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Collaborative Residual Metric Learning

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Collaborative Filtering

Matrix Completion

- Completing the elements of the user-item interaction matrix that are not 1

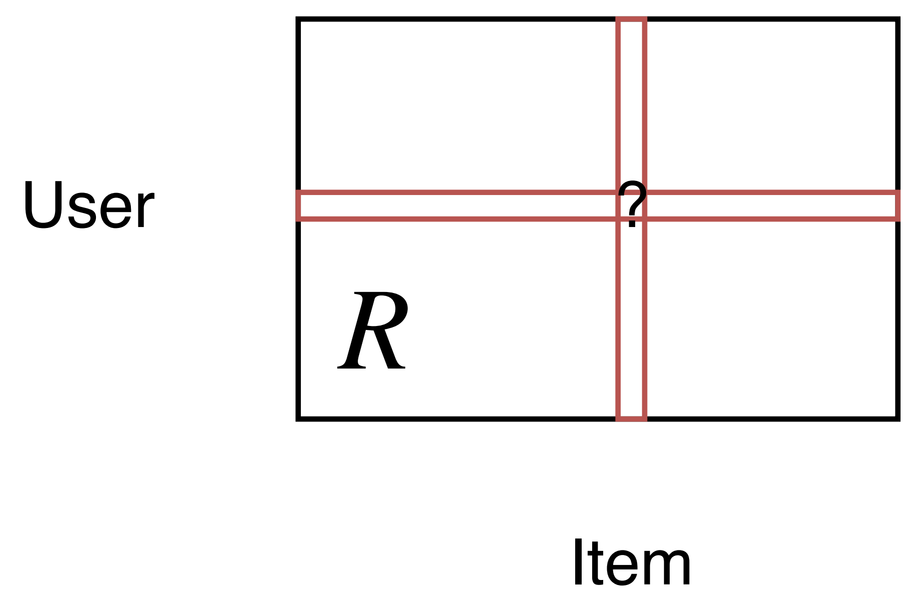
Link Prediction

- Predicting unconnected edges in user-item bipartite graphs

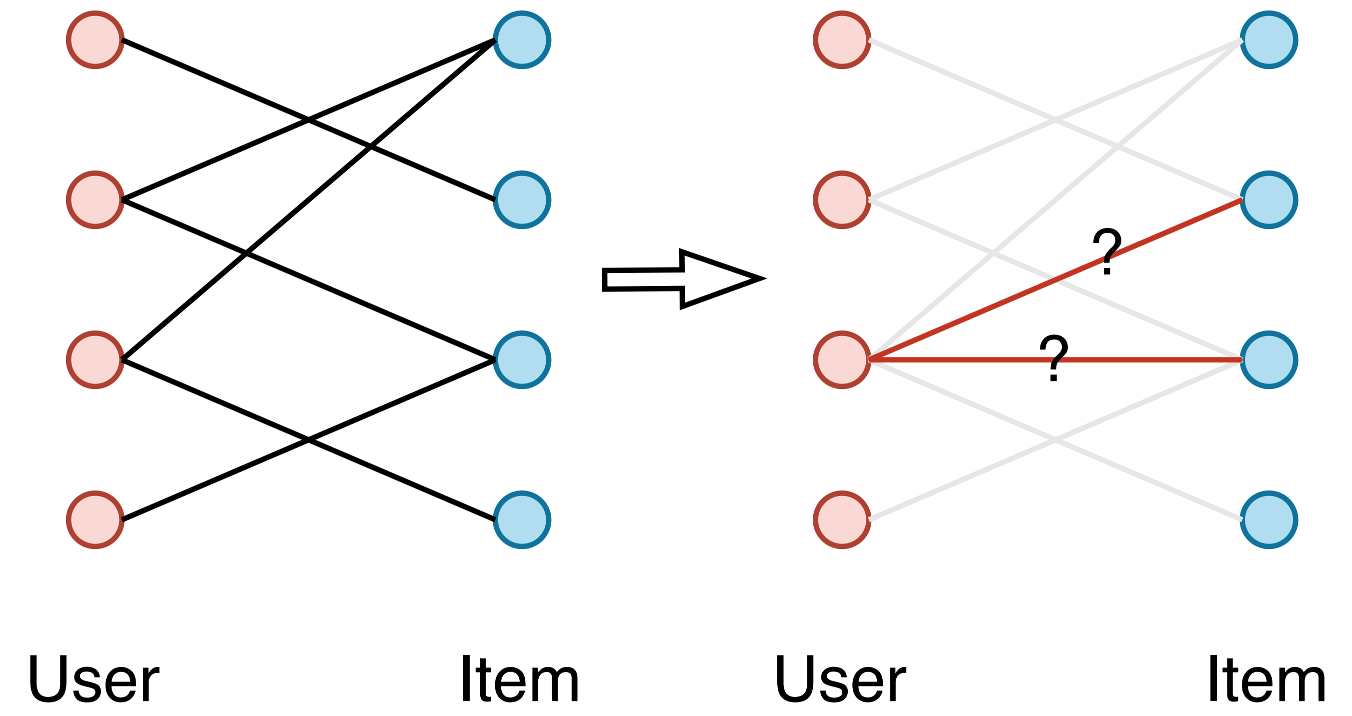
Nature of Recommendation Tasks

- Focus more on ranking of scores
- Focus more on top positions

Collaborative Filtering as a Matrix Completion Problem



Collaborative Filtering as a Link Prediction Problem



Type of Methods

Traditional Matrix Factorization

- Representing users and items with fixed-length vectors
- Both a trainable parameters ($N \propto |U| + |I|$)

Asymmetric Matrix Factorization

- Training only item vectors
- Representing users with the aggregation of item vectors
- No need to train when new users are added

