A Novel Trilateration Algorithm for RSSI-based Indoor Localization

Bo Yang, Luyao Guo, Ruijie Guo, Miaomiao Zhao, and Tiantian Zhao,

***Abstract*—This paper proposed a novel trilateration algorithm for indoor localization based on received signal strength indication (RSSI). Firstly, all the raw measurement data are preprocessed by a Gaussian filter to reducing the influence of measurement noise. Secondly, the transmit power and the path loss exponent are estimated by a novel least-squares curve fitting (LSCF) method in the RSSI-based localization. Thirdly, a novel trilateration algorithm is proposed based on the extreme value theory, which constructs a nonlinear error function depending on distances and anchor nodes position. To minimize the function, a Taylor series approximation can be used for reduce the computational complexity. And, an iteration condition is designed to further improve the positioning accuracy. Afterward, Bayesian filtering is used to smoothing the localization error, and decrease the influence of the process noise. Both the simulation and experimental results demonstrate the effectiveness of the proposed methodology.**

***Index Terms*—Received signal strength indicator, trilateration, Bayesian filter, least-squares curve fitting.**

I. Introduction

In recent years, wireless sensor networks (WSNs) have been applied in many applications, such as surveillance, environmental monitoring, disaster management and so on [1]. For most cases, it is difficult to determine the location for all the sensor nodes which are often arbitrarily distributed. Meanwhile, the sensor data without where they were collected are meaningless. Therefore, localization is one of the key technologies in WSNs.

Localization has many applications particularly in an indoor environment where the global navigation satellite system like GPS, GLONASS, GALILEO, BDS, etc. has poor coverage or is unavailable, because the satellite signal is blocked by roofs and walls of buildings [2]. Localization algorithms for an indoor environment can be generally classified into two categories: range-free algorithms [3], [4] and range-based algorithms [5], [6]. In range-free algorithms usually use only proximity information among adjacent sensor nodes and require no special hardware. Recently, applying machine learning algorithms for range-free based localization has attracted the interest of researchers in WSNs [3], [4]. However, the better localization accuracy needs more computational resources and processing time. In range-based algorithms, a sensor node can obtain physical measurements (e.g., distance or angular information) between itself and adjacent sensor nodes, and calculates its position according to the obtained physical measurements. Typical range-based algorithms for localization include time of arrival (TOA) [7], time difference of arrival (TDOA) [8], direction of arrival (DOA) [9] and received signal strength indicator (RSSI) [10]. There are several kinds of heuristic algorithms can be found in [5], [11]. The TOA, TDOA, DOA and heuristic algorithms are more accurate than the RSSI method, but they are more complex and not always practical which may resulting in lager amount of energy consumption in resource restrained networks. RSSI-based localization techniques are an excellent choice for low-power and low complexity of signal processing in a WSNs. These techniques have been widely investigated and received vast on-going interest [12].

In RSSI-based localization, the log-distance path loss model is the most popular channel model due to its simplicity. In the model, the RSSI is related with the transmit power and the path loss exponent. It has been shown that the location accuracy is highly dependent on the knowledge of the path loss exponent. In the literature of RSSI-based localization algorithms, it is usually assumed that the path loss exponent is accurately known a priori. However, such an assumption is an oversimplification for many practical scenarios [13]. The work in [14] presents the joint estimation of a parameter of radio propagation path-loss model and the position of the unknown target node. However, the iteration algorithm is needed, and the initial position estimate of the target node may not confirm convergence. For the critical radio propagation parameters of the transmit power and the path loss exponent, [15] uses maximum likelihood (ML) algorithm, [16] proposes linear least square (LLS) algorithm to estimate them. However, they are hard to implement in practice because the cost function of the above mentioned algorithms is highly nonlinear, and the function contains multiple local minimal and maximal values. In this paper, a novel algorithm of least squares curve fitting (LSCF) is proposed to estimate the critical parameters. It will obtain an only feasible solution.

In RSSI-based localization, least square trilateration algorithm is one of the simplest and most widely used method [17], [18]. However, the algorithm heavily depends on the ranging accuracy. In order to reduce the inconsistencies in the

Manuscript received XXX 00, 0000; revised XXX 00, 0000. This work was supported by National Natural Science Foundation of China (Grant Nos. U1610116, 61403240). Project 201810108011 supported by Nation/Provincial Training Program of Innovation and Entrepreneurship for Undergraduates. Key Research and Development Program of Shanxi (Grant No. 201903D121145).

The authors are with the School of Mathematical Sciences, Shanxi University, Taiyuan, Shanxi, 030006, China. e-mail: (yangbo@sxu.edu.cn; luyaoo.guo@gmail.com; 15525079541g@gmail.com; uristarlightrain@gmail.com; z15534079809@163.com).

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSEN.2020.2980966, IEEE Sensors Journal

RSSI-based method owing to calibrations, [19] proposed two weighted least square techniques to improving the robustness and accuracy of the position of a node. Due to the accuracy loss caused by linearization with the method of the linear least-square, [20] proposed a node positioning method based on nonlinear weighting least square to address the problem of positioning accuracy loss in the traditional least square linear equation. But the accuracy of the position depends of the number of anchor nodes. The increase in positioning accuracy through increase the of anchor nodes, which obviously growing the communication load. [21] proposed a nonlinear iterative least squares for wifi positioning that autonomously estimates access points location and propagation parameters of the weighted path loss model and that generates database by autonomous crowdsourcing. [22] propose a bilateral greed iteration localization method based on greedy algorithm. However, [21] and [22] need complex calculations. In order to obtain a better positioning accuracy, decrease the communication load, and reduce the complex of calculation. A novel trilateration algorithm is proposed to get the better positioning result.

In RSSI-based localization, the localization errors will become larger once facing the real localization environment with unknown or time-varying noise statistics. Bayesian filter techniques provide a powerful statistical tool to help manage measurement noise in localization [23], [24]. [25] utilized a Bayesian belief network to derive a posterior probability distribution over the target's location. [26] proposed a complete survey of Bayesian filtering techniques and their application in localization. [27] based on fuzzy logic and hierarchical voting, which aims to realize the positioning in mixed environments. In order to further improve the accuracy of RSSI-based localization, a Bayesian filtering is proposed under a mixed environment.

In this paper, a novel trilateration algorithm is proposed for RSSI-based indoor localization. Firstly, a preprocessing data method using Gaussian filter is considered to deal with the raw RSSI measurements. Secondly, a LSCF algorithm is designed to estimate the critical radio propagation parameters of transmit power and path loss exponent through the filtered data. Thirdly, a novel trilateration algorithm is proposed. To minimize the nonlinear position error function, a Taylor series approximation can be used for reduce the computational complexity. And, a iteration condition is designed to improve the positioning accuracy. Finally, localization accuracy is further improved by a Bayesian filtering. The performance of the proposed algorithm is evaluated by simulations and a experiment.

