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Residual Based Weighted Least Square Algorithm for Bluetooth/UWB Indoor Localization System

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Abstract: The recent growing interest for indoor Location-Based Services (LBSs) has created a need for more accurate and low-cost indoor localization system. The existing solutions are either low-cost with low-accuracy or high-accuracy with high-cost. A positioning solution for indoor localization system with Bluetooth and UWB is proposed in this paper. The system combines both advantages of Bluetooth and UWB, achieves higher precision than single Bluetooth system with relatively low cost. In order to get higher accuracy, we proposed a residual based weighted least square (RWLS) algorithm for the indoor localization system. The range error characteristics of Bluetooth and UWB are obtained by experiments. Simulation results of the system show that RWLS algorithm with Bluetooth/UWB indoor localization system can achieve a mean error of about 0.85 *m*.

Key Words: indoor localization, Bluetooth, UWB

1 Introduction

Indoor environment is the main area for people to live and work. The statistical data shows that the majority of people spend about 80%-90% of their time in indoor environment.[1] As the indoor environment becomes more and more complicated, indoor localization will offer great convenience for people's life, especially in complex indoor environment like shopping mall, parking lot and so on. Moreover, indoor localization can also offer enormous data to industry customers to provide better service for people.

Currently, the mainstream study about indoor environment contains inertia technology, WLAN technology, Bluetooth technology, ultra-wideband (UWB) technology and so on. Both of the Bluetooth and UWB attracted great attention in recent years. Bluetooth technology using in indoor localization is after the popularization of Bluetooth 4.0 version. As the 5.0 version which supports indoor positioning and navigation coming, the Bluetooth technology is more and more popular in indoor localization. The Topaz location system [2] is one of the results of these researches. Topaz can provide 2-D location information with an error range of around 2 *m*.

Bluetooth technology has the advantages like low cost, low power consumption and easy to dispose, but the precision of positioning is poor. UWB is a kind of Wireless Communication without carrier wave. Ultrasound signals are used by bats to navigate in the night, which inspire people to design a similar navigating system in the last hundreds of years [3]. Interests in UWB have greatly increased following the Federal Communications Commission's (FCC) decision to open up the bands from 3.1 to 10.6 and 22 to 29 GHz for UWB use in 2002 [4]. The Ubisense Company [2] provides a new real-time positioning system based on UWB technology. The Ubisense system results in a higher accuracy of about 15 *cm* in 3-D.

UWB signal has an operating range of up to 50–100 m and can detect sensors with accuracy of 15 *cm* [5]. UWB in indoor localization can realize high-precision of *cm*-level, but the cost is high. In addition, the large coverage range of each sensor results in that the UWB based positioning system is scalable [3].

These technologies used in indoor localization are both based on wireless sensor network, which can be divided into range-based methods and range-free methods. This paper focuses on the former one. System based on wireless sensor network is composed with mobile station (MS) and fixed points with coordinate. In Bluetooth indoor localization system, beacons with certain coordinate is used to send signals. As in UWB indoor localization system, anchors do the same job.

Indoor localization system with Bluetooth beacon is easy to arrange and has low cost, but has low accuracy. In contrast, UWB indoor localization system can be more accurate but the cost is high. In summary, it may be a great idea to integrating the low cost of Bluetooth indoor localization system with the high precision of UWB indoor localization system. In this paper, a new positioning method called residual based weighted least square algorithm (RWLS) is proposed, and the feasibility of Bluetooth/UWB indoor localization system is discussed.

The rest of this paper is organized as follows. In Section 2, the log distance path loss propagation model for Bluetooth is gotten from experiment, and the range error characteristics of Bluetooth and UWB are analyzed. Normal least square algorithm and a new positioning method called residual based weighted least square algorithm are introduced in Section 3. The simulation of the system with proposed parameters and positioning algorithm are detailed in Section 4. Finally, the conclusion of the paper is given in Section 5.

2 Range Error analysis of Bluetooth signal and UWB signal

The range error is one of the key points for the precision of indoor localization system. In this section, the range error characteristics of both Bluetooth and UWB signals are analyzed.

