CORNELL UNIVERSITY

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OPTIMIZATION MODELING IN FINANCE

Momentum Trading Optimization

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Abstract

In this project we combined simple momentum trading strategy and Markovitz portfolio optimization. On each rebalance day after we determine the list of stock to long/short, and we put this list of stock in the markovitz ooptimzation algorithm. In order to make our strategy more robust, we tested the parameters for simple momentum trading as well as the appropriate expected return and covariance matrix to be used in portfolio optimization.

We discovered that in the specific circumstance of our project, it is best to long/short the top/bottom 10%(decile) of all stocks in the pool, and it is best to use the 6 month look-back period for data to be plugged in the strategy. We also discovered that it is best to use top/bottom only strategy with simple return, and Fama French 3 factor covariance matrix as input for portfolio optimization.

I. Introduction

Momentum investing is a polular investment strategy in the market. It believes in the continuance of existing market trends and thus taking advantage of this trading signal. Specifically, to participate in momentum investing, a trader takes a long position in an asset that has shown an upward trending price, or the trader takes a short position in a security that has been in a downtrend. Introduced by Jegadeesh and Titman (1993), the basic equity price momentum strategy buys the top 10%(winners) and sells the bottom 10% (losers) of the past 6 months, resulting in an equally weighted zero-investment portfolio. It has historically delivered returns of 7% a year, on average. Therefore, people love this simple and successful trading strategy.

In this study, our goal is to improve upon this even-weighted momentum strategy by dynamically chaning the weights over time. Markowitz' portfolio theory, also called Mean-Variance model, becomes handy in assisting the selection of the most efficient portfolio weights by analyzing the tradeoff between expected returns and the risk. Then, our study becomes a quadratic optimization problem. We start with the raw momentum signals, measured by the past 6-month return. Next we try out different combinations of 3 expected returns and 4 estimations of the covariance matrix, and finding the optimal portfolio performance quantified by sharpe ratio. Finally, we explain all the difficulties we had to overcome and the solutions we came up with to create a convex quadratic optimization problem for this practical case.

The outline of this paper is as follows: Section 2 describes the existence of momentum effect both from theory and preliminary test results of dataset, Section 3 discusses the structure of Markovitz portfolio optimization and the various forms of return and risk, Section 4 describes the data and the division of traning and test sets, Section 5 determines momentum strategy parameters, Section 6 tests the developed momentum model with chosen weights, Section 7 concludes and Section 8 provides suggestions for further research.

II. Momentum Effect

A. Background

The existence of momentum effect in equity assets price movements have been studied and discussed many times over the years. Countless literature have touched upon the topic of stock return predictability on many variables, and momentum effect remains one of the most difficult one to explain (Jegadeesh and Titman 2001).

Momentum effect is in direct conflict with contrarian strategy, which is also known as the mean reversion strategy. In an influential paper, DeBondt and Thaler (1985) examine the returns of contrarian strategies that buy past losers and sell past winners and they found that in cases of very long term and very short term contrarian portfolios earned significantly positive returns.

In the study of Jegadeesh and Titman (JT) (1993), they examined the performance of momentum strategy with mid term holding periods of between 3 to 12 months. For US stocks during 1965 to 1989 most portfolios earned positive returns and all these returns are statistically significant. Their most successful portfolio is the 12 months look back and 3 months holding portfolio.

In this example we can see that momentum strategy is well founded and feasible.

B. Theory

According to Jegadeesh and Titman (JT) (2001) momentum strategies can be profitable in the following circumstances:

1) Stocks underreact to information
2) Past winners happen to be riskier to past losers
3) Premium for certain types of risk varies across time in a serially correlated fashion

Formally:

$$\begin{aligned} r_{it} &= \mu_i + b_i f_t + e_{it} \\ E(f_t) &= 0 \\ E(e_{it} &= 0 \\ Cov(e_{it}, f_t) &= 0, \forall i \\ Cov(e_{it}, e_{j,t-1}) &= 0, \forall i \neq j \end{aligned} \tag{1}$$

μ_i is the unconditional expected return on security i, r_{it} is the return on the security i, f_t is the unconditional unexpected return on a factor-mimicking portfolio, e_{it} is the firm specific component of return at time t, and b_i is the factor sensitivity of security i.

The cross-sectional covariance turns out to equal the expected profits to a trading strategy (Lehmann (1990) and Lo and MacKinlay (1990)):

$$E(r_{it} - \bar{r}_t)(r_{i,t-1} - \bar{r}_{t-1}) > 0$$

Where $\bar{r}_t$ is the cross sectional average.

Given the model, the profits can be decomposed into three terms:

$$E(r_{it} - \bar{r}_t)(r_{i,t-1} - \bar{r}_{t-1}) = \sigma_\mu^2 + \sigma_b^2 Cov(f_t, f_{t-1}) + Cov_i(e_{it}, e_{i,t-1})$$

where σ_μ^2 and σ_b^2 are the cross-sectional variances of expected returns and factor sensitivities respectively.