The remainder of this paper is organized as follows: In section II, the log-normal shadowing model and the data preprocessing by Gaussian filtering are given. Then, a LSCF algorithm is described to estimate the transmit power and the path loss exponent. Afterward, a novel trilateration localization algorithm is proposed. For improving the accuracy of localization, a Bayesian filtering is designed. In Section III, simulation and experiment results are presented to demonstrate the effectiveness of the proposed method. The conclusions are draw in section IV.

II. Novel trilateration localization algorithm and Bayesian filtering

A. Log-normal shadowing model

For RSSI-based indoor localization , it is prudent to describe the channel model used for generating RSSI values. Therefore, a well-known channel pathloss model, called the log-normal shadowing model (LNSM) as follows [28]

$$PL(d) = \overline{PL}(d_0) + 10n\lg(\frac{d}{d_0}) + X_\sigma \tag{1}$$

where $PL(d)$ is the path loss at distance d. $\overline{PL}(d_0)$ is the path loss at the reference distance d_0. The parameter n is the path loss exponent, which is affected by the surrounding environment. X_σ denotes the measurement noise which assumed a Gaussian random variable with a mean value of zero and a variance of σ.

The relationship between the RSSI and the path loss $PL(d)$ is shown below

$$RSSI = P_t - PL(d) \tag{2}$$

where, P_t indicates the received power. The following expressions can be obtained by (1) and (2) as

$$RSSI = A - 10n\lg d \tag{3}$$

where, $A = P_t - \overline{PL}(d_0) - X_\sigma$, $d_0 = 1$. The accurate relationship between a distance and the RSSI signal is obtained. The distance between a receiver node and a transmitter node can be calculated according to the sampling RSSI value as follows:

$$d = d_0 \cdot 10^{\frac{A-RSSI}{10n}} \tag{4}$$

It is well known that the RSSI measurements are sensitive to environmental influences. Based the model (4), the distance between an target node to be localized and an anchor node will be obtained based on the corresponding RSSI measurements. In order to reduce the measurement noise, a method of raw RSSI measurements data preprocessing will consider as following.

B. Data preprocessing

The RSSI measurements may differ greatly because of the influence of various environmental factors. The RSSI measurements can be assumed as Gaussian distribution with an average value of μ and a variance of σ_d. Let Y denotes the measured RSSI. The density function of Y can be obtained as follows:

$$P(Y) = \frac{1}{\sigma_d\sqrt{2\pi}} e^{-\frac{(Y-\mu)^2}{2\sigma_d^2}} \tag{5}$$

where

$$\begin{aligned} \sigma_d &= \sqrt{\tfrac{1}{m-1}\textstyle\sum_{i=1}^{m}(Y_i-\mu)^2} \\ \mu &= \tfrac{1}{m}\textstyle\sum_{i=1}^{m} Y_i \end{aligned} \tag{6}$$

m is the number of collected RSSI measurements. Y_i is the i-th RSSI measurement value. For the purpose of improve the accuracy of localization and decrease the influence of measurements noise, the posterior probability of RSSI distribution between $(\mu - \sigma_d \le Y_i \le \mu + \sigma_d)$ is 0.6826 from the characteristic of Gaussian distribution function. The geometric average value of RSSI is obtained by selecting the RSSI in this range, and then selecting the average value as the RSSI measurement value of the target node received from an anchor node.

Based on the model (3) and the preprocessing data of RSSI value, the distance of the target node to an anchor node will obtain. The two parameters of n and A depend on the environment. Therefore, it is necessary to determine the two parameters in the model (3) under a certain circumstance. The LSCF algorithm is proposed to determine the two parameters.

C. Parameter estimation

Generally, the parameters of A and n are known from empirical models, usually assume as a fixed value. In order to take into account environmental factors, the LSCF algorithm is proposed to determine the parameters.

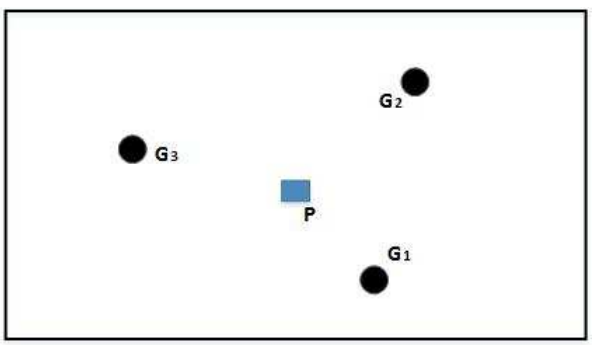

Fig. 1. The picture of nodes

As shown in Fig.1, a reference point P set in an experimental area, $G_i(i{=}1,2,3)$ are the anchor nodes. d_i is the distance between the anchor node G_i to the point P. The RSSI measurements of the reference point to the anchor node G_i is $RSSI_i$, and the raw data is preprocessing by Gaussian filtering. In the model (3), let $X_i{=}-10lg(d_i)$, $Y_i{=}RSSI_i$. So, $Z = (X_1,\ X_2,\ X_3)$. $Y(Z)$ is a discrete function defined on X_1, X_2, X_3, where $Y(Z) = (Y(X_1), Y(X_2), Y(X_3))$.

Assume a function of $F^*(Z) = A + nZ$, and a function of $F(Z) \in \phi$ that satisfied

$$\sum_{j=1}^{3}[Y(X_j) - F^*(X_j)]^2 = \min_{F\in\phi}\sum_{j=1}^{3}[Y(X_j) - F(X_j)]^2 \quad (7)$$

where $\phi = span\{\psi_0(Z), \psi_1(Z)\}$. Without loss of generality, we assume $\psi_0(Z) = (1, 1, 1)$, $\psi_1(Z) = Z$ is a basis vector in space of ϕ. Define a inner product $(f(Z), g(Z)) = \sum_{j=1}^{3} f_i(X_j)g(X_j)$. And $\|\cdot\|_2$ is L_2-norm. Therefore, the above problems can be translated into getting a $F(Z) \in \phi$ which satisfies

$$\|F^*(Z) - Y(Z)\|_2^2 = \min_{F\in\phi}\|F(Z) - Y(Z)\|_2^2 \quad (8)$$

Due the function $F(Z)$ is a vector in a space of ϕ. The function of $Y(Z)$ is a point out of the space of ϕ. The minimum value is to find a point in space ϕ which the distance between the point of $F(Z)$ and $Y(Z)$ is the shortest. So,

$$(F(Z) - Y(Z)) \perp \phi \quad (9)$$

We can obtain,

$$(F(Z) - Y(Z), \boldsymbol{\psi_i}) \;=\; 0, \quad i = 0, 1.$$

where, $\boldsymbol{\psi_i} = [\psi_i(X_1), \psi_i(X_2), \psi_i(X_3)]$. Due the function of $F(Z)$ is defined in the space of ϕ. It could be represented by a basis vectors of $\boldsymbol{\psi_0}$ and $\boldsymbol{\psi_1}$ in the space of ϕ. Assume, there are two parameters of α_1 and α_2. Satisfied