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In order to measure the distance of MS and fixed points, different measuring methods can be used, including time-of-arrival (TOA), time-difference-of-arrival (TDOA), angle-of-arrival (AOA), and received-signal-strength (RSS) [6]. Whereas the delays and angles are difficult to obtain, TOA, TDOA and AOA are inappropriate for Bluetooth indoor localization system. In a Bluetooth indoor localization system, the RSS indicator (RSSI) can be extracted directly. In order to measure the distance using RSSI, the first thing is to build a model to express the relationship between RSSI and the range.

Using a right model can make a great difference to ranging precision. In our experimental environment, it proves that the log distance path loss propagation model is appropriate to use, which is defined in Eq.(1).

$$PL(d) = PL(d_0) + 10 \times n \times \lg(\frac{d}{d_0}) + X_\sigma \tag{1}$$

where *PL(d)* (RSSI, in *dBm*) is the received signal power with distance *d*, $PL(d_0)$ is the power with reference distance d_0, usually $d_0 = 1m$, *n* is the path loss exponent and X_σ represents the noise, with zero mean and standard deviation (σ). According to Eq. (1), the relationship of RSSI and *d* is influenced by *n* and X_σ, which are related to real indoor environment. In order to get *n* and X_σ, a series of experiments has been done. It turns out that $n = 1.89$, and σ=3.86. The real measurement of RSSI and our path-loss propagation model with $n = 1.89$ are basically consistent as shown in Fig.1.

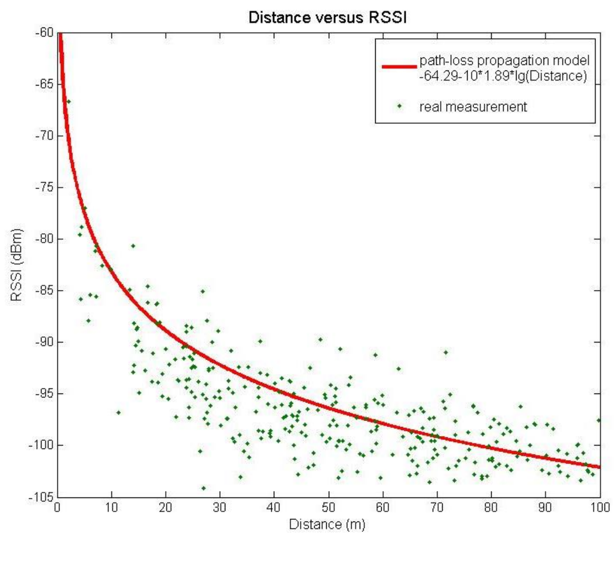

Fig.1. Relation between distance and RSSI with σ=3.86.

As seen in Fig.1, RSSI fluctuates greatly with the change of environment. A suitable filter to pre-process the RSSI data is necessary to get reliable RSSI. In this paper, a moving average filter is chosen to do this job.

The moving average filter is a calculation to get reliable data from the average of several real collected data points nearby. In this paper, the moving average filter can be represented as Eq. (2)

$$rssi(t) = \frac{\sum_{k=t-h}^{k=t+h} r(k)}{\omega}, h = \left\lfloor \frac{\omega}{2} \right\rfloor. \tag{2}$$

where *r(k)* represents the real collected data, ω means size of the window in moving average filter, which is often an odd number.

The moving average filter is simple and effective. Without Complex calculation, the running speed can be improved. Meanwhile, MAF can address data volatility properly. After the filter, the standard deviation σ of RSSI reduced to about 1.7 *dBm*.

In order to analyze the relationship between range error and real distance, depict the theoretically standard deviation of the measurement in Fig.2.