Linear Autoencoder

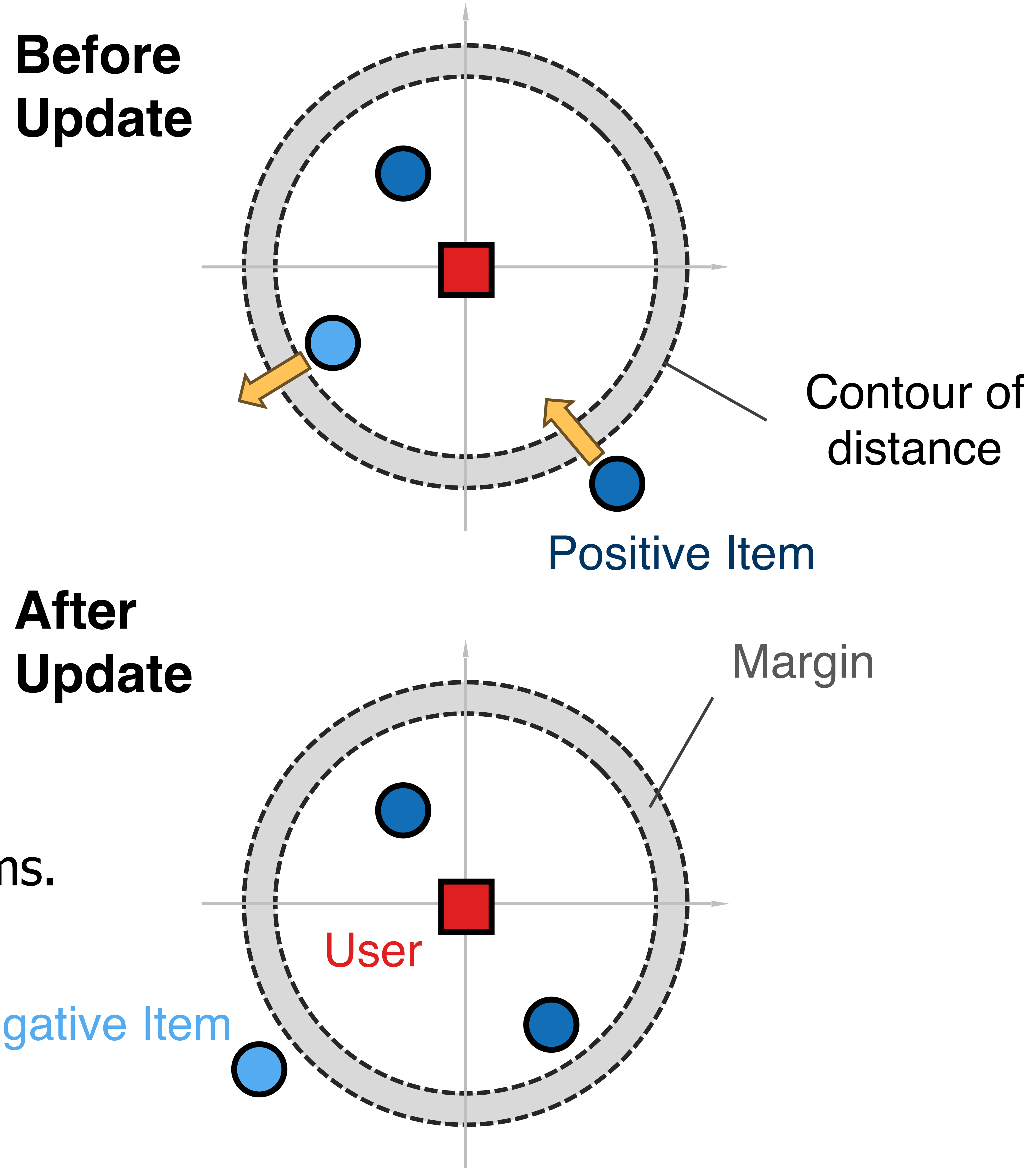
- Full-rank Extension of asymmetric matrix factorization
- Adding sparse constraint to low the cost of storage and inference
- SLIM (ICDM 2011), EDLAE (Netflix, NIPS 2020)

Metric Learning in Collaborative Filtering

Metric Learning

- The objective of metric learning is to learn a **valid distance metric** to pull similar nodes closer, push dissimilar nodes farther away:

- ☑ Non-negativity: $d(\mathbf{x}_i, \mathbf{x}_j) \geq 0$
- ☑ Identity: $d(\mathbf{x}_i, \mathbf{x}_j) \geq 0 \rightarrow i = j$
- ☑ Symmetry: $d(\mathbf{x}_i, \mathbf{x}_j) = d(\mathbf{x}_j, \mathbf{x}_i)$
- ☑ Triangle inequality: $d(\mathbf{x}_i, \mathbf{x}_j) \leq d(\mathbf{x}_i, \mathbf{x}_k) + d(\mathbf{x}_k, \mathbf{x}_j)$



Collaborative Metric Learning (CML)

- Updating vectors to decrease the Euclidean distance between users and interacted (similar) items, and the opposite for uninteracted items.
- Adopting triplet hinge loss, with a margin ζ