$$F(Z) = \alpha_1\boldsymbol{\psi_0} + \alpha_2\boldsymbol{\psi_1} \quad (10)$$

So,

$$\begin{cases} (\alpha_1\boldsymbol{\psi_0} + \alpha_2\boldsymbol{\psi_1} - \boldsymbol{Y}, \boldsymbol{\psi_0}) = 0 \\ (\alpha_1\boldsymbol{\psi_0} + \alpha_2\boldsymbol{\psi_1} - \boldsymbol{Y}, \boldsymbol{\psi_1}) = 0 \end{cases} \quad (11)$$

obtain

$$\begin{cases} \alpha_1(\boldsymbol{\psi_0}, \boldsymbol{\psi_0}) + \alpha_2(\boldsymbol{\psi_1}, \boldsymbol{\psi_0}) = (\boldsymbol{Y}, \boldsymbol{\psi_0}) \\ \alpha_1(\boldsymbol{\psi_0}, \boldsymbol{\psi_1}) + \alpha_2(\boldsymbol{\psi_1}, \boldsymbol{\psi_1}) = (\boldsymbol{Y}, \boldsymbol{\psi_1}) \end{cases} \quad (12)$$

So,

$$\begin{bmatrix} (\boldsymbol{\psi_0}, \boldsymbol{\psi_0}) & (\boldsymbol{\psi_0}, \boldsymbol{\psi_1}) \\ (\boldsymbol{\psi_1}, \boldsymbol{\psi_0}) & (\boldsymbol{\psi_1}, \boldsymbol{\psi_1}) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} (\boldsymbol{Y}, \boldsymbol{\psi_0}) \\ (\boldsymbol{Y}, \boldsymbol{\psi_1}) \end{bmatrix} \quad (13)$$

where

$$\begin{cases} \boldsymbol{\psi_i} &= [\psi_i(X_1), \psi_i(X_2), \psi_i(X_3)], \quad i = 0, 1 \\ (\boldsymbol{\psi_i}, \boldsymbol{\psi_k}) &= \Sigma_{j=1}^{3}\psi_i(X_j)\psi_k(X_j), \quad i = 0, 1, \quad k = 0, 1 \\ (\boldsymbol{Y}, \boldsymbol{\psi_i}) &= \Sigma_{j=1}^{3} Y(X_j)\psi_i(X_j), \quad i = 0, 1, \end{cases} \quad (14)$$

through the linear equations (13), α_1, α_2 can be solved uniquely. Let $A = \alpha_1$ and $n = \alpha_2$, the parameters of A and n can be obtained by the LSCF algorithm.

Remark 1: The parameters of A and n can be calculated according to the LSCF algorithm. Then, a distance between a target node and an anchor node can be calculated through the RSSI ranging model (3). Afterward, target's coordinates are received roughly by using traditional trilateration algorithm based on the calculated distance. Traditional trilateration algorithm cannot search a feasible solution when the chosen three anchor points are collinear, or the chosen three anchor points are non-collinear but three circles don't intersect at a single point or don't intersect at all which due to the estimation errors of the distances. So, a novel trilateration algorithm is given to consider the above mentioned problems.

D. A novel trilateration algorithm

The coordinates of three anchor nodes are (a_1, b_1), (a_2, b_2), (a_3, b_3). (a_0, a_0) are the coordinates of a target node, as shown in (Fig.2). d_1, d_2, d_3 represents the distance between (a_0, b_0) and (a_1, b_1), (a_2, b_2), (a_3, b_3), respectively. With d_1, d_2, d_3 as

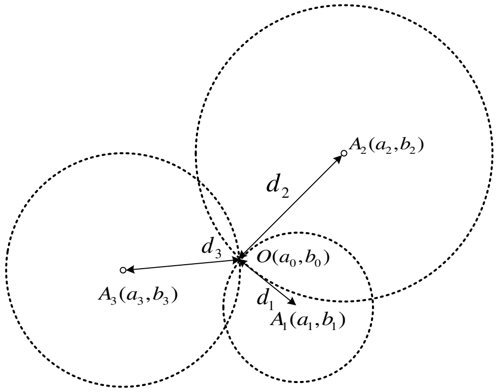

Fig. 2. Trilateration positioning schematic diagram

the radius, according to Pythagoras theorem, the target point position equation as:

$$\begin{cases} (a_1-a_0)^2+(b_1-b_0)^2=d_1^2 \\ (a_2-a_0)^2+(b_2-b_0)^2=d_2^2 \\ (a_3-a_0)^2+(b_3-b_0)^2=d_3^2 \end{cases} \tag{15}$$

$$\begin{cases} a_0=\dfrac{\begin{vmatrix} d_1^2-d_2^2-(a_1^2-a_2^2)-(b_1^2-b_2^2) & 2(b_2-b_1) \\ d_1^2-d_3^2-(a_1^2-a_3^2)-(b_1^2-b_3^2) & 2(b_3-b_1) \end{vmatrix}}{\begin{vmatrix} 2(a_2-a_1) & 2(b_2-b_1) \\ 2(a_3-a_1) & 2(b_3-b_1) \end{vmatrix}} \\ b_0=\dfrac{\begin{vmatrix} 2(a_2-a_1) & d_1^2-d_2^2-(a_1^2-a_2^2)-(b_1^2-b_2^2) \\ 2(a_3-a_1) & d_1^2-d_3^2-(a_1^2-a_3^2)-(b_1^2-b_3^2) \end{vmatrix}}{\begin{vmatrix} 2(a_2-a_1) & 2(b_2-b_1) \\ 2(a_3-a_1) & 2(b_3-b_1) \end{vmatrix}} \end{cases} \tag{16}$$

The position of the target node (a_0, b_0) can be obtained. The drawback of the traditional trilateration method is the denominator cannot be zero in (16). Based the traditional trilateration method, we have proposed a novel trilateration method.

We construct the following error function, so that the minimum value of the error function is the coordinates of the target node. The error function is

$$f(x,y)=\sum_{i=1}^{3}\left[\sqrt{(x-a_i)^2+(y-b_i)^2}-d_i\right]^2 \tag{17}$$

where (x, y) are the coordinates of the unknown nodes, a_i, b_i and d_i as same as the above mentioned in (15). Let $g_i(x, y) = \sqrt{(x-a_i)^2+(y-b_i)^2}$. Then $g_i(x, y)$ is the real distance from the target node to the anchor node A_i.

Since the function (17) is a nonlinear function, in order to find the extremum of the function, we approximate its nonlinear part $g_i(x, y)$ at the approximate coordinates of the target node (x', y') of the first-order Taylor expansion.