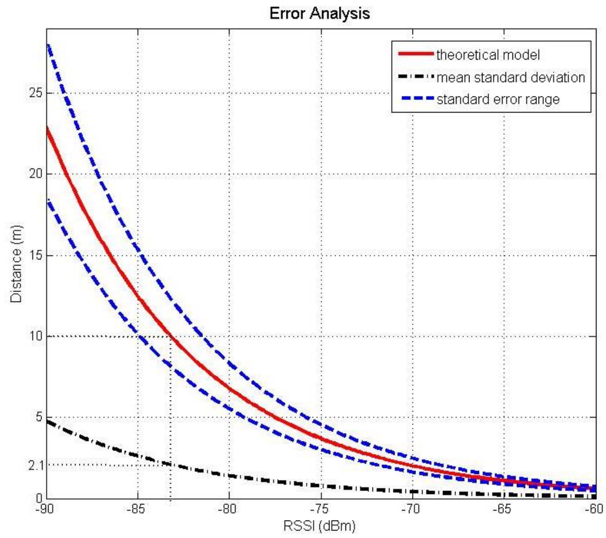

Fig.2. Error analysis

The variance of RSSI can have a huge impact on the measurement of distance, especially with a long distance, as shown in Fig.2. Within 10 meters, the range error keeps around 2 meters theoretically. Therefore, suppose that the measurement is effective within 10 meters. This processing can ensure the precision of localization.

In accurate indoor positioning, TOA estimation of the line-of-sight (LOS) environment using UWB is frequently used. After many times of test, the estimation of TOA in LOS environment results in small random errors, which are caused by paths arriving close to the detected first path and unsynchronized clocks. The measurement error model can be expressed as Eq.(3)

$$\hat{d} = d + X_\sigma. \tag{3}$$

where $\hat{d}$ represents the measured value, *d* represents the real distance, X_σ represents the noise, with zero mean and standard deviation (σ).

The effective range of UWB is 80*m*, and commercial UWB indoor localization systems have localization accuracy of 10–15 *cm* [7]. In our experiment, the UWB used has a measurement accuracy of 20 *cm*.

3 RWLS Algorithm in Indoor Localization System

Now the distance between the MS with both Bluetooth receiver and UWB receiver and Bluetooth beacons or UWB anchors are prepared. With the exact positions of Bluetooth beacons and UWB anchors, the location of MS could be simply found if the distance information is precise. However, errors exist in all measurements. And the system has inputs with different precisions. Two methods to get the optimal estimation are introduced.

A simple method to solve this problem is normal least square algorithm (LS). Suppose there are n of the Bluetooth beacons could provide effective information (within 10m far from MS), one UWB anchor and one MS in the system. Let (x, y) denotes the location of MS, denote (x_1, y_1) as the position of UWB anchor, d_1 is the distance between MS and UWB anchor and (x_{i+1}, y_{i+1}) ($1 \le i \le n$) is the location of the i-th Bluetooth beacon, d_{i+1} denotes the distance between MS and the i-th Bluetooth beacon. A set of formulas are derived from the geometrical relationship as shown in Eq.(4)

$$(x-x_i)^2+(y-y_i)^2=d_i^2. \tag{4}$$

In reality, the real distance is impractical to get. Only the estimation of distance $\hat{d}_i$ can be gotten from measurement. Take the first equation as minuend to do subtractions with the following equations, a linear system is obtained as

$$AX=b \tag{5}$$

where

$$A=\begin{bmatrix} x_1-x_2 & y_1-y_2 \\ x_1-x_3 & y_1-y_3 \\ \vdots & \vdots \\ x_1-x_{n+1} & y_1-y_{n+1} \end{bmatrix},$$

$$b=\begin{bmatrix} \hat{d}_2^2-\hat{d}_1^2+x_1^2-x_2^2+y_1^2-y_2^2 \\ \hat{d}_3^2-\hat{d}_1^2+x_1^2-x_3^2+y_1^2-y_3^2 \\ \vdots \\ \hat{d}_{n+1}^2-\hat{d}_1^2+x_1^2-x_{n+1}^2+y_1^2-y_{n+1}^2 \end{bmatrix}/2, \tag{6}$$

$$X=[x,y]^T.$$

By LS algorithm, the optimal solution is the estimate of MS position, represented as

$$\hat{X}=(A^TA)^{-1}A^Tb \tag{7}$$

As the estimated distance $\hat{d}_1$ is much more close to the real value, the solution is more close to the real location of MS than single Bluetooth indoor localization system.

In order to get higher accuracy, a better method is proposed, named Residual based Weighted Least Square (RWLS) algorithm.

In the normal LS method, the measurement value of each beacons or anchor is treated as equals. However, the credibility of different measurement is different. The RWLS algorithm evaluates the credibility of each measurements, and set it as the weight to influence the solution of equations.

