

Wilson-loop One-point Functions in ABJM Theory

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Based on Yunfeng Jiang, **JW** and Peihe Yang, [2306.nnnnn]
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Introductiion

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- In many cases, this correspondence is a strong-weak duality.
- So we can use weakly coupled gravity/string theory to compute quantities in strongly coupled gauge theory in the large N limit.
- The quantities include amplitudes, correlation functions of local operators, vacuum expectation values of loop operators, entanglement entropy...

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- The non-perturbative tools in the field theory side of gauge/gravity correspondence include **integrability**, supersymmetric localization, bootstrap...
- Integrability makes people be able to compute many quantities in the large N limit, even beyond the BPS sectors.

Integrability in AdS_5/CFT_4

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- Integrability is an important non-perturbative tool in AdS_5/CFT_4 correspondence.

Integrability in AdS_4/CFT_3

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- Almost every aspect of integrability in this case is more complicated and difficult.

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- Such states also appear in the Wilson-loop/domain wall one-point functions [Kristjansen, Vu, Zarembo, 21] and three-point functions involving two BPS determinant operators and one non-BPS single-trace operator in ABJM theory [Yang, Jiang, Komatsu, JW, 21].

Heisenberg XXX spin chain

- The Hilbert space of a closed XXX spin chain,

$$\mathcal{H} = \bigotimes_{i=1}^L \mathcal{H}_i, \quad \mathcal{H}_i \cong \mathbf{C}^2. \quad (1)$$

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- The Hamiltonian

$$H = J \sum_{j=1}^L \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + S_j^z S_{j+1}^z \right), \quad (2)$$

with periodic boundary condition,

$$S_{L+1}^\alpha = S_1^\alpha, \quad \alpha = x, y, z. \quad (3)$$

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- Here $U = T(0) = Q_1$ is a shift operator.

IBS for XXX chain

- The definition of IBS [Piroli, Pozsgay, Vernier, 17] for XXX chain is that the state $|B\rangle$ satisfying

$$Q_{2l-1}|B\rangle = 0, \quad l = 1, 2, \dots \quad (5)$$

- This is equivalent to

$$T(u)|B\rangle = T(-u)|B\rangle. \quad (6)$$

- For ABJM theory, since there are two sets of conserved charges, the definition of the integrable boundary states are more complicated. (More on this later.)

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- Eigenstates of integrable Hamiltonian can be labelled by Bethe roots, solutions to certain Bethe ansatz equations (BAEs).
- A selection rule for the overlap of an integrable boundary state and a Bethe state: the overlap is nonzero only when the Bethe roots satisfy certain pairing conditions.
- When this selection rule is satisfied, the overlap can often be expressed as a product of super-Gaudin-determinant and a prefactor. Great simplification!

ABJM theory

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- This theory should be low energy effective theory of N M2-branes putting at the tip of $\mathbb{C}^4/\mathbb{Z}_k$.

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- Two limits:
 - 't Hooft limit (planar limit): $N, k \rightarrow \infty, \lambda \equiv \frac{N}{k}$ fixed;
 - M-theory limit: $N \rightarrow \infty, k$ fixed.

Holographic dual

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Holographic dual

- When $N \gg k^5$, this theory is dual to **M-theory on $AdS_4 \times S^7/Z_k$** .
- When $k \ll N \ll k^5$, a better description is in terms of **IIA superstring theory on $AdS_4 \times CP^3$** .