The partial derivative of $g_i(x, y)$ with respect to x and y are

$$S_i=\frac{\partial g_i(x,y)}{\partial x}\Big|_{(x,y)=(x',y')}=\frac{x'-a_i}{\sqrt{(x'-a_i)^2+(y'-b_i)^2}}, \tag{18}$$

$$T_i=\frac{\partial g_i(x,y)}{\partial y}\Big|_{(x,y)=(x',y')}=\frac{y'-b_i}{\sqrt{(x'-a_i)^2+(y'-b_i)^2}}. \tag{19}$$

So $g_i(x, y)$ can be expressed as

$$g_i(x,y)\approx g_i(x',y')+S_i(x-x')+T_i(y-y') \tag{20}$$

By substituting (20) into (17) to get the following expression

$$f(x,y)\approx\sum_{i=1}^{3}\left[g_i(x',y')+S_i(x-x')+T_i(y-y')-d_i\right]^2 \tag{21}$$

To get the minimum value of $f(x, y)$, satisfy $\frac{\partial f(x,y)}{\partial x} = 0$ and $\frac{\partial f(x,y)}{\partial y} = 0$. So the partial derivative of $f(x, y)$ with respect to x and y are

$$\frac{\partial f}{\partial x}=2\sum_{i=1}^{3}\left[g_i(x',y')+S_i(x-x')+T_i(y-y')-d_i\right]S_i \tag{22}$$

$$\frac{\partial f}{\partial y}=2\sum_{i=1}^{3}\left[g_i(x',y')+S_i(x-x')+T_i(y-y')-d_i\right]T_i \tag{23}$$

The following linear equations can be obtained from extreme conditions.

$$\begin{pmatrix} S & U \\ U & T \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} \Sigma_{i=1}^{3}S_i(d_i-g_i)+Sx'+Uy' \\ \Sigma_{i=1}^{3}T_i(d_i-g_i)+Ux'+Ty' \end{pmatrix} \tag{24}$$

So, the position of the target node as follows

$$\begin{cases} x=\frac{T\prod_1-U\prod_2}{ST-U^2} \\ y=\frac{S\prod_2-U\prod_1}{ST-U^2} \end{cases} \tag{25}$$

where, $S = \sum_{i=1}^{3} S_i^2$, $T = \sum_{i=1}^{3} T_i^2$, $U = \sum_{i=1}^{3} S_iT_i$, $\prod_1 = S(d_i-g_i)+Sx'+Uy'$, $\prod_2 = T(d_i-g_i)+Ux'+Ty'$. Expressing the (25) as

$$\begin{cases} x=\alpha(x',y') \\ y=\beta(x',y') \end{cases} \tag{26}$$

In general, due the high-order terms are ignored in Taylor expansion, so the error is unavoidable. In order to further improve the positioning accuracy, it can be iteratively performed by the following formula.

$$\begin{cases} x_i=\alpha(x_{i-1},y_{i-1}) \\ y_i=\beta(x_{i-1},y_{i-1}) \end{cases} \tag{27}$$

Set the iteration termination condition $|x_i-x_{i-1}|+|y_i-y_{i-1}|<\xi$ and the maximum number of iterations to obtain the target position.

Remark 2: This step will avoid the singular problem of $ST-U^2$. On the one hand, we can change the point to $(x', y'+delta)$, where $0 < |delta| < 1$, when the randomly selected Taylor expansion point (x', y') happens to be collinear. On the other hand, if the chosen three anchor points are not collinear, the Taylor expansion point will choose a random value.

Remark 3: Due the mirror image, the solution space of the position is symmetrical. To deal with this problem, a limited area is setting to avoid the non-uniqueness of the potential function solution. In fact, the position of the anchor nodes in an real indoor environment are in alignment, not only for attractive but also minimizing the total cost under the condition of ensure the complete coverage for the network. Therefore, the solution space is predetermined by artificial decision in a real environment.

A terminal's coordinates are received roughly by using the novel trilateration algorithm. After obtaining the initial estimated position of the target node, the result is further refined by Bayesian filtering. It will further improve the positioning accuracy.

E. Bayesian filtering

The core idea of Bayesian filtering is to obtain a posterior probability density function for the complete description of state estimation of nonlinear systems based on the obtained measurement information.

Let $\boldsymbol{Y}^k$=$\{Y_1, Y_2, \cdots, Y_k\}$ represents the set of different RSSI measurements at the obtained k times. Y_k indicates the measured RSSI at the time of k. Z_k represents the possible location of the target node at the time of k. According to Bayesian formula, posterior probability can be obtained as the following equation [26]:

$$P_i(Z_k|\boldsymbol{Y}^k) = \frac{P_i(Y_k|Z_k)P_i(Z_k|\boldsymbol{Y}^{k-1})}{P_i(Y_k|\boldsymbol{Y}^{k-1})} \quad i = 1, \cdots, N, \tag{28}$$

where $P_i(Z_k|\boldsymbol{Y}^k)$ represents the posterior pdf. N is the number of anchor node. $P_i(Z_k|\boldsymbol{Y}^{k-1})$ is the prior pdf, indicating the prediction of the location of the target node when Y_k is not obtained. $P_i(Y_k|Z_k)$ is the likelihood function, representing the probability of the measured RSSI value Y_k when the location of the target node is at Z_k. Because $P_i(Y_k|\boldsymbol{Y}^{k-1})$ is an invariant relative to the pdf of the state vector of a target node, it can be regarded as a constant. Let $C_i = \frac{1}{P_i(Y_k|\boldsymbol{Y}^{k-1})}$.

Prediction step:

$$P_i(Z_k|\boldsymbol{Y}^{k-1}) = \int P_i(Z_k|Z_{k-1})P_i(Z_{k-1}|\boldsymbol{Y}^{k-1})dZ_{k-1} \tag{29}$$

$P_i(Z_k|Z_{k-1})$ denotes prior density, is depend on the state vector at time $k-1$. $P_i(Z_{k-1}|\boldsymbol{Y}^{k-1})$ is the posterior pdf at time $k-1$.

Update step:

$$P_i(Z_k|\boldsymbol{Y}^k) = \frac{P_i(Y_k|Z_k)P_i(Z_k|\boldsymbol{Y}^{k-1})}{\int P_i(Y_k|Z_k)P_i(Z_k|\boldsymbol{Y}^{k-1})dZ_k} \tag{30}$$

Thus, only three probabilities need calculating as following: (1) The prior pdf $P_i(Z_k|\boldsymbol{Y}^{k-1})$ assume as:

$$P_i(Z_k|\boldsymbol{Y}^{k-1}) = \frac{1}{\sigma_{b1}\sqrt{2\pi}} exp\{-\frac{D_1^2}{2\sigma_{b1}^2}\} \tag{31}$$

where $D_1 = \sqrt{(x-x')^2 + (y-y')^2}$, Z_k=(x, y), $\sigma_{b1}^2 = \sum_{i=1}^{k-1}[(x_i - x')^2 + (y_i - y')^2](k-2)^{-1}$, $x' = \frac{\sum_{i=1}^{k-1} x_i}{k-1}$, $y' = \frac{\sum_{i=1}^{k-1} y_i}{k-1}$, $Z_{k-1} = (x_{k-1}, y_{k-1})$.