In order to evaluate the credibility of measurement, the RWLS algorithm calculate the residual of the solutions with the measurement and without the measurement. The changes between these two groups of residuals can indicate the credibility of measurements.

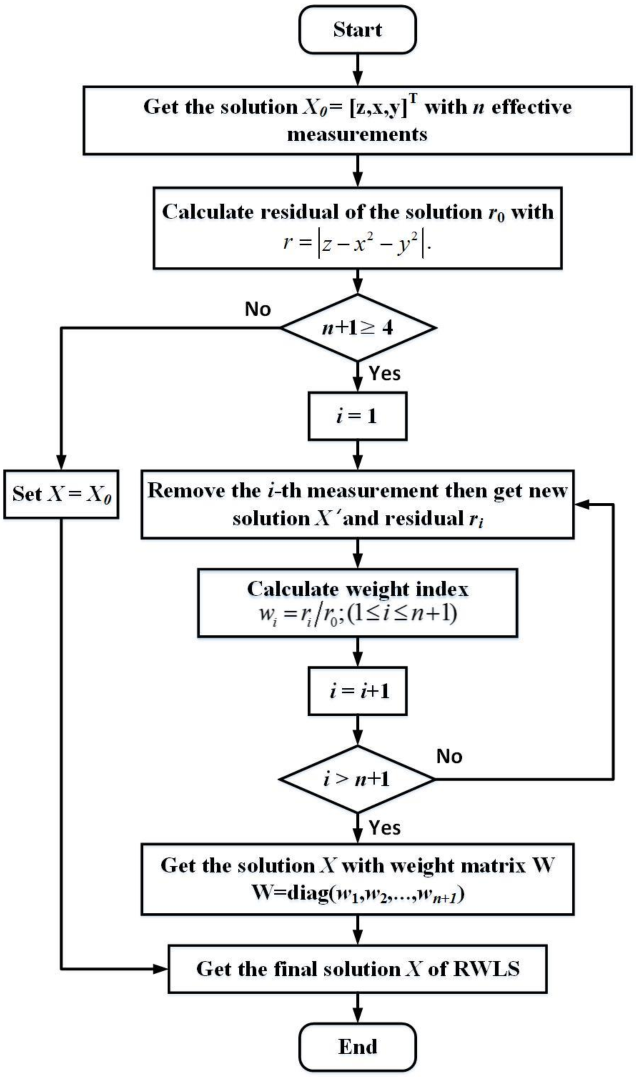

Fig.3.Flowchart of RWLS

In order to get the residual of solution, the equation should be changed. Different from the first method, a new unknown is introduced. The equation can be expressed as Eq. (8)

$$\begin{cases} x^2+y^2-2x_1x-2y_1y=\hat{d}_1{}^2-x_1{}^2-y_1{}^2 \\ x^2+y^2-2x_2x-2y_2y=\hat{d}_2{}^2-x_2{}^2-y_2{}^2 \\ \vdots \\ x^2+y^2-2x_{n+1}x-2y_{n+1}y=\hat{d}_{n+1}{}^2-x_{n+1}{}^2-y_{n+1}{}^2 \end{cases} \tag{8}$$

Suppose $z=x^2+y^2$, another linear system is obtained as $A'X=b'$, where

$$A'=\begin{bmatrix} 1 & -2x_1 & -2y_1 \\ 1 & -2x_2 & -2y_2 \\ \vdots & \vdots & \vdots \\ 1 & -2x_{n+1} & -2y_{n+1} \end{bmatrix}$$

$$b'=\begin{bmatrix} d_1{}^2-x_1{}^2-y_1{}^2 \\ d_2{}^2-x_2{}^2-y_2{}^2 \\ \vdots \\ d_{n+1}{}^2-x_{n+1}{}^2-y_{n+1}{}^2 \end{bmatrix} \tag{9}$$

$$X=[z,x,y]^T.$$

In this way, z will be calculated independently. Therefore, the difference between z and x^2+y^2 can indicates accuracy of the estimate more or less. It can be called as residual of the solution, represented as r.

$$r=\left|z-x^2-y^2\right|. \tag{10}$$

First, get preliminary estimate X_0 through Eq. (7). Get a standard indicator r_0 through Eq. (10). The indicator r_0 represents accuracy of the estimate with all the measurements. If there are only 3 effective reference points in the system, the position of MS could be gotten from the preliminary estimate X_0.

