section × height = base area × height

Now, we will learn how to find the volume of a cylinder by comparing a cylinder with a prism that has a regular polygonal base.

Investigation

Comparison between a Cylinder and a Prism

1. Fig. 14.9 shows **(a)** a regular pentagon, **(b)** a regular hexagon, **(c)** a regular 12-gon, and **(d)** a regular 16-gon inside a circle respectively. The polygons must be *regular.*

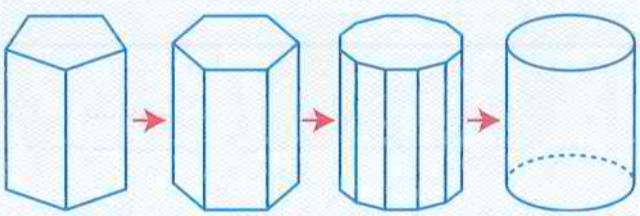


(a) **(b)** **(c)** **(d)**

Fig. 14.9

If the number of sides of a *regular* polygon is increased indefinitely, what will the polygon become?

2. Fig. 14.10 shows a sequence of *regular* right prisms, i.e. right prisms with regular polygonal bases.



(a) **(b)** **(c)** **(d)**

Fig. 14.10

If the number of sides of the regular polygonal base of a prism is increased indefinitely, what will the prism become?

In many ways, a cylinder is *like* a prism. However, a cylinder is *not* a prism because the base of a prism must be a polygon but the base of a cylinder is a circle. Although a regular polygon can become a circle if its number of sides is increased indefinitely, a polygon must have a *finite* number of sides and so a circle is *not* a polygon.

Since a cylinder is like a prism (see Fig. 14.10), by analogy, the formula for the volume of a cylinder should be the same as the formula for the volume of a prism. We have:

$$\begin{aligned}\text{Volume of a cylinder} &= \text{area of cross section} \times \text{height}\\ &= \text{base area} \times \text{height}\\ &= \pi r^2 h,\end{aligned}$$

where r = base radius and h = height of the cylinder.

RECALL

Area of a circle = πr^2

Worked Example 7

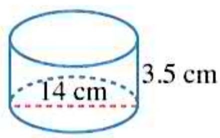
(Volume of a Cylinder)

The diameter of the base of a cylinder is 14 cm and its height is half of its base radius. Calculate the volume of the cylinder.

Solution:

Base radius = $14 \div 2 = 7$ cm

Height of the cylinder = $\frac{1}{2} \times 7 = 3.5$ cm



Volume of the cylinder $= \pi r^2 h$

$= \pi(7)^2(3.5)$

$= 539 \text{ cm}^3$ (to 3 s.f.)

PRACTISE NOW 7

SIMILAR QUESTIONS

Exercise 14C Questions 3–5, 7–9

1. The diameter of the base of a cylinder is 18 cm and its height is 2.5 times its base radius. Find the volume of the cylinder.
2. The volume of a cylindrical can of pineapple juice is 1000 cm^3 and the diameter of its base is 12 cm. Find the height of the can of pineapple juice.

Worked Example 8

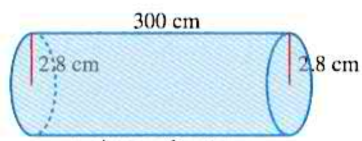
(Problem involving the Volume of a Pipe)

A pipe of radius 2.8 cm discharges water at a rate of 3 m/s. Calculate the volume of water discharged per minute, giving your answer in litres.

A glass is half filled with water. Without measuring the volume of the water, how do you determine that the volume of the water is *exactly* half the volume of the glass?

Solution:

Since water is discharged through the pipe at a rate of 3 m/s, i.e. 300 cm/s, in 1 second, the volume of water discharged is the volume of water that fills the pipe to a length of 300 cm as shown.



volume of water discharged in 1 second

In 1 second, volume of water discharged

= volume of pipe of length 300 cm

$= \pi r^2 h$

$= \pi(2.8)^2(300)$

$= 2352\pi \text{ cm}^3$

In 1 minute, volume of water discharged $= 2352\pi \times 60$

$= 443\ 000 \text{ cm}^3$ (to 3 s.f.)

$= 443\ l$

PRACTISE NOW 8

SIMILAR QUESTIONS

Exercise 14C Questions 10–11, 16

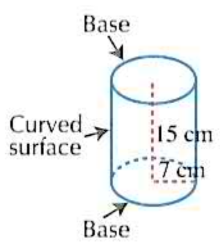
1. A pipe of radius 0.6 cm discharges petrol at a rate of 2.45 m/s. Find the volume of petrol discharged in 3 minutes, giving your answer in litres.

2. A pipe of diameter 0.036 m discharges water at a rate of 1.6 m/s into a cylindrical tank with a base radius of 3.4 m and a height of 1.4 m. Find the time required to fill the tank, giving your answer correct to the nearest minute.

Surface Area of a Cylinder

In Section 14.4, we have learnt that the total surface area of a prism is equal to the area of all the faces of the net. We shall now extend this concept to find the total surface area of a cylinder.

Fig. 14.11 **(a)** shows a *closed* cylinder which has a base radius of 7 cm and a height of 15 cm. Recall from the investigation in Section 14.2 that its corresponding net is as shown in Fig. 14.11 **(b)**.



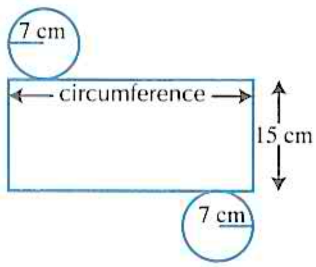


Fig. 14.11

The net of the cylinder consists of two __________ and one __________.
To find the total surface area of the cylinder, we need to know the area of the two circles and that of the rectangle.
The area of the two circles is $2\pi r^2 = 2 \times \pi \times 7^2 =$ __________.
How do we find the area of the rectangle? Notice that the length of the rectangle is the same as the circumference of the circular base, i.e. the length of the rectangle is $2\pi r = 2 \times \pi \times 7 =$__________. Hence, the area of the rectangle is __________.
$\therefore$ The total surface area of the cylinder in Fig. 14.11 **(a)** is __________.

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Thinking Time

Now consider a *closed* cylinder with a base radius of r and a height of h. Can you find a general formula for its total surface area?

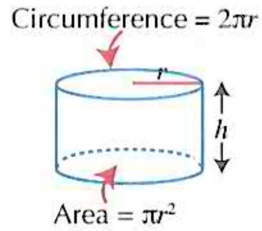


Fig. 14.12

To conclude, we have:

Total surface area of a *closed* cylinder $= 2 \times$ base area + curved surface area $= 2\pi r^2 + 2\pi rh$

Class Discussion

Total Surface Area of Other Types of Cylinders

We have just learnt how to find the total surface area of a closed cylinder. Discuss with your classmates how you can obtain the total outer surface area of

(a) an open cylinder,
(b) a pipe of negligible thickness,

by drawing the net of each of the two cylinders.
Hint: An open cylinder is open on one end while a pipe has two open ends.

Worked Example 9

(Surface Area of a Cylinder)

A closed metal cylindrical container has a base radius of 5 cm and a height of 12 cm.

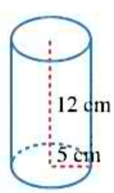
(i) Calculate the total surface area of the container.

The lid of the container is now removed. The exterior of the container, including the base, is painted green.

(ii) Express the area of the container that is painted as a percentage of the total surface area found in **(i)**.

Solution:

(i) Total surface area of the container $= 2\pi r^2 + 2\pi rh$
$= 2\pi(5)^2 + 2\pi(5)(12)$
$= 50\pi + 120\pi$
$= 170\pi$
$= 534$ cm² (to 3 s.f.)



(ii) Area of the container that is painted $= \pi r^2 + 2\pi rh$ (An open cylinder has only one base and a curved surface.)
$= \pi(5)^2 + 2\pi(5)(12)$
$= 25\pi + 120\pi$
$= 145\pi$
$= 456$ cm² (to 3 s.f.)

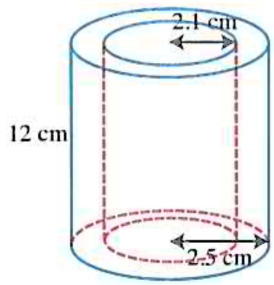
Required percentage $= \dfrac{145\pi}{170\pi} \times 100\%$
$= 85\dfrac{5}{17}\%$

PRACTISE NOW 9

SIMILAR QUESTIONS

Exercise 14C Questions 1(a)–(c), 2(a)–(d), 6, 12–15

1. A closed metal cylindrical can has a base radius of 3.5 cm and a height of 10 cm.
   (i) Find the total surface area of the can.
   The lid of the can is now removed. The exterior of the container, including the base, is painted purple.
   (ii) Find the ratio of the area of the can that is painted, to the total surface area found in **(i)**.

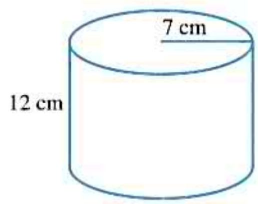
2. 

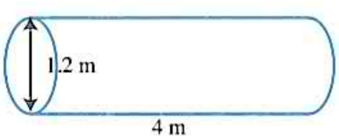
   The figure shows a section of a steel pipe of length 12 cm. The internal and external radii of the pipe are 2.1 cm and 2.5 cm respectively.
   (i) Show that the area of the cross section of the pipe is 1.84π cm^2.
   (ii) Find the internal curved surface area of the pipe.
   (iii) Hence, find the total surface area of the pipe.

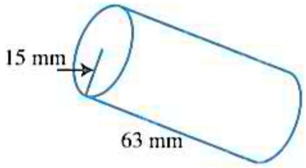
Exercise 14C

BASIC LEVEL

1. For each of the following closed cylinders, find **(i)** its volume, **(ii)** its total surface area.

   (a)


   (b)


   (c)


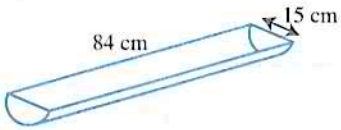
2. Complete the table for each closed cylinder.

	Diameter	Radius	Height	Volume	Total surface area
(a)			14 cm	704 cm^3	
(b)			20 cm	12 320 cm^3	
(c)	4 cm			528 cm^3	
(d)		4 m		1056 m^3	

3. The diameter of the base of a cylinder is 0.4 m and its height is $\frac{3}{4}$ of its base radius. Find the volume of the cylinder, giving your answer in litres.

4. 150 litres of water are poured into a cylindrical drum of diameter 48 cm. Find the depth of water in the drum.

5. The figure shows a drinking trough in the shape of a half-cylinder. Find its capacity in litres.

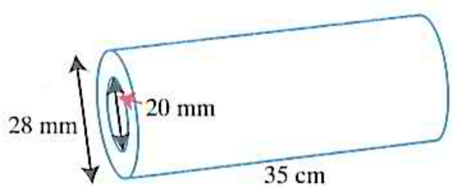


6. In a toy factory, 200 wooden closed cylinders of diameter 35 mm and height 7 cm have to be painted. What is the total surface area, in cm^2, that needs to be painted? (Take π to be 3.142.)

INTERMEDIATE LEVEL

7. A tank in the shape of a cylinder of diameter 2.4 m and height 6.4 m contains oil to the brim. Find the number of complete cylindrical containers of base radius 8.2 cm and height 28 cm which can be filled by the oil in the tank.

8. The figure shows a metal pipe of length 35 cm. The internal and external diameters of the pipe are 20 mm and 28 mm respectively. Find the volume of metal used in making the pipe, giving your answer in cubic centimetres.



9. A copper cylindrical rod of diameter 14 cm and length 47 cm is melted and recast into a wire of diameter 8 mm. Find the length of the wire, giving your answer in metres.

10. A pipe of diameter 2.4 cm discharges water at a rate of 2.8 m/s. Find the volume of water discharged in half an hour, giving your answer in litres.

11. A pipe of diameter 64 mm discharges water at a rate of 2.05 mm/s into an empty cylindrical tank of diameter 7.6 cm and height 2.3 m. Find the time required to fill the tank, giving your answer correct to the nearest minute.

12. An open rectangular tank of length 18 cm and breadth 16 cm contains water to a depth of 13 cm. The water is poured into a cylindrical container of diameter 17 cm. Find
   (i) the volume of water in the tank,
   (ii) the height of water in the cylindrical container,
   (iii) the surface area of the cylindrical container that is in contact with the water.

13. A closed steel cylindrical container has a diameter of 186 mm and its height is $\frac{1}{3}$ of its base radius.
   (i) Find the total surface area of the container, giving your answer in square centimetres.

   The lid of the container is now removed. The exterior of the container, including the base, is painted indigo.

   (ii) Express the area of the container that is painted as a fraction of the total surface area found in **(i)**.

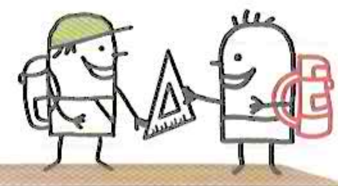
ADVANCED LEVEL

14. An open rectangular tank of length 32 cm and breadth 28 cm contains water to a depth of 19 cm. 2580 circular metal discs of diameter 23 mm and height 4 mm are dropped into the tank. Find
   (i) the new height of water in the tank,
   (ii) the surface area of the tank that is in contact with the water after the discs have been added.

15. A pipe of length 15 cm has an internal radius of 3.8 cm. The thickness of the pipe is 0.8 cm. Find the total surface area of the pipe.

16. On 5th June 2011, 124 mm of rainfall was recorded over an area of 28 km^2. If the rainwater falling onto the area was drained through two channels each with a cross-sectional area of 18 m^2 at a rate of 26.4 m/s, find the time, to the nearest minute, required for the channels to drain off the rain.

14.6 Volume and Surface Area of Composite Solids



In this section, we shall learn how to solve problems involving the volume and surface area of composite solids.

Worked Example 10

(Volume and Surface Area of a Composite Solid)

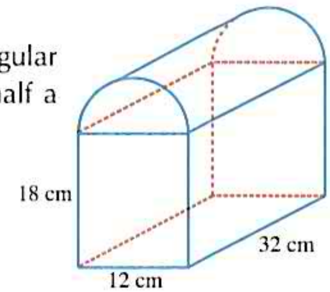
The figure shows a glass block made up of a rectangular prism of dimensions 32 cm by 12 cm by 18 cm and half a cylinder with a diameter of 12 cm.

Calculate

(i) the volume,

(ii) the total surface area,

of the glass block.



Solution:

(i) **Method 1:**

Volume of the rectangular prism $= 32 \times 12 \times 18$

$= 6912 \text{ cm}^3$

Volume of the half-cylinder $= \frac{1}{2}\pi(6)^2(32)$

$= 576\pi \text{ cm}^3$

$\therefore$ Volume of the glass block $= 6912 + 576\pi$

$= 8720 \text{ cm}^3$ (to 3 s.f.)

Method 2:

Cross-sectional area of the glass block $= 12 \times 18 + \frac{1}{2}\pi(6)^2$

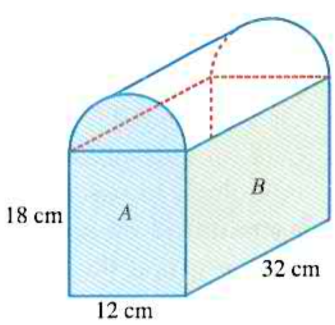
$= (216 + 18\pi) \text{ cm}^2$

$\therefore$ Volume of the glass block $= (216 + 18\pi) \times 32$

$= 8720 \text{ cm}^3$ (to 3 s.f.)

Which method do you prefer and why?

(ii)



Total surface area of the glass block

= 2 × area of region A + 2 × area of region B + area of base + curved surface area

$= 2 \times \left(12 \times 18 + \frac{1}{2}\pi(6)^2\right) + 2 \times 32 \times 18 + 32 \times 12 + \frac{1}{2} \times 2\pi(6)(32)$

$= 432 + 36\pi + 1152 + 384 + 192\pi$

$= 1968 + 228\pi = 2680 \text{ cm}^2$ (to 3 s.f.)

ATTENTION

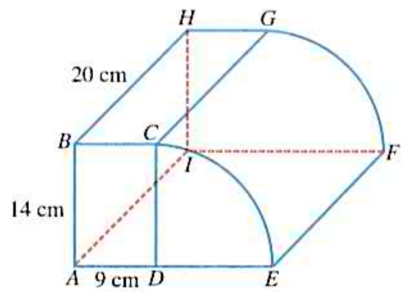
Do not find the sum of the total surface area of the rectangular prism and that of the half-cylinder to obtain the total surface area of the glass block.

PRACTISE NOW 10

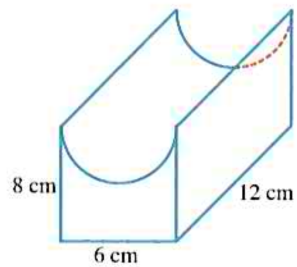
SIMILAR QUESTIONS

Exercise 14D Questions 1–8

1. The figure shows a closed container of a uniform cross section, which consists of a rectangle $ADCB$ and a quadrant DEC of a circle with centre D. Given that $AB = 14$ cm, $AD = 9$ cm and $AI = BH = CG = EF = 20$ cm, find
(i) the volume,
(ii) the total surface area,
of the container.



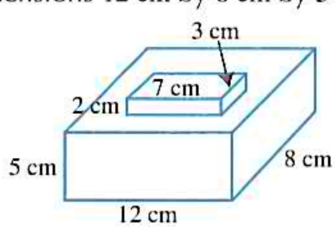
2. The figure shows a solid rectangular prism of dimensions 12 cm by 6 cm by 8 cm, with a half-cylinder of diameter 6 cm horizontally carved out of it. Find
(i) the volume,
(ii) the total surface area,
of the solid.



Exercise 14D

BASIC LEVEL

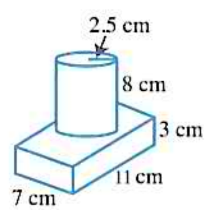
1. A solid is made up of a cuboid with dimensions 7 cm by 3 cm by 2 cm, and another bigger cuboid with dimensions 12 cm by 8 cm by 5 cm.



Find
(i) the volume,
(ii) the total surface area,
of the solid.

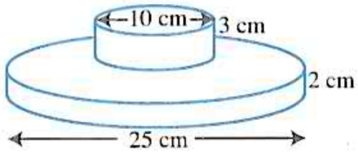
INTERMEDIATE LEVEL

2. A solid consists of a cylinder of a base radius of 2.5 cm and a height of 8 cm, and a cuboid with dimensions 11 cm by 7 cm by 3 cm.



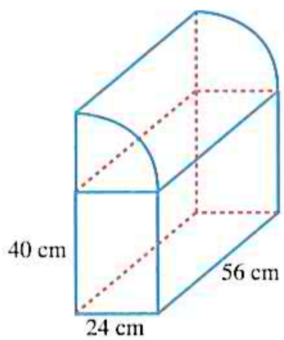
Find
(i) the volume,
(ii) the total surface area,
of the solid.

3. A solid is made up of a cylinder of diameter 10 cm and height 3 cm, and another bigger cylinder of diameter 25 cm and height 2 cm.



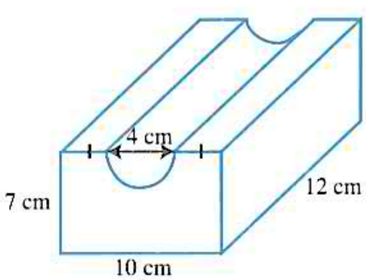
Find
(i) the volume,
(ii) the total surface area,
of the solid.

4. The figure shows a glass block made up of a rectangular prism with dimensions 56 cm by 24 cm by 40 cm and one-quarter of a cylinder with a base radius of 24 cm.



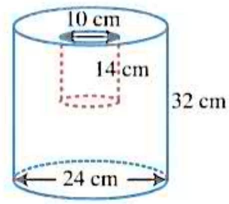
Find
(i) the volume,
(ii) the total surface area,
of the glass block.

5. The figure shows a solid cuboid of dimensions 12 cm by 10 cm by 7 cm, with a half-cylinder of diameter 4 cm horizontally carved out of it.



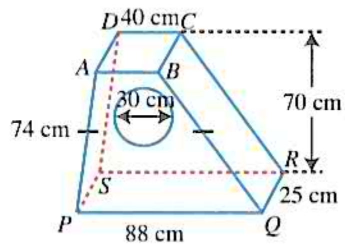
Find
(i) the volume,
(ii) the total surface area,
of the solid.

