ACM The Web Conference 2023
User Modeling and Personalization Track

Fine-tuning Partition-aware Item Similarities for Efficient and Scalable Recommendation

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Collaborative Filtering

Matrix Completion

- Completing the elements of the user-item interaction matrix that are not 1

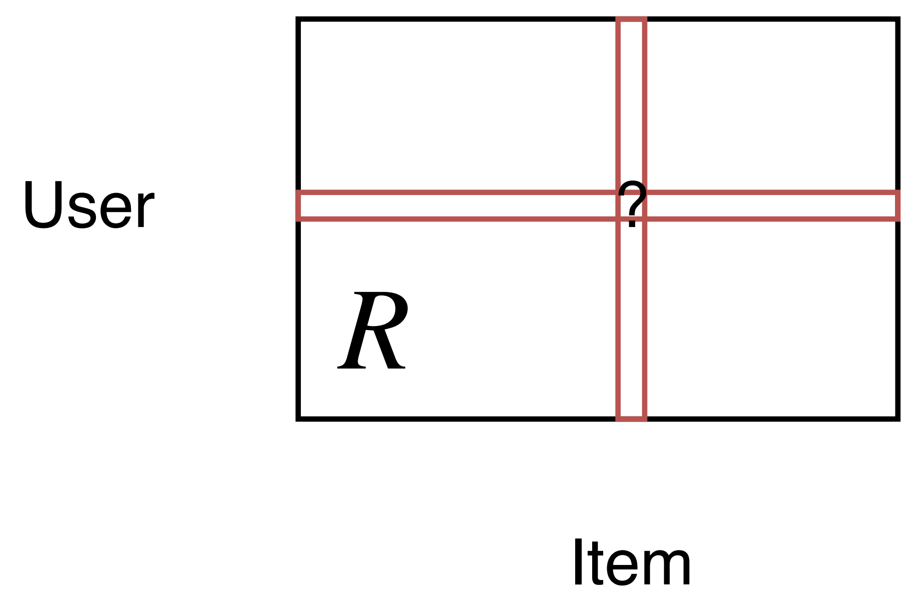
Link Prediction

- Predicting unconnected edges in user-item bipartite graphs

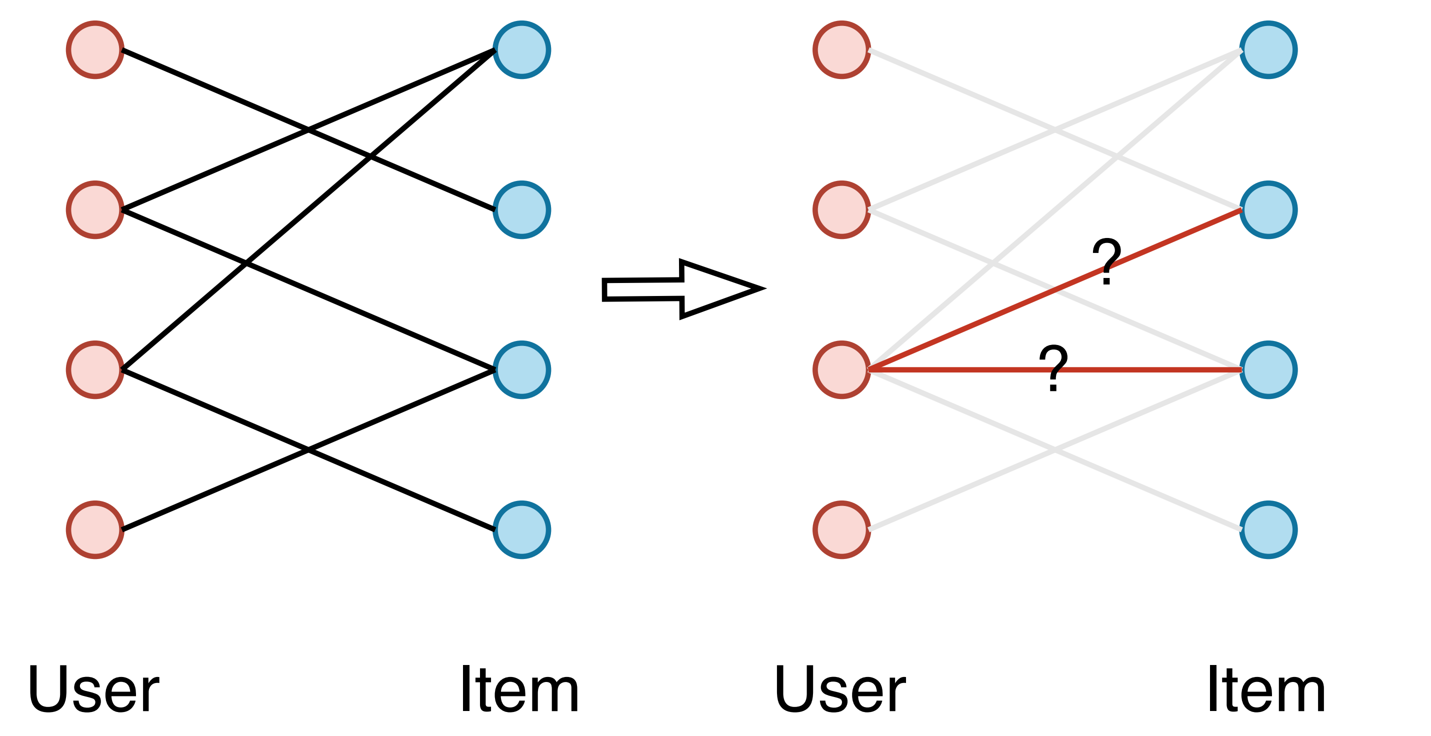
Nature of Recommendation Tasks

- Focus more on ranking of scores
- Focus more on top positions

Collaborative Filtering as a Matrix Completion Problem



Collaborative Filtering as a Link Prediction Problem



Type of Methods

Traditional Matrix Factorization

- Representing users and items with fixed-length vectors
- Both a trainable parameters ($N \propto |U| + |I|$)

Asymmetric Matrix Factorization

- Training only item vectors
- Representing users with the aggregation of item vectors