When there are at least 4 effective reference points in the system, remove the equation of one of the beacons or anchor and get new X and r_i ($1\le i\le$ n+1) through Eq. (7) and Eq. (10), the indicator r_i show the accuracy after the removal.

If the indicator r_i after the removal is larger than r_0, it means that the removed point has higher credibility than the average. And the greater the difference among r_0 and r_i, the higher credibility of the removed measurement. Therefore, the quotient between r_i and r_0 can indicates the credibility of the removed points. Choose the division between r_i and r_0 as the weight in LS algorithm, get a weight matrix shown in Eq.(11)

$$w_i=r_i/r_0;(1\le i\le n+1)$$

$$W=\begin{bmatrix} w_1 & & & \\ & w_2 & & \\ & & \ddots & \\ & & & w_{n+1}\end{bmatrix}. \tag{11}$$

Solve the Eq. (9) again with weighted least square, where

$$\hat{X}=(A^TWA)^{-1}A^TWb \tag{12}$$

RWLS concerns about different importance of the beacons and anchor, can achieve higher precision theoretically.

4 Simulation Results

In order to maximize the utilization of resources, an appropriate arrangement including arranging density and coordination method is imperative. According to preceding text, Bluetooth beacon has an effective coverage range of 10*m*, while UWB anchor does better with 80*m*.

Consider a scenario of 113*m*×113*m* as target area. Bluetooth beacons are interspersed every 6m with a total number of 400. UWB anchor is placed in the middle of the scenario. The MS is a receiver which can receive both Bluetooth signal and UWB signal. In this way, MS can get UWB signal and at least 3 signals from beacons everywhere in the scenario.

In order to prove the validity of this method, two group of simulation is conducted in the scenario. First group with single Bluetooth sensor while the other group with both Bluetooth and UWB.

Use the propagation model introduced in section 2.1, and set 5 as the window size of moving average filter. The Cumulative distribution functions of positioning errors of different methods are shown in Fig.4.

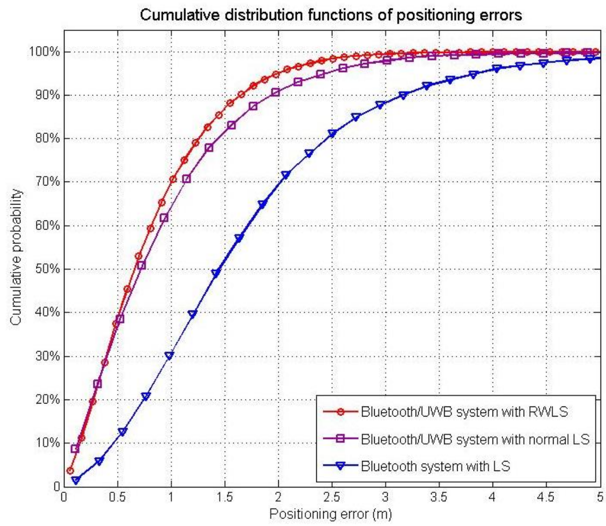

Fig.4. Cumulative distribution functions of positioning errors

It is obvious that the positioning precision of Bluetooth/UWB indoor localization system is much better than single Bluetooth indoor localization system. A comparison of two methods with Bluetooth/UWB indoor localization system is made in Fig.5.

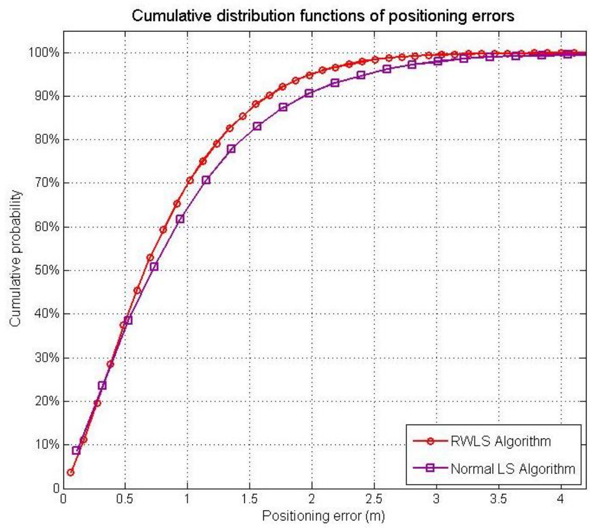

Fig.5. Comparison of two methods with Bluetooth/UWB indoor localization system

The comparison of RWLS and normal LS is expressed in Table 1 and Table 2.