6. The figure shows a solid cylinder of diameter 24 cm and height 32 cm. A cylinder of diameter 10 cm and height 14 cm is removed from the original cylinder.



(i) Find the volume of the remaining solid.
The remaining solid is to be painted on all its surfaces.
(ii) Find the area that will be covered in paint.

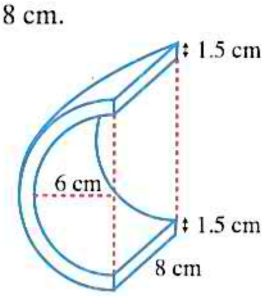
7. The figure shows a solid trapezoidal prism with a cylindrical hole of diameter 30 cm.



Given that $AB = DC = 40$ cm, $PQ = SR = 88$ cm, $PS = QR = 25$ cm, $AP = BQ = 74$ cm and the height of the solid is 70 cm, find
(i) the volume,
(ii) the total surface area,
of the solid.

ADVANCED LEVEL

8. A C-shaped solid with an internal radius of 6 cm and a uniform thickness of 1.5 cm has a height of 8 cm.



Find
(i) the volume,
(ii) the total surface area,
of the solid.

1. Conversion of Units

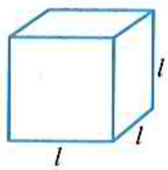
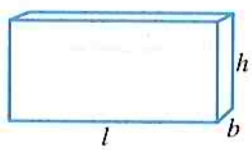
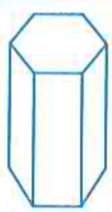
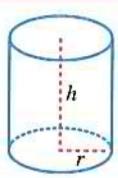
$1\ \text{m}l = 1\ \text{cm}^3$

$1\ l = 1000\ \text{m}l = 1000\ \text{cm}^3$

$1000\ l = 1\ \text{m}^3$

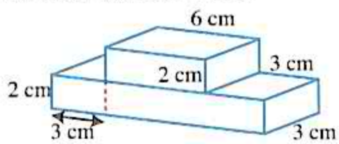
$1\ \text{m}^3 = 1\ 000\ 000\ \text{cm}^3$

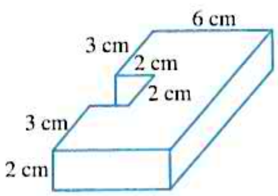
2. Volume and Total Surface Area of Solids

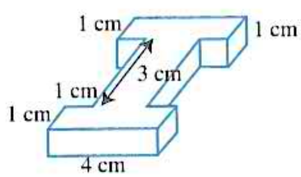
Name	Figure	Volume	Total surface area
Cube		l^3	$6l^2$
Cuboid		$l \times b \times h$	$2(lb + lh + bh)$
Prism		Area of cross section × height = base area × height	Total area of the lateral faces + 2 × base area = perimeter of the base × height + 2 × base area
Closed Cylinder		$\pi r^2 h$	$2\pi r^2 + 2\pi rh$

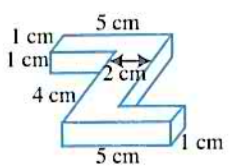
Review Exercise 14

1. Each of the following figures is made up of two or more rectangular prisms. For each of the following prisms, find
   (i) its volume,
   (ii) its total surface area.

   (a)


   (b)


   (c)


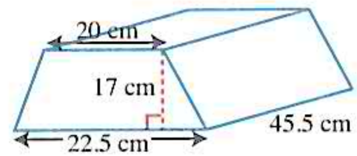
   (d)


2. A rectangular brick measures 18 cm by 9 cm by 6 cm. Find the number of bricks required to build a rectangular wall 4.5 m wide, 18 cm thick and 3.6 m high.

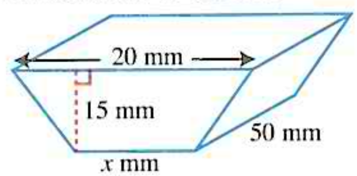
3. A rectangular block of metal is 256 mm long, 152 mm wide and 81 mm high. If the metal block is melted to form a cube, find the length of each side of the cube.

4. Find the total surface area of a cube that has a volume of 343 cm^3.

5. The figure shows a sketch of the world's largest gold bar that is 45.5 cm long. It is a solid prism with uniform cross section of a trapezium.



   (i) Find its volume.
   (ii) The mass of the gold bar is 250 kg. Find the volume of a gold bar with a mass of 200 g, leaving your answer in mm^3.
   (iii) Suppose the manufacturer of the gold bar decides to mould it into smaller identical pieces of gold bar, each weighing 200 g and with dimensions as shown.



   Find the value of x.

6. A cylindrical barrel of diameter 70 cm and height 80 cm is filled to the brim with water. A hole at the bottom drains away 0.2 litres of water every minute. Find the time taken for the water level in the barrel to drop by 6 cm.

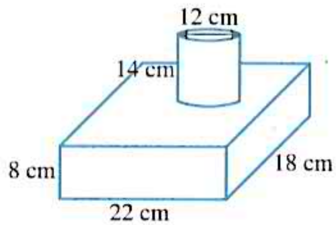
7. A cylindrical pail of a base radius of 32 cm contains water to a height of 25 cm.
   (i) Find the volume of water in the pail.
   2000 metal cubes of sides 2 cm are added to the pail one at a time.
   (ii) Find the new height of water in the pail.

8. A metal pipe has a length of 8.9 cm. The internal and external diameters of the pipe are 4.2 cm and 5 cm respectively.

(i) Find the volume of metal used in making the pipe.

(ii) If the metal costs \$8 per kg and the density of the metal is 2700 kg/m^3, find the cost of the pipe.

9. A solid consists of a cylinder of diameter 12 cm and height 14 cm, and a cuboid with dimensions 22 cm by 18 cm by 8 cm.



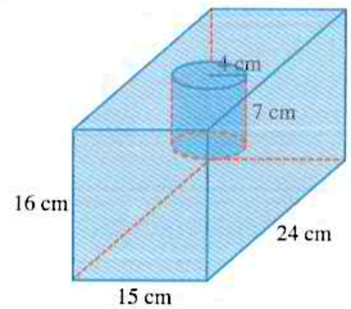
Find

(i) the volume,

(ii) the total surface area,

of the solid.

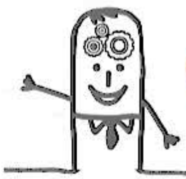
10. The figure shows a solid cuboid with dimensions 24 cm by 15 cm by 16 cm. A cylinder with a base radius of 4 cm and a height of 7 cm is removed from the cuboid.



(i) Find the volume of the remaining solid.

The remaining solid is to be painted on all its surfaces.

(ii) Find the area that will be covered in paint.



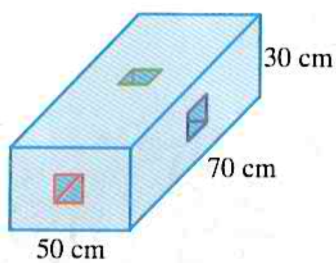
Challenge Yourself

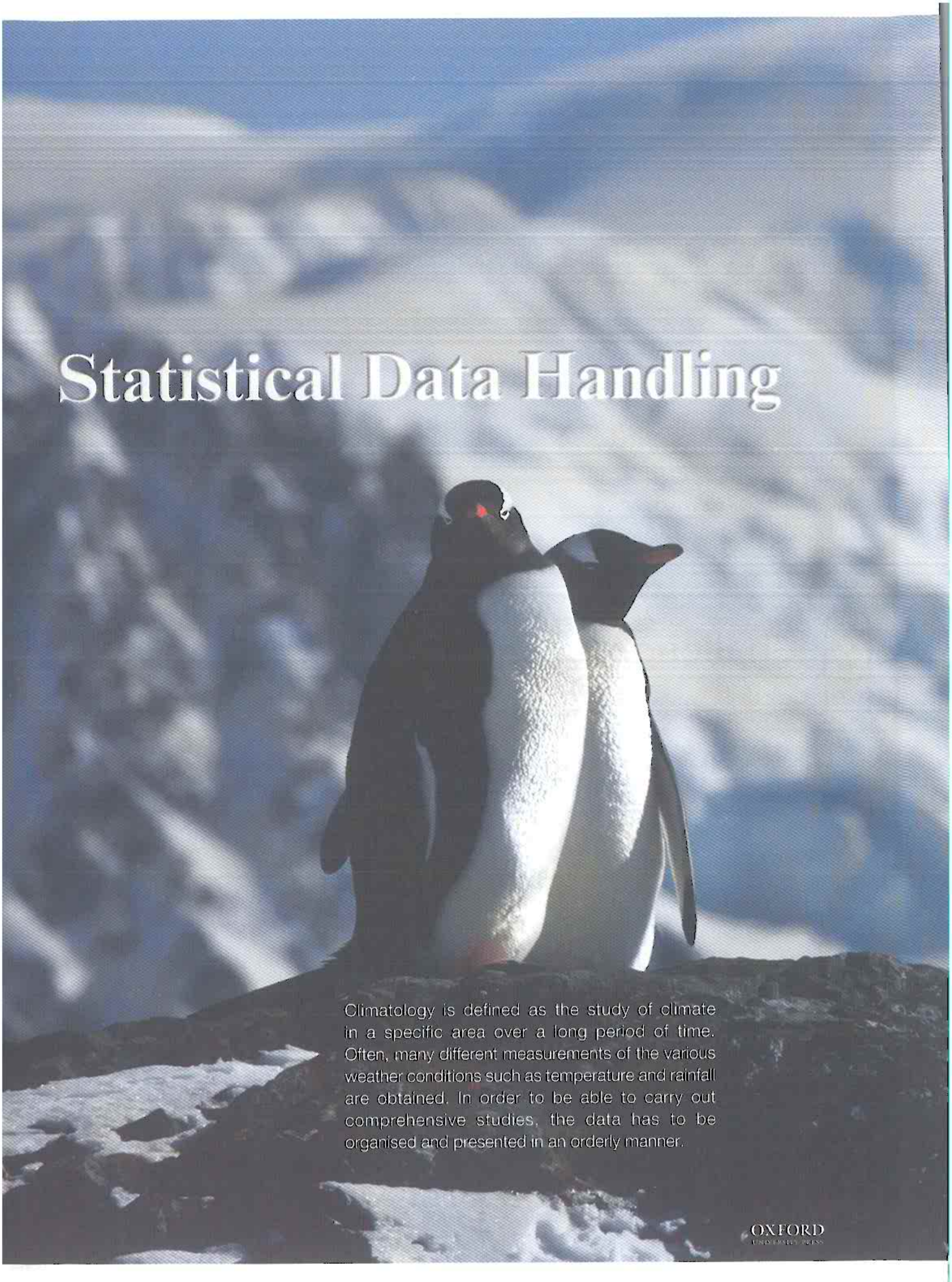
A cuboid that is 70 cm long, 50 cm wide and 30 cm high has square holes of length 10 cm in the centre of each of the faces of the cuboid. The holes cut through the length of each face of the solid. Find

(i) the volume,

(ii) the surface area,

of the solid.





Statistical Data Handling

Climatology is defined as the study of climate in a specific area over a long period of time. Often, many different measurements of the various weather conditions such as temperature and rainfall are obtained. In order to be able to carry out comprehensive studies, the data has to be organised and presented in an orderly manner.

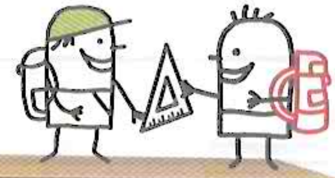
Chapter Fifteen

LEARNING OBJECTIVES

At the end of this chapter, you should be able to:

- collect, classify and tabulate data,
- construct and interpret data from pictograms, bar graphs, pie charts and line graphs,
- evaluate the purposes and appropriateness of the use of different statistical diagrams,
- explain why some statistical information or diagrams can lead to a misinterpretation of data.

15.1 Introduction to Statistics



According to a study, about 54% of the population of Singapore use a smartphone, which makes Singapore the country with the highest proportion of smartphone users in the world. Do you know how the figure of 54% was obtained?

Figures like 54% are obtained through a process of collection, organisation, display and interpretation of data in various ways. To summarise, the four stages of a statistical study are:

Collection → Organisation → Display → Interpretation

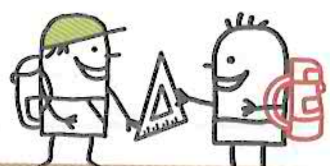
The above study shows an example of how statistics may be applied to daily life. The use of statistics is essential in areas such as climatology, economics, biology and population census as it allows us to gain insights into problems in the real world. Statistics also enables people to make informed decisions so that they are able to formulate plans wisely. Have you ever wondered how statistical data are collected, summarised and presented?

In primary school, we have learnt statistical diagrams such as pictograms, bar graphs (or bar charts), pie charts and line graphs. In this chapter, we will discuss the advantages and disadvantages of each of these diagrams, and how to choose an appropriate diagram given a certain situation.

Story Time

Sir William Petty (1623–1687) was one of the first to develop human statistical and census methods. He was from a humble family background but by the age of 35, he was at the forefront of the progressive sciences. He broke his leg on board a ship while working as a cabin boy in his teens. After this setback, he applied to study in Caen, where he supported himself by teaching English. Search on the Internet to find out more about him and his contributions. Are we able to apply such perseverance in our lives?

15.2 Pictograms and Bar Graphs



A school canteen vendor is asked to sell fruits to the students. The vendor would like to know which fruit the students like most.

1. **Collection of Data**

   The vendor conducts a survey in the school. A total of 500 students are surveyed.

   Two levels in the school are selected as the sample group for the survey conducted by the school canteen vendor. Are they representative of the entire school? Explain your answer.

   Each student can select only one fruit of his choice from apples, honeydew, pears, watermelons and oranges.

INFORMATION

There are different methods of data collection:

- Conducting surveys, e.g. in Singapore, a population census is conducted every ten years to find out various characteristics of the population distribution
- Conducting experiments, e.g. experiments are carried out to find out the average lifespan of a particular brand of energy-saving lightbulbs
- Observations, e.g. scientists often use observations to study behavioural patterns of different species

2. **Organisation of Data**

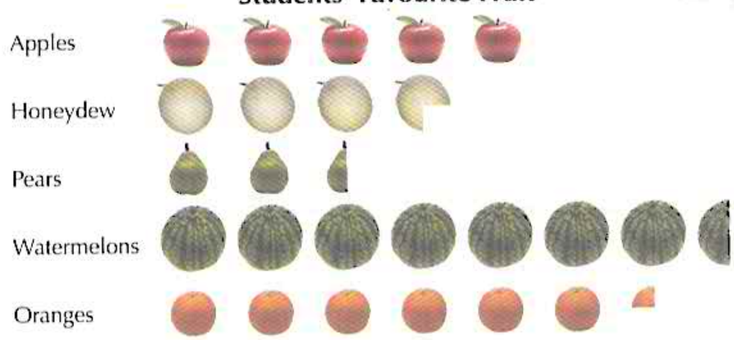
After collecting data from his sample group, the vendor organises the data in a table form as shown in Table 15.1.

Fruit	Apples	Honeydew	Pears	Watermelons	Oranges
Number of students	100	75	50	150	125

Table 15.1

3. **Display of Data**

The data collected is displayed using a **pictogram** (see Fig. 15.1) and a **bar graph** (see Fig. 15.2).



Each figure represents 20 students.

Fig. 15.1

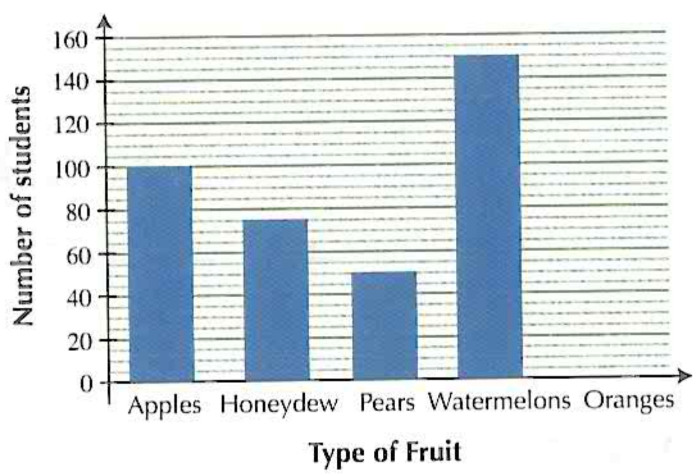


Fig. 15.2

Using Table 15.1, complete the bar graph in Fig. 15.2.

4. **Interpretation of Data**

- From the pictogram in Fig. 15.1 and the bar graph in Fig. 15.2, it is clear that the most popular fruit among the selected group of students is the ____________, and the least popular fruit is the ____________.
- If the canteen vendor decides to sell three types of fruits to the students, which three should he choose? Explain your answer.

ATTENTION

- In a pictogram, a figure is used to represent a category.
- A legend is included to show the number represented by each figure.
- The number of figures in Fig. 15.1 is proportional to the number of students who like each fruit.
- An advantage of a pictogram is that it is more colourful and appealing while a disadvantage is that it is difficult to use icons to represent exact values.

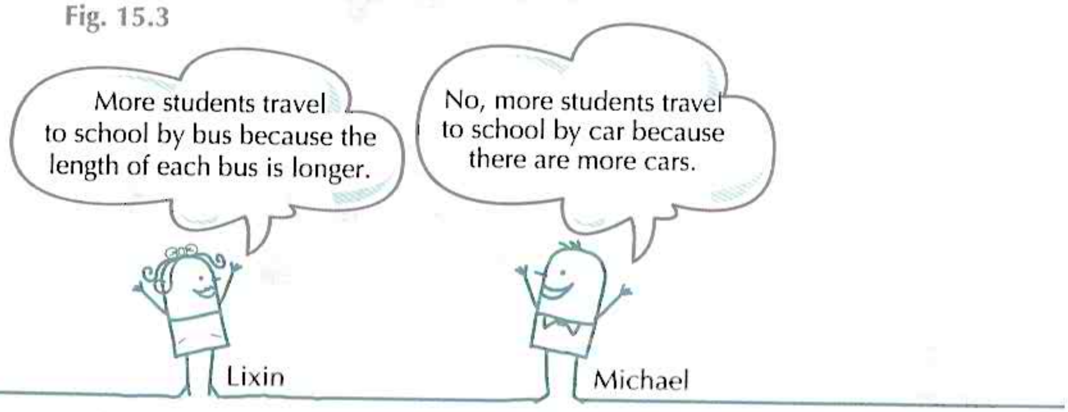
ATTENTION

- The bars in a bar graph must be of the same width.
- The space between the bars allows for ease of distinction between each category.
- The height of each bar in Fig. 15.2 is proportional to the number of students who like each fruit.
- An advantage of a bar graph is that the data sets with the lowest and the highest frequencies can be easily identified while a disadvantage is that if the frequency axis does not start from 0, the displayed data may be misleading.

Thinking Time

A survey is conducted among a group of students who travel to school either by bus or by car. Fig. 15.3 shows a pictogram that displays the data collected.

Fig. 15.3



1. Who is correct, Lixin or Michael? Explain your answer.
2. How do you modify the pictogram in Fig. 15.3 to avoid a misinterpretation of the data?

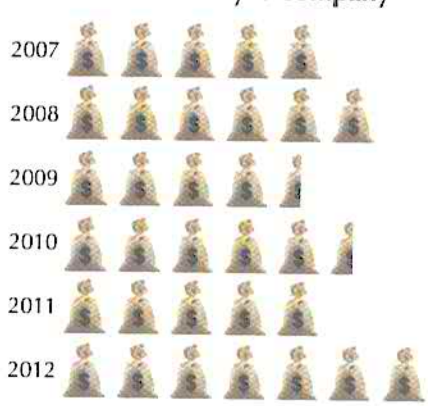
PRACTISE NOW

SIMILAR QUESTIONS

Exercise 15A Questions 1–6

1. The pictogram shows the profits earned by a company in each year from 2007 to 2012.

Profits Earned by a Company



Each  represents \$1 000 000.

(a) What was the profit earned by the company in
   (i) 2010,   (ii) 2012?

(b) In which year did the company earn the least profit? How much did the profit decrease that year as compared to the previous year?

2. A company owns seven electrical shops. Study the bar graph.

(a) Complete the bar graph using the data given in the table.

	Shop 6	Shop 7
November	64	70
December	88	96

(b) Find the total number of television sets sold in the seven shops in
  (i) November, (ii) December.

(c) Express the total number of television sets sold in the seven shops in November as a percentage of the total number of television sets sold in the seven shops in November and December.