(2) The likelihood $P_i(Y_k|Z_k)$ assume as:

$$P_i(Y_k|Z_k) = \frac{1}{\sigma_{b2}\sqrt{2\pi}} exp\{-\frac{[g_i(x,y) - d_i]^2}{2\sigma_{b2}^2}\} \tag{32}$$

where $g_i(x, y) = \sqrt{(x-a_i)^2 + (y-b_i)^2}$. (a_i, b_i) represents the coordinates of an anchor node. d_i represents the distance from the target node to each receiver, σ_{b2} same as σ_d as shown in (6).

(3) In this paper, we assume that the information is exchanged independently between two nodes. Bayesian estimation can be calculated according to posterior pdf. According to (28)-(32), the pdf of the target node position from different anchor nodes can be obtained:

$$p(x, y) = \prod_{i=1}^{N} P_i(Z_k|\boldsymbol{Y}^k) \tag{33}$$

where

$$\begin{aligned} p(x,y) \quad &= \quad \left(C_i \frac{1}{\sigma_{b1}\sigma_{b2}(\sqrt{2\pi})^2}\right)^N \\ &\times exp\left\{\sum_{i=1}^{N}\left[-\frac{D_1^2}{2\sigma_{b1}^2} - \frac{(D_2 - d_i)^2}{2\sigma_{b2}^2}\right]\right\} \end{aligned}$$

$D_2 = g_i(x, y)$. The problem of obtain the coordinates of Z_k=(x^*, y^*) that satisfied $p(x^*, y^*)$=max $p(x, y)$ is a multivariate nonlinear extremum problems. There are many intelligent algorithms to solve this problem. And, it is difficult to obtain the posterior pdf in the real situation. These issues will be the future directions need to consider, not be discussed in detail in this paper.

III. Experiment and simulation analysis

In this section, the performance of the proposed novel trilateration algorithm is evaluated. The experiment was conducted on the hall of the first floor in the Mathematical Sciences Building on the campus of Shanxi University. The hardware devices are designed for the localization system are described as follows.

A. Introduction of Hardware part

In the system, two types of nodes are used, transmitter and receiver node. In the experiment, both of nodes which are using a wireless communication module CC2530. Two printed circuit board (PCB) modules have been designed. The structure of these nodes are shown in Fig. 3.

Power consumption is critical importance issue in WSNs. The total power consumption of nodes depends on the number of function implemented, transmission time and transmission rate. Because the transmitter nodes are only used to transmit data within a short period of time. So, a transmitter node is supplied with a battery-powered model. The receiver node is supplied with AC power mode due it needs to calculating the distance through the measurements and positioning the target by the proposed algorithm.

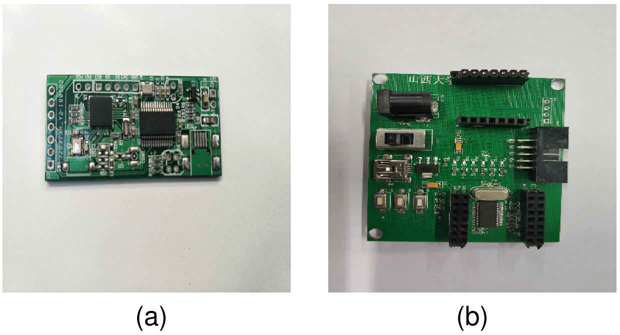

Fig. 3. (a) The PCB of a transmitter node. (b) The PCB of a receiver node.

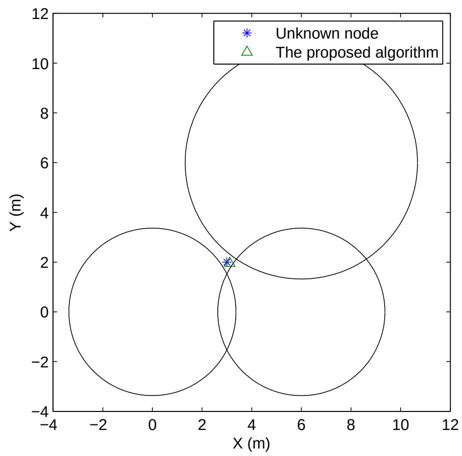

Fig. 4. The position of three circles not intersect at a single

B. Simulation 1

The purpose of the experiment is to verify the efficiency of the proposed novel trilateration method, when the three circles don't intersect at a single. Fig.4 shows that due to measurement error induce the three circles intersect but not each two circles in the indoor environment. Where the blue asterisk is the position of the unknown node. And, the green triangle is the positioned point by the proposed algorithm.

Fig.5 shows the situation of three anchor nodes are collinear. Where the blue asterisk denotes the position of the unknown node. The red triangle denotes the unknown node's position. The dotted line in the figure indicates that the obtained solution will not exist in this constrained area. Thought the Fig. 4 and Fig. 5, it can find that the proposed method could obtain the position of an unknown target although the anchor nodes are collinear or the three circles can't intersect at one point. In the next simulation, we will evaluate the accuracy and effectiveness of our proposed algorithm by making several comparisons with other related methods, such as least square, weighted nonlinear least square, weighted hyperbolic algorithm and weighted circular algorithm.

C. Simulation 2

The Root Mean Square Error (RMSE) be used as a criterion for analyzing the localization accuracy. ([29])

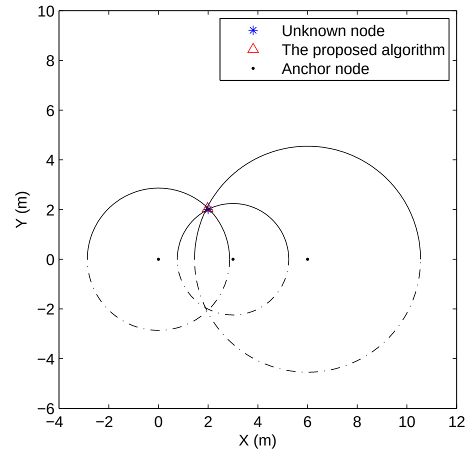

Fig. 5. The position of three anchors are collinear

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}[(\hat{x}_i - x_i)^2 + (\hat{y}_i - y_i)^2]}{N}} \quad (34)$$

where $(\hat{x}_i,\ \hat{y}_i)$ are the estimated coordinates of the node i. $(x_i,\ y_i)$ are the corresponding true coordinates. N is the total number of unknown nodes.