- No need to train when new users are added

Linear Autoencoder

- Full-rank Extension of asymmetric matrix factorization
- Adding sparse constraint to low the cost of storage and inference
- SLIM (ICDM 2011), EDLAE (Netflix, NIPS 2020)

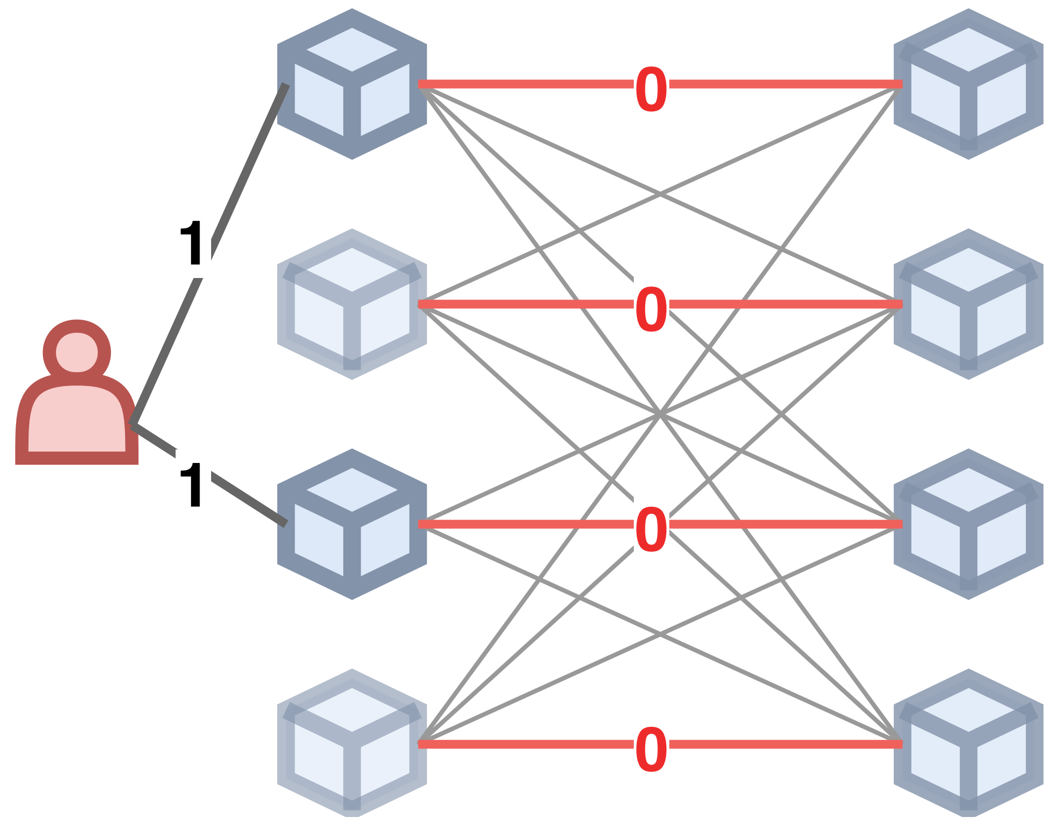
Ning, Xia, and George Karypis. "Slim: Sparse linear methods for top-n recommender systems." *ICDM 2011.*
Steck, Harald. "Autoencoders that don't overfit towards the identity." *NIPS 2020.*

Linear Autoencoder

Problem Formulation

$$\arg\min_{C} \|R - RC\|_F^2 + \frac{\theta_2}{2}\|C\|_F^2 + \theta_1\|C\|_1, \quad s.t.\, diag(C) = 0$$

- C — Weight matrix
- $\|R - RC\|_F^2$ — Linear autoencoder with hidden layer removed
- $\frac{\theta_2}{2}\|C\|_F^2$ — L2 Regularization
- $\theta_1\|C\|_1$ — L1 Regularization
- $diag(C) = 0$ — Diagonal-zero constraint (Avoid $C = I$)



Efficiency and Scalability Problem

- Computational complexity $O(|I|^2)$

Steck, Harald, et al. "Admm slim: Sparse recommendations for many users." *WWW 2020*.