(d) (i) Express the total number of television sets sold in Shop 7 in November and December as a percentage of the total number of television sets sold in the seven shops in November and December.
  (ii) The company would like to close down one shop due to insufficient cash flow. Based on the number of television sets sold in November and December, the manager proposed to close down Shop 7. Do you agree with the manager? Explain your answer.

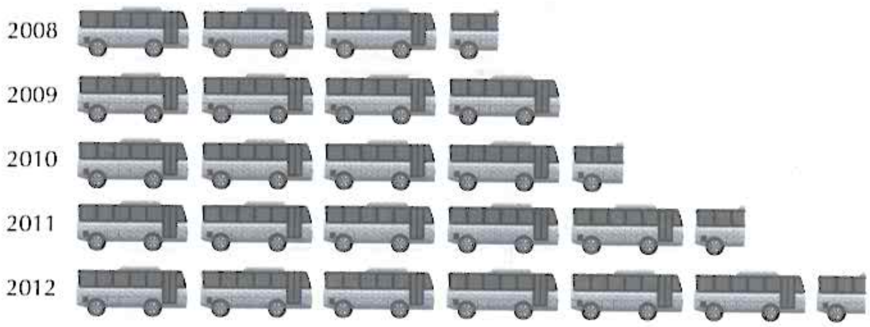
(e) In which month did the company perform better in terms of sales? Explain your answer.

Exercise 15A

BASIC LEVEL

1. The pictogram illustrates the number of buses registered with the Registry of Vehicles each year from 2008 to 2012.

Number of Registered Buses



Each  represents 40 000 buses.

(i) In which year was the greatest number of buses registered? Estimate the number of buses registered in that year.

(ii) Estimate the total number of buses registered from 2008 to 2012.

(iii) In 2010, the registration fee for each bus was \$1000. Estimate the total amount the Registry of Vehicles collected in that year.

(iv) Estimate the percentage increase in the number of buses registered from 2011 to 2012.

2. The table shows the number of students who play volleyball, basketball and tennis respectively.

Sport	Volleyball	Basketball	Tennis
Number of students	40	60	50

(i) Complete the pictogram.

Students who Play Volleyball, Basketball or Tennis

Volleyball

Basketball

Tennis

Each circle represents 10 students.

(ii) Find the ratio of the number of students who play volleyball to the number of students who play tennis.

(iii) Express the number of students who play tennis as a percentage of that who play basketball.

3. The table shows the number of copies of a newspaper distributed to households in each year from 2008 to 2012.

Year	2008	2009	2010	2011	2012
Number of copies (in thousands)	250	275	290	315	280

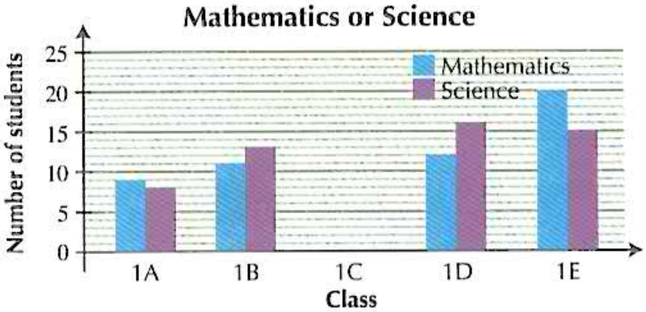
Use a bar graph to illustrate the above information.

INTERMEDIATE LEVEL

4. Study the table and bar graph.

Class	Class 1A	Class 1B	Class 1C	Class 1D	Class 1E
Number of students who score a distinction in Mathematics	9	11	16	12	20
Number of students who score a distinction in Science	8		12		15

Students who Score a Distinction in Mathematics or Science



(a) Complete the table and the bar graph.

(b) Find the total number of students in the 5 classes who score a distinction in
(i) Mathematics, (ii) Science.

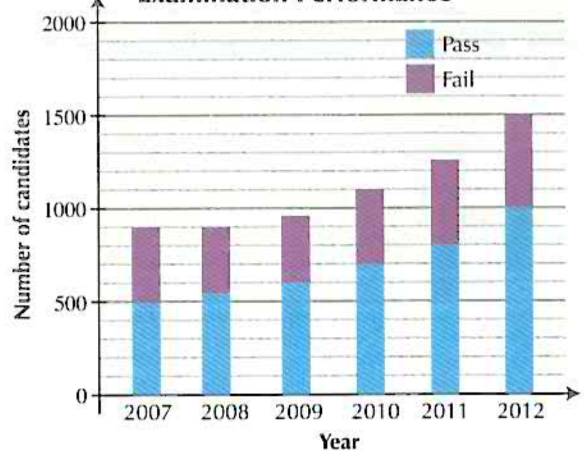
(c) Express the number of students who score a distinction in Mathematics in Class 1D as a percentage of the total number of students who score a distinction in Mathematics in the 5 classes.

(d) If there are 40 students in Class 1D, find the percentage of students in the class who score a distinction in Science.

(e) Is Jun Wei correct to say that there are 35 students in Class 1E? Explain your answer.

5. The graph shows the number of candidates who sat for an examination in each year from 2007 to 2012.

Examination Performance



(i) State the number of candidates who sat for the examination in 2009.

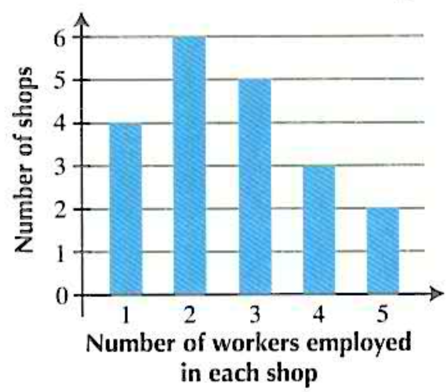
(ii) State the number of candidates who failed the examination in 2012.

(iii) Express the number of candidates who failed the examination in 2012 as a percentage of the total number of candidates who failed the examination in the six years.

(iv) Comment on the trend in the percentage of successful candidates throughout the six years. Provide a reason for this trend.

6. The bar graph illustrates the results of a survey conducted on shops in a housing estate.

Number of Workers in the Shops

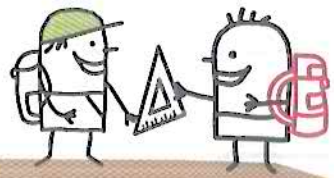


(i) Find the total number of workers employed in the housing estate.

(ii) Express the number of shops hiring 3 or more workers as a percentage of the total number of shops in the housing estate.

(iii) Suggest why some shops employ more workers than others.

15.3 Pie Charts



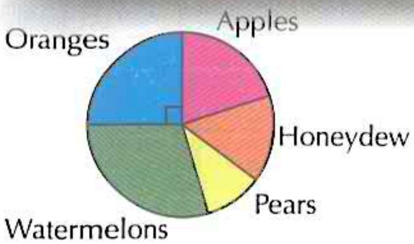


Fig. 15.4

A **pie chart** is constructed by dividing a circle into different sectors.

The data collected by the school canteen vendor on students' favourite fruit, as mentioned in the beginning of Section 15.2, can be represented using a pie chart as shown in Fig. 15.4. The pie chart is divided into five different sectors and from the pie chart, we can see that $\frac{1}{4}$ of the students like oranges. We would not be able to infer this from a bar graph immediately.

We use a pie chart when we want to show the relative size of each data set in proportion to the entire data set. In such situations, the actual numerical value of individual data set is of less importance.

Construction of a Pie Chart

We will now learn how to construct a pie chart based on the data provided in Table 15.1.

The circle represents the total number of students surveyed, i.e. 500 students.

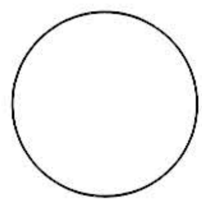


Fig. 15.5

The number of students a sector represents is proportional to the angle at the centre of the circle. For example, 100 students like apples most. Thus the angle of the sector representing the number of students who like apples most $= \frac{100}{500} \times 360° = 72°$.

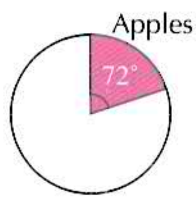


Fig. 15.6

ATTENTION

The angle at the centre of the circle is 360°.

Table 15.2 shows the size of the angle of each sector.

Fruit	Angle of Sector
Apples	$\frac{100}{500} \times 360° = 72°$
Honeydew	$\frac{75}{500} \times 360° = 54°$
Pears	$\frac{50}{500} \times 360° = 36°$
Watermelons	$\frac{150}{500} \times 360° = 108°$
Oranges	$\frac{125}{500} \times 360° = 90°$

Table 15.2

Using the calculated angles in Table 15.2, we obtain the pie chart as shown in Fig. 15.7.

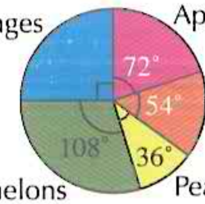


Fig. 15.7

ATTENTION

- As long as the angles are accurate, the size of the pie chart does not matter.
- The angle of each sector in Fig. 15.7 is proportional to the number of students who like each fruit.
- The sum of the angles of all the sectors should add up to 360°.
- An advantage of a pie chart is that the relative size of each data set in proportion to the entire set of data can be easily observed while a disadvantage is that the exact numerical value of each data set cannot be determined directly.

PRACTISE NOW

SIMILAR QUESTIONS

Exercise 15B Question 1

The table shows Farhan's expenditure on a holiday.

Item	Food	Shopping	Hotel	Air Ticket	Others
Amount spent	\$1000	\$1200	\$400	\$1200	\$200

Construct a pie chart using data from the table.

Worked Example 1

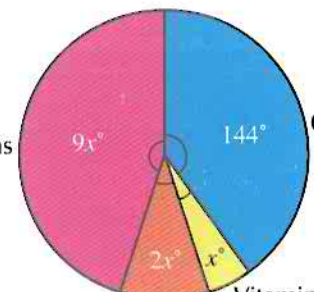
(Problem involving a Pie Chart)

The pie chart shows the nutritional composition of a food product.

(i) Calculate the value of x.

(ii) Express the amount of fat as a percentage of the components of the food product.

(iii) Given that a food product contains 120 g of carbohydrates, calculate the mass of the food product.



Solution:

(i) $144° + 9x° + 2x° + x° = 360°$ (∠s at a point)

$$\begin{aligned} 12x° &= 360° - 144° \\ &= 216° \\ x° &= 18° \\ \therefore x &= 18 \end{aligned}$$

(ii) Angle of sector representing the amount of fat in the food product $= 2 \times 18° = 36°$

$$\therefore \text{Required percentage} = \frac{36°}{360°} \times 100\% = 10\%$$

(iii) The angle of the sector representing carbohydrates is 144° and this constitutes 120 g.

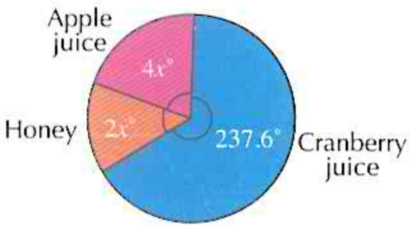
$$\therefore \text{Mass of the food product} = \frac{360°}{144°} \times 120 = 300 \text{ g}$$

PRACTISE NOW 1

SIMILAR QUESTIONS

Exercise 15B Questions 1–4, 6–7

1. The pie chart shows the composition of a jar of fruit punch.

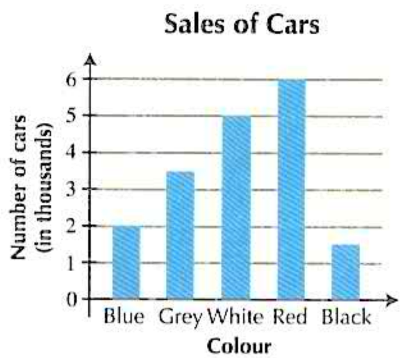


(i) Find the value of x.

(ii) Express the amount of apple juice in the fruit punch as a percentage of all the components in the fruit punch.

(iii) Given that a jar of fruit punch contains 759 ml of cranberry juice, find the amount of fruit punch in the jar.

2. The bar graph shows the number of cars of different colours sold in one year in a city.

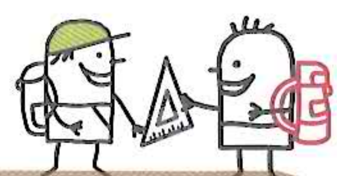


(i) Which is the least popular colour?

(ii) If the information is illustrated on a pie chart, find the angle of each sector.

(iii) Huixian says that there is a mistake in the bar graph because 3.5 grey cars and 1.5 black cars do not exist in the real world. Do you agree with her? Explain your answer.

15.4 Line Graphs

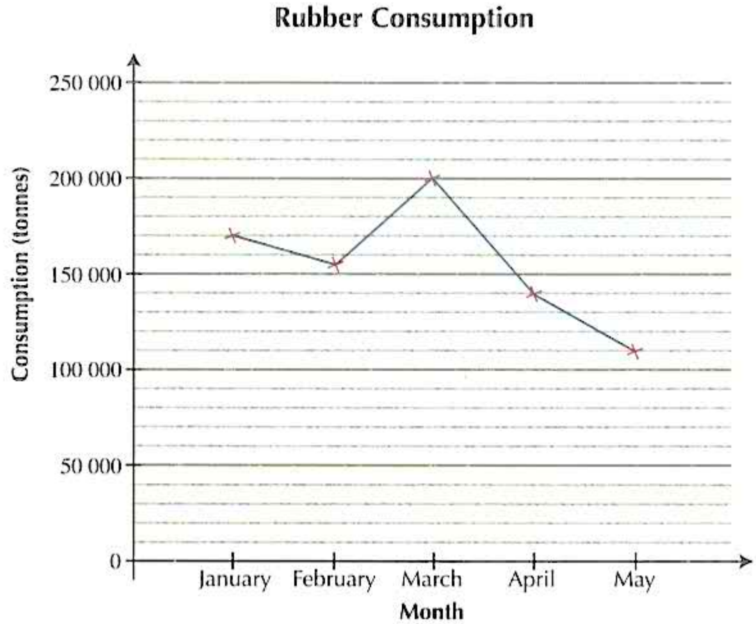


A **line graph** is preferred when we want to observe a rising or falling trend in a set of data over a period of time. To obtain a line graph, we can plot the points on a graph paper and connect each successive point by line segments.

Worked Example 2

(Problem involving a Line Graph)

The line graph shows the rubber consumption of the automobile industry in a country from January to May of a particular year.



(i) In which month was the rubber consumption the lowest?

(ii) Using data from the line graph, construct a table showing the rubber consumption over the five months.

(iii) Calculate the percentage increase in the rubber consumption from February to March.

(iv) Suggest a reason for the increase in rubber consumption from February to March.

ATTENTION

An advantage of a line graph is that intermediate values can be easily obtained while a disadvantage is that these intermediate values may not be meaningful.

Solution:

(i) The rubber consumption was the lowest in May.

(ii)

Month	January	February	March	April	May
Consumption (tonnes)	170 000	155 000	200 000	140 000	110 000

(iii) Percentage increase in the rubber consumption from February to March

$= \frac{200 - 155}{155} \times 100\%$

$= 29.0\%$ (to 3 s.f.)

(iv) The increase in rubber consumption from February to March may be caused by an increase in sales as more rubber tyres were needed for the production of more vehicles.

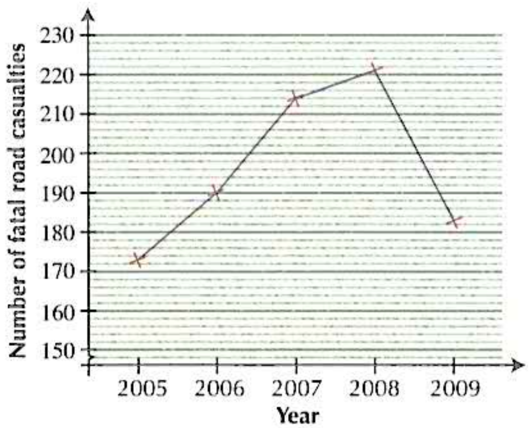
PRACTISE NOW 2

SIMILAR QUESTIONS

Exercise 15B Questions 5, 8–9

The line graph shows the number of fatal road casualties in each year from 2005 to 2009.

Fatal Road Casualties

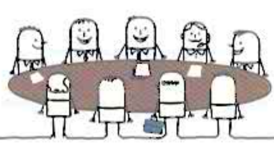


(i) In which year was the number of fatal road casualties the highest?

(ii) Using data from the line graph, construct a table showing the number of fatal road casualties over the five years.

(iii) Find the percentage decrease in the number of fatal road casualties from 2008 to 2009.

(iv) Suggest a reason for the decrease in the number of fatal road casualties from 2008 to 2009.



Class Discussion

Comparison of Various Statistical Diagrams

Work in pairs.

1. Suggest some other advantages and disadvantages of

(a) a pictogram,

(b) a bar graph,

(c) a pie chart,

(d) a line graph.

2. For each of the following scenarios, choose the most suitable statistical diagram to display the data. Justify your answer.

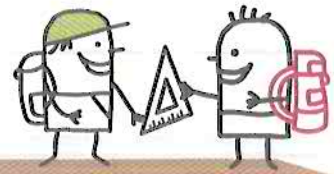
(a) The distribution of birthday months in a class of 40 students.

(b) The population of Singapore from the year 2004 to the year 2013.

(c) The number of Secondary 1 students who travel to school by the various modes of transport, i.e. by bus, by car, by Mass Rapid Transit (MRT) and by walking.

(d) The relative proportions of Secondary 1 students who prefer the different drinks, i.e. iced lemon tea, orange juice, soya bean drink, isotonic drink and plain water.

15.5 Statistics in Real-World Contexts



In this section, we shall consolidate what we have learnt in the previous sections by illustrating how the four stages of statistical studies are carried out in real life.

Consider the following scenario:

> The student council is in charge of organising Games' Day in a school. They would like to find out the sport which the student population likes most in order to facilitate the organisation of the event. Only three sports would be played on that day.

1. Collection of Data

The student council conducts a questionnaire survey among 600 students in the school. An example of the questionnaire is as shown.

> **Questionnaire**
>
> From the list below, put a tick next to the sport which you like most. You should only select one sport.
>
> ☐ Soccer ☐ Captain's ball ☐ Basketball
>
> ☐ Hockey ☐ Netball

2. Organisation of Data

The student council consolidates the results of the survey as shown in Table 15.3.

Sport	Tally	Number of students
Soccer	𝍸 |	176
Captain's ball	𝍸 ||||	144
Basketball	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	
Hockey		
Netball	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	92
	Total	600

Table 15.3

Note: The use of tallies is a structured way of organising results obtained. For convenience during counting, tallies are grouped in fives (𝍸) with the fifth tally crossing the first four tallies.

Complete Table 15.3.

3. **Display of Data**

After considering each of the following, the student council decides to use a bar graph to display the data.

- A pictogram may include values that require a fraction of the icon to be used.
- A bar graph is a useful statistical diagram as the sports which the students like the most and the least can be recognised at one glance; and yet maintaining its accuracy at the same time.
- A pie chart can be used as the sport which the students like the most can be easily identified by looking at the relative size of each sector. However, it may be quite difficult to compare the numerical values of the data sets, especially when the values are close to one another.
- A line graph is not suitable as the data collected in this case do not show trends over a period of time.

A spreadsheet has been used to generate the bar graph as shown in Fig. 15.8.

Students' Favourite Sport

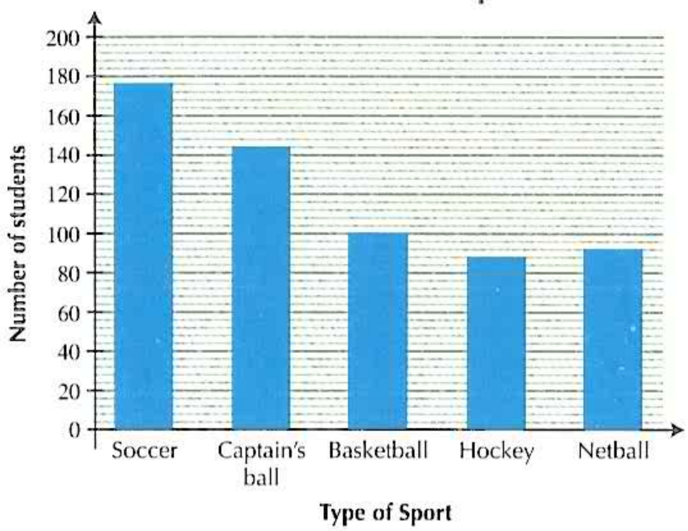


Fig. 15.8

Internet Resources

Search on the Internet for instructions on how to create statistical diagrams using a spreadsheet.