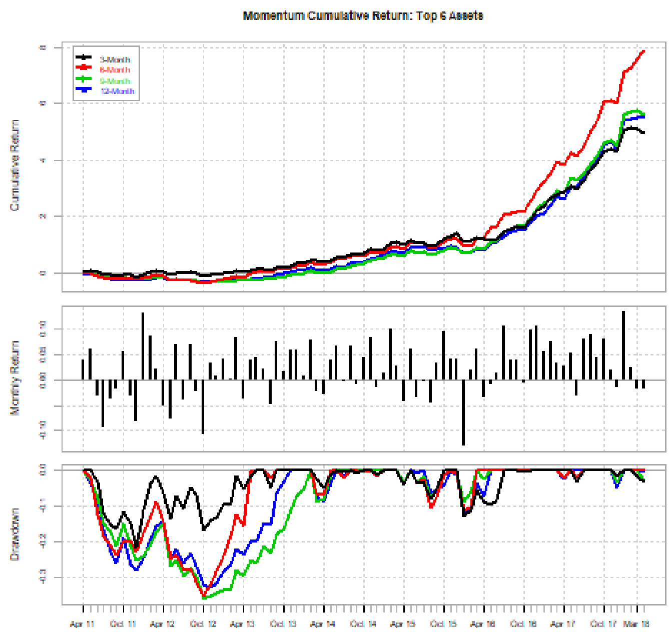
The study indicates that the cross-sectional differences in expected returns under the CAPM and the Fama-French three-factor model cannot account for the momentum profits. If the serial covariance of factor related returns were to contribute to momentum profits, then the factor realizations should be positively serially correlated.

Next we will attempt some simple momentum strategies to examine the profitability of the strategy using data from recent years.

C. Preliminary Test of Simple Momentum Strategy

In order to see the existence of momentum effect, here is a simple momentum strategy:

1) Data: 2010 to 2018 technology sector large cap stocks (62)



2) Choose the top 6 performers given the previous 3, 6, 9, or 12 months' return, hold for a month and rebalance the portfolio

In order to see that momentum effect works, I chose a long horizon (8 years) in order for insignificant variations to wear off. To compare the effect of taking different look back periods, we set 4 different look back periods: 3, 6, 9, and 12 months, as an examination for the result of the experiments of Jegadeesh and Titman (JT) (1993). And we can clearly see that over 8 years, all momentum portfolios have positive returns and the 6-month look back portfolio has a cumulative return of 800% overall, which is a validation of the study of Jegadeesh and Titman (JT) (1993), despite using data of much more present period. Instead of 12 month look back creates the best result, here 6-month look back seems to have the most promising returns.

From the drawdown plot and the portfolio return evolution we can see that prior to 2014 the large cap tech sector seems to be going through a relatively volatile movement and the momentum portfolio is suffering from significant of up to 30%. The 3-month look back portfolio seems to be least affected by it. Hence when constructing the strategy, choose between the 3, 6, 12 month lookback period.

III. Markovitz Portfolio Optimization

A. Intuition

For the optimization of the momentum trading strategy, it is natural to take the portfolio optimization approach since the basic momentum strategy consist of a portfolio that rebalances every fixed period of time, and we can optimize the portfolio that consists of longing the best performing stocks (long only) or longing the best and shorting the worst performing ones (long-short portfolio).

However the standard markovitz may not suffice. According to Jegadeesh and Titman (JT) (2001), in practice portfolio optimization is often known as error maximization. Reason is that standard portfolio optimization magnifies the errors in the return and risk input due to estimation and other errors.

As shown in the study of Jegadeesh and Titman (JT) (2001) and many others, predicted inputs including expected returns and risk can be modified to reduce the impact of potential errors, and here we proposes the following different inputs for expected return and risk estimation:

Expected Return

1) Monthly Sum Realized Returns
2) Characteristic Returns
3) Simple Recent Returns

Covariance Matrix

1) Sample covariance matrix
2) Single index factor model covariance matrix
3) Fama French Factor model covariance matrix
4) Fama French covariance shrinkage

B. Objective Function

For 100% long-short zero investment portfolio:

$$max_x \quad \mu^T x - \lambda x^T V x$$

$$s.t. \quad e^T x = 0$$

$$e^T x \mathbb{1}_{x>0} = 1$$

$$|x| \leq w$$

This is an optimization problem with penalizing element, controlled by λ which is a risk averseness parameter that measures the trade off between return and risk. Because of the last constraint (100% long & 100% short) this optimization problem is not convex, hence we introduced two new vectors z1 and z2 st. z1+z2=x, where z1 is the long portfolio and z2 the short portfolio, and impose the restrictions sum(z1) = 1 and sum(z2) = -1.

C. Expected Return

We computed expected returns in four different ways as return inputs to optimize our portfolio.

1) Monthly Sum Realized Returns: First, we base our expected returns f_t, on the n (3/6/9) months sum realized returns, lagged by month to deal with short-term reverals (suggested by Jegadeesh and Titman, 1995).

$$f_{it} = \sum_{t*=t-n-1}^{t-2} r_{it*} \forall i \epsilon I_t$$

The monthly sum realized return will result in the risk of error maximization as the optimzier may assign a large weight on the stocks with highest and lowest expected returns.In order to avoid error maximization, expected return should be taken over a long period of time, however in our case since our lookback period only ranges from 3 months to 12 months, the shortage of data will lead to a non-satisfactory estimation. Therefore, we introduce an adjustion of expected return to only reflect the momentum's top and bottom phonomen.