Bosonic 1/6-BPS circular WLs

- We consider the loops along $x^\mu = (R \cos \tau, R \sin \tau, 0), \tau \in [0, 2\pi]$.
- The construction is the following,

$$W_{1/6}^B = \text{Tr} \mathcal{P} \exp \left(-i \oint d\tau \mathcal{A}_{1/6}^B(\tau) \right), \quad (7)$$

$$\hat{W}_{1/6}^B = \text{Tr} \mathcal{P} \exp \left(-i \oint d\tau \hat{\mathcal{A}}_{1/6}^B(\tau) \right), \quad (8)$$

$$\mathcal{A}_{1/6}^B = A_\mu \dot{x}^\mu + \frac{2\pi}{k} R_I^J Y^I Y_J^\dagger |\dot{x}|, \quad (9)$$

$$\hat{\mathcal{A}}_{1/6}^B = \hat{A}_\mu \dot{x}^\mu + \frac{2\pi}{k} R_I^J Y_J^\dagger Y^I |\dot{x}|, \quad (10)$$

with $R_J^I = \text{diag}(i, i, -i, -i)$. [Drukker, Plefka, Young, 08][Chen, JW, 08][Rey, Suyama, Yamaguchi, 08]

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- when the parameters in the construction take special values, the fermionic 1/6-BPS WL becomes half-BPS WL [Drukker, Trancanelli, 09] or bosonic 1/6-BPS WL.
- We focus a class of fermionic 1/6-BPS WL with R -matrix above replaced by $U_I^J = \text{diag}(i, i - 2\bar{\alpha}^1\beta_1, -i, -i)$. (Here and below, we omit the details about the suitable coupling of WLs with fermions.)

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- When $\bar{\alpha}^1\beta_1 = i$, the WL becomes half-BPS and now $U_I^J = \text{diag}(i, -i, -i, -i)$.

Local operators

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- When C is symmetric and traceless, \mathcal{O}_C is a chiral primary operator.
- Here we take \mathcal{O}_C to be a generic local operator which is eigen-operator of the planar two-loop anomalous dimension matrix.

Wick contraction

- At tree-level, the correlator $\langle W(\mathcal{C})_{1/6}^B \mathcal{O}_C(0) \rangle$ only gets contributions from

$$\oint \cdots \oint d\tau_{1>2>\cdots>L} \left(\frac{2\pi}{k} \right)^L \langle \text{tr}(R^{\tilde{J}_1}_{\tilde{I}_1} Y^{\tilde{I}_1}(x_1) Y^{\dagger}_{\tilde{J}_1}(x_1) \cdots \\ R^{\tilde{J}_L}_{\tilde{I}_L} Y^{\tilde{I}_L}(x_L) Y^{\dagger}_{\tilde{J}_L}(x_L)) C^{J_1 \cdots J_L}_{I_1 \cdots I_L} \text{tr}(Y^{I_1}(0) Y^{\dagger}_{J_1}(0) \cdots \\ Y^{I_L}(0) Y^{\dagger}_{J_L}(0)) \rangle, \quad (11)$$

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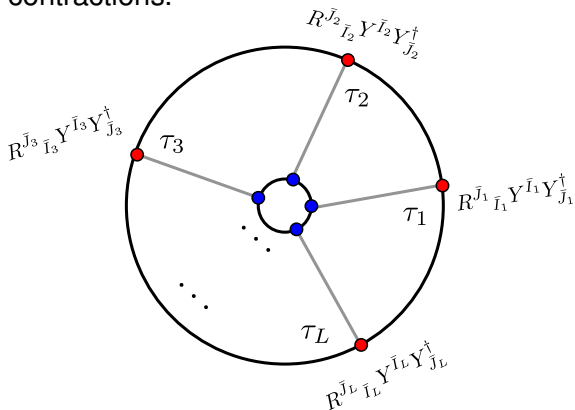
- At tree-level, the correlator $\langle W(\mathcal{C})_{1/6}^B \mathcal{O}_C(0) \rangle$ only gets contributions from

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- where $x_i = (R \cos \tau_i, R \sin \tau_i, 0)$, $i = 1, \cdots, L$, and

$$\oint \cdots \oint d\tau_1 > \tau_2 > \cdots > \tau_L = \int_0^{2\pi} d\tau_1 \int_0^{\tau_1} d\tau_2 \cdots \int_0^{\tau_{L-1}} d\tau_L. \quad (12)$$

- In the large N limit, we only take into account planar Wick contractions.



Planar Wick
contractions between the local operator and the Wilson loop.