4. **Interpretation of Data**

From the bar graph in Fig. 15.8, the student council decides to choose ____________, ____________ and ____________ to be played on Games' Day.

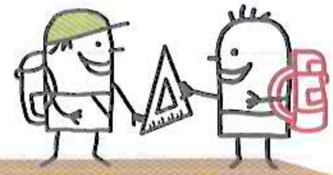
Performance Task

Consider the following scenario:

> Your schoolmates have been complaining about the lack of food stalls in the new school canteen. As a member of the student council, your teacher has assigned you to find out the possible types of stalls that the school can have in the school canteen.

Work in groups of four to write a report to recommend some possible types of food stalls that the school can engage for the school canteen. The report should also consist of a questionnaire, a table, an appropriate statistical diagram drawn using a suitable software and an interpretation of the data collected.

15.6 Evaluation of Statistics



As we have learnt in the previous sections, collection, organisation, display and interpretation of data are the four components of statistical study. In this section, we will discuss some problems in each of these components.

Class Discussion

Evaluation of Statistics

Part I: Collection of Data

Read the article and answer the questions.

> NEWS
>
> **Zidane Named Best European Footballer in Last 50 Years**
>
> **PARIS:** In a UEFA (Union of European Football Associations) website poll in 2004, Zinedine Zidane was named Europe's best footballer in the past 50 years. He obtained 123 582 votes, followed by Franz Beckenbauer with 122 569 votes and Johan Cruyff with 119 332 votes.

1. Do you know who the three footballers are? Your teacher will take a poll in your class to find out the number of students who know each of the three footballers.
2. How did UEFA conduct the poll? Were the voters who took part in the poll representative of all football fans? Explain your answer.
3. If older football fans were to participate in the poll, do you think Zidane would have come in first place? Justify your opinion.
   Hint: Zidane was famous in the 1990s while Beckenbauer and Cruyff were at the peak of their careers in the 1970s.
4. What lesson can you learn about the choice of a sample during data collection?

Part II: Organisation of Data

Read the article and answer the questions.

> NEWS
>
> **Most Number of Complaints Received Against Banks and Insurance Firms for the First Time**
>
> **QASVILLE:** The Customers Organisation of Qasville revealed that they have received the most number of complaints against banks and insurance firms for the first time, i.e. 1416 complaints in 2012. Coming in second was timeshare companies with 1238 complaints, followed by motor vehicle companies with 975 complaints.

1. Which three types of companies received the most number of complaints?
2. A statistician commented, 'It is possible that timeshare companies receive the most number of complaints.' Discuss with your classmates why this comment may be true.
   Hint: How were the data (number of complaints) organised?
3. What lesson can you learn about the organisation of statistical data?

Part III: Display of Data

The bar graph in Fig. 15.9 shows the number of light bulbs sold by 5 companies in a week. Study the bar graph and answer the questions below.

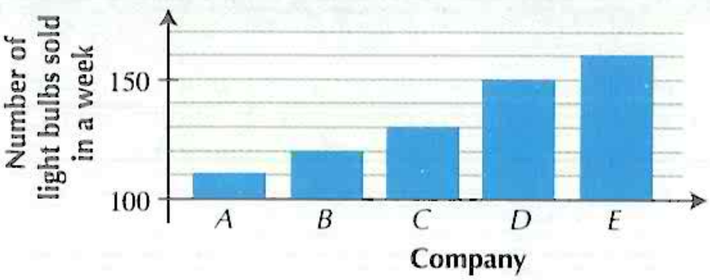


Fig. 15.9

1. Company E claims that it has sold twice as many light bulbs as Company C. Is the claim valid? Explain your answer.
2. What lesson can you learn about the display of statistical data?

Part IV: Interpretation of Data

Read the article and answer the questions.

NEWS

Employees' Satisfaction

QASVILLE: In a survey conducted among 300 employees of a company, only 40% of them were not satisfied with working in the company. Therefore, the survey concluded that the employees were satisfied with the company and that the company was a good place to work in.

1. How did the survey arrive at the conclusion as stated in the article?
2. Although only 40% of the employees were not satisfied with working in the company, can you conclude that the employees were satisfied with the company and that the company was a good place to work in?
3. In order to pass a law in the Singapore Parliament that results in a constitutional amendment, at least two-thirds of the elected Members of Parliament must agree on it. Why is it that a simple majority is not enough in this case? Explain your answer.
4. What lessons can you learn about the interpretation of statistical data?

Part V: Ethical Issues

From Parts I – IV, we have seen some examples of statistical abuse. Poor use of statistics can be found everywhere, e.g. magazines and advertisements. Have you ever encountered such instances? Discuss with your classmates why people make use of statistics to mislead others. Why is it not ethical for them to do so?

SIMILAR QUESTIONS

Exercise 15B Questions 10–13

Exercise 15B

BASIC LEVEL

1. The table shows the number of students who travel to school by bus, by car, by bicycle and on foot respectively.

Mode of transport	Bus	Car	Bicycle	Foot
Number of students	768	256	64	192

Construct a pie chart using data from the table.

2. A survey is conducted to find out which of the four ice-cream flavours, chocolate, yam, mango and vanilla, the students in a class prefer. The pie chart shows the results of the survey.

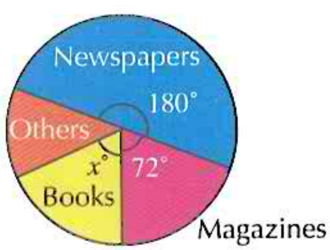
(i) If one-quarter of the class prefers yam, state the angle of the sector that represents this information.

(ii) Find the angle of the sector that represents the number of students who prefer vanilla.

(iii) Express the number of students who prefer vanilla as a percentage of the total number of students in the class.

(iv) If 5 students prefer mango, find the total number of students in the class.

3. The pie chart shows the sources of revenue of a publishing company.



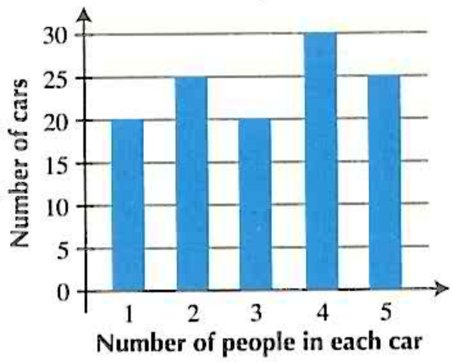
(i) Express the revenue the company earns from newspapers as a percentage of the total revenue of the company.

(ii) Express the revenue the company earns from magazines as a percentage of the total revenue of the company.

(iii) If the revenue the company earns from books is $17\frac{1}{2}\%$ of the total revenue of the company, find the value of x.

4. The bar graph illustrates the results of a survey conducted on cars at a traffic junction.

Number of People in the Cars



(i) Find the total number of cars in the survey.

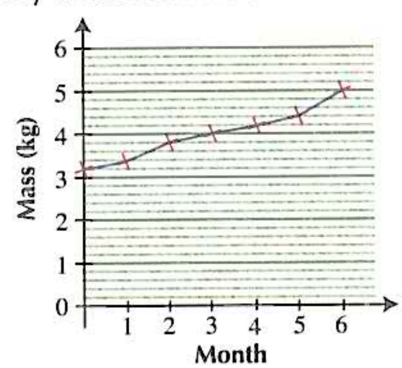
(ii) Find the total number of people in all the cars.

(iii) Express the number of cars with 4 or more people as a percentage of the total number of cars surveyed.

(iv) If the information is illustrated on a pie chart, find the angle of each sector.

5. The line graph shows the mass of a baby from birth to 6 months.

Baby's Mass from Birth to 6 Months

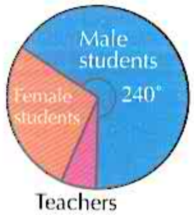


(i) Using data from the line graph, construct a table showing the mass of the baby from birth to 6 months.

(ii) Find the percentage increase in the mass of the baby from the 4th to 6th month.

INTERMEDIATE LEVEL

6. The pie chart shows the distribution of students and teachers in a school.



(a) If there are five times as many female students as teachers in the school, find the angle of the sector that represents the number of teachers in the school.

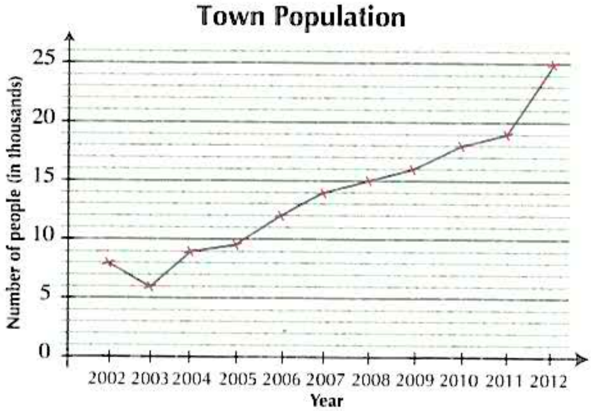
(b) If there are 45 teachers in the school,
   (i) find the number of female students in the school,
   (ii) find the number of male students in the school.

(c) If $\frac{2}{3}$ of the teachers are women, express the number of females in the school as a percentage of the total school population.

7. A factory produces three products, A, B and C, in the ratio $1 : x : 5$. When this information is illustrated on a pie chart, the angle of the sector that represents the quantity of C produced is 120°. Find the value of x.

8. The line graph shows the number of people in a town in each year from 2002 to 2012.

Town Population



(i) Between which two years did the town have the greatest increase in the number of people?
(ii) Using data from the line graph, construct a table showing the number of people in the town over the 11 years.
(iii) Find the percentage increase in the number of people in the town from 2009 to 2012.
(iv) Suggest a reason for the sudden increase in the number of people in 2012.

9. The table shows the temperature of a patient taken every 3 hours.

Time (hours)	1500	1800	2100	0000	0300	0600	0900
Temperature (°C)	39	39	39.5	37.5	39	38	37

(i) Use a line graph to illustrate the above data.
(ii) From the line graph which you have drawn in (i), estimate the temperature of the patient at 1700 hours and 0100 hours respectively.

ADVANCED LEVEL

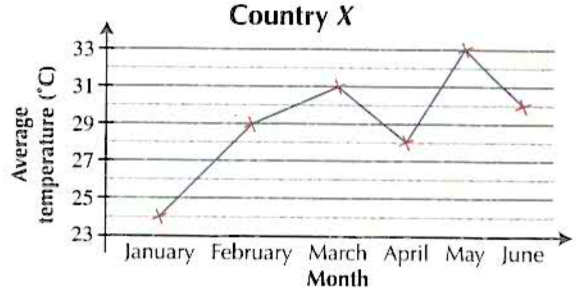
10. Kate and Khairul each conducts a survey among 200 Singaporeans to find out whether they like shopping. The table shows the data that they have collected.

	Kate's data	Khairul's data
Like shopping	128	29
Neutral	47	24
Dislike shopping	25	147
Total	200	200

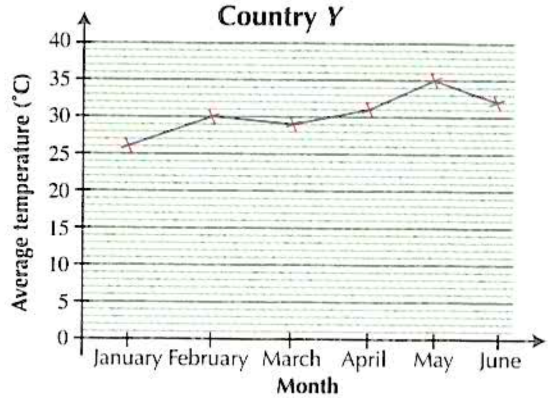
Suggest two reasons to explain the discrepancy between the two sets of data.

11. The line graphs show the average temperatures of two countries X and Y in each month from January to June.

Country *X*



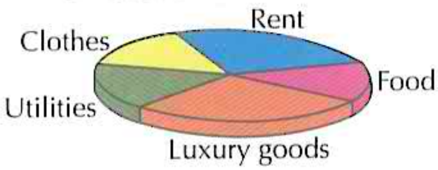
Country *Y*



Nora says that the temperatures in Country X change more drastically than those in Country Y. Do you agree with her? Explain your answer.

12. The monthly expenditure of Raj is presented in a 3-dimensional pie chart and a 2-dimensional pie chart respectively.

3-Dimensional Pie Chart



2-Dimensional Pie Chart

(i) Based on the 3-dimensional pie chart, which item does Raj spend the most on?

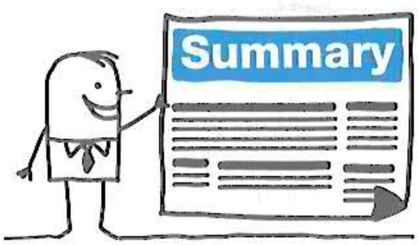
(ii) Based on the 2-dimensional pie chart, which items does Raj spend the most on?

(iii) Suggest a reason to explain the discrepancy between your answers in **(i)** and **(ii)**.

13. The table shows the number of accidents involving cars and motorcycles in each of the months from April to June in Singapore.

	April	May	June
Cars	211	194	257
Motorcycles	23	31	38

Amirah concludes that it is more dangerous to drive a car than to ride a motorcycle because there are more accidents involving cars. Do you agree with her? Provide two reasons for your answer.



1. The four stages of a statistical study are:

Collection → Organisation → Display → Interpretation

2. The data collected are usually organised in a table and displayed using statistical diagrams such as a **pictogram**, a **bar graph**, a **pie chart** or a **line graph**.
3. The choice of an appropriate statistical diagram depends on the type of data collected and the purpose of collecting the data.
   - A pictogram is most suitable when the data has to be presented in a lively and interesting manner and when the data do not include values that require a fraction of the icon to be used.
   - A bar graph is most suitable when we want to compare between the data sets and when we are interested to know the exact numerical values.
   - A pie chart is most suitable when we want to show the relative size of each data set in proportion to the entire data set.
   - A line graph is most suitable when we want to show trends over a period of time.

Review Exercise 15

1. The pictogram shows the average weekly pocket money that students in each of the four classes receive.

Average Weekly Pocket Money Received by Students

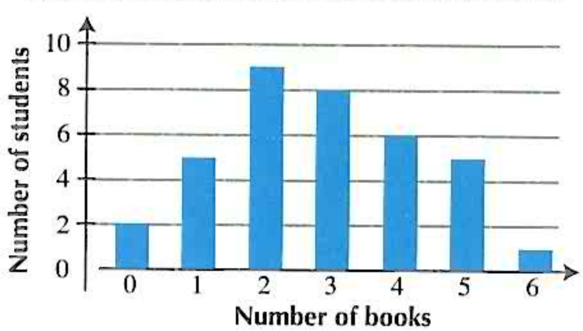
Each [banknote] represents $10.

   (i) Find the ratio of the average weekly pocket money received by students in Class 1D to that of the students in Class 1B.

   (ii) Express the average weekly pocket money received by students in Class 1C as a percentage of that of the students in Class 1A.

2. A survey is conducted to find out the average number of books that the students in a class read in a month. The bar graph shows the results of the survey.

Number of Books Students Read in a Month



   (i) Find the total number of books read by the students in the class in a month.

   (ii) Express the number of students who read more than 4 books as a percentage of the total number of students in the class.

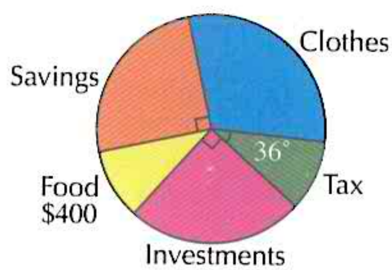
   (iii) If the information is illustrated on a pie chart, find the angle of the sector that represents the number of students who read fewer than 3 books.

3. The table shows the percentage of students who are enrolled in the Science course, the Engineering course and the Business course in a university respectively.

Type of course	Science	Engineering	Business	Arts
Percentage of students	25%	30%	15%	?

If the rest of the students are enrolled in the Arts course, construct a pie chart using data from the table.

4. The pie chart shows Devi's monthly expenditure.



If the amount she spends on clothes is three times as much as that she spends on food,

   (i) express the amount she spends on food as a percentage of that she saves,

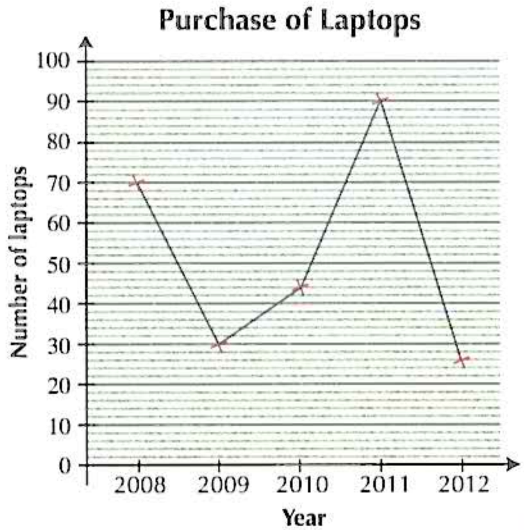
   (ii) find her annual income.

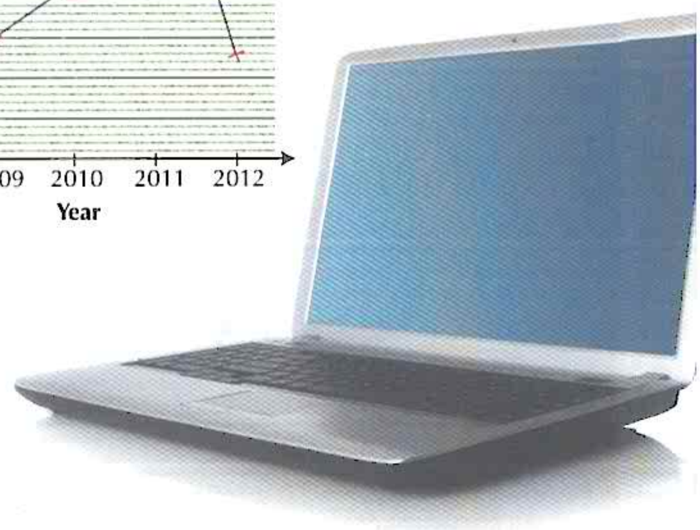
5. The line graph shows the number of laptops purchased by a company in each year from 2008 to 2012.

(i) Using data from the line graph, construct a table showing the number of laptops purchased by the company over the five years.

(ii) Find the percentage decrease in the number of laptops purchased by the company from 2008 to 2009.

(iii) Suggest a reason for the decrease in the number of laptops purchased by the company from 2008 to 2009.



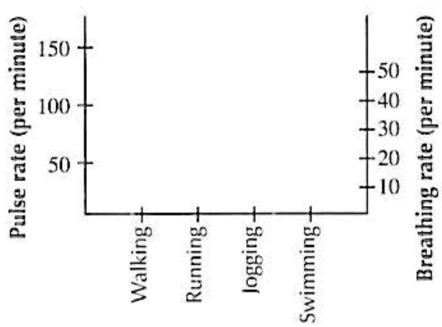


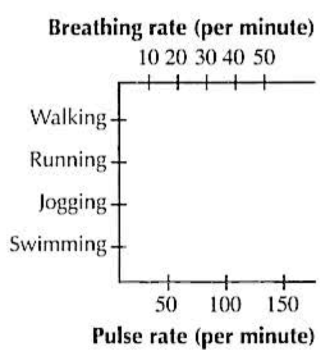
Challenge Yourself

The table shows Raj's pulse rate and breathing rate after 10 minutes of each activity.

Activity	Pulse rate (per minute)	Breathing rate (per minute)
Walking	85	25
Running	145	45
Jogging	110	35
Swimming	120	40

Which of the following is a better way to display the data using a bar graph? Hence, draw the bar graph.