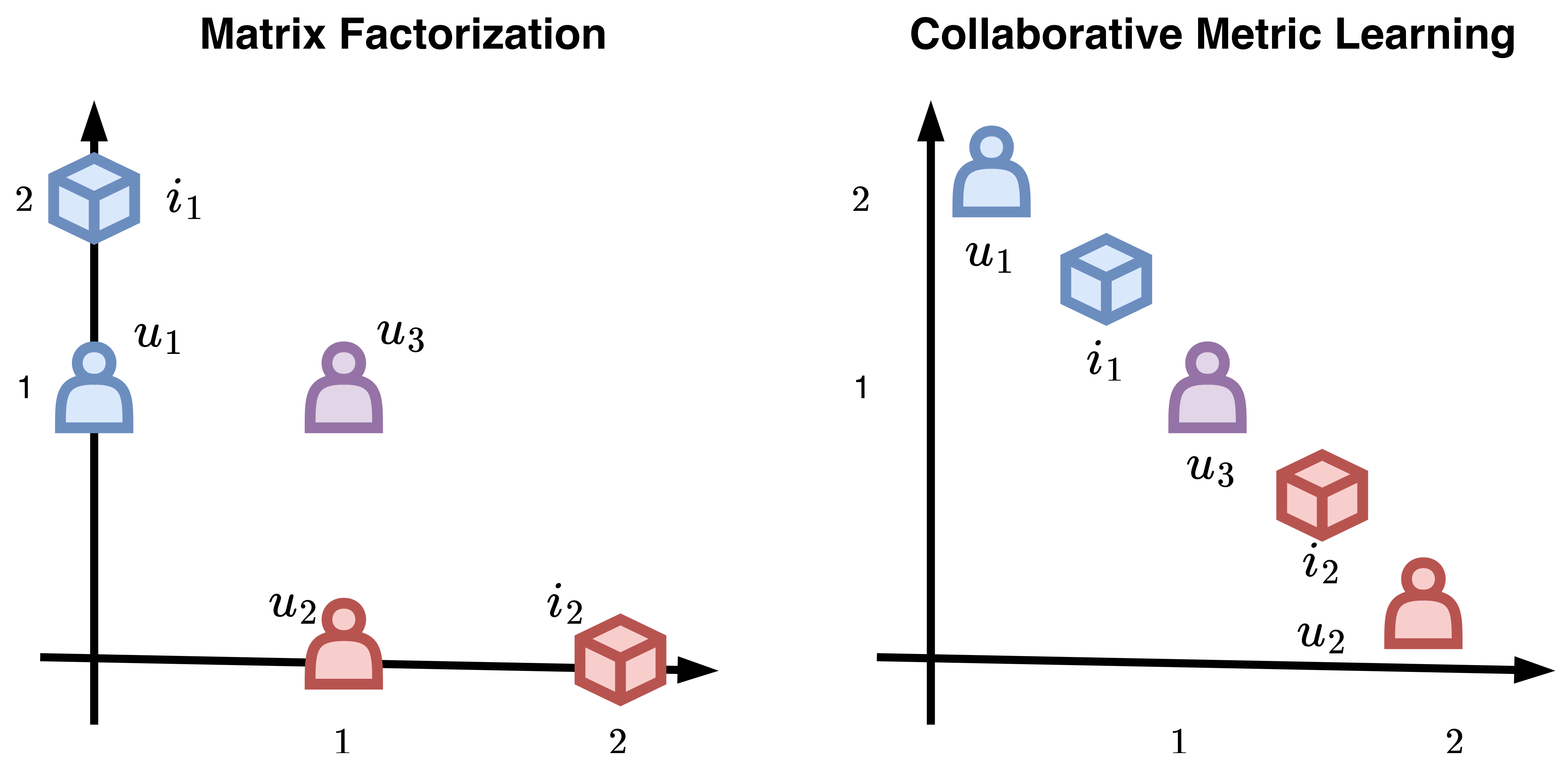
$$L = (d^2(\mathbf{e}_u, \mathbf{e}_i) - d^2(\mathbf{e}_u, \mathbf{e}_j) + \zeta)_+$$

Cheng-Kang Hsieh, Longqi Yang, Yin Cui, Tsung-Yi Lin, Serge Belongie, and Deborah Estrin. 2017. Collaborative Metric Learning. In WWW'17

Metric Learning in Collaborative Filtering

Propagation of Similarity

- MF is not reliable on capturing u-u and i-i similarity
- CML shows the capability of propagating similarities through triangle inequality



Special Case of Metric Learning

- Generalized Mahalanobis (GM) Distance

$$d(\mathbf{x}_i, \mathbf{x}_j) = \sqrt{(\mathbf{x}_i - \mathbf{x}_j)^T \mathbf{W} (\mathbf{x}_i - \mathbf{x}_j)}$$

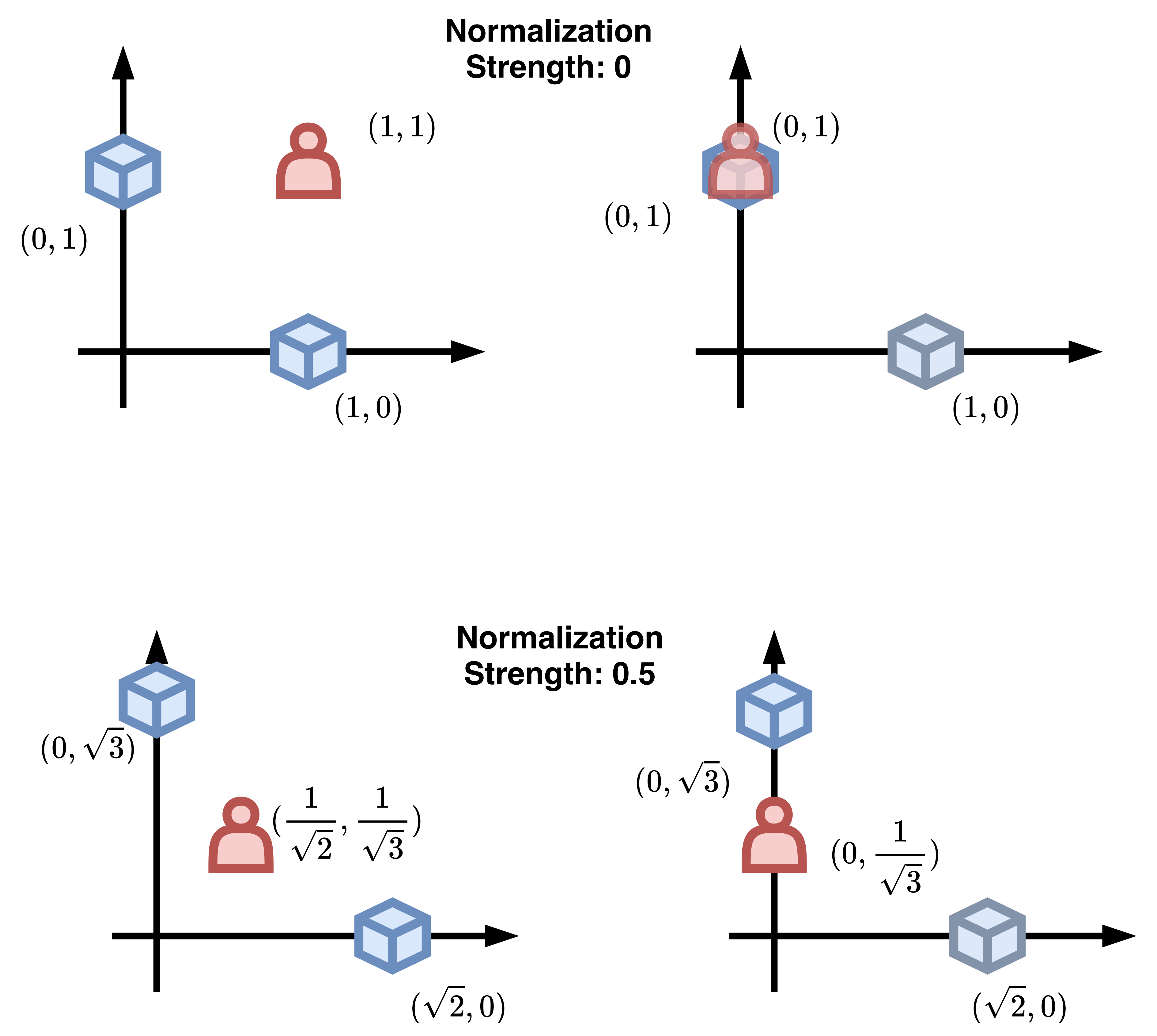
- W must be symmetric positive semidefinite (PSD)
- CML learns a rank-d weight matrix $\mathbf{W} \in \mathbb{R}^{(|U|+|I|)\times(|U|+|I|)}$
- Cannot generalize to methods like asymmetric matrix factorization