Motivation - Start from GCN

LightGCN

- Graph convolutional layers with linear activation
- Increasing layers - low efficiency and over-smooth problem

$$E^{(l+1)} = (\tilde{D}^{-\frac{1}{2}} A \tilde{D}^{-\frac{1}{2}}) E^{(l)}$$

$$E = \frac{1}{L+1} \sum_{l=0}^{L} E^{(l)}$$

UltraGCN

- Approximation of infinite layers of LightGCN
- Convergence status ($E^{(l+1)} \approx E^{(l)}$)
- Convert graph convolution to the problem of maximizing the weighted inner product of vectors
  - User - Item interaction graph ($\tilde{R} = D_U^{-\frac{1}{2}} R D_I^{\frac{1}{2}}$)
  - Item - Item adjacency graph ($\tilde{Q} = \tilde{R}^T \tilde{R} / R^T R$)

$$e_u = \frac{1}{d_u + 1} e_u + \sum_{i \in \mathcal{N}_u} \frac{1}{\sqrt{d_u + 1}\sqrt{d_i + 1}} e_i$$

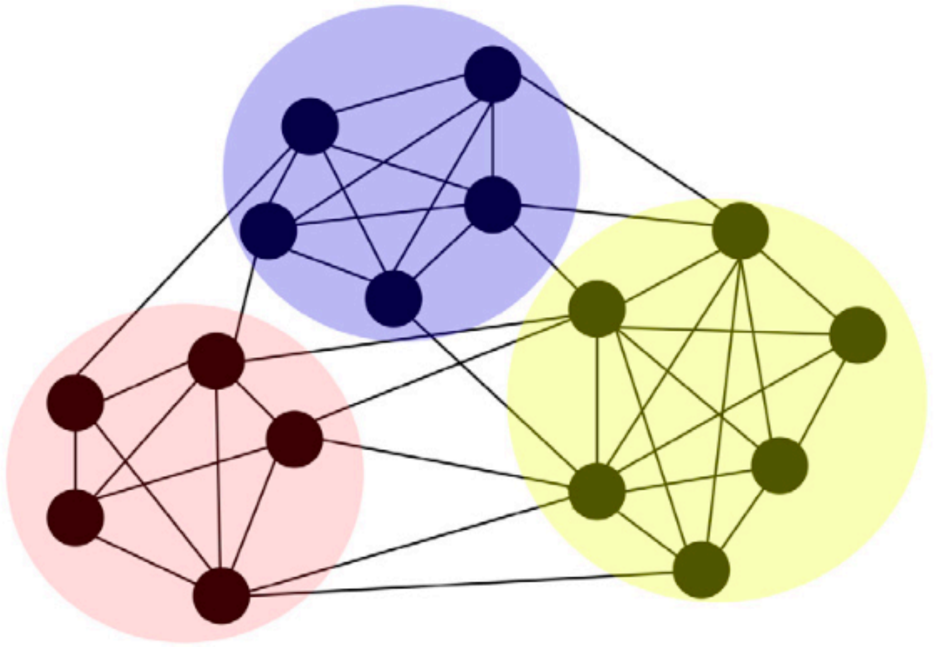
$$\beta_{u,i} = \frac{1}{d_u} \sqrt{\frac{d_u + 1}{d_i + 1}}$$

$$\omega_{i,j} = \frac{Q_{ij}}{\sum_k Q_{ik}} \cdot \frac{\sum_k Q_{jk}}{\sum_k Q_{ik} - Q_{ii}}$$

He, Xiangnan, et al. “LightGCN: Simplifying and powering graph convolution network for recommendation." *SIGIR 2020.*
Mao, Kelong, et al. "UltraGCN: ultra simplification of graph convolutional networks for recommendation." *CIKM 2021.*

Sampling in UltraGCN

- I - I adjacency graph is the 2-hop relations of U - I graph
- Much more denser than U - I graph