Table 1: Position Error of LS and RWLS with Different Distance between MS and UWB Anchor

Distance from UWB anchor	LS error (*m*)	RWLS error (*m*)
10*m*	1.31	1.09
30*m*	0.99	0.84
50*m*	1.01	0.79
70*m*	1.14	0.94

Table 2: Comparison of Position Error with LS and RWLS

Positioning algorithm	Min error (m)	Max error (m)	Mean error (m)	Variance (m^2)
LS	0.0030	7.54	1.00	0.59
RWLS	0.0028	4.79	0.85	0.39

From Table 1, it's obvious that the positioning error is irregular with the varying of distance from UWB anchor. Therefore the accuracy is irrelevant to the distance from UWB anchor. Table 2 shows that the RWLS algorithm does a better job than normal LS algorithm in minimum error, maximum error, mean error and variance of the error.

In order to analyze the feasibility of Bluetooth/UWB indoor localization system, a contrast contains cost and accuracy is made. After market research, an UWB anchor costs about two orders of magnitude more than a Bluetooth beacon. In the simulation, 400 Bluetooth beacons and one UWB anchor is used. The cost of Bluetooth/UWB indoor localization system only increases 8.3% than single Bluetooth indoor localization system. And the number can be deduced with larger target space.

As seen from data above, RWLS reduced 15% of the error than normal LS algorithm, and reduced 52.8% of the error than single Bluetooth indoor localization system with a cost increase of only 8.3%. The Bluetooth/UWB indoor localization system is suitable for large space with few obstructions.

5 Conclusion

An indoor localization system with Bluetooth and UWB sensors is designed in this paper. The system combines the advantages of both Bluetooth and UWB, thereby realize the goal of high precision and relatively low cost. The range error characteristics of both Bluetooth and UWB are analyzed to design the replace of beacons and anchor. A moving average filter is taken to smooth the RSSI. A Residual based Weighted Least Square algorithm is proposed to locate the MS. It proves by simulation that Bluetooth and UWB indoor localization system with RWLS has higher precision than the system with single Bluetooth sensor, and the cost only increases 8.3%. Therefore, the system is appropriate to a large space with few obstructions, like super parking lots.

The precision of RWLS algorithm is hopeful to be further improved. In addition, inertial sensors also will be used in the system to improve the precision of localization in our future work.

References

[1] Y.Zhang and Y.Zhao, "Design implementation of an air quality monitoring system for indoor environment based on microcontroller," *International Journal of Smart Home*, 9(11):301-312, 2015.

[2] Y.Zhang, "Research on the Key Techniques of Indoor Localization," Anhui: *University of Science and Technology of China*, 2014

[3] Y. Gu, A. Lo, "A Survey of Indoor Positioning System for Wireless Personal Network", in *IEEE Communications Surveys &Tutorials*, 11 (2): 13-32,2009

[4] FCC, Washington, DC, "The first report and order regarding ultra-wideband transmission systems," FCC 02-48, ET Docket 98-153, 2002.

[5] Ubisense, Cambridge, U.K., "Hardware datasheet," 2006.

[6] F. Gustafsson and F. Gunnarsson, "Mobile positioning using wireless networks: Possibilities and fundamental limitations based on available wireless network measurements," *IEEE Signal Process*, 22 (4): 41–53, Jul. 2005.

[7] M. R. Mahfouz, C. Zhang and B. C. Merkl, "Investigation of High-Accuracy Indoor 3-D Positioning Using UWB Technology," *IEEE Transactions on Microwave Theory and Techniques*, 56 (6):1326-1330, 2008

[8] S.Wang, Y.Hou, "Error Analysis of Least Squares Method and Optimization for WSN," *Journal of System Simulation*, 21(19):6211-6215,6220, Oct, 2009.