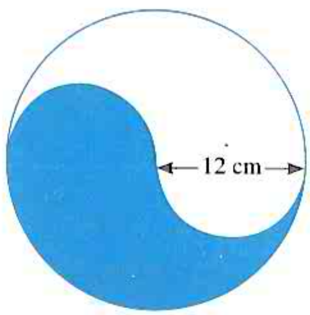




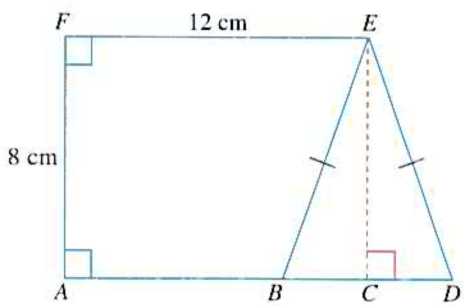
D1 Revision Exercise

1. The perimeter of a quadrant of a circle is 71.4 cm. Find its area.

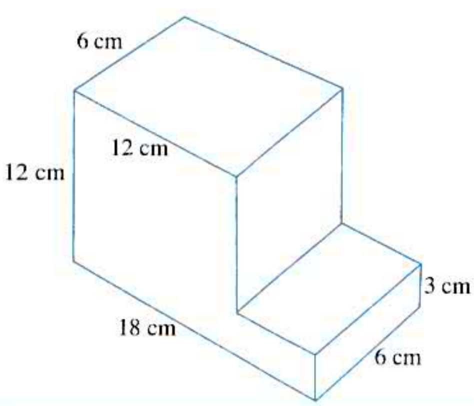
2. The perimeter of the shaded region is made up of an arc of a semicircle of radius 12 cm and the arcs of two semicircles of diameter 12 cm. Find the perimeter and the area of the shaded region.



3. In the figure, BDE is an isosceles triangle with an area of 24 cm². If $AF = 8$ cm and $EF = 12$ cm, find the area of the trapezium $ABEF$.

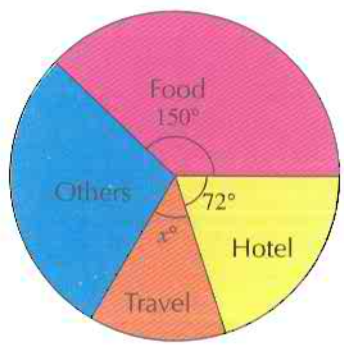


4. Find the volume and the total surface area of the solid.



5. A closed rectangular box measuring 72 cm by 54 cm by 48 cm externally, is made of wood 1.5 cm thick. Find
   (i) the capacity of the box in litres,
   (ii) the volume of wood used in making the box,
   (iii) the mass of the box if the density of the wood used is 0.9 g/cm³.
   $\left(\text{Density} = \frac{\text{Mass}}{\text{Volume}}\right)$

6. The pie chart shows the cost breakdown of a holiday.

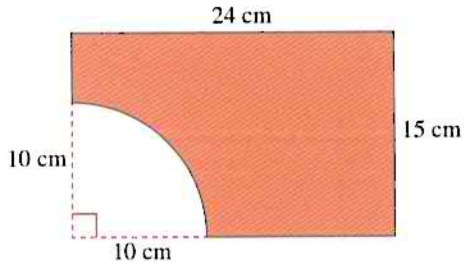


   (a) Find the percentage of the cost that was spent on
   (i) food,
   (ii) hotel.
   (b) If 15% of the total cost was spent on travel, find the value of x.

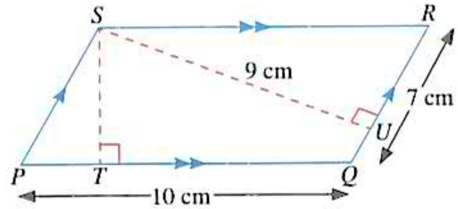
D2 Revision Exercise

1. A photograph measuring 40 cm by 25 cm is framed up with a uniform margin of width 4 cm all around it. Find the area of the margin.

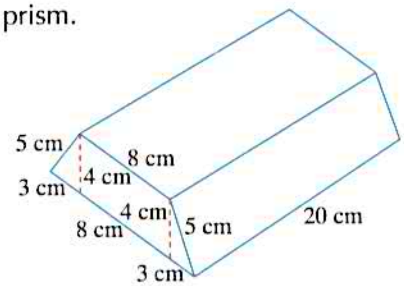
2. In the figure, a quadrant of a circle of radius 10 cm is removed from a rectangle 15 cm wide and 24 cm long. Find
   (i) the perimeter,
   (ii) the area,
   of the figure.



3. The figure shows a parallelogram $PQRS$ where $PQ = 10$ cm and $QR = 7$ cm. If $SU = 9$ cm, find the length of ST.

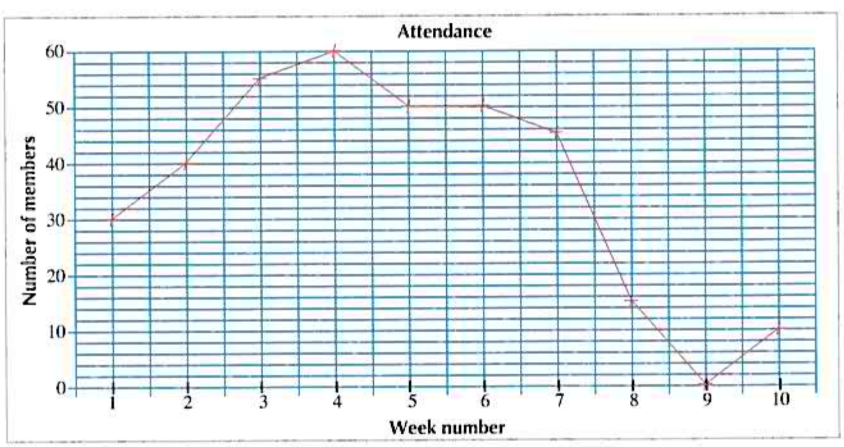


4. The cross section of the prism is a trapezium. Find the volume and the total surface area of the prism.



5. The internal and external radii of a hollow metal cylinder are 5 cm and 6 cm respectively. Find the mass of the cylinder if its height is 2.4 m and its density is 7.6 g/cm^3.

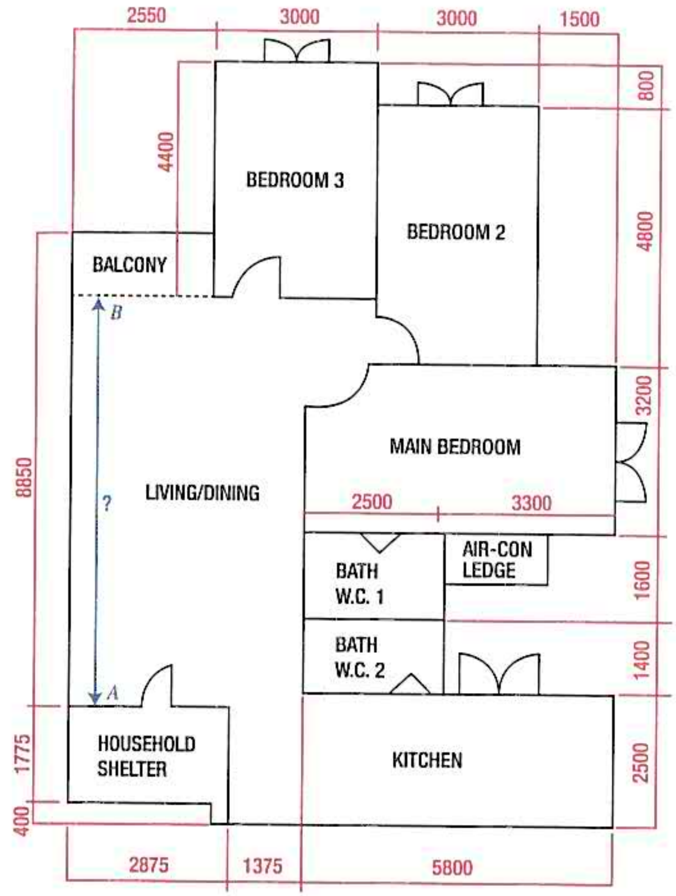
6. The line graph shows the number of Drama Club members attending the weekly meeting during a school term.



   (i) In which week was the attendance the greatest?
   (ii) The Drama Club stopped its weekly meeting during a week in which the school examination was held. Which week was it?
   (iii) Express the drop in attendance from the 7th week to the 8th week as a percentage of the number of members who attended the meeting in the 7th week.
   (iv) Suggest a reason for the low attendance in the 8th week.

PROBLEM 1: Floor Area

The figure shows the floor plan of a 5-room flat that Mr Lee is interested to purchase. Assume that all adjacent walls are perpendicular to each other.



Find out what W.C. in the floor plan stands for.

(i) What do you think is a suitable unit for the measurements in the floor plan?

(ii) Mr Lee wants to know the length of the living room from the balcony to the household shelter, i.e. the length of AB. Can you show him how to find the answer?

(iii) The property agent tells Mr Lee that the floor area of this flat is 110 m². If Mr Lee pays \$500 000 for this flat, find its price per square metre, correct to the nearest dollar.

(iv) Mr Lee finds that the price of a unit in a nearby condominium is \$1000 psf, where psf stands for 'per square foot' and 1 foot $\approx$ 30.48 cm. How much more expensive is the condominium unit as compared to the flat that Mr Lee is interested to purchase?

PROBLEM 2: Scuba Diving

Divers use diving cylinders, which usually contain an oxygen-enriched air mix, to help them stay underwater for long periods of time. Although the exterior of a diving cylinder is not cylindrical as shown in Fig. **(a)**, its internal compartment, which contains air, is cylindrical as seen in Fig. **(b)**.

Fig. (a)

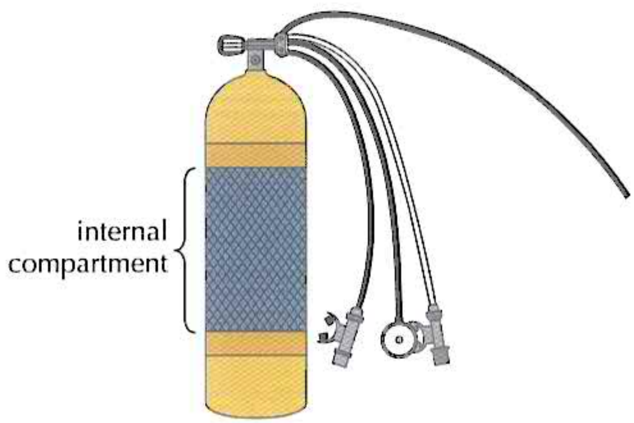


Fig. (b)

The volume of a diving cylinder is often measured by the volume of water that it can hold. A diver intends to stay underwater at a depth of 15 m for an hour. He wants to determine whether a particular diving cylinder, with an internal compartment of radius 6.75 cm and height 85 cm, is suitable.

(i) Find the volume of water that the diving cylinder can contain, giving your answer in litres.

(ii) As gas can be compressed, diving cylinders can hold a greater volume of gas than water. Given that

$$\text{volume of gas} = \frac{\text{volume of cylinder} \times \text{pressure in cylinder}}{\text{atmospheric pressure}},$$

where the pressure in the cylinder is 200 bars and the atmospheric pressure is 1.01 bars, find the volume of gas that the diving cylinder can hold, giving your answer in litres.

(iii) The volume of gas that a diver consumes is given by

$$\text{volume of gas consumed} = \text{breathing rate} \times \text{duration} \times \text{ambient pressure}.$$

Assuming that the diver's breathing rate is 20 litres per minute and that for every 10 m underwater, the ambient pressure increases by 1 bar from the atmospheric pressure, find the duration the diver can stay underwater.

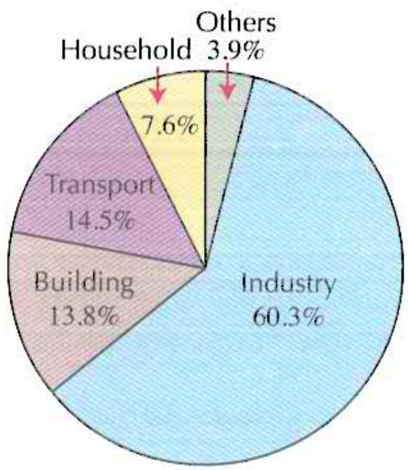
(iv) Hence, determine whether the diving cylinder is suitable for the diver. Explain your answer.

PROBLEM 3: Climate Change

Climate change is defined as significant variations in the statistical observations of the weather conditions, which occur over an extended period of time.

(a) An indication of climate change is the overall increase in the annual mean surface temperature. Studies show that the annual mean surface temperature in Singapore has increased from 26.8 °C in 1948 to 27.6 °C in 2011. Find the percentage increase in the annual mean surface temperature in Singapore from 1948 to 2011, giving your answer correct to 2 decimal places

(b) One of the contributors of climate change is greenhouse gas emissions. Singapore's emissions were 41 million tonnes (MT) in 2005. Singapore's Business-As-Usual (BAU) emissions are expected to reach 77.2 MT in 2020. The pie chart shows a projection of Singapore's BAU emissions for the year 2020 for various sectors.



INFORMATION

The projection of Singapore's BAU emissions for the year 2020 is based on the amount of greenhouse gases that would be emitted without the implementation of any measures.

Source: National Climate Change Secretariat, Prime Minister's Office (Singapore)

Note: Others include waste, water and other electricity use.

(i) Which of the above accounts for the majority of Singapore's projected emissions in 2020? Find the amount of emissions contributed by that sector.

(ii) Give two reasons for the likely increase in emissions from 2005 to 2020.

(c) Find out some measures that have been put in place by the Singapore government to reduce emissions and to mitigate the effects of climate change.

PROBLEM 4: Smartphone Price Plans

Smartphone price plans usually include a fixed monthly component and a variable component depending on the monthly usage. The user is entitled to a certain amount of free talk time and data usage.

In Singapore, there are three telecommunication companies providing mobile services. In groups of three or four, work out how you can advise your classmates to choose one of the following plans.

Price Plan	A	B	C
Monthly subscription	$36	$39.90	$38
Free local incoming calls	Unlimited		
Free local outgoing calls	100 minutes If the outgoing calls exceed 100 minutes, the excess usage is charged at $0.002675/second.		
Free local data bundle	12 GB If the data usage exceeds 12 GB, the excess usage is charged at $0.032/10kB and is capped at $30.	12 GB If the data usage exceeds 12 GB, the excess usage is charged at $0.0054/2kB and is capped at $30.	12 GB If the data usage exceeds 12 GB, the excess usage is charged at $0.0034/kB and is capped at $30.
Local short messaging service (SMS)	Unlimited free SMS for students		

Guiding Questions

(a) Are there any assumptions that you may need to make?

(b) Other than the basic monthly subscription fee, what are some other factors you should consider?

(c) How do you solve this problem? Explain your method clearly.

PROBLEM 5: Fun Fair

Your school is organising a fun fair. Your class decides to make sugar cookies using the following ingredients to sell at the fun fair.

List of Ingredients for Sugar Cookies (makes 48 cookies)

- 350 g all-purpose flour
- 1 teaspoon baking soda
- 130 g butter, softened
- 300 g white sugar
- 1 egg

Decide on the number of cookies your class should make and the selling price of the cookies in order to maximise your profit.

You may need to consider the following:

- Total cost of making the cookies
- Budget
- Fundraising target

Guiding Questions

(a) Are there any other assumptions that you may need to make?

(b) How do you determine the number of cookies that should be made?

(c) What are some factors to consider before pricing the cookies?

(d) How do you solve this problem? Explain your method clearly.

(e) If you were to carry out the fundraising plan according to your solution, are there any other considerations that may affect your profit?

Practise Now Answers

Chapter 1

Practise Now 1

1. Composite; prime

Practise Now 2

1. 32
2. 29

Practise Now 3

1. $2 \times 3^2 \times 7$
2. $7^2 \times 11$

Practise Now 4

1. 28
2. 84

Practise Now 5

1. 14
2. 21

Practise Now 6

11; 5

Practise Now 7

1. **(a)** 231
   (b) 0.3582
2. 125.7 cm

Practise Now 8

No; yes

Practise Now 9

1. 28
2. 14
3. 84

Practise Now 10

45

Practise Now 11

1. 360
2. 2520
3. 8

Practise Now 12

540

Practise Now 13

1. 2.00 a.m.
2. **(i)** 14 cm
   (ii) 37

Chapter 2

Practise Now (Page 28)

1. **(i)** 2013, 6
   (ii) –5, –17
   (iii) $2013, 1.666, \frac{3}{4}, 6$
   (iv) $-5, -\frac{1}{2}, -3.8, -17, -\frac{2}{3}$
2. **(a)** –43.6 °C
   (b) –423 m
   (c) –1
   (d) –\$10 000

Practise Now (Page 30)

1. **(b)** <
   (c) >
   (d) >
2. $-5, -3.8, -1\frac{1}{2}, 0, \frac{3}{4}, 1.666, 4$

Practise Now (Page 34)

(a) 7
(b) –3
(c) –2
(d) –14
(e) 14
(f) –14
(g) –2
(h) –27

Practise Now 1

–6 °C

Practise Now (Page 36)

(a) 11
(b) –11
(c) 2
(d) –2
(e) –4
(f) 40
(g) –36
(h) 2
(i) –3
(j) –18

Practise Now 2

1. 28 °C
2. –165 m; 479 m

Practise Now (Page 40)

(a) –12
(b) –20
(c) 8
(d) 21
(e) 10
(f) 18
(g) –30
(h) –36
(i) 40
(j) 200

Practise Now (Page 41)

(a) –4
(b) –5
(c) 2
(d) –2
(e) –4
(f) 4

Practise Now 3a

(a) ±8
(b) –3
(c) 6

Practise Now 3b

(a) –27
(b) –64
(c) 6
(d) –2

Practise Now 4a

(a) –30
(b) –55

Practise Now 5

(a) $3\frac{9}{10}$
(b) $-2\frac{11}{12}$

Practise Now 6

(a) 6
(b) $1\frac{2}{3}$

Practise Now 7a

(a) $-1\frac{7}{8}$
(b) $1\frac{9}{40}$

Practise Now 8

(a) 32.544
(b) 48.23

Practise Now 9

(a) 2.3
(b) 12.3

Practise Now 10

(a) 33.6
(b) -2.2
(c) -0.115
(d) -0.5

Practise Now 11

0.583

Chapter 3

Practise Now 1

1. (a) 3 409 730
   (b) 3 409 700
   (c) 3 410 000
   (d) 3 410 000
2. 11 649 999; 11 550 000

Practise Now 2

1. (a) 78.5
   (b) 78
   (c) 78.47
   (d) 78.470
2. No

Practise Now 3

$175

Practise Now (Page 65)

1. (a) 3
   (b) 4
   (c) 1
   (d) 2

Practise Now (Page 65)

(a) 3
(b) 4
(c) 5
(d) 6

Practise Now (Page 65)

(a) 2
(b) 3
(c) 6
(d) 4

Practise Now (Page 66)

(a) 2
(b) 3
(c) 3
(d) 4

Practise Now (Page 66)

(a) 3
(b) 5
(c) 2

Practise Now 4

1. (a) 3750
   (b) 0.004 710
   (c) 5000
   (d) 0.10, 0.100
2. 8

Practise Now 5

(i) 10.2 m
(ii) 41.0 m

Practise Now 6

1. Not reasonable
2. (a) 19
   (b) 40
3. $\frac{240}{80}$ hours

Practise Now 7

S$3.75

Practise Now 8

Option A

Practise Now 9

$66\frac{2}{3}\%$

Chapter 4

Practise Now 1

1. (a) 28
   (b) $-1\frac{1}{2}$
2. $12\frac{1}{4}$

Practise Now (Page 88)

(a) $7x$
(b) $-x$
(c) x
(d) $-7x$

Practise Now (Page 89)

(a) x
(b) $7x$
(c) $-7x$
(d) $-x$

Practise Now (Page 90)

(a) $6x - 2$
(b) $-x + 2$
(c) $x + 3y$
(d) $x - 9y$

Practise Now 2

1. (a) $10x - y$
   (b) $-3x + 3y$
   (c) $-12x - 6y$
   (d) $\frac{1}{6}x$
   (e) $\frac{9}{8}y$
2. (i) $-2p - 3q + 4r$
   (ii) 16

Practise Now (Page 93)

(a) $-3x - 2$
(b) $-3x + 2$
(c) $3x + 2$
(d) $-2x - y + 4$

Practise Now (Page 93)

(a) $-2x + 2$
(b) $x - 4$
(c) $x + 3y$
(d) $-3x + 7y$

Practise Now (Page 94)

(a) $15x$
(b) $-15x$
(c) $-15x$
(d) $15x$

Practise Now 3

(a) $3x + 6$
(b) $-5x + 20y$
(c) $-ax - 2ay$

Practise Now (Page 96)

(a) $4x + 1$
(b) $-x + 8$
(c) $-4x - y$
(d) $-7x - 2y$

Practise Now 4

1. (a) $26x + 4y$
   (b) $7x - 4y$
   (c) $5x - 8$
2. (i) $p + 5$
   (ii) $3p + 15$
   (iii) $5p + 38$
   (iv) $5p + 11$