Graph Sampling

- Select Top-k neighbors in I - I graph

$$\arg\max_{j \in I}^{k} \ \omega_{i,j}$$

Graph Partitioning

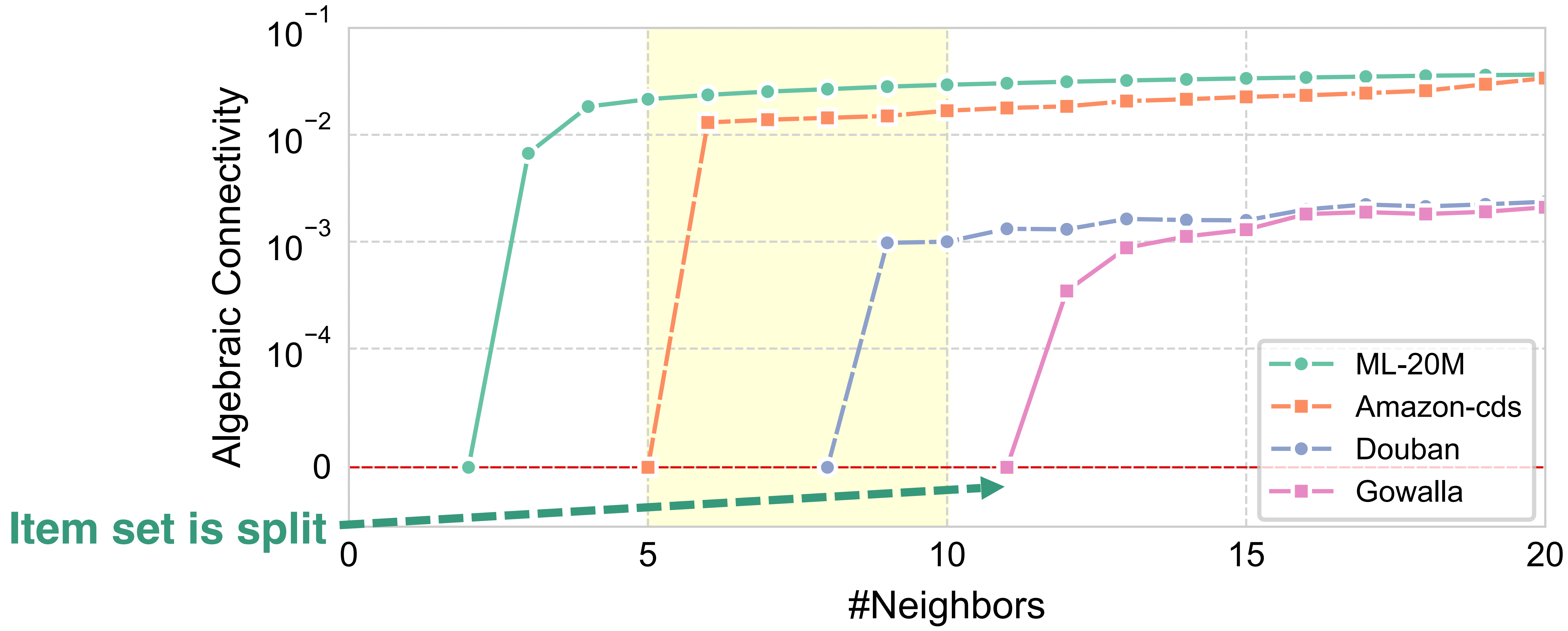
- Modularity maximization
- Graph sampling in UltraGCN can be considered as a local graph partitioning process

$$M(\mathcal{G}) = \frac{1}{2w} \sum_{i,j}^{\mathcal{E}} [A_{ij} - \frac{d_i d_j}{2w}] \mathbf{1}_{c_i = c_j}$$

$$M(\mathcal{G}_i) = \frac{1}{2d_i} \sum_{i,j} [\omega_{ij} - \frac{d_j}{2}] \mathbf{1}_{c_i = c_j} = \frac{1}{4d_i} \sum_{i,j} \omega_{ij} \mathbf{1}_{c_i = c_j}$$

Newman, Mark EJ, and Michelle Girvan. "Finding and evaluating community structure in networks." *Physical review E, 2004.*
Shen, Yifei, et al. "How powerful is graph convolution for recommendation?." *CIKM 2021*.

Community Structure in UltraGCN



Algebraic Connectivity (Fielder value) in the sampled I - I graph in UltraGCN

Our motivation

- Using global graph partitioning strategy to replace local one in UltraGCN
  - Neither real-world samples nor partitioning results are ideal
- Proposing new training strategy to alleviate information loss brought by partitioning

FPSR: Recursive Graph Partitioning

- Constructing full I - I graph requires high cost
- Using spectral partitioning without constructing the graph