2) Characteristic Returns: We change the expected return to plus one for the top decile, minus one for the bottom decile and zero for the intermediate deciles. The characteristic returns f_t^*become

$$f_{it}^* = \begin{cases} 1 & f_{it} > F_t^{-1}(0.1) \\ 0 & F_t^{-1}(0.1) < f_{it} < F_t^{-1}(0.9) \\ -1 & f_{it} < F_t^{-1}(0.9) \end{cases}$$

Where F_t is the sample cumulative distribution function of the monthly sum realized returns. By having all equal expected returns for the stocks in the top and bottom decile, we reduce the probability that the optimizer put extreme weight on very few stocks, and thus reduce the error maximization. In fact, equal expected returns for stocks results in a minimum variance portfolio, which potentially improve the performance of the optimized portfolio.

3) Simple Recent Returns: Apart from the two expected returns above, we would also use a simple recent return $r_{i(t-2)}$ in our strategies for the top and bottom only portfolio. In order to avoid the short-term reverals, we compute the monthly return of each stock, lagged by one month, which is

$$r_{i(t-2)} = P_{i(t-2)}/P_{i(t-3)} - 1$$

D. Covariance Matrix

As we know from the mean-variance optimization, the covariance matrix of stock returns, as one of the two fundamental parts, represents risk control. Portfolio management is complex in the real world, as many asset managers are not allowed to short any stock, or they are typically measured against the benchmark of an equity market index with fixed weights. Therefore, picking up the appropriate covariance matrix of stock returns is essential to our optimization problem.

1) Sample Covariance: The traditional method is to gather a history of past stock returns and then compute their sample covariance matrix, V_t^S.

However, we will see that this method is problematic. When the number of stocks within the portfolio is large, especially relative to the number of historical return observations available, the sample covariance matrix is estimated with a lot of error. For example, we have 302 US stocks in our dataset, and we look back using the history record of 6 months, hence the sample covariance matrix is not positive definite and its inverse does not exist. Here comes the problem that we need the inverse of the covariance matrix for the closed form solution of our initial portfolio.

2) Factor Model: The reduction in the number of samples is a primary reason that factor models are used.

a) Fama and French Risk Factor: According to Martens and Van Oord's study on "Hedging the time-varying exposures of momentum returns" (2008), through time the momentum strategy has significant time-varying risk exposures to the well-known Fama and French (1993) risk factors, i.e. market, size and value-growth. Therefore, we apply a factor covariance matrix using the Fama and French 3-factor model:

$$r_{t,n} = \alpha_n + \beta 1_n RPM_t + \beta 2_n SMB_t + \beta 3_n HML_t + \epsilon_{t,n} \tag{2}$$

where

- RPM_t is the difference between returns on a diversified market portfolio and a risk-free return in month t.
- SMB_t (small minus big) is the difference between the return to a portfolio of small capitalization stocks and the return to a portfolio of large capitalization stocks in month t.

- HML_t (high minus low) is the difference between the return to a portfolio of high Book-to-Market stocks and the return to a portfolio of low B/M stocks in month t.

At the end of each month, we estimate equation (2) for each available stock based on data for the past 24 months, with a minimum requirement of 12 available months. This results in estimates for the factor loading matrix $B = [\beta 1_n, \beta 2_n, \beta 3_n]$. Then, the covariance matrix provided by this model:

$$V_t^{FF} = B_t F_t B_t^{'} + \Delta_t \tag{3}$$

where

- B_t is the (Nx3) matrix with the three estimated equity risk factor loadings for each of the N stocks using data up to month t–1.
- F_t is the (3x3) sample covariance matrix of the equity risk factors using the past 24 months.
- Δ_t is the (NxN) diagonal matrix with the residual variances of the stocks.

b) Fama and French Shrinkage: Using shrinkage on the factor exposures can make the covariance matrix more robust, because it helps reduce bias. Therefore we assume a hierarchical model for the risk model, in particular we assume a flat prior in the random coefficient model for the parameters in (2). This risk model becomes:

$$r_{t,n} = \alpha_n + \beta_n f_t + \epsilon_{t,n} \tag{4}$$

$$\alpha_n \sim N(\alpha, \sigma_\alpha^2), \beta_n \sim N(\beta, \Sigma_\beta), \epsilon_{t,n} \sim N(0, \sigma_\epsilon^2)$$

We assume flat priors for all parameters. Such estimators contain relatively little bias, but the estimation error increases. Then, we need to find a compromise between the sample covariance matrix and a highly structured estimator. For the convenience of our project, instead of looking for the optimal tradeoff, we choose to use an intuitive shrinkage on β_n:

$$\beta_n^{Shrink} = [(B'B)/\sigma_\epsilon + \Sigma_\beta^{-1}]^{-1}[(B'B)\beta_n/\sigma_\epsilon + \Sigma_\beta^{-1}\bar{\beta}] \tag{5}$$

where

- σ_ϵ is the standard error of the OLS regression of (2)
- β_n is the three factor exposures of stock n estimated in (2) using OLS
- Σ_β is the cross-sectional covariance matrix of all these estimated OLS betas
- $\bar{\beta}$ is the crosssectional average of the OLS betas

This results in a large shrinkage towards the cross-sectional beta for stocks with a large standard error of regression, the stocks that have most likely erroneous betas. Betas with a large estimation error are shrunk more towards the cross-sectional average than stocks with a low standard error of regression.