Cheng-Kang Hsieh, Longqi Yang, Yin Cui, Tsung-Yi Lin, Serge Belongie, and Deborah Estrin. 2017. Collaborative Metric Learning. In WWW'17

Collaborative Signals

Representing Users with Items

- User feature: $\mathbf{P} \in \mathbb{R}^{|U|\times|I|} = \mathbf{D}_I^{-t}\mathbf{R}$
- Item feature: $\mathbf{Q} \in \mathbb{R}^{|I|\times|I|} = \mathbf{D}_I^{t}$
- Preference score: $y_{ui} = \mathbf{p}_u^T\mathbf{C}\mathbf{q}_i$



Fitting More Methods by Introducing Normalization Strength

Methods	t	C
• Asymmetric Matrix Factorization	0	$\mathbf{E}_I^T\mathbf{E}_I$
• Linear Autoencoder	0	$\mathbf{W}_{sp,diag\_0}$
• Graph Filtering Model	0.5	$\mathbf{V}\mathbf{V}^T$

Incorporate Signal-based Models with ML

Linear Autoencoder not Working Well with ML

☑ Symmetry: can be added as a constraint to the optimization

☐ PSD: almost impossible to satisfy because of the diagonal zero constraint, $diag(\mathbf{W}) = 0$

Focus instead on the difference of distances

- Recommendation task focuses on the relative relationship of preference scores (distance)

$$\begin{aligned}
\Delta D^2 &= d^2(\mathbf{p}_u, \mathbf{q}_i) - d^2(\mathbf{p}_u, \mathbf{q}_j) \\
&= \mathbf{q}_i^T\mathbf{W}\mathbf{q}_i - \mathbf{q}_j^T\mathbf{W}\mathbf{q}_j - 2\mathbf{p}_u^T\mathbf{W}(\mathbf{q}_i - \mathbf{q}_j) + (\mathbf{p}_i^T\mathbf{W}\mathbf{p}_i - \mathbf{p}_i^T\mathbf{W}\mathbf{p}_i) \\
&= W_{ii}(d_i^{2t} - 2R_{ui}) - W_{jj}(d_j^{2t} - 2R_{uj}) - 2\underline{\mathbf{p}_u^T\mathbf{H}(\mathbf{q}_i} - \mathbf{q}_j) \\
&= W_{ii}(d_i^{2t} - 2R_{ui}) - W_{jj}(d_j^{2t} - 2R_{uj}) + \Delta Y
\end{aligned}$$

y_{ui} in signal-based models

where $H = W - diag(W)$ is called the ***Hollow matrix***.

- Eliminate redundant terms in the GM distances
- Separate diagonal and non-diagonal entries

Incorporate Signal-based Models with ML

Finding Alternative Solutions

- $\mathbf{W} = \mathbf{H} + \mathbf{X}$ can always be PSD when $\mathbf{X} = \omega \mathbf{D}_I^{-2t}$, $\mathbf{H}$ is the hollow matrix.
- Derive the relationships between ΔD^2 and ΔY (Preference Residual):

$$\begin{aligned}\Delta D^2 - \Delta Y = &\ W_{ii}(d_i^{2t} - 2R_{ui}) - W_{jj}(d_j^{2t} - 2R_{uj}) \\ = &\ 2\omega(d_i^{-2t}R_{uj} - d_j^{-2t}R_{ui})\end{aligned}$$

A. When $R_{ui} = R_{uj} = 0$

(item i and j are both uninteracted)

$$\Delta D^2 - \Delta Y = 0$$

- Δ Distance = Preference Residual
- Critical to the model inference process

B. When $R_{ui} = 1, R_{uj} = 0$

(item i is interacted, item j is uninteracted)

$$\Delta D^2 - \Delta Y = 2\omega d_i^{-2t}$$

- The bias is always positive
- Useful in the model training process

How does t affect the recommendations?