We randomly select 50 locations as the coordinates of the unknown nodes, and use the algorithm to locate them in turn. The positioning result is shown in Fig. 6. In Fig. 7, the real coordinates almost coincides with the theoretical coordinates, and the ranging error of each point is small, so the accuracy of this algorithm is improved remarkably. It can be seen that the error of the algorithm in this paper is controlled within 2 m through Fig 7.

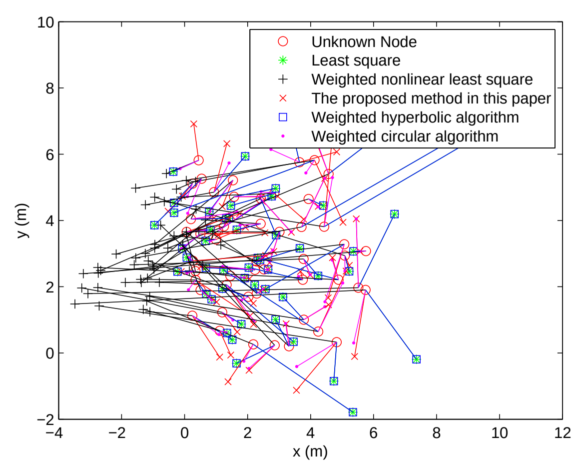

Fig. 6. The analytical diagram of the actual coordinates and the theoretical coordinates

Fig. 8 depicts the relationship between the average RMSE and the standard variance of measurement RSSI. The average RMSE of these algorithms increase as increase the standard variance of measurement RSSI.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSEN.2020.2980966, IEEE Sensors Journal

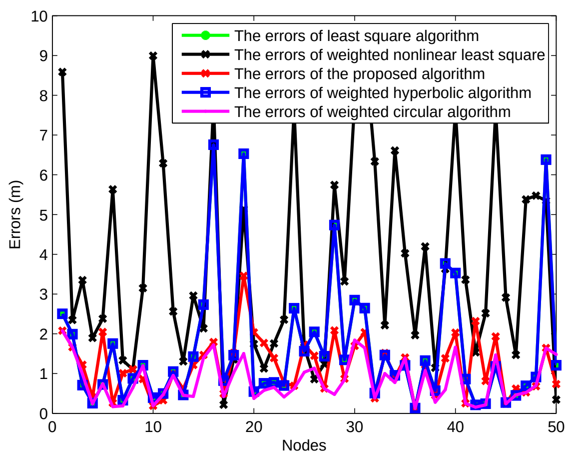

Fig. 7. Error analysis result diagram

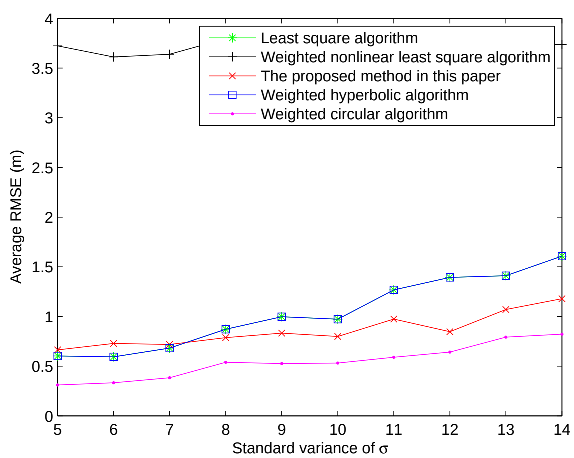

Fig. 8. Average RMSE versus the standard variance of measurement RSSI

The performance of the proposed algorithm is weaken than the weighted circular algorithm. But it is better than the other compared algorithms. We have also observed that the weighted circular technique has a very good performance and provides better localization results than the other technique, but it needs much higher computational costs due to used a gradient search method for minimizing the error functions. In real environment, the wireless sensor network nodes generally have scarce computational and networking resources. Due to the accuracy-cost trade off, the novel trilateration algorithm can be applied in most applications. The proposed algorithm presents significantly improved the localization accuracy compared with the other algorithms.

The running time of different algorithms as shown in Fig. 9. The average running time of the proposed algorithms is 1.1×10^{-4} s, which is lower than the weighted circular algorithm 1.1×10^{-3} s, high than the weighted hyperbolic algorithm 3.2×10^{-5} s, least square algorithm 2.5×10^{-5} s and weighted nonlinear least square algorithm 3.0×10^{-5} s. The average energy consumption of the proposed algorithm is lower than the weighted circular algorithm. It can indirectly reflect the energy consumption of the proposed algorithm compares the other algorithms.

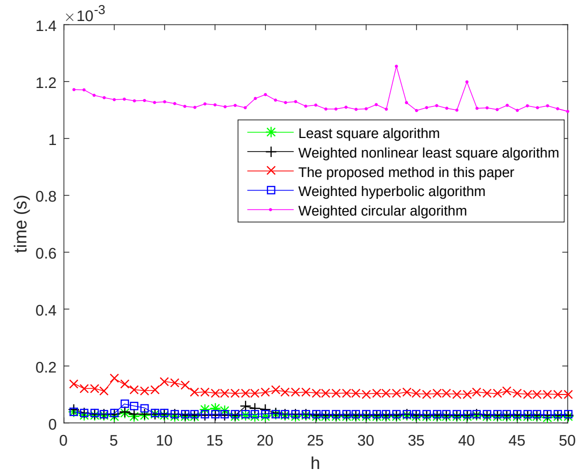

Fig. 9. Runing time of different algorithms

The empirical cumulative distribution functions (CDF's) corresponding to distance error of the three algorithms as shown in Fig. 10. At a probability of 80%, the localization error of the proposed algorithms is 1.696 m, which is high than the weighted circular algorithm (1.433 m), lower than the localization errors of weighted hyperbolic algorithm and least square algorithm (2.45 m) and weighted nonlinear least square algorithm (6.08 m). The localization accuracy of the proposed algorithm is better than those of the two other algorithm. In the next, the method will apply to a experiment situation.