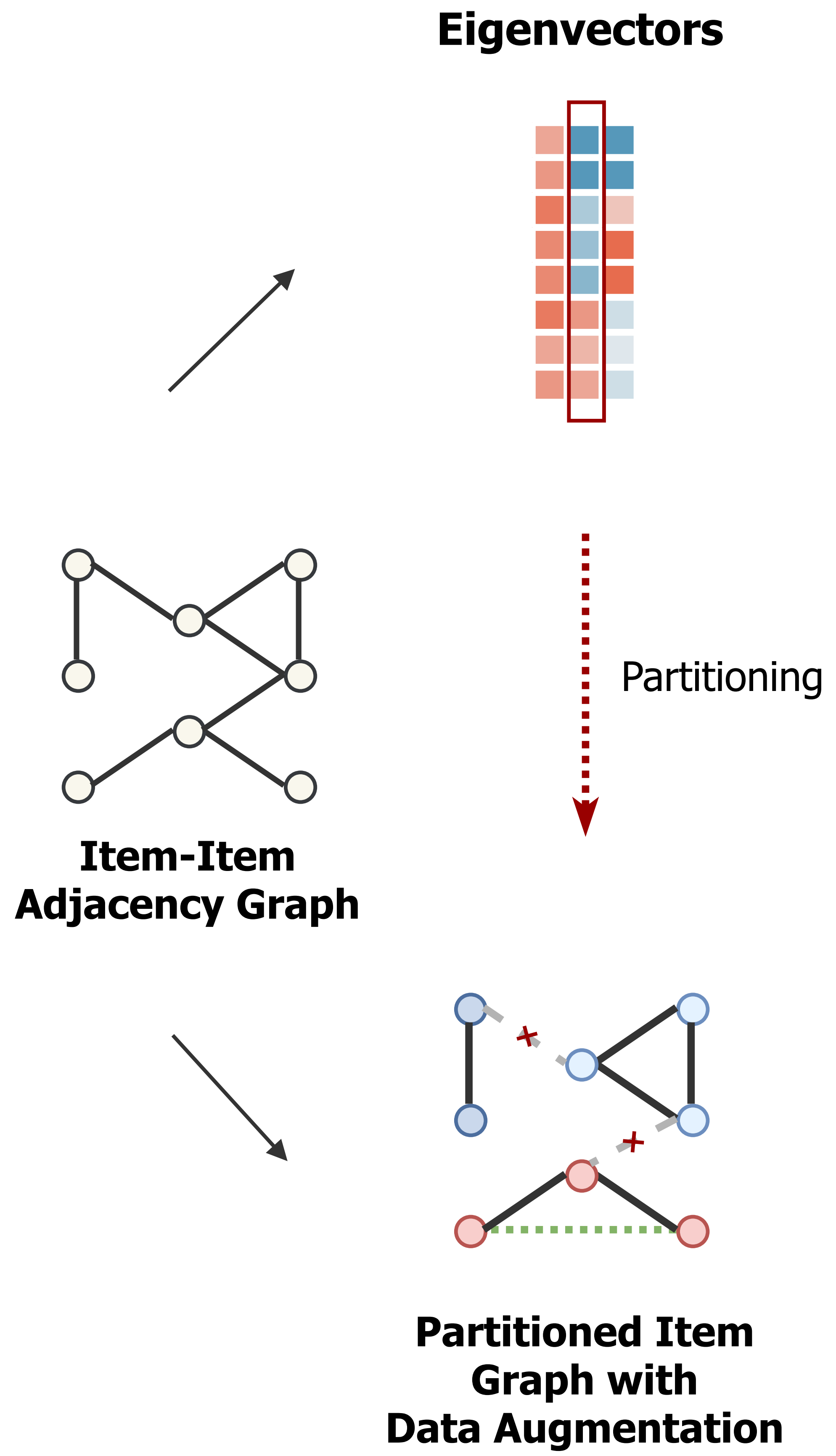
Fielder Vector

- Eigenvector corresponded to the second smallest eigenvectors in Laplacian matrix
- Equal to right singular vector corresponded to the second largest singular values in $\tilde{R}$
- Performing Truncated SVD on highly sparse $\tilde{R}$

$$i \in \begin{cases} I_1, & sign(\mathbf{v}_i) = +1, \\ I_2, & sign(\mathbf{v}_i) = -1. \end{cases}$$

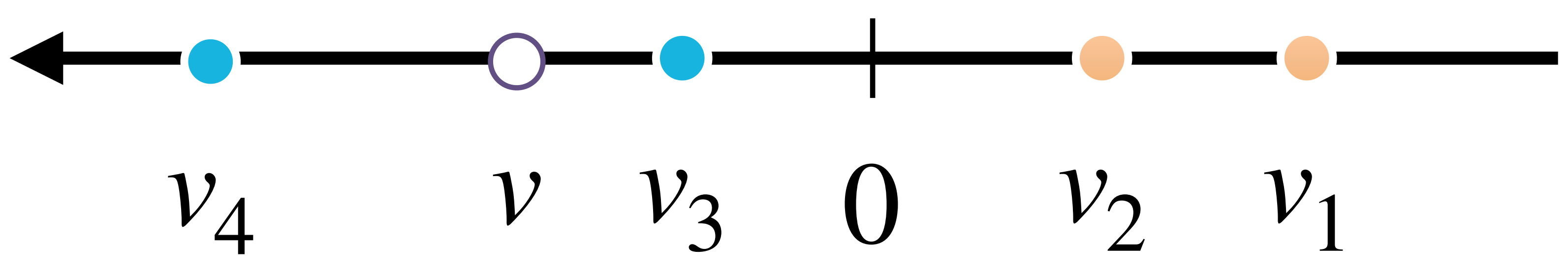
Recursive Partitioning

- Expecting the ratio between partition size and item size to be small
- Recursively performing partitioning process when partition size is not small enough



FPSR: Aggregate Global-level information

Fielder Vector



$$vv_4 > vv_3 > 0 > vv_2 > vv_1$$

- Implied item similarity information
- Adding to the learning of autoencoder through inner product
- Extending to Top-k singular vectors $V \in \mathbb{R}^{|I| \times k}$