Practise Now 5

(a) $\frac{1}{6}x - \frac{3}{20}y$
(b) $y - 6x$

Practise Now 6

1. (a) $\frac{7x - 19}{6}$
   (b) $\frac{-5x + 22}{12}$
2. (a) $\frac{-2x + 11}{12}$
   (b) $\frac{13x + 11}{9}$

Practise Now 7

(a) $-5(2x - 5)$
(b) $9a(2 - 6y + 4z)$

Chapter 5

Practise Now (Page 111)

(a) 4
(b) 13
(c) −10
(d) −1

Practise Now (Page 112)

(a) 5
(b) 1
(c) −2
(d) −3

Practise Now (Page 113)

(a) −7
(b) 3
(c) 4
(d) 1

Practise Now (Page 114)

(a) 2
(b) 5
(c) 1
(d) $-\frac{6}{7}$

Practise Now 1

1. (a) −5
   (b) 2
   (c) −3
   (d) −2
   (e) $1\frac{2}{7}$
2. (a) 2
   (b) 2.05

Practise Now 2

(a) −8
(b) $5\frac{5}{6}$
(c) $-\frac{5}{7}$

Practise Now 3

(a) 5
(b) $2\frac{1}{2}$

Practise Now 4

(a) 50 N
(b) 1000 kg

Practise Now 5

1. $1\frac{17}{24}$
2. $\frac{7}{18}$

Practise Now 6

(i) $A = \frac{1}{2}\pi r^2$
(ii) 39.275 cm^2

Practise Now 7

1. 4 and 20
2. 15

Practise Now 8

$16\frac{1}{2}$

Practise Now 9

1. (a) $x > 5$
   (b) $x \leqslant -4$
2. 2

Practise Now 10

12

Chapter 6

Practise Now (Page 143)

1. (i) 5
   (ii) −1
2. (i) $-\frac{2}{5}$
   (ii) $\frac{4}{5}$

Practise Now 1

1. (ii) 2.5

Practise Now 2

(a) 3
(b) $-\frac{1}{2}$
(c) $1\frac{1}{2}$
(d) −6

Practise Now 3

(a) (i) \$4.50
    (ii) \$6
    (iii) \$8
(b) 4.50, 6, 8

Practise Now 4

(a) 20 minutes
(b) 9 km
(c) (i) $\frac{9}{10}$
    (ii) 0
    (iii) $-\frac{4}{5}$
    (iv) 0
    (v) $-\frac{5}{7}$

Chapter 7

Practise Now 1

1. (a) 28, 33
   (b) −50, −56
   (c) 1215, 3645
   (d) −18, 6
2. (a) 22, 29
   (b) 15, 11

Practise Now 2

(i) 23
(ii) 58

Practise Now 3

1. (a) $4n + 1$
   (b) $5n + 2$
   (c) $6n - 4$
   (d) $3n - 2$
2. (i) 23, 27
   (ii) $4n - 1$
   (iii) 199

Practise Now 4

1. **(ii)** $2 + 5 \times 4 = 22$; $2 + 6 \times 4 = 26$; $4n + 2$
   (iii) 8054
2. **(i)** $72 = 8 \times 9$
   (ii) 10

Practise Now 5

(ii) 6, 12; 7, 14; $n + 1$, $2n + 2$
(ii) 54; 110
(iii) 59; 60

Chapter 8

Practise Now 1

(a) **(i)** $\frac{9}{20}$
(ii) $3\frac{1}{20}$
(iii) $\frac{11}{200}$
(iv) $\frac{61}{700}$
(b) **(i)** 85%
(ii) 2320%

Practise Now 2

(a) **(i)** 0.12
(ii) 4.13
(iii) 0.236
(iv) 0.0625
(b) **(i)** 76%
(ii) 278.9%

Practise Now 3

1. **(i)** 37.5%
   (ii) 62.5%
2. $66\frac{2}{3}\%$

Practise Now (Page 201)

1. **(a)** \$2.65
   (b) \$100.80
2. \$115

Practise Now 4

1. 1455
2. 441

Practise Now 5

Village A

Practise Now 6

1. **(a)** \$350
   (b) **(i)** $66\frac{2}{3}\%$
   (ii) 50%
2. **(a)** 24
   (b) 6%

Practise Now 7

$4\frac{2}{13}\%$ decrease

Practise Now 8

50

Practise Now 9

1. \$600
2. \$125 000

Practise Now 10

1. \$3450
2. \$120 000

Chapter 9

Practise Now 1

(i) 33 : 20
(ii) 20 : 53

Practise Now 2

(a) 2 : 15
(b) 27 : 40
(c) 3 : 10

Practise Now 3

$3\frac{11}{15}$

Practise Now 4

1. 609
2. \$900

Practise Now 5

(i) 10 : 12 : 27
(ii) 10 : 27

Practise Now 6

\$360

Practise Now 7

Lixin

Practise Now 8

1. **(a)** \$2.44
   (b) **(i)** $614\frac{4}{5}$ km
   (ii) \$185.25
2. 60

Practise Now 9

06 00, Saturday

Practise Now 10

12 h 26 min

Practise Now 11

1. **(i)** 40.32 km/h
   (ii) 11.2 m/s
2. $11\,458\frac{1}{3}$ m
3. 80 km/h

Practise Now 12

1. **(i)** 13.5 m/s
   (ii) 81 000 cm/min
2. 2.93

Practise Now 13

$16\frac{11}{289}$ km/h

Practise Now 14

5 hours

Practise Now 15

356

Chapter 10

Practise Now (Page 259)

(a) Acute
(b) Reflex
(c) Obtuse
(d) Obtuse
(e) Reflex
(f) Acute

Practise Now 1

1. **(a)** $a = 58$
   (b) $b = 20$
2. $c = 16$

Practise Now 2

1. $a = 22$
2. $b = 45$

Practise Now 3

(i) 37°
(ii) 37°

Practise Now 4

$a = 10$, $b = 25$

Practise Now (Page 269)

(a) **(i)** $\angle a$ and $\angle m$, $\angle b$ and $\angle n$, $\angle c$ and $\angle o$, $\angle d$ and $\angle p$, $\angle e$ and $\angle i$, $\angle f$ and $\angle j$, $\angle g$ and $\angle k$, $\angle h$ and $\angle l$
(ii) $\angle c$ and $\angle m$, $\angle d$ and $\angle n$, $\angle g$ and $\angle i$, $\angle h$ and $\angle j$
(iii) $\angle c$ and $\angle n$, $\angle d$ and $\angle m$, $\angle g$ and $\angle j$, $\angle h$ and $\angle i$

Practise Now 10
(i) 108°
(ii) 36°
(iii) 162°
(iv) 9°
(v) 20

Chapter 12
Practise Now 3
(i) 11.3 cm
(ii) 4.0 cm

Practise Now 4
(i) 77°
(ii) 5.3 cm

Practise Now 5
(iii) Y, Z, XY, XZ

Practise Now 6
1. 12.2 cm
2. 12.3 cm

Practise Now 7
1. 71°
2. 74°

Practise Now 8
(i) 7.0 cm
(ii) 54°

Chapter 13
Practise Now 1
(a) 160 000 cm^2
(b) 0.0357 m^2

Practise Now 2
1. 273 m^2
2. $201\frac{1}{2}$ m^2

Practise Now 3
(i) 94.0 cm
(ii) 462 cm^2
(iii) 322 cm^2

Practise Now 4
(i) 168 m^2
(ii) 74 m

Practise Now 5
24 m

Practise Now 6
1. 192 m^2
2. 40 cm^2

Practise Now 7
(i) 36.4 m^2
(ii) 29.7 m

Practise Now 8
(i) 6 m
(ii) 7.2 m

Practise Now 9
1200 m^2

Chapter 14
Practise Now 1
(a) (i) 10 000 000 cm^3
(ii) 10 000 000 m*l*
(b) (i) 0.165 m^3
(ii) 165 *l*

Practise Now 2
1. (i) 52
(ii) 4446
2. 22 cm

Practise Now 3
3 312 000 cm^3

Practise Now 4
1. (i) 400 cm^3
(ii) 340 cm^2
2. (i) 1.152 *l*
(ii) 544 cm^2
3. 54 cm^2

Practise Now 5
1. 160 m^3
2. 4.5

Practise Now 6
(i) 162 cm^3
(ii) 180 cm^2

Practise Now 7
1. 5730 cm^3
2. 8.84 cm

Practise Now 8
1. 49.9 *l*
2. 520 minutes

Practise Now 9
1. (i) 297 cm^2
(ii) 47 : 54
2. (ii) 158 cm^2
(iii) 358 cm^2

Practise Now 10
1. (i) 5600 cm^3
(ii) 1920 cm^2
2. (i) 406 cm^3
(ii) 445 cm^2

Chapter 16
Practise Now (Pages 394 – 395)
1. (a) (i) \$5 500 000
(ii) \$7 000 000
(b) 2009; \$1 500 000
2. (b) (i) 384
(ii) 594
(c) $39\frac{43}{163}\%$
(d) (i) $16\frac{476}{489}\%$
(ii) No
(e) December

Practise Now 1
1. (i) 20.4
(ii) $22\frac{2}{3}\%$
(iii) 1150 m*l*
2. (i) Black
(ii) 40°, 70°, 100°, 120°, 30°
(iii) No

Practise Now 2
(i) 2008
(ii) $17\frac{43}{221}\%$

Answers

Exercise 1A

1. **(a)** Composite
(b) Prime
(c) Prime
(d) Composite

2. **(a)** $2^3 \times 3^2$
(b) 11×17
(c) $2^4 \times 3 \times 7$
(d) $2 \times 3^2 \times 5 \times 7$

3. **(a)** 42
(b) 24
(c) 15
(d) 12

4. 99

5. 28

6. **(a)** 8
(b) 9
(c) 6
(d) 9

7. **(a)** 9291
(b) 1.0024
(c) 9

8. 63.2 cm

9. 169 cm^2

10. **(a)** Composite
(b) Prime
(c) Composite
(d) Prime

11. 38

12. 43

Exercise 1B

1. **(a)** 6
(b) 12
(c) 15
(d) 1

2. **(a)** 120
(b) 462
(c) 324
(d) 85 680

3. 14

4. 198

5. 30 096

6. 9

7. **(i)** 9
(ii) 19, 7, 3

8. **(i)** 240 s
(ii) 20 minutes

9. **(a)** True
(b) True
(c) False, e.g. 4
(d) True
(e) True

10. 7, 21 or 63

11. **(a)** True
(b) False, e.g. 4
(c) True
(d) True

12. **(i)** 16 cm
(ii) 12

13. **(i)** 34
(ii) 10

14. **(i)** 42

15. **(i)** 210
(ii) 1.26 m

Review Exercise 1

1. **(a)** 35
(b) 24

2. **(a)** 8
(b) 7

3. **(a)** Composite
(b) Prime

4. **(i)** 84
(ii) 83 160

5. 55, 110, 165, 220, 330 or 660

6. **(i)** 27
(ii) 4, 3, 2

7. 6.03 p.m.

8. **(i)** 7 May
(ii) Every 12 days

Challenge Yourself – Chapter 1

1. **(i)** 11, 12, 1, 2, 3, 4
(ii) 2

3. **(i)** 6; 2520
(iii) Yes
(iv) No

4. $m + n - \text{HCF}(m, n)$

5. **(i)** 12
(ii) $n - \text{HCF}(m, n)$

Exercise 2A

1. **(i)** 10 001, 4

   (ii) −12, −2017

   (iii) $\frac{1}{5}$, 4.33, 10 001, 4

   (iv) −0.3, $-\frac{5}{7}$, −12, $-1\frac{1}{2}$, −2017

2. **(a)** 30 m above sea level

   (b) −35

   (c) An anticlockwise rotation of 30°

   (d) A speed of 45km/h of a car travelling West

3. **(a)** <

   (b) <

   (c) <

   (d) >

   (e) <

   (f) >

5. **(a)** −13, −3, 23, 30, 230

   (b) −10, −0.5, $-\frac{3}{20}$, 15, 150

6. **(a)** −273.15 °C

   (b) −86 m

7. **(a)** >

   (b) >

   (c) <

   (d) >

Exercise 2B

1. **(a)** 4

   (b) 3

   (c) −6

   (d) −8

   (e) 6

   (f) −16

   (g) −8

   (h) −7

2. **(a)** 7

   (b) 8

   (c) −11

   (d) −6

   (e) 9

   (f) −3

   (g) −11

   (h) 9

3. **(a)** 0

   (b) −17

   (c) 1

   (d) −9

   (e) 18

   (f) −10

   (g) 9

   (h) 1

4. **(a)** 12

   (b) −7

   (c) −12

   (d) −46

   (e) 5

   (f) −37

   (g) −11

   (h) −12

5. **(a)** 35

   (b) −30

   (c) −8

   (d) 4

   (e) −6

   (f) −35

   (g) 29

   (h) −38

6. −4 °C

7. 23 °C

8. −51 m; 189 m

9. **(i)** 5

   (iii) 4 years

Exercise 2C

1. **(a)** −27

   (b) −32

   (c) 35

   (d) 6

   (e) 14

   (f) 0

2. **(a)** −3

   (b) −8

   (c) 4

   (d) −7

   (e) −3

   (f) 6

3. **(a)** ±9

   (b) ±4

   (c) ±5

   (d) ±10

4. **(a)** 9

   (b) 2

   (c) −3

   (d) Not possible

5. **(a)** −8

   (b) −125

   (c) −1000

   (d) −216

5. (a) 3
(b) -4
(c) 2
(d) -6

7. (a) -75
(b) -3
(c) -130
(d) 43
(e) 21
(f) 5
(g) 2
(h) -80
(i) -2
(j) 4

9. (a) 40
(b) 105
(c) -18
(d) 0
(e) 28
(f) -9
(g) $51\frac{1}{4}$
(h) $1\frac{1}{3}$

11. 4

Exercise 2D

1. (a) $-1\frac{1}{4}$
(b) $2\frac{7}{8}$
(c) $\frac{7}{10}$
(d) $-7\frac{5}{6}$

3. (a) $2\frac{1}{2}$
(b) $1\frac{1}{2}$
(c) $1\frac{1}{2}$
(d) $1\frac{1}{3}$

5. (a) $-1\frac{3}{5}$
(b) $-\frac{2}{25}$
(c) $-1\frac{5}{16}$
(d) -11
(e) $-3\frac{1}{3}$
(f) $\frac{8}{15}$

7. (a) 17.664
(b) 19.56
(c) 0.0216
(d) 0.49

8. (a) 2.7
(b) 11
(c) 57.1
(d) 2.9

9. (a) 8.2
(b) 1.3
(c) -3.2
(d) -12.1

10. (a) $\frac{3}{20}$
(b) $2\frac{17}{20}$
(c) $-1\frac{6}{7}$
(d) $-8\frac{11}{24}$
(e) $-1\frac{9}{20}$

12. (a) $\frac{1}{7}$
(b) -1
(c) $9\frac{1}{8}$
(d) $-1\frac{3}{16}$
(e) $\frac{4}{9}$
(f) $-5\frac{1}{10}$

14. (a) $-\frac{1}{25}$
(b) -7
(c) -0.36
(d) -0.03

15. (a) 16.934
(b) -2.085
(c) -5.842
(d) 7.288

16. $4\frac{17}{28}$ hours

17. $6\frac{1}{12}$

18. $\frac{12}{35}$

Review Exercise 2

1. (a) $>$
(b) $>$
(c) $<$
(d) $<$

2. (a) $5.5, 4, \frac{29}{33}, -\frac{3}{4}, -2.365$
(b) $10\frac{1}{2}, 5.855, \frac{5}{8}, -2\pi, -8$

3. (a) 67
(b) -28
(c) -15
(d) -36

4. (a) -84
(b) 120
(c) -40
(d) $1\frac{1}{4}$

5. (a) 56
(b) 10
(c) -24
(d) 6
(e) 54
(f) 28
(g) -26
(h) 3
(i) -102
(j) -67

6. 3

7. (a) $5\frac{2}{5}$

(b) $3\frac{5}{12}$

(c) $-13\frac{3}{4}$

(d) $-\frac{3}{10}$

(e) 6

(f) $\frac{3}{13}$

(g) $-2\frac{5}{12}$

(h) $-10\frac{11}{12}$

8. $\frac{598}{1225}$

9. (a) −101

(b) 490.4

(c) −0.48

(d) −3.01

Challenge Yourself – Chapter 2

1. $x = 3, y = -2$

2. (a) 3, 4; 5; 6; 1, 2; 8, 8

(b) 4, 8; 3, 4; 1, 2; 2, 2; 2, 2

3. (b) $(3 + 3) \div 3 + 3 - 3 = 2$

(c) $3 + 3 - 3 - 3 + 3 = 3$

(d) $(3 + 3 + 3 + 3) \div 3 = 4$

(e) $3 + 3 \div 3 + 3 \div 3 = 5$

(f) $3 + 3 + (3 - 3) \times 3 = 6$

Exercise 3A

1. (a) 698 400

(b) 698 000

(c) 700 000

2. (a) 45.7

(b) 46

(c) 45.740

3. (i) 160 m

(ii) 1500 m^2

4. (a) 4.9 m

(b) 10 cm

(c) $11.00

(d) 6.49 kg

5. No

6. 5 077 499; 5 076 500

7. No

Exercise 3B

1. (a) 5

(b) 5

(c) 3

2. (a) 730

(b) 503.9

(c) 0.003 019

(d) 6400; 6400

(e) 10.0

(f) 8.08

3. 4, 5 or 6

4. (a) 0.010 10

(b) 200 000

(c) 2013.7

(d) 9.0

5. 6

6. (i) 16.2 cm

(ii) 65.0 cm

7. (i) 21.6 m

(ii) 1470 m^2

8. 6

9. 21 249; 21 150

10. (i) 0.000 000 7

(ii) 0

(iii) No

Exercise 3C

1. Not reasonable; 7.03

2. (a) 80 000; 78 507

(b) 3; 2.99

3. (i) 3.6; 30

(ii) 0.12

4. $\frac{270}{9}$ l

5. 1 : 2

6. $4000

7. S$10

8. Option A

9. $\frac{80}{100} \times \$80$, $\frac{90}{100} \times \$70$

10. S$30

Review Exercise 3

1. (a) 6500

(b) 6000

(c) 6480.0

2. (i) 4.8; 40

(ii) 0.12

3. 0.0245 kg

4. S$5.25

5. $(3 \times 110 + 2 \times 150 + 5 \times 80)$ g

6. $\frac{28}{4}$

7. $\frac{80}{100} \times \$85$, $\frac{90}{100} \times \$76$

8. Option B

Challenge Yourself – Chapter 3

1. 987×123 is greater than 988×122.