Non-negativity Constraint

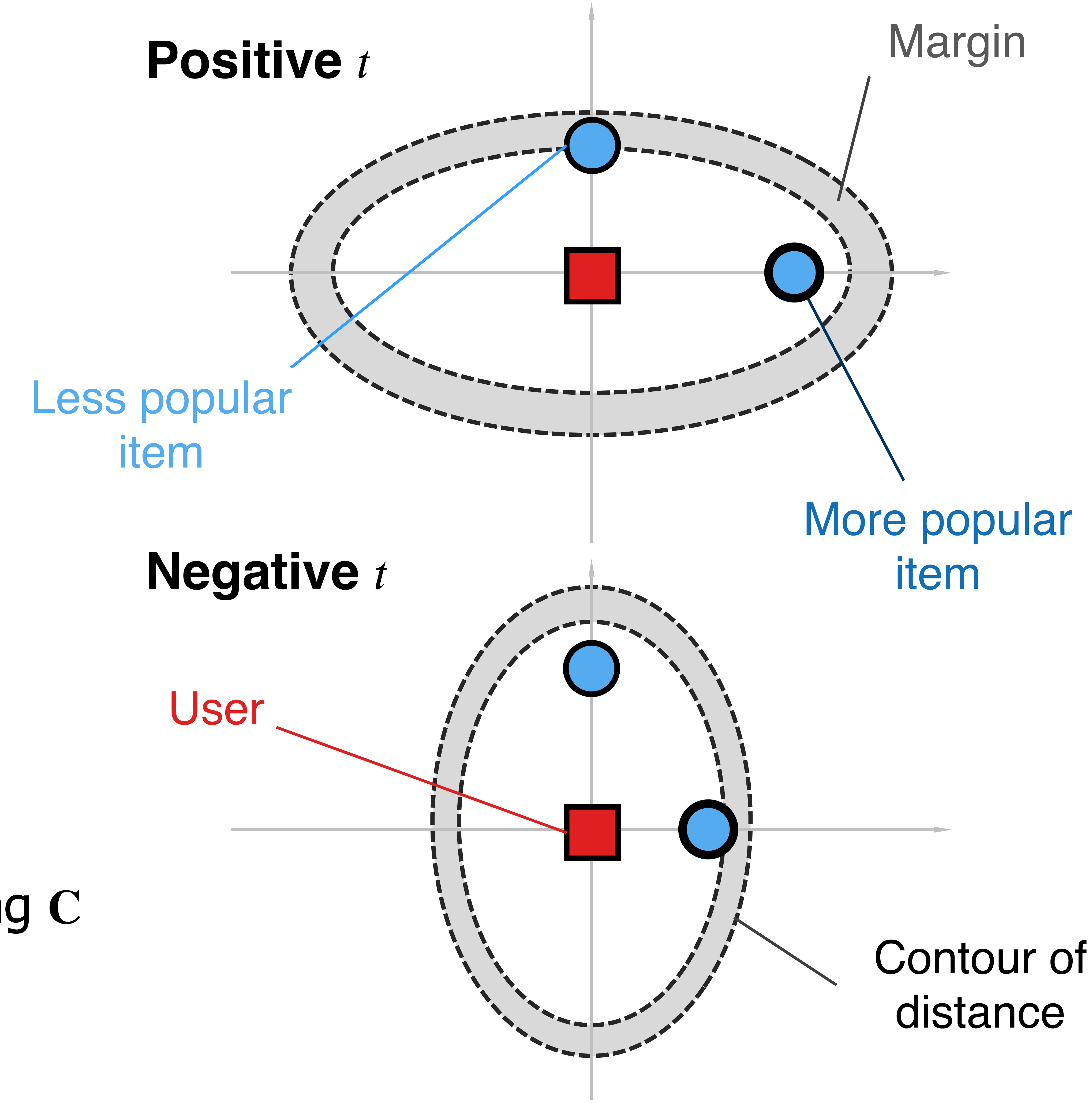
- General form of signal-based model with non-negativity constraint and l_2 regularization:

$$\underset{\mathbf{W}}{\text{minimize}} \;\; \mathscr{L}(\mathbf{W}) + \|\mathbf{W}\|_F^2, \;\; s.t.\, \mathbf{W} \geq 0$$

- Preference score for each (u, i) pair:

$$y_{ui} = \sum_{j \in I_u^+} \left(\frac{d_i}{d_j}\right)^t W_{ji}$$

- Counterfactual results: Larger t will increase the chance of popular items to be recommended
- Improve the **novelty** of the recommendations by decreasing $\mathbf{C}$



Collaborative Residual Metric Learning (CoRML)

Triplet Residual Margin Loss

Triplet Margin Loss

$$L = \sum_{u \in U} \sum_{(i^+, i^-) \in (I_u \times I \backslash I_u)} (d_{ui^+}^2 - d_{ui^-}^2 + \zeta)_+$$

- Replace $d_{ui^+}^2 - d_{ui^-}^2$ (ΔD^2) with ΔY
- Replace ζ with $2\omega d_{i^+}^{-2t}$ as an adaptive margin

Triplet Residual Margin Loss

$$L_{TRM} = \sum_{u \in U} \sum_{(i^+, i^-) \in (I_u \times I \backslash I_u)} (y_{ui^-} - y_{ui^+})_+$$

$$= \sum_{u \in U} \Big(\sum_{i^+ \in I_u} \alpha_{ui^+} y_{ui^+} + \sum_{i^- \notin I_u} \beta_{ui^-} y_{ui^-} \Big)$$

$$\alpha_{ui^+} = \sum_{i^- \notin I_u} - \frac{\delta(y_{ui^-} > y_{ui^+})}{|I| - |I_u|}, \ \beta_{ui^-} = \sum_{i^+ \in I_u} \frac{\delta(y_{ui^-} > y_{ui^+})}{|I_u|}$$

Approximated Ranking Weights

- α and β are are coefficients dynamically updated by the ranking of the value of y_{ui} for user u
- Use numerical value to approximate ranking

$$\tilde{\alpha}_{ui^+} = \phi y_{ui^+} - 1, \tilde{\beta}_{ui^-} = \phi y_{ui^-}$$

Scaling factor

$$\phi_u = \epsilon \Big(\frac{d_u}{max_{u \in U} d_u} \Big)^{-t_u}$$

- Global scaling (ϵ): rescale y_{ui^+} to map $\tilde{\alpha}_{ui^+}$ to negative values, and keep $\tilde{\beta}_{ui^-}$ positive
- User-degree scaling (t_u): reduce the effects of different number of non-zero entries in the collaborative signal of each user

Loss Function

$$L_{CoRML} = \sum_{u \in U} \sum_{i \in I} y_{ui}(\phi_u y_{ui} - R_{ui}) = tr(\mathbf{Y}^T(\mathbf{\Phi Y} - \mathbf{R}))$$

Hybrid Preference Score

$$\mathbf{Y} = \mathbf{R}(\lambda \underbrace{\mathbf{D}_I^{-t}\mathbf{H}\mathbf{d}_I^t}_{} + (1-\lambda)\underbrace{\mathbf{D}_I^{-\frac{1}{2}}\mathbf{G}\mathbf{D}_I^{\frac{1}{2}}}_{})$$

Extension from linear autoencoder with adjustable t

Extension from graph signal model $\mathbf{G} = (\mathbf{V}\mathbf{V}^T - diag(\mathbf{V}\mathbf{V}^T))_+$