Partition-aware Autoencoder

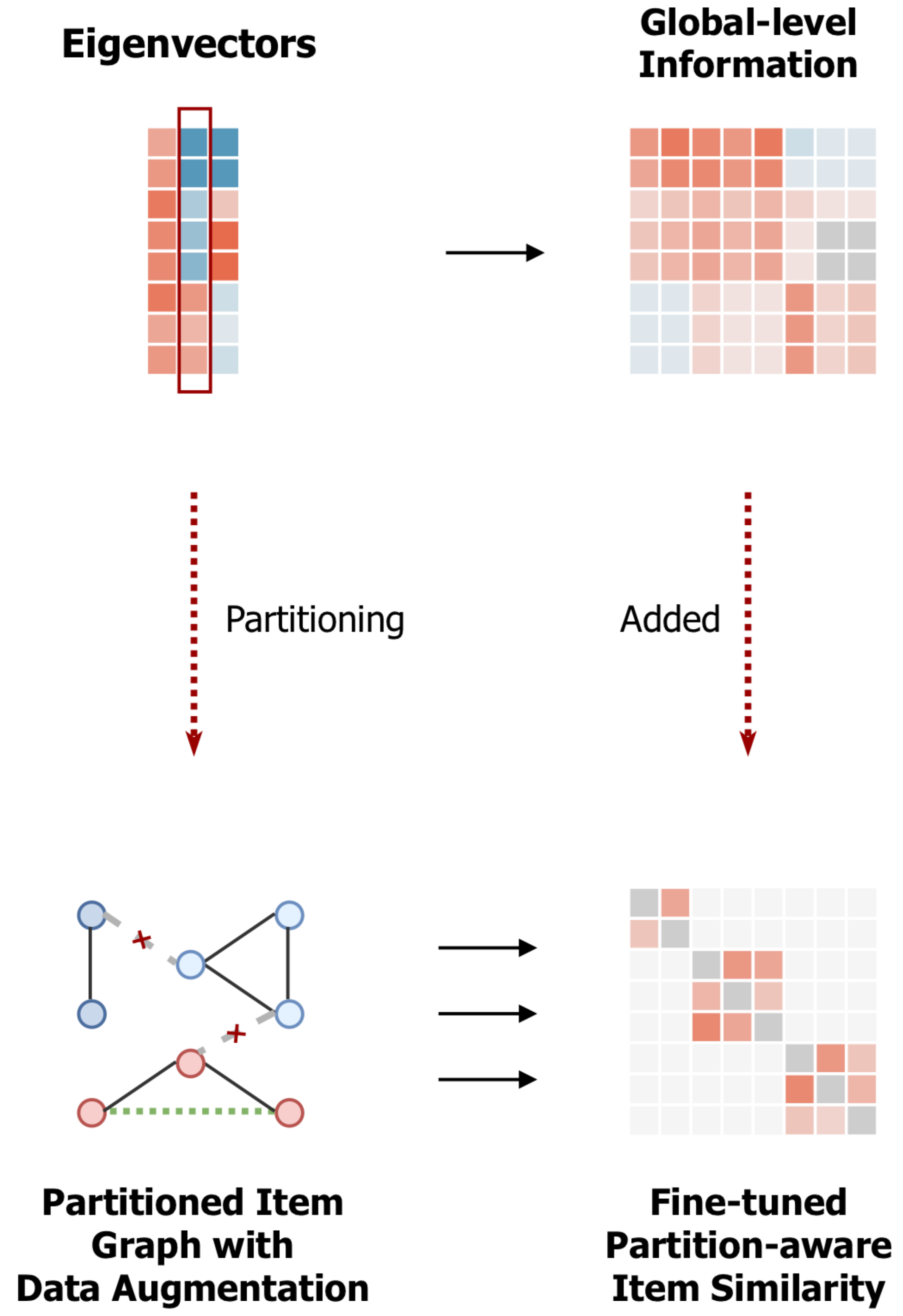
- V is the solution of the constraint optimization problem:

$$\arg\min_{U,V} \|\tilde{R} - UV^T\|_F^2, \; s.t. \; V^TV = I$$

- Aggregating global-level information

$$C = \underbrace{\lambda D_I^{-\frac{1}{2}} VV^T D_I^{\frac{1}{2}}}_{} + S$$

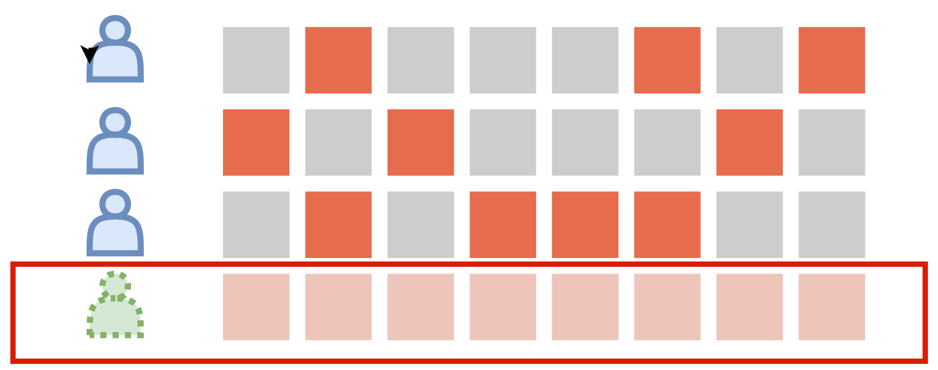
W



FPSR: Local Prior Knowledge of Partitions

- Items in the same partition can be considered having potential similarity
- Adding prior knowledge by inserting virtual users in R

$$\mathbf{1}_{n,i} = \begin{cases} 1, & \text{if } i \in I_n, \\ 0, & \text{otherwise.} \end{cases}$$