Wick contraction

- One can easily obtain

$$\langle W(\mathcal{C})_{1/6}^B \mathcal{O}_C(0) \rangle = \frac{\lambda^{2L} k^L}{(L-1)!(2R)^{2L}} C_{I_1 \dots I_L}^{J_1 \dots J_L} R_{J_L}^{I_L} \cdots R_{J_1}^{I_1}, \quad (13)$$

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- where $\lambda \equiv \frac{N}{k}$ is the 't Hooft coupling of ABJM theory and the tree-level propagators of the scalar fields

$$\langle Y^{I\alpha}_{\bar{\beta}}(x) Y_J^\dagger{}^{\bar{\gamma}}{}_{\rho}(y) \rangle = \frac{\delta_J^I \delta_\rho^\alpha \delta_{\bar{\beta}}^{\bar{\gamma}}}{4\pi|x-y|}, \quad (14)$$

have been used.

Boundary state

- In the spin chain language, we can introduce the following boundary state

$$|\mathcal{B}_{1/6}^B\rangle = |\mathcal{B}_R\rangle, \quad (15)$$

where, for a four-dimensional matrix R , we define the boundary state $|\mathcal{B}_R\rangle$ as

$$|\mathcal{B}_R\rangle \equiv R^{I_1}_{J_1} R^{I_2}_{J_2} \cdots R^{I_L}_{J_L} |I_1, J_1, \cdots, I_L, J_L\rangle = (R^I_J |I, J\rangle)^{\otimes L}, \quad (16)$$

which is a two-site state.

Overlap

- Then the above correlation function can be expressed as

$$\langle W(\mathcal{C})_{1/6}^B \mathcal{O}_C(0) \rangle = \frac{\lambda^{2L} k^L}{(L-1)!(2R)^{2L}} \langle \mathcal{B}_{1/6}^B | \mathcal{O}_C \rangle, \quad (17)$$

where $|\mathcal{O}_C\rangle$ is the spin chain state corresponding to the operator \mathcal{O}_C .

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where $|\mathcal{O}_C\rangle$ is the spin chain state corresponding to the operator \mathcal{O}_C .

- Our convention for the Hermitian conjugation and the overlap of the spin chain states is

$$(\langle I_1 \bar{J}_1 \cdots I_L \bar{J}_L |)^{\dagger} = |I_1 \bar{J}_1 \cdots I_L \bar{J}_L \rangle, \quad (18)$$

$$\begin{aligned} \langle I_1 \bar{J}_1 \cdots I_L \bar{J}_L | M_1 \bar{N}_1 \cdots M_L \bar{N}_L \rangle &= \delta_{I_1 M_1} \delta^{J_1 N_1} \cdots \\ &\delta_{I_L M_L} \delta^{J_L N_L} \end{aligned} \quad (19)$$

Norm

- Let us define the normalization factor $\mathcal{N}_{\mathcal{O}}$ using the two-point function of \mathcal{O} and \mathcal{O}^\dagger as

$$\langle \mathcal{O}(x) \mathcal{O}^\dagger(y) \rangle = \frac{\mathcal{N}_{\mathcal{O}}}{|x - y|^{2\Delta_{\mathcal{O}}}}, \quad (20)$$

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where $\Delta_{\mathcal{O}}$ is the conformal dimension of \mathcal{O} .

- At tree level and the planar limit, we have

$$\mathcal{N}_{\mathcal{O}} = \left(\frac{N}{4\pi} \right)^{2L} L \langle \mathcal{O} | \mathcal{O} \rangle. \quad (21)$$

WL one-point function

- We define the Wilson-loop one-point function as

$$\langle\langle \mathcal{O} \rangle\rangle_{W(C)} \equiv \frac{\langle W(C) \mathcal{O} \rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}} . \quad (22)$$