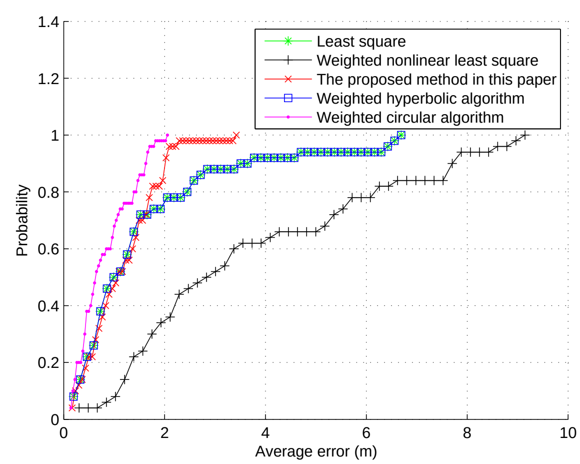

Fig. 10. Average RMSE versus CDF

D. Experiment

To validate the generality of the proposed method, a mixed experiment is designed as follows, the structure of the experiment as shown in 11. A, B, C, D denotes anchor nodes. E, F, G denotes unknown target responsible receive RSSI values, respectively. There is a obstruction in the experiment.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSEN.2020.2980966, IEEE Sensors Journal

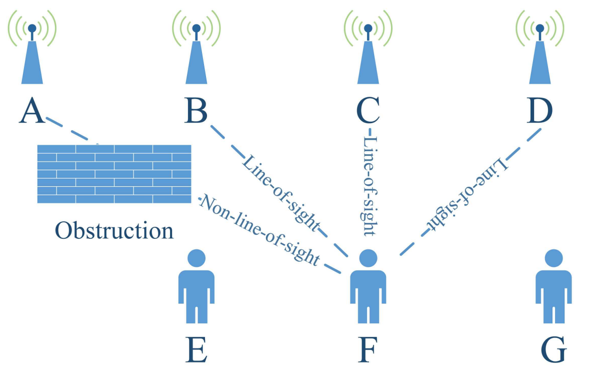

Fig. 11. The experiment structure in LOS and NLOS

The monitoring area covered by the experiment is 8 m $\times$ 8 m with the coordinates from (0,0) to (8,8). Anchor nodes are placed at A(1,5), B(3,5), C(5,5), D(7,5). The target node is placed at E(3,2), F(5,2) and G(7,2), respectively. The radio propagation parameters A and n of log-distance path loss model can be determined by the least squares curve fitting. In the indoor NLOS and LOS environment, the target node is placed at 0.5 m, 0.8 m, 1 m, and 1.5 m from the anchor node A(1,5) and C(5,5). Meanwhile, 30 RSSI measurements are obtained for each distance.

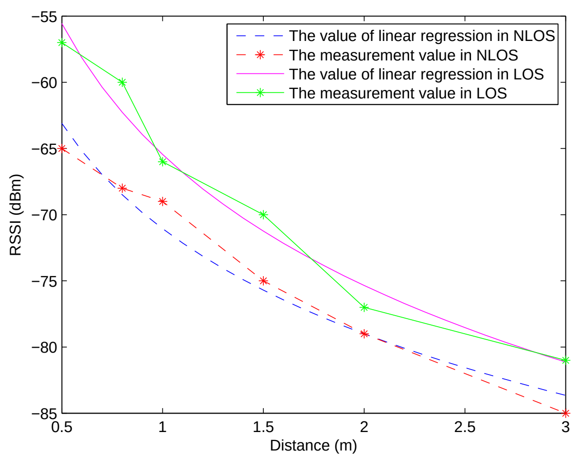

Fig. 12. The relation between RSSI and distance

Fig. 12 shows that both the linear regression value in (3) and the actual measured RSSI value, which decrease gradually with the increase of distance d, and it is found that the larger of the distance, the slower of the attenuation. In Fig. 12, it can easily find that the collected RSSI measurements more higher in LOS environment than in NLOS environment.

We have set up three locations as the coordinates of unknown nodes, and use different algorithms to locate them in turn. The positioning results are shown in Fig. 13. However, the least square and weighted hyperbolic algorithm cannot obtain a feasible solution due to the anchor nodes aligned in a line. The order of the location accuracy, nonlinear weighted least square is the worst, the proposed method is middle, the weighted circular algorithm is the best. Meanwhile, the

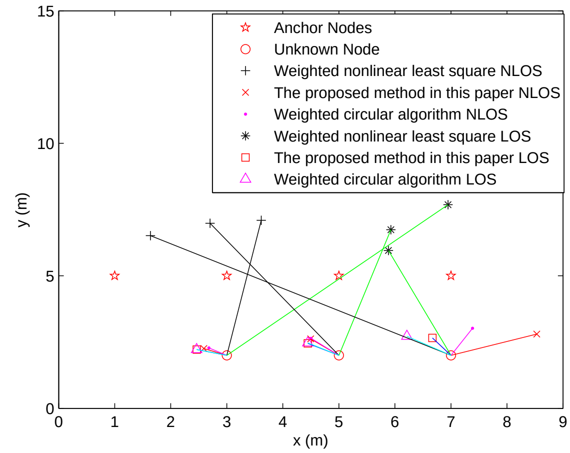

Fig. 13. The analytical of the actual coordinates and the theoretical coordinates in NLOS and LOS

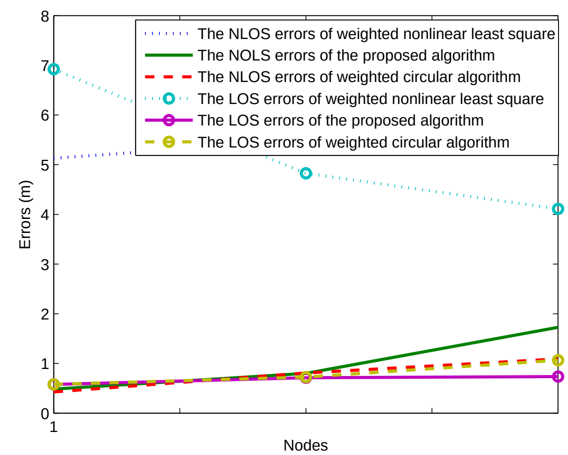

Fig. 14. Error analysis result diagram in NLOS and LOS

location performance is better in LOS than in NLOS environment when applying the same algorithm. Corresponding, the real coordinates almost coincides with the theoretical coordinates, and the ranging error of each point is smaller of the proposed algorithm in Fig. 14. So, the accuracy of the proposed algorithm is improved remarkably. It can be seen that the error of the algorithm is controlled within 1 m in Fig. 14.

IV. Conclusion

In this paper, we proposed a novel trilateration localization algorithm for RSSI-based indoor localization. A data preprocessing is designed by a Gaussian filter. The critical radio propagation parameters of the transmit power and the path loss exponent are obtained by a LSCF algorithm. A nonlinear error function is constructed by distances and anchor nodes positions. By solving the extremum of the error equation, a novel trilateration algorithm is proposed to avoid the nonuniqueness of a solution based on the on extreme value theory. To minimize the error function, a Taylor series approximation can be used to reduce the computational complexity. For the high-order terms are ignored in Taylor expansion, an iterative process is performed to further improve the positioning accuracy. The proposed algorithm can weaken the influence of collinear of anchor nodes, will improve the utilization of anchor nodes in the indoor environment. Finally, Bayesian filtering is used to improve the accuracy of the target position. Simulation and a real experiment results show that the effectiveness and the improvements of localization accuracy of the proposed method.

REFERENCES

[1] H. Liu, H. Darabi, P. Banerjee, and J. Liu, "Survey of wireless indoor positioning techniques and systems," *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)*, vol. 37, no. 6, pp. 1067–1080, Nov. 2007.