Optimization Problem

$$\min_{\boldsymbol{H}} tr\left(\boldsymbol{Y}^T(\boldsymbol{\Phi}\boldsymbol{Y} - \boldsymbol{R})\right),$$

$$s.t. \quad diag(\boldsymbol{H}) = 0, \boldsymbol{H} \geq 0, \boldsymbol{H} = \boldsymbol{H}^T$$

- Optimized through Alternating Directions Method of Multipliers (ADMM)

CoRML

Experiments

Performance Comparison

Dataset

- 4 real-world public datasets

Evaluation Metrics

- NDCG@K
- MRR@K

Baselines

- CML models
- Signal-based Models
- GCN models

Dataset	Metric	CML	L-CML	DPCML	SLIM	EASE	RecVAE	GFCF	UltraGCN	SimGCL	CoRML
Pinterest	NDCG@5	0.0509	0.0594	0.0563	0.0488	0.0558	0.0516	0.0620	0.0572	0.0616	***0.0655**
	NDCG@10	0.0665	0.0766	0.0724	0.0630	0.0704	0.0668	0.0785	0.0729	0.0783	***0.0824**
	NDCG@20	0.0897	0.1021	0.0965	0.0841	0.0921	0.0895	0.1031	0.0962	0.1031	***0.1076**
	MRR@5	0.1018	0.1186	0.1133	0.0957	0.1125	0.1024	0.1239	0.1146	0.1237	***0.1306**
	MRR@10	0.1164	0.1343	0.1283	0.1084	0.1262	0.1164	0.1390	0.1292	0.1390	***0.1458**
	MRR@20	0.1261	0.1444	0.1381	0.1171	0.1353	0.1258	0.1488	0.1387	0.1488	***0.1556**
Gowalla	NDCG@5	0.0853	0.0985	0.0999	0.1100	0.1211	0.0890	0.1174	0.1108	0.1229	***0.1317**
	NDCG@10	0.0953	0.1093	0.1087	0.1156	0.1268	0.0978	0.1257	0.1181	0.1295	***0.1383**
	NDCG@20	0.1125	0.1281	0.1261	0.1302	0.1412	0.1140	0.1440	0.1348	0.1460	***0.1554**
	MRR@5	0.1533	0.1743	0.1811	0.1912	0.2186	0.1613	0.2121	0.2001	0.2235	***0.2334**
	MRR@10	0.1682	0.1899	0.1957	0.2043	0.2323	0.1752	0.2269	0.2144	0.2377	***0.2479**
	MRR@20	0.1768	0.1984	0.2040	0.2118	0.2393	0.1832	0.2352	0.2225	0.2454	***0.2558**
Yelp2018	NDCG@5	0.0483	0.0574	0.0556	0.0535	0.0611	0.0525	0.0587	0.0585	0.0646	***0.0690**
	NDCG@10	0.0521	0.0617	0.0592	0.0554	0.0628	0.0558	0.0617	0.0621	0.0676	***0.0716**
	NDCG@20	0.0629	0.0742	0.0709	0.0644	0.0722	0.0663	0.0731	0.0737	0.0795	***0.0832**
	MRR@5	0.1007	0.1188	0.1156	0.1117	0.1277	0.1106	0.1236	0.1234	0.1349	***0.1435**
	MRR@10	0.1149	0.1345	0.1304	0.1245	0.1413	0.1247	0.1380	0.1385	0.1499	***0.1586**
	MRR@20	0.1241	0.1443	0.1399	0.1327	0.1496	0.1336	0.1472	0.1478	0.1594	***0.1679**
ML-20M	NDCG@5	0.2319	0.2731	0.2620	0.2785	0.3025	0.3045	0.2718	0.2365	0.2675	***0.3189**
	NDCG@10	0.2326	0.2689	0.2588	0.2710	0.2934	0.3033	0.2671	0.2280	0.2644	***0.3103**
	NDCG@20	0.2486	0.2832	0.2725	0.2813	0.3036	0.3204	0.2799	0.2369	0.2794	***0.3212**
	MRR@5	0.3761	0.4341	0.4190	0.4478	0.4829	0.4777	0.4356	0.3919	0.4310	***0.4967**
	MRR@10	0.3932	0.4494	0.4347	0.4621	0.4963	0.4923	0.4506	0.4063	0.4466	***0.5098**
	MRR@20	0.4002	0.4554	0.4409	0.4677	0.5014	0.4976	0.4566	0.4124	0.4527	***0.5149**