In addition, shrinkage estimator is always positive definite. When the number of the stocks N exceeds the number of past returns used in the estimation process, the sample covariance matrix is singular. This would imply that threre exist stock portfolios with zero risk. On the other hand, our shinkage estimator is a convex combination of an estimator that is positive definite (the shrinkage target) and an estimator that is positive semi-definite (the sample covariance matrix). It is therefore positive definite.

c) Single Index Model: Using only the market factor from the Fama and French factors, a shrunk version of the Fama and French 3-factor covariance matrix. We can view it as the CAPM model. To simplify analysis, SIM assumes that there is only one macroeconomic factor that causes the systematic risk affecting all stock returns and this factor can be represented by the rate of return on a market index.

$$r_{t,n} = \alpha_n + \beta_n RPM_t + \epsilon_{t,n} \tag{6}$$

Similiar to Fama-French, covariance matrix is calculated by equation (3).

IV. Data Collection

A. Raw Data

In the study of Jegadeesh and Titman (JT) (2001), they used US stock data of size between 1500 to 2500, which provided opportunity for error maximization. In our study of the subject, we wish to have a large enough sample size to maintain objectivity and reveal meaningful information, all the while being small enough to avoid the complexity of rank issues due to not having enough observation and the requirement for semidefinite matrices.

We screened the list of all US companies with the following criteria:

1) IPO date before 2000: in order to obtain enough recent data (2000 to 2018) to conduct a meaningful research and analysis on stock momentum
2) Market Capitalization of more than 100 Million: to ensure that the company has significant size to capture more general characteristics of the market and closer to real investment
3) Stocks with same amount of entries: to avoid missing data and ease of manipulation

The above criteria gave us a list of 302 stock entries

B. Training & Test Set

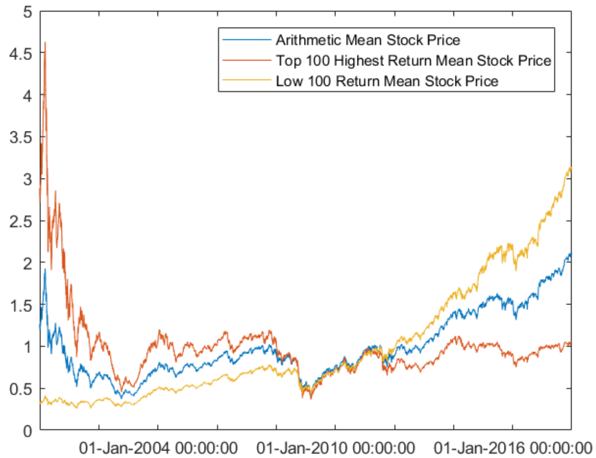
We will be separating the dataset by time segment, and we will be running back-tests on training set to determine parameter, and then compare the statistics on test set to calibrate the best parameter and the best methods for momentum trading.

Given that the financial crisis in 2008 and subsequent years have out of the ordinary volatility, we decided to skip 3 years in our study.

- Training Set:
  Jan-01-2000 to Dec-31-2004
  Jan-01-2001 to Dec-31-2014
- Test Set:
  Jan-01-2005 to Dec-31-2007
  Jan-01-2015 to Dec-31-2017

C. Preliminary Analysis

To visualize and understand the characteristics of the dataset, we made the following plot with the equally weighted average price (by mean return) over 18 years. Note that the companies with the highest average return reached their peak over during the early 2000 (dot com bubble period), and then slowly declined, best for shorting strategy, while most other companies (on the lower return end) followed a more natural upward trending curve, which provides a strong source of momentum. For our portfolio construction, we will be using all of these data to provide a more general and robust result.



V. Implementation

A. Methodology

In this section we will be testing and comparing different models with different parameters.

1) Momentum Strategy Parameters:

- Percentile: long the top d percent of best performing stock and short the lowest d percent of worst performing portfolio
- Horizon: the frequency of rebalances (months)
- Number of samples: length of look-back period, number of months' previous data to collect for stock selection

2) Markovitz Optimization: We will be focusing the following variations of portfolio optimizations

- Expected Return: 3 methods
- Covariance Matrix: 4 methods
- lambda: the risk averseness parameter
- w: the constraint on the weight of a single asset

B. Determining Momentum Strategy Parameters

In this section we will be choosing the parameters by testing these parameters on simple momentum strategy: zero investment long-short portfolio, long top 10% short lowest 10% using simple expected return and simple covariance matrix. Using lambda = 5, w = 0.15

1) Run Portfolio From 2001 to 2004:

Table I: Simple momentum strategy with different percentile and number of samples
(Training Set 1: Jan-01-2000 to Dec-31-2004)

	1	2	3	4
Percentile	deciles	deciles	vigintiles	vigintiles
Number of Samples	6 months	9 months	6 months	9 months
Average Return	1.49%	0.11%	-0.21%	-1.1%
Volatility	0.1854	0.1576	0.2148	0.1931
Sharpe Ratio	**0.2775**	0.0252	-0.0345	-0.1985

Figure 1: Comparison of Cumulative Returns for 4 portfolios

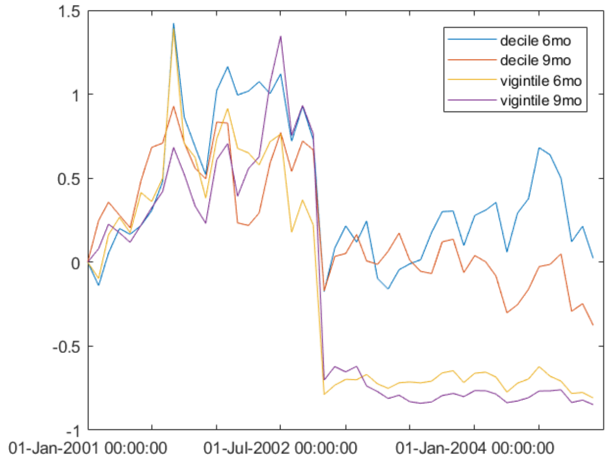
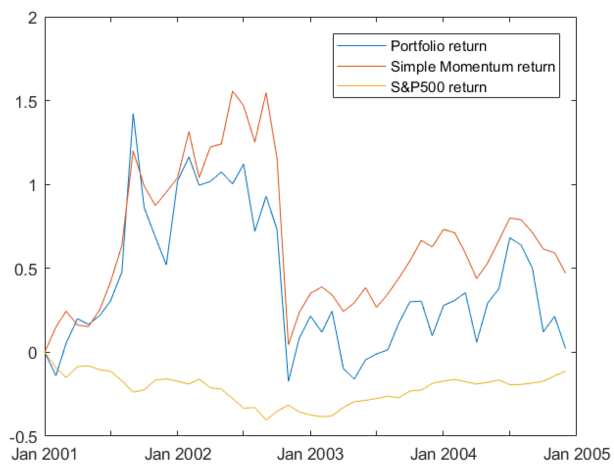


Figure 2: Cumulative Returns: Markovitz Optimization Vs. Equally Weighted Momentum Vs. S&P500



We can see from the training data result table1 and figure1 that out of all the parameters, using top and bottom deciles (10%) and 6 months of look-back period provides the strategy with best result, hence we will be using these 2 parameters for the testing set of 2005 to 2007.

Note that these decisions are time dependent and must be cross validated by testing using same parameters and on different datasets.

We can see that in comparison to the the benchmark portfolio (equally weighted momentum)

2) Run Portfolio From 2011 to 2014:

Table II: Simple momentum strategy with different percentile and number of samples
(Training Set 2: Jan-01-2011 to Dec-31-2014)

	1	2	3	4
Percentile	deciles	deciles	vigintiles	vigintiles
Number of Samples	6 months	9 months	6 months	9 months
Average Return	0.19%	-2.42%	0.55%	-1.3%
Volatility	0.0881	0.0956	0.0964	0.1061
Sharpe Ratio	0.0747	-0.877	**0.1962**	-0.4237

Figure 3: Comparison of Cumulative Returns for 4 portfolios

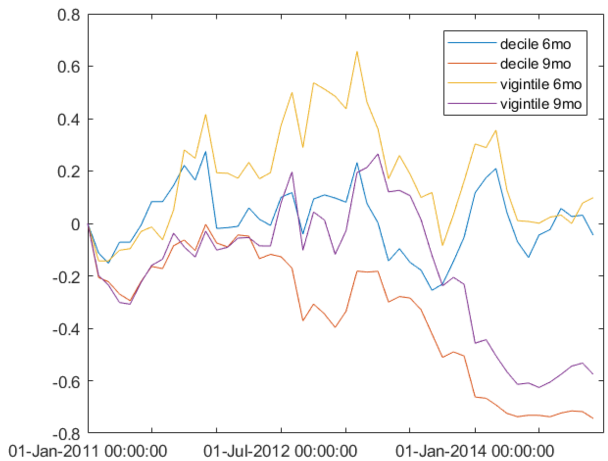
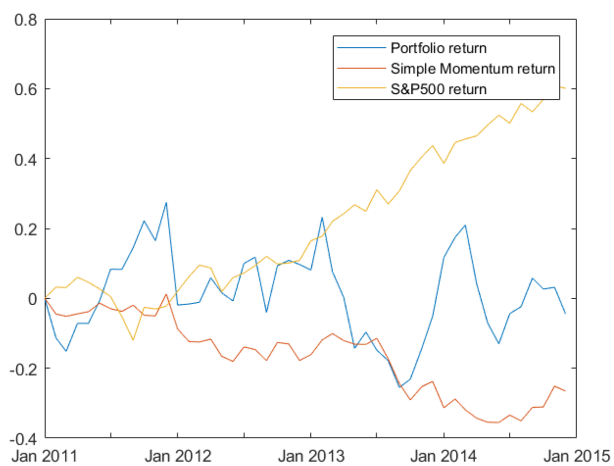


Figure 4: Cumulative Returns: Markovitz Optimization Vs. Equally weighted Momentum



From table II we can see that using vigintiles and 6 months look-back has the best risk to return profile while other portfolios struggles. However in comparison with the sp500 cumulative return we can see that momentum strategy is not very profitable.