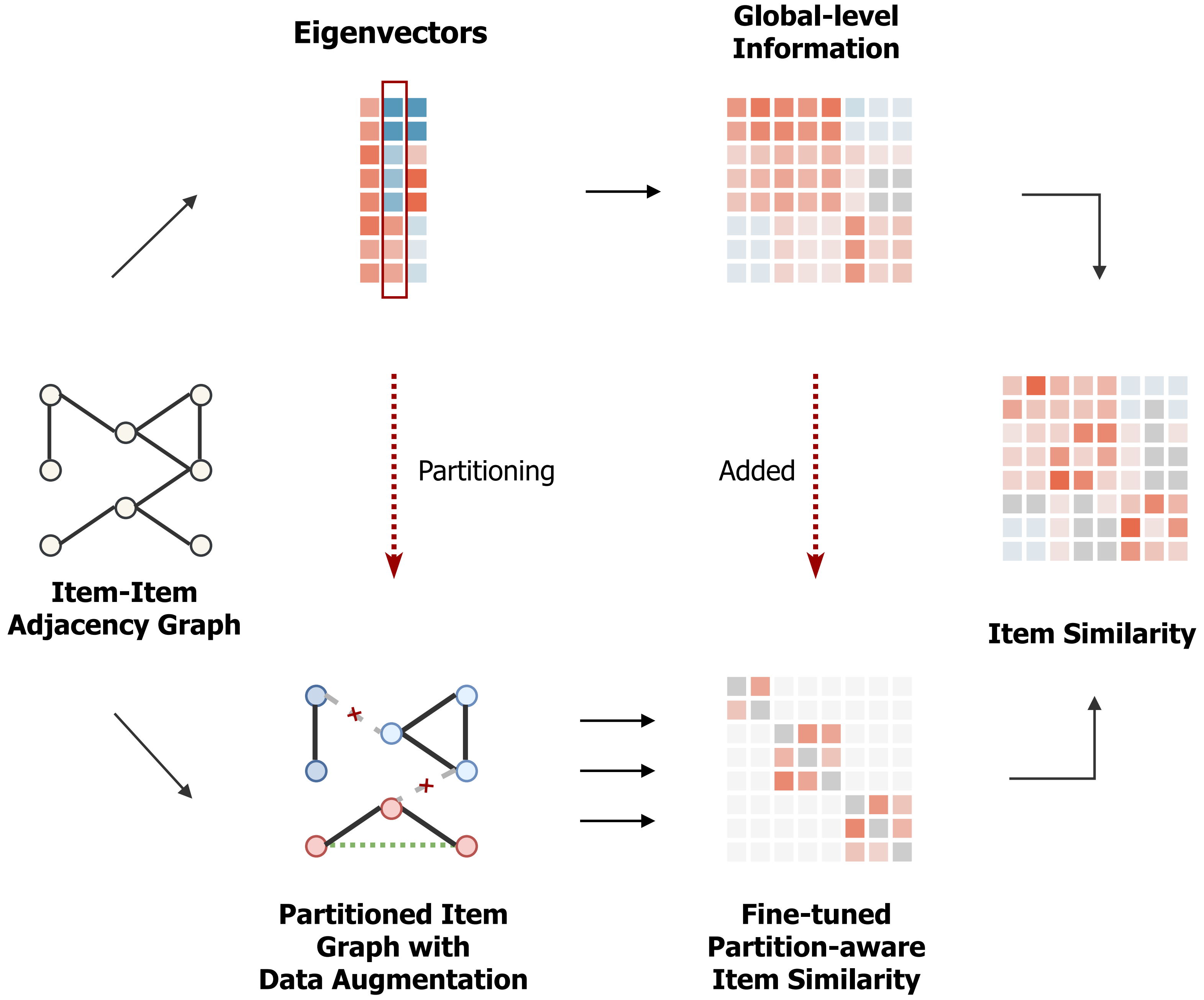


FPSR

$$\underset{S}{\operatorname{argmin}} \ \frac{1}{2}\|R - R(\lambda W + S)\|_F^2 + \frac{\theta_2}{2}\|D_I^{\frac{1}{2}}(\lambda W + S)\|_F^2$$

$$+\theta_1\|S\|_1 + \sum_n \frac{\eta}{2}\|\mathbf{1}_n^T - \mathbf{1}_n^T(\lambda W + S)\|_F^2$$

$$s.t. \ \ diag(S) = 0, \ S \geq 0, \ S_{ij|\mathcal{G}(i)\neq\mathcal{G}(j)} = 0,$$



Optimization

Alternating Direction Method of Multipliers (ADMM)

$$\underset{Z_n, S_n}{\operatorname{argmin}} \frac{1}{2} tr(Z_n^T \hat{Q} Z_n) - tr((I - \lambda W_n^T)\hat{Q} Z_n) + \theta_1 \|S_n\|_1$$
$$s.t.\ diag(Z_n) = 0,\ S_n \geq 0,\ Z_n = S_n,$$

Virtual users are not really inserted to R, but inserted by adding η

$$\hat{Q} = R_n^T R_n + \theta_2 D_I + \eta$$

Updating Rules

$$Z_n^{(t+1)} = (\hat{Q} + \rho I)^{-1}(\hat{Q}(I - \lambda W_n) + \rho(S_n^{(t)} - \Phi_n^{(t)}) - diagMat(\mu)),$$
$$S_n^{(t+1)} = (Z_n^{(t+1)} + \Phi_n^{(t)} - \frac{\theta_1}{\rho})_+,$$
$$\Phi_n^{(t+1)} = \Phi_n^{(t)} + Z_n^{(t+1)} - S_n^{(t+1)}.$$