2. 2000 kg

Exercise 4A

1. **(a)** $ab + 5y$

(b) $f^3 - 3$

(c) $6kq$

(d) $\frac{2w}{3xy}$

(e) $3x - 4\sqrt{z}$

(f) $\frac{2p}{5q}$

2. **(a)** 52

(b) $3\frac{1}{2}$

(c) 136

(d) $\frac{1}{2}$

3. **(a)** 69

(b) 57

(c) $-\frac{7}{15}$

(d) $-1\frac{7}{15}$

4. **(a)** $-x - 1$

(b) $7x + 7y$

(c) $4xy + 8x$

(d) $-2x + 5y - 4z$

5. **(a)** $2x - y$

(b) $-10a + 6b$

(c) $-11c + 12d$

(d) $8hk$

6. **(a)** $(a + b)^2 - \sqrt[3]{3xy}$

(b) $(20x + 500y)$ cents

7. **(a)** $2\frac{1}{4}$

(b) $2\frac{1}{10}$

(c) $\frac{3}{8}$

(d) $\frac{6}{275}$

8. **(a)** $-3x - 3y$

(b) $-10x + 5y$

(c) $x + 14y$

(d) $-5x + 9y$

9. **(a)** $\frac{7}{12}x$

(b) $\frac{1}{15}y$

(c) $\frac{6}{35}a$

(d) $\frac{11}{12}b$

10. **(i)** $11p - 2q - 5r$

(ii) 50

11. **(i)** $12m + 5$

(ii) $15m + 10$

12. $\$(11w + 12m)$

13. **(a)** $\frac{5}{2}a$

(b) $\frac{2}{5}b$

(c) $\frac{2}{7}c$

Exercise 4B

1. **(a)** $-x - 5$

(b) $-4 + x$

(c) $6y + 14$

(d) $16y - 40$

(e) $24a - 32b$

(f) $-3c - 18$

(g) $-4d + 24$

(h) $2ax - 2ay$

2. **(a)** $5a + 7b$

(b) $19p + 84q$

(c) $-4a + 7b$

(d) $7x - 9y + 10z$

3. $4x + 20$

4. $(10x - 6y)$ cents

5. $\$(19x + 42n + 15)$

6. **(a)** $-2u + 15v$

(b) $-5a + 3b$

(c) $11m - 8n$

(d) $13x + 38$

(e) $-9a + 27b$

(f) $9p - 14q$

(g) $-5x + 9y - 6$

(h) $-5p + 6q + 20$

(i) $18a + 60b - 52c$

(j) $-24x - 28y$

7. **(a)** $8x - 2$

(b) $4x - y - 4z$

(c) $-12p - 13q + 20rs$

(d) $2a + 2b - 9c - 4d$

8. **(a)** $-6a - 16$

(b) $25c + 5d$

9. $\$\left(\frac{1800m - mb + 2000f}{f}\right)$

Exercise 4C

1. **(a)** $\frac{1}{12}x + \frac{1}{10}y$

(b) $\frac{8}{3}a - \frac{26}{35}b$

(c) $-\frac{23}{72}c - \frac{7}{12}d$

(d) $\frac{3}{2}f - \frac{5}{12}h - \frac{67}{20}k$

2. **(a)** $3a + \frac{11}{2}b - \frac{9}{2}c$

(b) $2x - 3$

(c) $4p - 2$

(d) $16x + 2$

3. (a) $\frac{9}{10}x$

(b) $\frac{1}{12}a$

(c) $\frac{17h+7}{35}$

(d) $\frac{x-4}{8}$

(e) $\frac{23x-3}{10}$

(f) $\frac{5y+3}{12}$

(g) $\frac{a-11}{8}$

(h) $\frac{7}{12}q$

4. (a) $3(4x-3)$

(b) $-5(5y+7)$

(c) $9b(3-4y)$

(d) $4a(2x+3-z)$

(e) $2m(2-3y-9z)$

5. (a) $-6x+3y$

(b) $-3p+4q$

6. (a) $\frac{41x+13}{6}$

(b) $\frac{-9x+7}{10}$

(c) $\frac{-17z+18}{20}$

(d) $\frac{-14p-25q}{6}$

(e) $\frac{-9a+8b}{15}$

(f) $\frac{23x-6}{20}$

(g) $-\frac{13}{12}a$

(h) $1\frac{2}{3}$

(i) $\frac{9}{14}a$

(j) $\frac{17x+32}{6}$

7. (a) $5x(1+2b+2c)$

(b) $3x(-y+2z)$

(c) $2x(-6y-7)$

(d) $3a(5b-8)$

(e) $4y(2x-7)$

8. (a) $\frac{32p-41q}{14}$

(b) $\frac{-41a+173b}{30}$

(c) $\frac{-21f-23h+16k}{12}$

(d) $\frac{96+7x-10y-27z}{24}$

Review Exercise 4

1. (a) 27

(b) 8

(c) 16

(d) −18

(e) 3

(f) 81

2. $2\frac{22}{35}$

3. (a) $7ab-3xy$

(b) $-18p-8q$

(c) $a+11b$

(d) $14x-26y$

(e) $60f-68h$

(f) $3x+32y$

4. (a) $\frac{5x+15}{12}$

(b) $\frac{-3x+y}{24}$

(c) $\frac{22a-26b}{15}$

(d) $\frac{-5f+34h-21k}{30}$

(e) $\frac{-4p+103q}{30}$

(f) $\frac{-25x+37y-120}{42}$

5. (a) $7q(3p+2-4r)$

(b) $4(x-2y+4z)$

6. (a) $5x$ cents

(b) $30x$ cents

(c) $\frac{65}{7}x$ cents

7. $\frac{xy}{180}$ km

8. (a) $155y$ seconds

(b) $(23\,400z+8400)$ seconds

9. (i) $\$(27.5x+20y)$

(ii) \$271.25

10. (i) $(4p+2q+9)$ marks

(ii) $(6p+3q-9)$ marks

(iii) $3(2p+q-3)$

Challenge Yourself – Chapter 4

2. $x=2, y=3, z=6$

Revision Exercise A1

1. (a) 6

(b) 182

2. (i) 5

(ii) 158 760

3. (i) 21

(ii) 5

(iii) 6

4. (i) {−2, 6}; {−1, 5}; {1, 3}

(ii) {−2, −1}; {1, 2}

(iii) {−1, 5, 6}; {1, 3, 6}; {1, 4, 5}; {2, 3, 5}

5. (a) 900

(b) 30

(c) 2

6. $\frac{630}{7\times 60}$ m/s

7. (a) −2

(b) 3

(c) 14

(d) 18

8. $\$\left(\frac{11a+6b}{50}\right)$

Revision Exercise A2

1. (a) 18
   (b) 20 944
2. (a) –96
   (b) 20
3. 180
4. (i) –2 °C
   (ii) 4 °C increase
5. (a) $2\frac{11}{12}$
   (b) (i) –44.030
   (ii) 0.313
6. 85
7. (i) (4×3) m^2
   (ii) $\$(4 \times 3 \times 90)$
8. $-10x + 13$

Exercise 5A

1. (a) 7
   (b) –14
   (c) 22
   (d) 4
   (e) 1.2
   (f) 6
   (g) –3.7
2. (a) –7
   (b) 6
   (c) 5
   (d) 3
   (e) 1
   (f) 3
   (g) 2.05
   (h) 0.6
   (i) $\frac{13}{28}$
   (j) $\frac{5}{8}$
3. (a) 1
   (b) –17
   (c) –4
   (d) 4
4. (a) 1
   (b) 4
   (c) $2\frac{4}{5}$
   (d) –10
   (e) 2.25
   (f) $-\frac{11}{15}$
   (g) –3
   (h) 24
   (i) 2
   (j) $\frac{2}{21}$
   (k) 38
   (l) $-13\frac{1}{2}$
   (m) 13
   (n) $-1\frac{6}{7}$
5. (a) 21
   (b) –8
   (c) 3
   (d) 24
   (e) 4
   (f) 10
6. (a) 9
   (b) 5
   (c) $2\frac{16}{25}$
   (d) $-1\frac{1}{32}$
7. (a) $2\frac{1}{2}$
   (b) 19
8. (a) –2
   (b) $-5\frac{2}{3}$
   (c) $\frac{8}{19}$
   (d) $-\frac{10}{13}$
   (e) 4
9. (a) 4
   (b) $-1\frac{1}{2}$
   (c) $-\frac{1}{2}$
   (d) $-2\frac{2}{7}$
   (e) $2\frac{4}{9}$
   (f) $\frac{1}{8}$
10. (a) 3
    (b) $1\frac{7}{8}$
    (c) $7\frac{11}{26}$
    (d) $-1\frac{1}{3}$
    (e) 50
    (f) $-13\frac{1}{2}$
    (g) 18
    (h) $-\frac{1}{3}$
11. (a) 1
    (b) $\frac{9}{10}$
    (c) 1
    (d) $\frac{1}{13}$
    (e) $47\frac{1}{2}$
    (f) $2\frac{5}{12}$
12. $x = \frac{19}{20}$ is the solution.
13. $\frac{3}{4}$
14. $-\frac{8}{9}$

Exercise 5B

1. $33\frac{1}{5}$
2. $1\frac{2}{5}$
3. (i) 1386
   (ii) 7

4. (i) 450 cm^2

(ii) 6 cm

5. (a) $P = xyz$

(b) $S = p^2 + q^3$

(c) $A = \frac{m+n+p+q}{4}$

(d) $T = 60a + b$

6. 3

7. $3\frac{1}{4}$

8. $24\frac{1}{20}$

9. $11\frac{1}{2}$

10. $\frac{1}{3}$

11. 3

12. $-6\frac{1}{27}$

13. $7\frac{1}{2}$

14. $-\frac{18}{49}$

15. (i) $S = 3n + 6$

(ii) -309

16. (i) $T = cd + \frac{ef}{100}$

(ii) 577.50

17. (i) 56.7 °C

(iii) −79.8 °F

Exercise 5C

1. 8700 kg

2. 17

3. 19, 15, 13

4. 18

5. 14

6. 28

7. 12

8. 28

9. 8

10. 25

11. \$12.80

12. 40

13. $7\frac{19}{23}$ km/h

14. $\frac{9}{14}$

15. 25

Exercise 5D

1. (a) $>$

(b) $<$

(c) $\geqslant$

(d) $\leqslant$

(e) $>$

(f) $<$

2. (a) $x \leqslant 6$

(b) $x \geqslant 15\frac{1}{2}$

(c) $y < -12$

(d) $y > -4\frac{4}{5}$

(e) $x < 7$

(f) $x \geqslant -10\frac{1}{2}$

(g) $y \leqslant -2\frac{1}{2}$

(h) $y > -2\frac{2}{9}$

3. 7

4. $1\frac{1}{7}$

5. 2

6. −37

7. −3, −2, −1, 0, 1, 2, 3

Review Exercise 5

1. (a) 2

(b) 15

(c) $2\frac{3}{7}$

(d) 20

(e) $20\frac{1}{2}$

(f) 4

(g) 7

(h) $-\frac{2}{5}$

(i) −3

(j) $-\frac{52}{55}$

2. (a) $x < -1\frac{7}{18}$

(b) $y \geqslant -2\frac{2}{5}$

3. −1

4. 3

5. −9

6. 24

7. 17

8. (i) $1437\frac{1}{3}$

(ii) 3

9. $7\frac{1}{2}$

10. 15 and 17

11. 130 kg

12. 25

13. 22

14. 9

15. 25 cents

16. $3\frac{3}{5}$ km

17. $\frac{9}{11}$

18. 18

19. 20

20. 441

21. 20

22. 6

23. 10

Challenge Yourself – Chapter 5

1. No solution

2. 1

3. $A = 3, B = 5, C = 6, D = 8$

4. $A = 2, B = 4, C = 7, D = 9$

xercise 6A

$A(-4, -3)$, $B(-2, 4)$, $C(3, 4)$, $D(4, 2)$, $E(1, 1)$, $F(3, -3)$

(i) 17

(ii) –3

(i) –3

(ii) 10

(a) Rectangle

(b) Rhombus

(c) Isosceles triangle

(d) Quadrilateral

(e) Trapezium

24 units^2

The points lie on a straight line.

(a) (i) $-1\frac{2}{3}$

(ii) $1\frac{1}{3}$

(b) (i) 1

(ii) $-\frac{3}{4}$

xercise 6B

(b) They are parallel lines.

(b) They are parallel lines.

(b) They are parallel lines.

(ii) $a = 2$, $b = 12$, $c = 1.5$

xercise 6C

(a) (i) $105

(ii) $90

(iii) $70

(b) 105, 90, 70

(a) (i) 27 km

(ii) 47 km

(b) $5.60

3. (i) 100, 200, 300, 400

(iii) $390

(iv) 72

Review Exercise 6

1. (a) Rectangle

(b) Square

(c) Trapezium

(d) Kite

2. (a) $A(-5, 0)$, $B(-4, 3)$, $C(-3, 4)$, $D(0, 5)$, $E(3, 4)$, $F(4, 3)$, $G(4, -3)$, $H(3, -4)$, $I(-3, -4)$, $J(-4, -3)$

(b) (i) H

(ii) G

3. (i) $46\frac{1}{2}$

(ii) $8\frac{1}{2}$

(iii) $-3\frac{1}{2}$

4. (i) 5

(ii) –10

(iii) 70

5. (ii) $a = -2$, $b = 0$

Challenge Yourself – Chapter 5

–5 or 7

Exercise 7A

1. (a) 39, 44

(b) 40, 32

(c) 384, 768

(d) 50, 25

(e) 16, –4

(f) –288, 576

(g) –87, –94

(h) –50, –40

(i) 50, 60

(j) 80, 87

(k) 324, 972

2. (a) 9, 15

(b) 12, 8

(c) –33, –32

(d) 88, 85

(e) 21, 28

3. (a) –67, –131

(b) 8, 13

(c) 144, 196

(d) –216, 343

(e) 81, 243

Exercise 7B

1. (a) $6n + 1$

(b) $3n - 7$

(c) $7n + 53$

(d) $-3n + 17$

2. (i) 15

(ii) 21

(iii) 105

3. (i) 18, 21

(ii) $3n$

(iii) 315

4. (i) 30, 34

(ii) $4n + 6$

(iii) 806

5. (i) $3 + 1 = 4$, $4 + 1 = 5$, $5 + 1 = 6$, $6 + 1 = 7$

(ii) 50

(iii) 100

6. (ii) 2, 3, 4, 5, $n - 1$

(iii) 29

7. (a) 3, 9, 19, 33

(b) (i) $2n^2 - 1$

(ii) 301 087

8. (ii) 16, 25, 36, 49, $(n+1)^2$

(iii) 441

(iv) 10

9. (i) $54 = 6 \times 9$

(ii) 13

10. (i) $1 + 3 + 5 + 7 + 9 + 11$

$= 36 = 6^2 = (5+1)^2$

(ii) $a = 25, c = 13, d = 12$

11. (a) (i) $\frac{8-2}{2} = 3$, $\frac{10-2}{2} = 4$,

$\frac{12-2}{2} = 5$, $\frac{14-2}{2} = 6$

(ii) $2(3) + 2 = 8$,
$2(4) + 2 = 10$,
$2(5) + 2 = 12$,
$2(6) + 2 = 14$

(b) (i) 9

(ii) 14

(c) (i) 46

(ii) 74

12. (i) $\frac{4 \times (4-1)}{2} = 6$, $\frac{5 \times (5-1)}{2} = 10$,

$\frac{6 \times (6-1)}{2} = 15$, $\frac{7 \times (7-1)}{2} = 21$

(ii) 190

13. (i) 1 5 10 10 5 1

(ii) $1 + 5 + 10 + 10 + 5 + 1$

$= 32 = 2^5$; 2^{n-1}

14. (a) 1, 2, 3, 4, 5, 6; 3, 5, 7, 9, 11, 13; 4, 7, 10, 13, 16, 19; 10, 12, 14, 16, 18, 20

(b) (i) 19

(ii) 26 cm

(iii) $2n + 1$

(iv) $(2n + 8)$ cm

15. (i) $\frac{2}{8 \times 9 \times 10} = \frac{1}{8} - \frac{2}{9} + \frac{1}{10}$

(ii) $\frac{2}{10 \times 11 \times 12} = \frac{1}{660}$

(iii) $p = 19$

16. (a) (i) 11, 13

(ii) 24, 28

(iii) 84, 112

(iv) 85, 113

(b) $13^2 + 84^2 = 85^2$
$15^2 + 112^2 = 113^2$

17. (i) 7, 12; 8, 14; $n + 2$, $2n + 2$
(ii) 23; 48
(iii) 31, 33

18. (i) 5 (ii) 8
(iii) 89

Review Exercise 7

1. (i) 53, 44

(ii) 28, 40

(iii) $\frac{1}{27}, \frac{1}{81}$

(iv) 121, 169

2. (i) 64, 81

(ii) $(n+2)^2$

(iii) 729

3. (ii) $5 \times 5 + 1 = 26$, $5n + 1$

(iii) 281

(iv) It is not possible.

4. (ii) $15 = \frac{5 \times 6}{2}$; $\frac{1}{2}n(n+1)$

(iii) 3003

(iv) 11

5. (i) $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3$

$= 784 = (1 + 2 + 3 + 4 + 5 + 6 + 7)^2$

(ii) 14 400

(iii) 8

6. (i) 37

(ii) $2n + 2$

(iii) 362

Challenge Yourself – Chapter 7

1. 7

2. $\frac{1}{2}n(n-1)$

3. (i) 4, 9

4. (i) 11, 18

(ii) For $n \geqslant 3$, $T_n = T_{n-1} + T_{n-2}$.

(iii) Lucas Numbers (which is different from Lucas Sequence)

5. (i) 10, 12

(ii) For $n \geqslant 4$, $T_n = T_{n-2} + T_{n-3}$.

(iii) Perrin Numbers (or Perrin Sequence)

Revision Exercise B1

1. (a) 2.4

(b) $\frac{5}{8}$

2. (a) $x > 5$

(b) $y \leqslant -1\frac{3}{5}$

3. $-\frac{23}{30}$

4. 21

5. 5 hours

7. (i) $9n - 3$

(ii) 18

(i) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 8^2$

(ii) 12

Revision Exercise B2

1. (a) $1\frac{10}{11}$

   (b) $-\frac{1}{2}$

2. (a) $x \geqslant -7\frac{6}{7}$

   (b) $y > -6$

3. 42

4. 12

5. (ii) $(-2, 0)$

6. (b) 25 units^2

7. (i) $-3n + 47$

   (ii) 20

8. (i) $6^2 - 2 \times 6 = 24$

   (ii) 9

Exercise 8A

1. (a) $\frac{7}{25}$

   (b) $1\frac{29}{50}$

   (c) $\frac{31}{250}$

   (d) $\frac{33}{500}$

2. (a) 0.04

   (b) 6.33

   (c) 0.0002

   (d) 0.337

3. (a) 60%

   (b) 90%

   (c) 4.8%

   (d) 120%

   (e) 48%

   (f) 124%

4. (a) 78%

   (b) 25%

   (c) 7%

   (d) 9.5%

   (e) 135%

   (f) 260%

5. (a) \$35

   (b) 3.6 m

6. (i) $52\frac{12}{19}$ %

   (ii) $47\frac{7}{19}$ %

7. 84

8. Jun Wei

9. (a) $11\frac{19}{21}$ %

   (b) 75%

   (c) 300%

   (d) 1.5%

   (e) 67%

   (f) 125%

   (g) $16\frac{2}{3}$ %

   (h) 30%

10. (a) $\frac{837}{10\,000}$ ml

    (b) $460\frac{11}{16}$ m

    (c) 84 l

    (d) \$1

11. Bronze, silver, gold

12. 5610

13. They have the same value.

14. 8.61%

15. 45%

Exercise 8B

1. (a) 81

   (b) 63

   (c) 66

   (d) 135

2. (a) 85

   (b) 28

   (c) 140

   (d) 240

3. 25%

4. (i) \$650

   (ii) 40%

5. 25%

6. \$86 400

7. 300

8. \$245 000

9. \$65

10. 2496

11. 16.64%

12. 3% increase

13. \$680 000

14. \$20 000

15. \$64 000

16. 120%

Review Exercise 8

1. $1785\frac{5}{7}$ %

2. (i) \$291.20

   (ii) \$1164.80

3. $\frac{3}{40}$

4. Huixian

5. 464

6. 300

7. 10 000

8. $17\frac{11}{17}$ %

Challenge Yourself – Chapter 8

1. 250

2. 32.8%

Exercise 9A

1. **(a)** 30 : 7

   (b) 35 : 72

   (c) 9 : 17

   (d) 145 : 280 : 26

   (e) 16 : 36 : 15

   (f) 11 : 21 : 60

2. **(a)** 96

   (b) $\frac{16}{25}$

3. 15 : 16

4. **(i)** 25 : 40 : 44

   (ii) 8 : 5

   (iii) 10 : 11

5. **(i)** 14 : 25

   (ii) 25 : 39

6. **(i)** 9 : 50

   (ii) 3 : 9 : 925

7. $154

8. **(i)** 544

   (ii) $953.70

9. **(a)** 20 : 3

   (b) 12 : 53

   (c) 12 : 15 : 8

   (d) 4 : 30 : 45

   (e) 12 : 33 : 55

10. **(a)** $\frac{9}{20}$

    (b) $x = 2\frac{1}{2}$, $y = 6\frac{3}{4}$

11. **(i)** 6 : 16 : 9

    (ii) 16 : 9

12. **(i)** 80

    (ii) 10

13. $150 500, $236 500, $107 500

14. 7

15. $450

16. $\frac{40}{89}$

Exercise 9B

1. **(a)** 30

   (b) $0.19

   (c) $1600

   (d) $4\frac{8}{13}$ kg

2. Vishal

3. $55.20

4. **(i)** $0.06

   (ii) $27.60

5. **(i)** 743.4 km

   (ii) $339.51

6. **(i)** 350 g

   (ii) 18 m^2

7. **(i)** 269 °C

   (ii) $31\frac{5}{8}$ minutes

8. 15

9. **(i)** $1680

   (ii) $2040

10. 737

Exercise 9C

1. **(a)** 08 00

   (b) 21 42

   (c) 00 00

   (d) 02 42

2. **(a)** 3.30 a.m.

   (b) 11.12 p.m.

   (c) 7.15 p.m.

   (d) 12.00 a.m.