3) Conclusion: From the testing of our portfolio during the 2 period we can see that the momentum profile has changed a lot over 10 years. Within the comparison of the model, using 6 months look-back is the optimal period (in comparison to 9 months). We decided not to use 3 months data since 3 months does not offer enough data point to form an educated estimation and longer period does not provide a more time specific information. On the other hand it also reviews how sensitive the results are to parameters, hence we must use caution when making changes and isolate independent variables.

In choosing whether to long-short top-bottom decile or vigintile, we decided to go with decile since it is the optimal option for training set 1 and the second best option from training set 2.

We will be using deciles and 6 month look-back as our parameter for momentum for all following back-tests. We will also be rebalancing monthly as it is the common approach.

C. Strategy Outline

For each of the four covariance matrix, we construct four optimized portfolios and test if they outperform the standard momentum portfolio. We use the mean-variance optimization method and build a zero net investment long-short portfolio.

We consider the impact of characteristic momentum phenomenon in two ways:

(1) We adjust the expected return to characteristic.

(2) We only invest in the stocks in the top and bottom deciles.

It is a trade-off problem to whether invest in the whole stock pool. If we reduce our investment universe to only the top and bottom decile stocks, we give up the benefit of diversification. However, allocations the intermediate decile stocks reduces the realized return of the strategy as less weight are assigned to the top and bottom deciles.

These two ways to take into account momentum's top and bottom only characteristic lead to four different portfolios in total:

- Full-Raw: We invest in all the stock pool and use the monthly sum realized returns f_{it} as expected returns.
- Full-101: We optimize over all stocks, but use the characteristic returns f^*_{it} as expected returns.
- Top/Bottom-Raw: We reduce the investment universe to only the stocks in the top and bottom decile of momentum's ranking and use these stocks' simple recent returns $r_{i(t-2)}$ as expected returns.
- Top/Bottom-101: We only invest in the top and bottom decile stocks, and use the characteristic returns f^*_{it}, which is adjusting the expected returns to plus one for the top decile and minus one for the bottom decile.

  In addition, we show the results for three choices of for λ to indicate different levels of risk tolerance. When λ = 0.5, the investor tends to be risk-loving, who does not fully care about risk. When λ = 1, the investor is risk-neutral, holding a neutral attitude to risk. When λ = 3, the investor is risk-avoiding, rather worried about risk than caring about gains.

D. Training Set Jan-01-2000 to Dec-31-2004

1) Sample Covariance Matrix: Since we use monthly data, for 6 months lookback we will not be able to conduct optimization using sample covariance matrix since it is out of rank, hence we do the top/bottom only for this matrix.

Table III: Optimization of the momentum strategy with different expected returns: Sample Covariance Matrix

Returns Covariances	Top/Bottom-Raw Sample	Top/Bottom-101 Sample
λ_1 = 0.5	0.3004	0.2809
λ_2 = 1	**0.3161**	0.2832
λ_3 = 3	0.3140	0.2746

From results we find that using sample mean we get higher performance than 101 approach, and that risk-neutral portfolio (same risk tolerance as the equally weighted momentum portfolio) has the highest return. We will turn to other models to compare performance.

2) Single Factor Model Covariance Matrix: Single factor covariance matrix has a better overall sharpe performance compared to simple covariance.

Table IV: Optimization of the momentum strategy with different expected returns: Single Factor Model

Returns Covariances	Full-Raw SIM	Full-101 SIM	Top/Bottom-Raw SIM	Top/Bottom-101 SIM
λ_1 = 0.5	0.0992	-0.2837	0.5041	-0.1590
λ_2 = 1	0.1055	-0.2821	**0.5148**	-0.1590
λ_3 = 3	0.1074	-0.2820	0.5035	-0.1590

We can see here that when we use the entire universe of stock in our portfolio, performance dropped down significantly. Both the full and top/bottom 101 portfolio has negative return, meaning the 101 mean may not be suitable for the portfolio optimization.

3) Fama French 3 Factor Covariance:

Table V: Optimization of the momentum strategy with different expected returns: Fama French 3 Factor Covariance

Returns Covariances	Full-Raw FF	Full-101 FF	Top/Bottom-Raw FF	Top/Bottom-101 FF
λ_1 = 0.5	0.0989	-0.0668	0.5073	-0.0946
λ_2 = 1	0.1033	-0.0654	0.5119	-0.0944
λ_3 = 3	0.1157	-0.0655	**0.5157**	-0.0943

Overall, 3 factor model is very similar to the single factor model.