Detailed Analysis

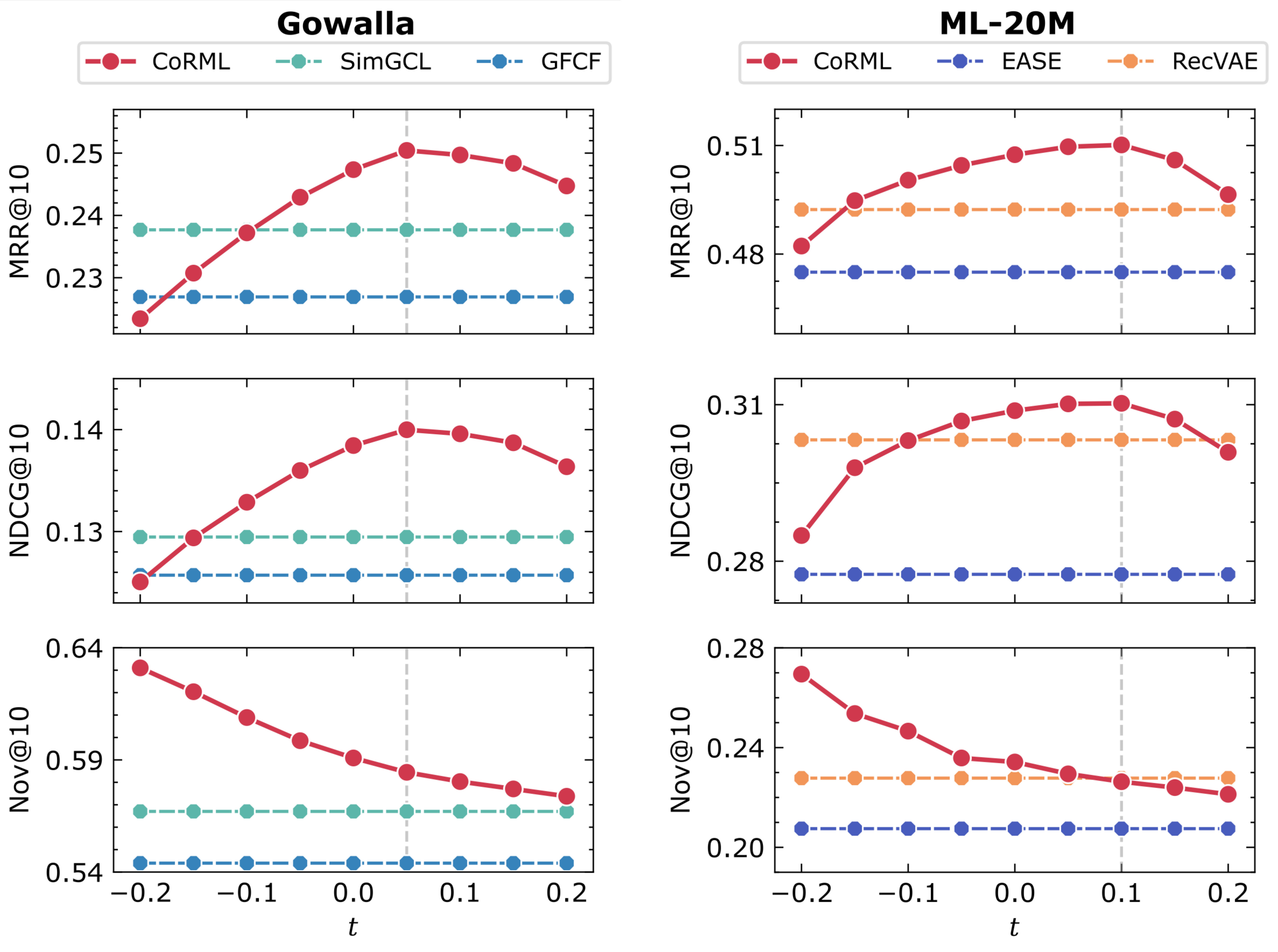
Mitigating Popularity Bias

Novelty

- Measure the popularity of top-K items recommended

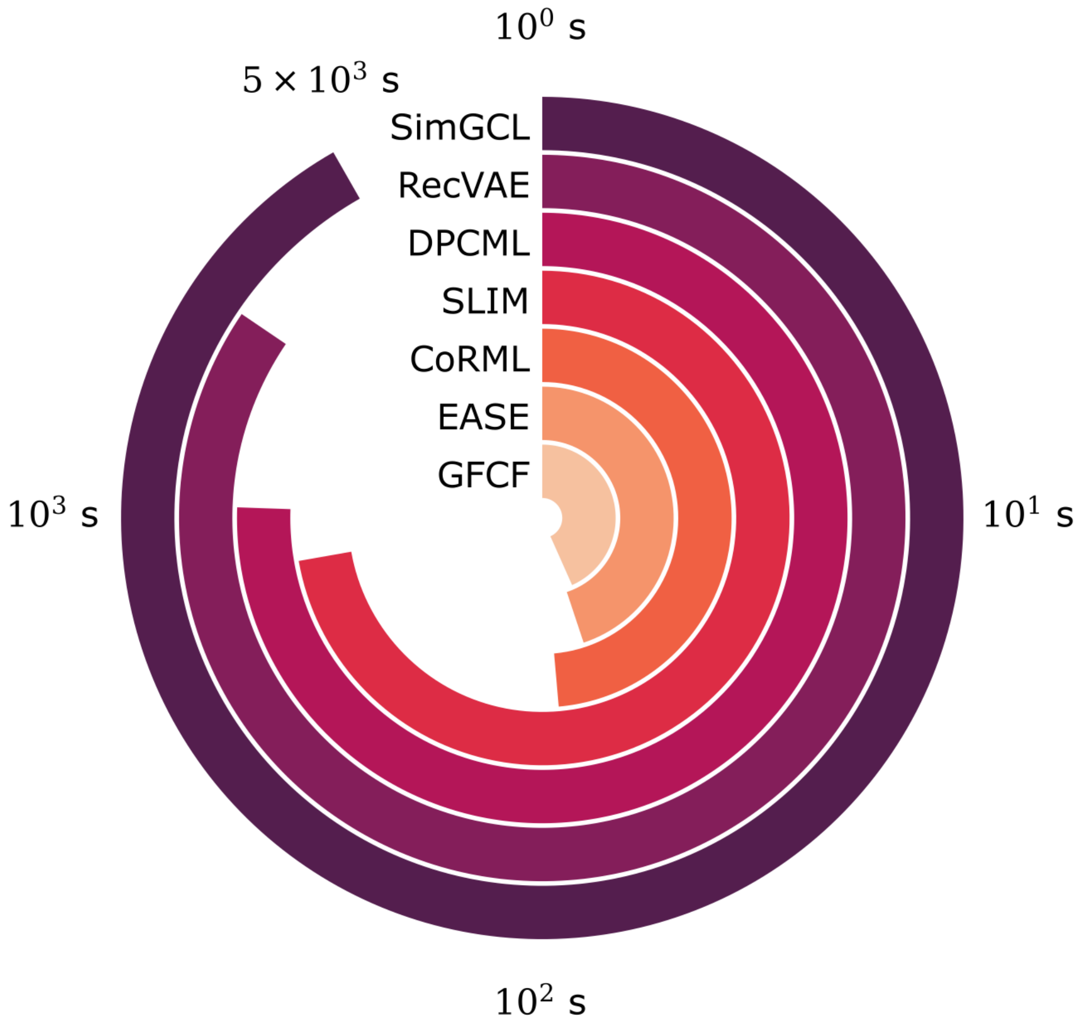
$$Nov@K = \frac{1}{|U|K} \sum_{u}^{|U|} \sum_{i}^{K} -\frac{1}{\log_2 |U|} \log_2 \frac{d_i}{|U|}$$

- Smaller normzalization strength increases the novelty of recommendation
- Accuracy and novelty are not just trade-off