3. **(a)** 03 35

   (b) 06 35 the next day

   (c) 2 h 15 min

   (d) 12 h 28 min

   (e) 10 h 40 min

   (f) 22 35

4. 6 h 37 min

5. 07 13, Tuesday

6. 10 51

7. **(i)** 7 h 10 min

   (ii) 05 30

8. **(a)** 4 h 55 min

   (b) 9 h 20 min

   (c) 6 h 50 min

   (d) 6 h 25 min

   (e) 13 h 15 min

Exercise 9D

1. **(i)** 49.2 km/h

   (ii) $13\frac{2}{3}$ m/s

2. 360 000 m

. (a) 504 km/h

(b) 1134 km/h

(c) $14\frac{13}{25}$ km/h

(d) $4\frac{1}{2}$ km/h

. (a) $\frac{13}{20}$ m/s

(b) $101\frac{17}{18}$ m/s

(c) $\frac{1}{6}$ m/s

(d) $1433\frac{1}{3}$ m/s

. $10\frac{3701}{7200}$

. $88\frac{4}{5}$ km/h

. (i) 4 s

(ii) $7\frac{1}{2}$ m/s

. 16 m/s

. $84\frac{6}{19}$ km/h

10. 25 207

11. 280 m

12. 37 800 m

Review Exercise 9

1. 9 : 8

2. (i) 9 kg, 15 kg, 21 kg

(ii) $10.80

3. (i) 45

(ii) 12

4. (i) $9

(ii) 46

5. (i) 09 51

(ii) 88 km/h

6. (i) 12 30

(ii) 3 h 20 min; 58.5 km/h

7. $21\frac{30}{131}$ km/h

8. (i) $\frac{1}{50}x$ s

(ii) $\frac{50x}{x+1500}$ m/s

9. $148\frac{3}{4}$ km

10. 39

Challenge Yourself – Chapter 9

1. 18 s

2. Vishal, 1 m

Exercise 10A

1. (a) $a = 79, b = 106, c = 98$

(b) $d = 50, e = 228$

(c) $f = 117, g = 45$

(d) $h = 243, i = 94, j = 56$

2. (a) Obtuse

(b) Reflex

(c) Acute

(d) Reflex

(e) Acute

(f) Obtuse

3. (a) $72°$

(b) $44°$

(c) $37°$

(d) $26°$

4. (a) $144°$

(b) $168°$

(c) $78°$

(d) $9°$

5. (a) $a = 147$

(b) $b = 65$

(c) $c = 20$

(d) $d = 25$

6. (a) 49

(b) 30

7. (a) $a = 106$

(b) $b = 30$

(c) $c = 11.25$

(d) $d = 11$

8. (i) $48°$

(ii) $42°$

9. (a) $a = 47$

(b) $b = 18, c = 126$

10. (a) 90

(b) 60

11. $30°, 60°, 120°, 150°$

12. (a) $a = 11, b = 45$

(b) $c = 23, d = 69, e = 111$

(c) $f = 6, g = 23$

(d) $h = 22, i = 22, j = 44$

13. (i) $x = 22, y = 16$

(ii) $132°$; $278°$

Exercise 10B

1. (a) (i) $B\hat{X}R$ and $D\hat{Z}R$, $A\hat{X}R$ and $C\hat{Z}R$, $A\hat{X}S$ and $C\hat{Z}S$, $B\hat{X}S$ and $D\hat{Z}S$, $B\hat{W}P$ and $D\hat{Y}P$, $A\hat{W}P$ and $C\hat{Y}P$, $A\hat{W}Q$ and $C\hat{Y}Q$, $B\hat{W}Q$ and $D\hat{Y}Q$

(ii) $A\hat{X}S$ and $D\hat{Z}R$, $B\hat{X}S$ and $C\hat{Z}R$, $A\hat{W}Q$ and $D\hat{Y}P$, $B\hat{W}Q$ and $C\hat{Y}P$

(iii) $A\hat{X}S$ and $C\hat{Z}R$, $B\hat{X}S$ and $D\hat{Z}R$, $A\hat{W}Q$ and $C\hat{Y}P$, $B\hat{W}Q$ and $D\hat{Y}P$

(b) No

(c) No

2. (a) $a = 117, b = 117, c = 63, d = 78$

(b) $e = 31, f = 66$

(c) $g = 83, h = 69$

(d) $i = 45, j = 60$

3. (a) $a = 38, b = 34$
   (b) $c = 20, d = 70$
   (c) $e = 18$
   (d) $f = 29$
4. (a) $a = 104$
   (b) $b = 106$
5. (a) $a = 140$
   (b) $b = 32$
   (c) $c = 275$
6. (i) 86°
   (ii) 141°
7. (i) 68°
   (ii) 54°
8. (i) 52°
   (ii) 56°
   (iii) 262°
9. $x = 59.5, y = 43.1$
10. $x = 147, y = 59$
11. 46°
12. $w + x = y + z$

Review Exercise 10

1. (a) $a = 16, b = 48$
   (b) $c = 112, d = 29$
2. (a) $a = 36$
   (b) $b = 36$
3. (a) $a = 23$
   (b) $b = 33, c = 22$
4. (a) $a = 124$
   (b) $b = 23, c = 39$
   (c) $d = 29$
   (d) $e = 56$
   (e) $f = 65$
   (f) $g = 19$
5. (i) 128°
   (ii) 54°
6. (i) 154°
   (ii) 144°
7. (i) 44°
   (ii) 58°
8. 118°
9. $w + z + 180 = x + y$

Challenge Yourself – Chapter 10

1. $\frac{1}{2}(n + 1)(n + 2)$
2. 260°
3. 67

Exercise 11A

1. (a) 100°, scalene triangle, obtuse-angled triangle
   (b) 70°, isosceles triangle, acute-angled triangle
   (c) 60°, equilateral triangle, acute-angled triangle
   (d) 90°, scalene triangle, right-angled triangle
2. (a) 100°
   (b) 6°
   (c) 150°
   (d) 56°
3. (a) $a = 51$
   (b) $b = 24$
   (c) $c = 20$
   (d) $d = 22.5$
   (e) $e = 56$
   (f) $f = 60$
4. (a) $a = 102$
   (b) $b = 40, c = 55$
   (c) $d = 70, e = 22$
5. 45°
6. (i) 35°
   (ii) 125°
7. (a) $a = 25, b = 97$
   (b) $c = 138$
8. (a) $a = 58, b = 127$
   (b) $c = 130$
9. (a) $a = 35, b = 108$
   (b) $c = 39, d = 77$
   (c) $e = 24, f = 96, g = 48$
   (d) $h = 112, i = 65, j = 47$
10. 100
11. (i) 104°
    (ii) 76°
12. (i) 78°
    (ii) 52°
13. (i) 101°
    (ii) 42°
14. 15°
15. 72°

Exercise 11B

1. (a) $a = 36, b = 36$
   (b) $c = 51, d = 51$
2. (a) $a = 106, b = 48$
   (b) $c = 20, d = 40$
3. (a) $a = 40, b = 58$
   (b) $c = 114$
4. (a) $a = 37, b = 127$
   (b) $c = 67.5, d = 22.5$

5. (a) $a = 114, b = 33$

(b) $c = 38, d = 104$

(c) $e = 42, f = 48$

6. (i) 64°

(ii) 26°

7. (i) 115°

(ii) 60°

8. (i) 88°

(ii) 23°

9. 6

10. $x = 30, y = 114$

11. (i) 65°

(ii) 46°

12. (i) 59°

(ii) 31°

13. (i) 110°

(ii) 28°

14. (i) 54°

(ii) 72°

(iii) 36°

15. (i) 31°

(ii) 97°

16. (i) 58°

(ii) 96°

Exercise 11C

1. (a) 1620°

(b) 1800°

(c) 2340°

(d) 3240°

2. (a) $a = 110$

(b) $b = 66$

(c) $c = 83$

(d) $d = 30$

3. (a) (i) 720°

(ii) 120°

(b) (i) 2880°

(ii) 160°

4. (a) 165°

(b) 170°

5. (a) 4

(b) 8

(c) 30

(d) 90

6. (a) 9

(b) 20

(c) 45

(d) 72

7. 162°

8. (i) 30

(ii) 30°

9. 8

10. 21

11. 77.1°

12. (i) 162°

(ii) 153°

13. (i) 120°

(ii) 60°

(iii) 60°

(iv) 108°

(v) 72°

(vi) 24°

14. (i) 12

(ii) 135°

(iii) 120°

15. (i) 10

(ii) 126°

(iii) 144°

16. 360°

17. 180°

19. (i) Equilateral triangle and square

Review Exercise 11

1. (a) $a = 22.5$

(b) $b = 244, c = 26$

2. (a) $a = 70, b = 20$

(b) $c = 39, d = 63$

(c) $e = 66, f = 66$

(d) $g = 70, h = 40$

(e) $i = 22, j = 24$

(f) $k = 39, l = 63$

3. (a) $a = 60$

(b) $b = 52, c = 31, d = 97$

(c) $e = 122, f = 58, g = 64$

4. (a) $a = 35$

(b) $b = 30, c = 90$

5. (i) 40°

(ii) 32°

6. (i) 56°

(ii) 31°

7. 150°

8. 18

9. 18

10. (i) 157.5°

(ii) 112.5°

11. 13

12. 15

13. 10

Challenge Yourself – Chapter 11

1. 150°
2. 30°
3. Yes
4. (ii) Yes
5. $\frac{1}{2}n(n-3)$

Exercise 12A

3. 9.4 cm
4. 7.5 cm
5. 53°
7. 9.1 cm
8. (i) 11.6 cm
   (ii) 5.9 cm
9. (i) 52°
   (ii) 3.9 cm
10. (i) 52°
    (ii) 8.0 cm
11. (i) 71°
    (ii) 4.2 cm
12. (i) 7.5 cm
    (ii) 7.2 cm
13. (iii) X, Y, XY, YZ
14. (i) 10.9 cm
    (iii) 4.7 cm
17. (ii) Diameter

Exercise 12B

1. 16.9 cm
2. 12.8 cm
3. 10.1 cm, 6.5 cm
4. 133°
5. 171°
6. (i) 7.1 cm
   (ii) 66°
7. 3.9 cm, 6.9 cm
8. 11.7 cm, 10.9 cm
9. (i) 8.4 cm
   (ii) 7.1 cm
10. (i) 5.4 cm
    (ii) 4.5 cm
11. (iii) 7.0 cm
12. (i) 128°
    (ii) 5.5 cm
13. (i) 109°
    (ii) 4.1 cm
14. (i) 8.6 cm
    (ii) 6.5 cm
    (iii) 51°

Review Exercise 12

1. (i) 5.4 cm
   (ii) 3.3 cm
2. (i) 46°
   (ii) 7.9 cm
3. (iii) Rhombus
4. (i) 7.1 cm
   (ii) 6.5 cm
5. (i) 121°
   (ii) 6.5 cm
6. (iii) Square
   (iv) 9.5 cm

Challenge Yourself – Chapter 12

1. Circumcircle
2. Incircle

Revision Exercise C1

1. 400
2. 5236
3. 7 : 8
4. 1219
5. (i) 6 hours 25 minutes
   (ii) 693 km
6. (i) 110°
   (ii) 14°
7. (i) 44
   (ii) 84°
   (iii) 21°
8. (ii) 5.6 cm

Revision Exercise C2

1. 8%
2. 30
3. 7 : 20
4. $77\frac{1}{2}$
5. $58\frac{1}{2}$ km/h
6. $x = 154$, $y = 72$
7. 26
8. 7.7 cm, 11.7 cm

Exercise 13A

1. (a) 400 000 cm^2
   (b) 0.0016 m^2
   (c) 300 cm^2
   (d) 2.8 m^2
2. (a) 14 cm
   (b) 65 cm

. $77.25\ m^2$

. (a) 20 cm, 62.8 cm, 314 cm^2

(b) 1.8 m, 11.3 m, 10.2 m^2

(c) 56.0 mm, 28.0 mm, 2460 mm^2

(d) 28.0 cm, 14.0 cm, 88.0 cm

. 0.56 m

. (a) (i) 96.0 cm

(ii) 661 cm^2

(b) (i) 39.9 cm

(ii) 89.3 cm^2

(c) (i) 56.5 cm

(ii) 191 cm^2

. (i) 28.6 m

(ii) 50.4 m^2

. 350 m^2

. 235.75 m^2

10. 157.5 m^2

11. 17.6 cm

12. (i) 113 m^2

(ii) $4838.05

13. (i) 22.4 cm

(ii) 35.0 cm^2

14. (i) 83.8 cm

(ii) 236 cm^2

15. (i) 42.2 m

(ii) 28.5 m^2

16. 0.250 cm^2

17. 99.0 minutes

Exercise 13B

1. (a) 84 cm^2

(b) 7 m

(c) 5.5 mm

2. (a) 54 cm^2

(b) 14 m

(c) 13 mm

3. (i) 54 cm^2

(ii) 32 cm

4. 14 m

5. (i) 416.25 cm^2

(ii) 89.5 cm

6. (i) 18 m

(ii) 11.7 m

7. 60 cm^2

8. 351 m^2

9. (a) 320 cm^2

(b) 364 cm^2

10. 184 cm^2

11. 175 cm^2

12. (i) 80 cm^2

(ii) 20 cm^2

Review Exercise 13

1. (a) 659 cm^2

(b) 220 cm^2

(c) 1040 cm^2

(d) 105 cm^2

2. (i) 88.0 cm

(ii) 154 cm^2

3. 576 cm^2

4. (i) 225 m^2

(ii) 79.6 m

5. 12 cm^2

6. 18 cm^2

7. 0.038 225 ha

8. (i) 12

(ii) $x = 8, y = 4$

9. 0.455 m

10. 15

Challenge Yourself – Chapter 13

1. 18.5 cm or 37 cm

2. (i) 180 cm

(ii) 180 cm

3. 125 cm^2

Exercise 14A

1. (a) (i) 4 000 000 cm^3

(ii) 500 000 cm^3

(b) (i) 0.25 m^3

(ii) 0.0678 m^3

2. (a) (i) 840 000 cm^3

(ii) 840 000 m*l*

(b) (i) 0.002 560 m^3

(ii) 2.56 *l*

3. (a) (i) 480 cm^3

(ii) 376 cm^2

(b) (i) 420 cm^3

(ii) 358 cm^2

(c) (i) 115 200 mm^3

(ii) 27 360 mm^2

(d) (i) $7\frac{1}{2}$ cm^3

(ii) $41\frac{1}{2}$ cm^2

(e) (i) $\frac{21}{64}$ cm^3

(ii) $3\frac{43}{160}$ cm^2

(f) (i) 4.095 cm^3

(ii) 19.26 cm^2

4. (a) 2160 mm^3, 1284 mm^2
(b) 8 cm, 158 cm^2
(c) 2.5 cm, 89.5 cm^2
(d) 8 m, 432 m^2

5. (i) 16
(ii) 105

6. 0.190 m

7. 4.1 m

8. 5.352 m^3

9. 6480 cm^3

10. (i) 4.8 *l*
(ii) 0.142 m^2

11. (i) 112 *l*
(ii) 1.16 m^2

12. 96 cm^2

13. 614.125 cm^3

14. (i) 456 000
(ii) $25 080 000
(iii) $836

15. 0.0495 m^3

16. 138.6 *l*

17. (i) 5 cm
(ii) 540 cm^3

18. (i) 26 m^2, 78 m^3; 25 m^2, 75 m^3; 36 m^2, 64.8 m^3
(ii) No

Exercise 14B

1. (a) 369 840 cm^3
(b) 16 644 cm^3
(c) 960 cm^3
(d) 1152 cm^3
(e) 770 cm^3
(f) 4725 cm^3

2. (a) 6 cm^2, 42 cm^3
(b) 14 cm, 693 cm^3
(c) 32 cm, 240 cm^2
(d) 400 cm, 95.94 cm^2

3. 102 480 m^3

4. (a) (i) 180 cm^3
(ii) 264 cm^2
(b) (i) 153 cm^3
(ii) 250 cm^2

5. (i) 2000 m^3
(ii) 1490.25 m^2

Exercise 14C

1. (a) (i) 1850 cm^3
(ii) 836 cm^2
(b) (i) 4.52 m^3
(ii) 17.3 m^2
(c) (i) 44 500 mm^3
(ii) 7350 mm^2

2. (a) 8.00 cm, 4.00 cm, 453 cm^2
(b) 28.0 cm, 14.0 cm, 2990 cm^2
(c) 2 cm, 42.0 cm, 553 cm^2
(d) 8 m, 21.0 m, 629 m^2

3. 18.8 *l*

4. 82.9 cm

5. 7.42 *l*

6. 19 200 cm^2

7. 4895

8. 1.06 cm^2

9. 144 m

10. 2280 *l*

11. 26 minutes

12. (i) 3744 cm^3
(ii) 16.5 cm
(iii) 1110 cm^2

13. (i) 725 cm^2
(ii) $\frac{5}{8}$

14. (i) 23.8 cm
(ii) 3750 cm^2

15. 834 cm^2

16. 61 minutes

Exercise 14D

1. (i) 522 cm^3
(ii) 432 cm^2

2. (i) 388 cm^3
(ii) 388 cm^2

3. (i) 1220 cm^3
(ii) 1230 cm^2

4. (i) 79 100 cm^3
(ii) 12 100 cm^2

5. (i) 765 cm^3
(ii) 563 cm^2

6. (i) 13 400 cm^3
(ii) 3760 cm^2

7. (i) 94 300 cm^3
(ii) 16 800 cm^2

8. (i) 255 cm^3
(ii) 427 cm^2

Review Exercise 14

1. (a) (i) 108 cm^3
(ii) 168 cm^2
(b) (i) 88 cm^3
(ii) 152 cm^2

(c) (i) 14 cm^3

(ii) 50 cm^2

(d) (i) 16 cm^3

(ii) 58 cm^2

3000

147 mm

294 cm^2

(i) 16 436.875 cm^3

(ii) 13 149.5 mm^3

(iii) $15\frac{49}{750}$

115 minutes

(i) 80 400 cm^3

(ii) 30.0 cm

(i) 51.4 cm^3

(ii) $1 110 000

(i) 4750 cm^3

(ii) 1960 cm^2

0. (i) 5410 cm^3

(ii) 2140 cm^2

Challenge Yourself – Chapter 14

) 92 000 cm^3

i) 18 400 cm^2

Exercise 16A

(i) 2012; 260 000

(ii) 960 000

(iii) $180 000 000

(iv) $18\frac{2}{11}$ %

(ii) 4 : 5

(iii) $83\frac{1}{3}$ %

(a) 13, 16

(b) (i) 68

(ii) 64

(c) $17\frac{11}{17}$ %

(d) 40%

(e) No

5. (i) 950

(ii) 500

(iii) $20\frac{20}{49}$ %

6. (i) 53

(ii) 50%

Exercise 16B

2. (i) 90°

(ii) 100°

(iii) $27\frac{7}{9}$ %

(iv) 36

3. (i) 50%

(ii) 20%

(iii) 63

4. (i) 120

(ii) 375

(iii) $45\frac{5}{6}$ %

(iv) 60°, 75°, 60°, 90°, 75°

5. (ii) $19\frac{1}{21}$ %

6. (a) 20°

(b) (i) 225

(ii) 540

(c) $31\frac{13}{27}$ %

7. 9

8. (i) 2011 and 2012

(iii) $56\frac{1}{4}$ %

9. (ii) 39 °C, 38 °C

11. No

12. (i) Luxury goods

(ii) Rent and luxury goods

13. No

Review Exercise 16

1. (i) 2 : 1

(ii) 175%

2. (i) 102

(ii) $16\frac{2}{3}$ %

(iii) 160°

4. (i) 40%

(ii) $48 000

5. (ii) $57\frac{1}{7}$ %

Revision Exercise D1

1. 314 cm^2

2. 75.4 cm; 226 cm^2

3. 84 cm^2

4. 972 cm^3; 684 cm^2

5. (i) 158.355 *l*

(ii) 28 269 cm^3

(iii) 25 442.1 g

6. (a) (i) $41\frac{2}{3}$%

(ii) 20%

(b) 54

Revision Exercise D2

1. 584 cm^2

2. (i) 73.7 cm

(ii) 281 cm^2

3. 6.3 cm

4. 880 cm^3; 728 cm^2

5. 63 000 g

6. (i) 4^{th}

(ii) 9^{th}

(iii) $66\frac{2}{3}$ %

Problems in Real-World Contexts

1. (i) mm

   (ii) 7725 mm

   (iii) $4545

   (iv) 2.37 times

2. (i) 12.2 l

   (ii) 2410 l

   (iii) 48.0 minutes

   (iv) No

3. (a) 2.99%

   (b) (i) Industry; 46.5516 MT