4) Factor Shrinkage Covariance:

Table VI: Optimization of the momentum strategy with different expected returns: Factor Shrinkage Covariance

Returns Covariances	Full-Raw Shrinkage	Full-101 Shrinkage	Top/Bottom-Raw Shrinkage	Top/Bottom-101 Shrinkage
λ_1 = 0.5	0.0991	-0.3561	0.5016	-0.3561
λ_2 = 1	0.0997	-0.3582	0.5042	-0.3507
λ_3 = 3	0.1008	-0.0195	**0.5232**	-0.3507

5) Conclusion: During the time period of 2001 to 2004, the best models for covariance matrices are the factor models, with a sharpe ratio of 0.5, where the simple covariance matrix model have only 0.3.

The momentum portfolios with all stocks in the universe and specific momentum signal as input for markovitz all have unfavorable results, meaning it is not the best practice during this time frame.

It seems that the 3 factor covariance matrix model has the best overall result (higher sharpe for the 101 method). And we will recommend 3 factor covariance model and simple mean for the training set base on our test results.

E. Training Set Jan-01-2010 to Dec-31-2014

With the above test result, we can deduce that using full list of stocks in the portfolio will not be beneficial to the performance and the 101 method also yields unsatisfactory results. Hence for the following training part, we will be using Top/Bottom sample mean method only, and we will be comparing across different covariance matrix in this section.

1) Results: See table VII

Table VII: Optimization of the momentum strategy with different Covariance Matrices

Returns Covariances	Top/Bottom-Raw Sample	Top/Bottom-Raw SIM	Top/Bottom-Raw FF	Top/Bottom-Raw Shrinkage
λ_1 = 0.5	-0.1459	-0.1860	-0.1830	-0.1972
λ_2 = 1	-0.0782	-0.1957	-0.1933	-0.1835
λ_3 = 3	**-0.0545**	-0.2012	-0.1919	-0.1951

2) Conclusion: All results here yield negative sharpe's ratio, meaning performance is not optimal, our version of momentum strategy is not optimal for this time period. However for our purpose we will only determining the best covariance matrix to use for optimization.

We can see that sample covariance matrix is the best use here, while the factor models have even worse performance. In the test set testing section, we will be further examining the properties of these matrices. And we choose to use sample covariance matrix for this period of time.

VI. Testing And Validation

Determined Parameters

- zero investment long-short momentum portfolio
- Decile (top & bottom 10%) and 6 months look-back
- Top/bottom only
- Simple return as expected return
- Factor model covariance for the first test period, sample covariance for the second test period. More evidence required to determine the better option.

A. Test Set: Jan-01-2005 to Dec-31-2007

Table VIII: Optimization of the momentum strategy with different Covariance Matrices

Returns Covariances	Top/Bottom-Raw Sample	Top/Bottom-Raw SIM	Top/Bottom-Raw FF	Top/Bottom-Raw Shrinkage
λ_1 = 0.5	**0.8551**	0.7492	0.7608	0.7426
λ_2 = 1	0.6683	0.7586	0.7732	0.7489
λ_3 = 3	0.4518	0.8168	**0.8366**	0.8047

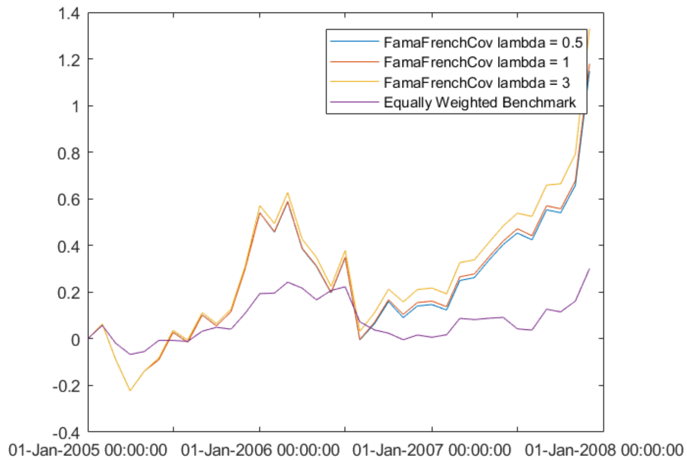
B. Test Set: Jan-01-2015 to Dec-31-2017

Table IX: Optimization of the momentum strategy with different Covariance Matrices

Returns Covariances	Top/Bottom-Raw Sample	Top/Bottom-Raw SIM	Top/Bottom-Raw FF	Top/Bottom-Raw Shrinkage
λ_1 = 0.5	0.2164	0.1516	0.1570	0.1443
λ_2 = 1	0.2710	0.1610	0.1603	0.1482
λ_3 = 3	-0.0329	0.1638	**0.1727**	0.1699

C. Observation

Figure 5: Cumulative Returns for Top/Bottom-Raw + Fama-French portfolio (year 2005-2007)