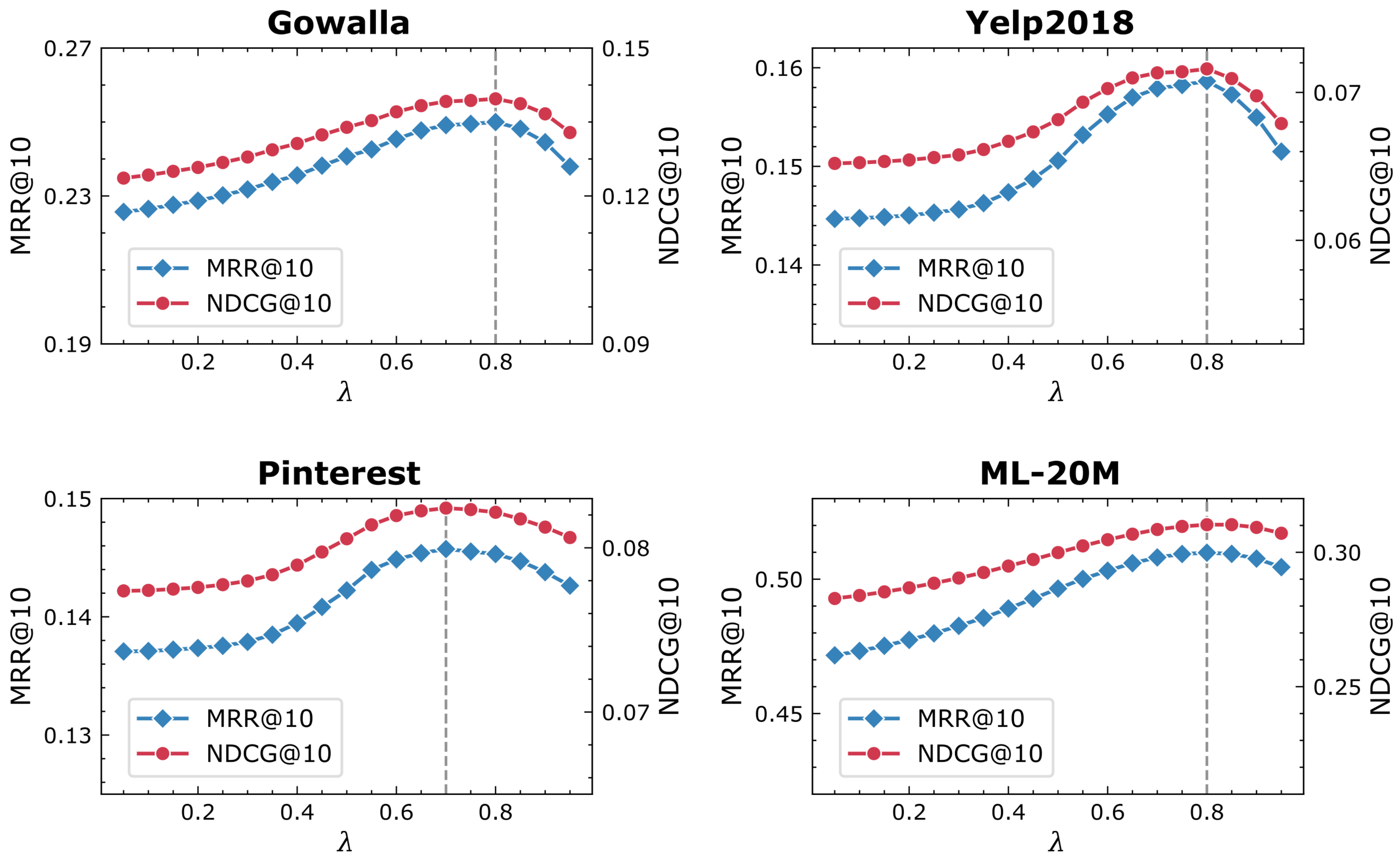


Tao Zhou, Zoltán Kuscsik, Jian-Guo Liu, Matúš Medo, Joseph Rushton Wakeling, and Yi-Cheng Zhang. 2010. Solving the apparent diversity-accuracy dilemma of recommender systems. Proceedings of the National Academy of Sciences 107, 10 (2010), 4511–4515.

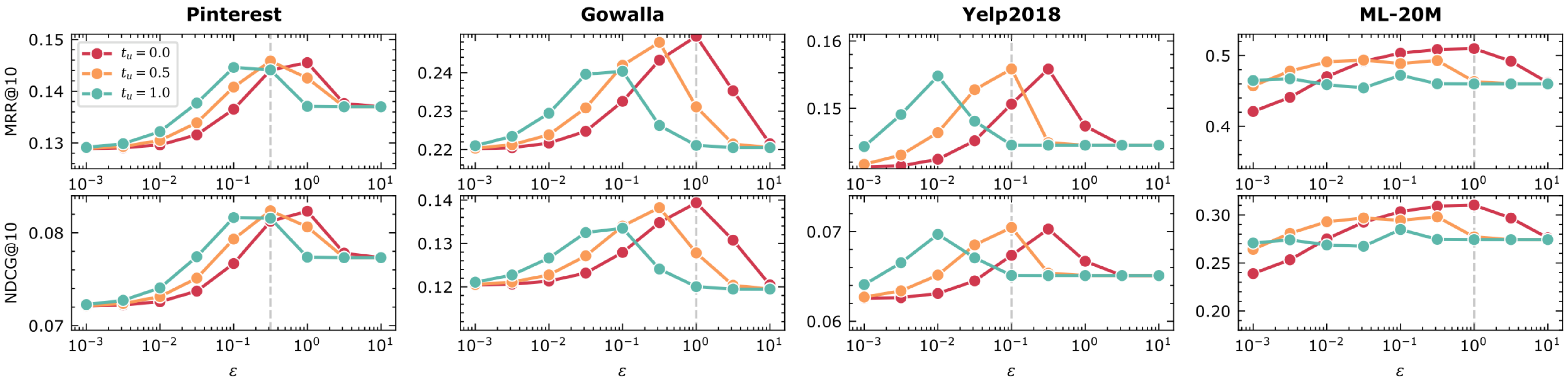
Detailed Analysis



Training Time



Effect of λ



Effect of scaling factors

Thank you!

The code is available at GitHub: *Joinn99/CoRML*

Paper

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Code