Lagrange Multiplier

$$\mu = diag((\hat{Q} + \rho I)^{-1}(\hat{Q}(I - \lambda W_n) + \rho(S_n^{(k)} - \Phi_n^{(k)}))) \oslash diag((\hat{Q} + \rho I)^{-1})$$

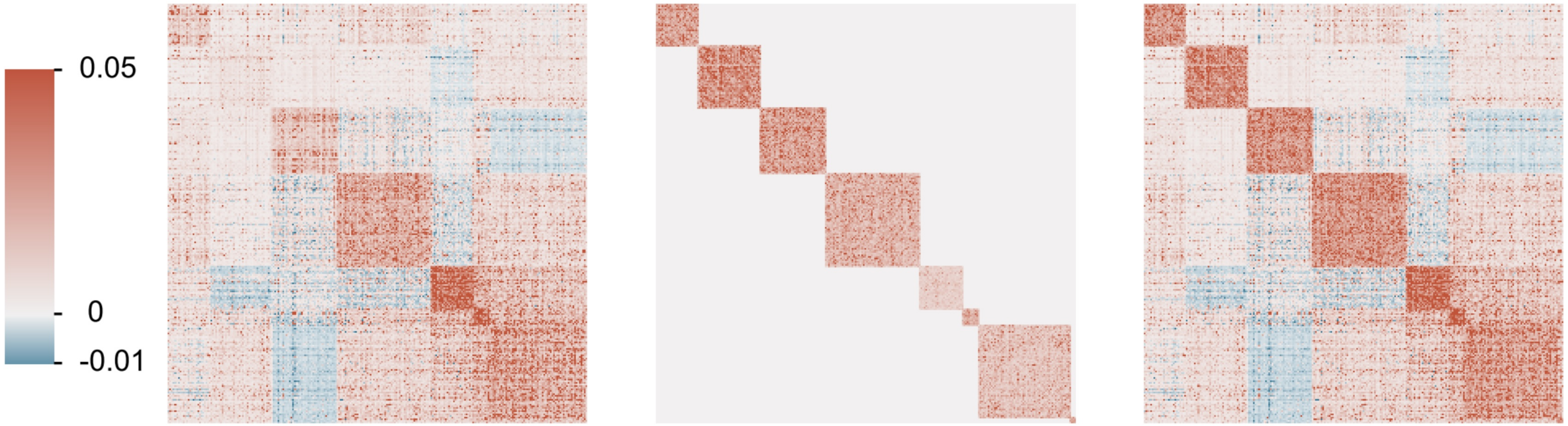
Experiments

- **Yelp2018** Dataset
  - 31,668 Users
  - 38,048 Items
  - 1,561,406 Interactions
- **Douban** Dataset
  - 13,024 Users
  - 22,347 Items
  - 792,062 Interactions

Dataset	Metric	GF-CF	LightGCN	SimGCL	UltraGCN	EASE	BISM	FPSR
Yelp2018	**Recall@20**	*0.0697*	*0.0653*	*0.0681*	*0.0683*	*0.0657*	*0.0662*	***0.0703***
	NDCG@20	*0.0571*	*0.0532*	*0.0556*	*0.0561*	*0.0552*	*0.0559*	***0.0584***
	#Params	-	*4.46M*	*4.46M*	*4.46M*	*1448M*	*191M*	*3.27M*
	Time (s)	*23*	*~74000*	*~2900*	*617*	*22*	*783*	*35*
Douban	**Recall@20**	*0.1719*	*0.1571*	*0.1699*	*0.1925*	*0.2038*	***0.2158***	*0.2095*
	NDCG@20	*0.1365*	*0.1206*	*0.1346*	*0.1556*	*0.1786*	*0.1889*	***0.1950***
	#Params	-	*5.04M*	*5.04M*	*5.04M*	*499M*	*158M*	*2.14M*
	Time (s)	*19*	*~22000*	*~2700*	*~5900*	*16*	*410*	*33*

Performance Comparison (Partial)

All experiments are conducted on with the same Intel(R) Core(TM) i9-10900X CPU @ 3.70GHz machine with one Nvidia RTX A6000 GPU.



Global-level information | Fine-tuned item similarity | Weighted sum

Visualization of item similarity matrix learned in Yelp2018 dataset

Dataset	Amazon-cds		Douban		Gowalla		Yelp2018	
Metrics	R@20	N@20	R@20	N@20	R@20	N@20	R@20	N@20
w/o global-level information	0.1540	0.0873	0.2046	0.1909	0.1883	**0.1566**	0.0702	0.0582
w/o local prior knowledge	0.1542	0.0887	0.2085	0.1945	0.1785	0.1486	0.0662	0.0556
w/o regularization	0.1539	0.0870	0.2084	0.1943	0.1832	0.1501	0.0692	0.0574
FPSR	**0.1576**	**0.0896**	**0.2095**	**0.1950**	**0.1884**	**0.1566**	**0.0703**	**0.0584**

Ablation analysis

Detailed Analysis

Thanks!

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