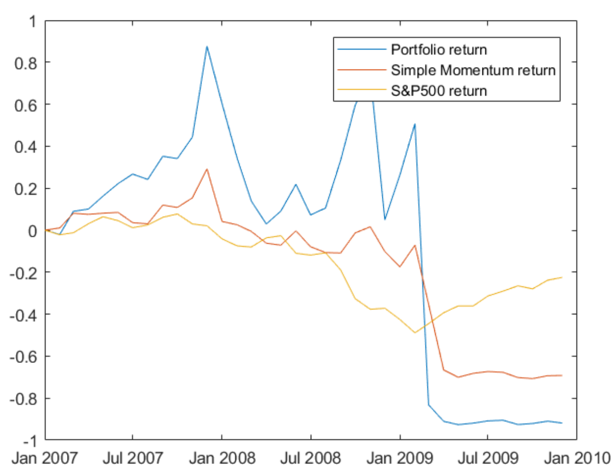
For the first test set data, best outcome comes from sample covariance, however on average Fama French covariance matrix has the best performance, which is in accordance with our training results above.

For the second test set data, again, sample covariance matrix yields the highest result, and Fama French has the best performance.

As a result, we believe that using Fama French Covariance matrix is a good option for covariance matrix in portfolio optimization.

D. Crisis Analysis

Figure 6: Cumulative Returns on Year 2007-2009



We didn't measure the VaR mathematically, however, we test how our strategies would work during extremely bad time. Thw worst scenario in our sample period is the year 2008 when the financial crisis happened. From our test we find that simple momentum strategy underperformed the market when crushes happened, while our optimized portfolio performed even worse.

It is disappointing that our portfolio failed to avoid momentum crushes. Partly because we didn't set any stop loss signals in our strategies, and we would work on further to minimize the tail risk.

VII. Conclusion

We started this project trying to construct a practical momentum trading strategy. Traditional momentum strategies are simple and and easy to implement. In order to capture the momentum effect as much as possible, we tried 4 different portfolio strategies, and even in the absence of transaction cost, the very diversified portfolio with all stocks have unsatisfactory return performance in contrast to using only top and bottom stocks. We discovered that it is sufficient to use simple return and part of the equity to invest.

Other than momentum effect, we also try to optimize the strategy by using different covariance matrix. Our matrices are the sample covariance matrix and the factor model derived covariance matrices. Introducing factor model indeed made our test results better in terms of performance.

However, our test results with the optimized portfolio are more volatile compared to the equally weighted benchmark portfolio. Which means that our strategy could be further optimized using methods other than portfolio optimization, for example we can introduce maximum draw-down limit and a stop-loss parameter to liquidate when there's a sudden drop.

VIII. Future Work

Weight Constraint

In our study, we applied 15% as weight constraint on individual stock, to some extent avoid error maximization caused by individual stocks. However, a variety choice of weight constraints should be tested to find the optimal one for our portfolio.

Transaction Cost

In our project, we didn't consider transaction cost, so our return is higher than it actually would be. With transaction cost, our strategies would be more complicated but it will provide a more realistic evaluation of performance.

Explore More Robust Strategies

In order to avoid crushes when the market goes through crisis, we can incorporate a stop-loss mechanism and liquidate the portfolio when losses are significant. We can also put more restrictions on the risk tolerance setup.

Make Use of Correlation

We could consider the correlation of stocks within the same industry and construct diversified portfolio using companies with distinctly different profile, which will be convenient to incorporate more features in the model.

IX. Reference

Jegadeesh, Narasimhan and Titman, Sheridan, Momentum (October 23, 2001). University Of Illinois Working Paper. pp 1-2.

Lo, Andrew and Craig MacKinlay, A. (1990). When Are Contrarian Profits Due To Stock Market Overreaction?. Review of Financial Studies.

DeBondt, W., Thaler, R., 1985. Does the stock market overreact? Journal of Finance 40, 793–805.

DeBondt, W., Thaler, R., 1987. Further evidence of overreaction and stock market seasonality. Journal of Finance 42, 557 – 581.

Jegadeesh, N., Titman, S., 1993. Returns to buying winners and selling losers: implications for stock market efficiency. Journal of Finance 48, 65–91.

Chan, L.K.C., Jegadeesh, N., Lakonishok, J., 1996. Momentum strategies. Journal of Finance 51, 1681–1713.

Chan, K., Hameed, A., Tong, W., 2000. Profitability of momentum strategies in the international equity markets. Journal of Financial and Quantitative Analysis 35, 153–172.

Rouwenhorst, K.G., 1998. International momentum strategies. Journal of Finance 53, 267–284.

Jegadeesh, N., Titman, S., 2001. Profitability of momentum strategies: an evaluation of alternative explanations. Journal of Finance 56, 699–720.

Lewellen, J., 2002. Momentum and autocorrelation in stock returns. Review of Financial Studies 15, 533–563.

Patro, D.K., Wu, Y., 2004. Predictability of short-horizon equity returns in international equity markets. Journal of Empirical Finance 11, 553–584.

Chopra, N., Lakonishok, J., Ritter, J.R., 1992. Measuring abnormal performance: do stocks overreact? Journal of Financial Economics 31, 235–268.

Richards, A.J., 1997. Winner–loser reversals in national stock market indices: can they be explained? Journal of Finance 52, 2129–2144.