[2] V. Bianchi, P. Ciampolini, and I. D. Munari, "RSSI-based indoor localization and identification for ZigBee wireless sensor networks in smart homes," *IEEE Transactions on Instrumentation and Measurement*, vol. 68, no. 2, pp. 566–575, Feb. 2019.

[3] Z. E. Khatab, A. Hajihoseini, and S. A. Ghorashi, "A fingerprint method for indoor localization using autoencoder based deep extreme learning machine," *IEEE Sensors Letters*, vol. 2, no. 1, pp. 1–4, Mar. 2018.

[4] G. Bhatti, "Machine learning based localization in large-scale wireless sensor networks," *Sensors*, vol. 18, p. 4179, Nov. 2018.

[5] A. Abolfathi Momtaz, F. Behnia, R. Amiri, and F. Marvasti, "NLOS identification in range-based source localization: Statistical approach," *IEEE Sensors Journal*, vol. 18, no. 9, pp. 3745–3751, May. 2018.

[6] X. Liu, S. Su, F. Han, Y. Liu, and Z. Pan, "A range-based secure localization algorithm for wireless sensor networks," *IEEE Sensors Journal*, vol. 19, no. 2, pp. 785–796, Jan. 2019.

[7] I. Guvenc and C. Chong, "A survey on TOA based wireless localization and NLOS mitigation techniques," *IEEE Communications Surveys Tutorials*, vol. 11, no. 3, pp. 107–124, 3rd Quarter 2009.

[8] K. C. Ho, X. Lu, and L. Kovavisaruch, "Source localization using TDOA and FDOA measurements in the presence of receiver location errors: Analysis and solution," *IEEE Transactions on Signal Processing*, vol. 55, no. 2, pp. 684–696, Feb. 2007.

[9] S. Qin, Y. D. Zhang, and M. G. Amin, "Generalized coprime array configurations for direction-of-arrival estimation," *IEEE Transactions on Signal Processing*, vol. 63, no. 6, pp. 1377–1390, Mar. 2015.

[10] D. Konings, F. Alam, F. Noble, and E. M. Lai, "Device-free localization systems utilizing wireless RSSI: A comparative practical investigation," *IEEE Sensors Journal*, vol. 19, no. 7, pp. 2747–2757, Apr. 2019.

[11] M. Wu, N. Xiong, and L. Tan, "Adaptive range-based target localization using diffusion Gauss-Newton method in industrial environments," *IEEE Transactions on Industrial Informatics*, vol. 15, no. 11, pp. 5919–5930, Nov. 2019.

[12] R. Niu, A. Vempaty, and P. K. Varshney, "Received-signal-strength-based localization in wireless sensor networks," *Proceedings of the IEEE*, vol. 106, no. 7, pp. 1166–1182, Jul. 2018.

[13] W. Li and Y. Jia, "Distributed target tracking by time of arrival and received signal strength with unknown path loss exponent," *IET Signal Processing*, vol. 9, no. 9, pp. 681–686, Dec. 2015.

[14] X. Li, "RSS-based location estimation with unknown pathloss model," *IEEE Transactions on Wireless Communications*, vol. 5, no. 12, pp. 3626–3633, Dec. 2006.

[15] A. Coluccia and F. Ricciato, "On ML estimation for automatic RSS-based indoor localization," in *IEEE 5th International Symposium on Wireless Pervasive Computing 2010*, May. 2010, pp. 495–502.

[16] J. Yang and Y. Chen, "Indoor localization using improved RSS-based lateration methods," in *GLOBECOM 2009 - 2009 IEEE Global Telecommunications Conference*, Nov. 2009, pp. 1–6.

[17] P. Cotera, M. Velazquez, D. Cruz, L. Medina, and M. Bandala, "Indoor robot positioning using an enhanced trilateration algorithm," *International Journal of Advanced Robotic Systems*, vol. 13, no. 3, p. 110, Jan. 2016.

[18] S. Safavi, U. A. Khan, S. Kar, and J. M. F. Moura, "Distributed localization: A linear theory," *Proceedings of the IEEE*, vol. 106, no. 7, pp. 1204–1223, Jul. 2018.

[19] P. Tarrío, A. M. Bernardos, and J. R. Casar, "Weighted least squares techniques for improved received signal strength based localization," *Sensors*, vol. 11, no. 9, pp. 8569–8592, Sep. 2011.

[20] F. Xiao, M. Wu, H. Huang, R. Wang, and S. Wang, "Novel node localization algorithm based on nonlinear weighting least square for wireless sensor networks," *International Journal of Distributed Sensor Networks*, vol. 8, no. 11, p. 803840, Nov. 2012.

[21] Y. Zhuang, Z. Syed, J. Georgy, and N. El-Sheimy, "Autonomous smartphone-based WiFi positioning system by using access points localization and crowdsourcing," *Pervasive and Mobile Computing*, vol. 18, pp. 118 – 136, Apr. 2015.

[22] J. Yang, Y. Li, and W. Cheng, "An improved geometric algorithm for indoor localization," *International Journal of Distributed Sensor Networks*, vol. 14, no. 3, p. 1550147718767376, Mar. 2018.

[23] G. V. Zàruba, M. Huber, F. A. Kamangar, and I. Chlamtac, "Indoor location tracking using RSSI readings from a single Wi-Fi access point," *Wireless Networks*, vol. 13, no. 2, pp. 221–235, Apr. 2007.

[24] M. Uradzinski, H. Guo, X. Liu, and M. Yu, "Advanced indoor positioning using Zigbee wireless technology," *Wireless Personal Communications*, vol. 97, no. 4, pp. 6509–6518, Dec. 2017.

[25] T. Roos, P. Myllymäki, H. Tirri, P. Misikangas, and J. Sievänen, "A probabilistic approach to WLAN user location estimation," *International Journal of Wireless Information Networks*, vol. 9, no. 3, pp. 155–164, Jul. 2002.

[26] V. Fox, J. Hightower, L. Liao, D. Schulz, and G. Borriello, "Bayesian filtering for location estimation," *IEEE Pervasive Computing*, vol. 2, no. 3, pp. 24–33, Jul. 2003.

[27] L. Cheng, J. Hang, Y. Wang, and Y. Bi, "A fuzzy c-means and hierarchical voting based RSSI quantify localization method for wireless sensor network," *IEEE Access*, vol. 7, pp. 47 411–47 422, Apr. 2019.

[28] T. S. Rappaport, "Characterization of uhf multipath radio channels in factory buildings," *IEEE Transactions on Antennas and Propagation*, vol. 37, no. 8, pp. 1058–1069, Aug. 1989.

[29] Q. Luo, Y. Peng, J. Li, and X. Peng, "RSSI-based localization through uncertain data mapping for wireless sensor networks," *IEEE Sensors Journal*, vol. 16, no. 9, pp. 3155–3162, May. 2016.