- Then for $W_{1/6}^B$ we have

$$\langle\langle \mathcal{O} \rangle\rangle_{W(C)_{1/6}^B} = \frac{\pi^L \lambda^L}{R^{2L} (L-1)! \sqrt{L}} \frac{\langle \mathcal{B}_{1/6}^B | \mathcal{O} \rangle}{\sqrt{\langle \mathcal{O} | \mathcal{O} \rangle}} . \quad (23)$$

- The computation of the Wilson loop one-point function thus amounts to the calculation of

$$\frac{\langle \mathcal{B}_{1/6}^B | \mathcal{O} \rangle}{\sqrt{\langle \mathcal{O} | \mathcal{O} \rangle}} , \quad (24)$$

which will be done by integrability in some cases.

ABJM spin chain

- The operator $\mathcal{O}_C = C_{I_1 \dots I_L}^{J_1 \dots J_L} Y^{I_1} Y_{J_1}^\dagger \dots Y^{I_L} Y_{J_L}^\dagger$ can be mapped to a state $|C\rangle := C_{I_1 \dots I_L}^{J_1 \dots J_L} |I_1 \bar{J}_1 \dots I_L \bar{J}_L\rangle$ on an alternative closed $SU(4)$ spin chain with length $2L$.

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- The Hilbert space of this chain is $\mathbf{C}^{8L} = \bigotimes_{i=1}^{2L} \mathbf{C}^4$.

ABJM spin chain

- The operator $\mathcal{O}_C = C_{I_1 \dots I_L}^{J_1 \dots J_L} Y^{I_1} Y_{J_1}^\dagger \dots Y^{I_L} Y_{J_L}^\dagger$ can be mapped to a state $|C\rangle := C_{I_1 \dots I_L}^{J_1 \dots J_L} |I_1 \bar{J}_1 \dots I_L \bar{J}_L\rangle$ on an alternative closed $SU(4)$ spin chain with length $2L$.
- The Hilbert space of this chain is $\mathbf{C}^{8L} = \bigotimes_{i=1}^{2L} \mathbf{C}^4$.
- The odd site of the chain is in the $\mathbf{4}$ representation of $SU(4)$, while the even site is in the $\bar{\mathbf{4}}$ representation.

Hamiltonian

- The planar two-loop anomalous dimensional matrix can be map to the following Hamiltonian on the above chain ([Minahan, Zarembo, 08][Bak, Rey, 08]),

$$\mathbb{H} = \frac{\lambda^2}{2} \sum_{l=1}^{2L} (2 - 2P_{l,l+2} + P_{l,l+2}K_{l,l+1} + K_{l,l+1}P_{l,l+2}) , \quad (25)$$

where P_{ab} and K_{ab} are permutation and trace operators acting on the a -th and b -th sites. We denote the set of orthonormal basis of the Hilbert space at each site by $|i\rangle$, $i = 1, \dots, 4$. The two operators act as

$$P|i\rangle \otimes |j\rangle = |j\rangle \otimes |i\rangle, \quad K|i\rangle \otimes |j\rangle = \delta_{ij} \sum_{k=1}^4 |k\rangle \otimes |k\rangle . \quad (26)$$

Integrability

- In the algebraic Bethe ansatz (ABA) approach, we introduce the following R-matrices

$$\begin{aligned} R_{12}^{\bullet\bullet}(u) &= R_{12}^{\circ\circ}(u) = u + P_{12} \equiv R_{12}(u), \\ R_{12}^{\bullet\circ}(u) &= R_{12}^{\circ\bullet}(u) = -u - 2 + K_{12} \equiv \bar{R}_{12}(u), \end{aligned} \quad (27)$$

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- where \bullet denotes the states in the 4 representation of $SU(4)_R$, while \circ denotes the states in the $\bar{4}$ representation.
- These R -matrices satisfy a set of Yang-Baxter equations and the following crossing symmetry relation,

$$R_{12}(u)^{t_1} = \bar{R}_{12}(-u - 2), \quad \bar{R}_{12}(u)^{t_1} = R_{12}(-u - 2). \quad (28)$$

Integrability

- Using these R -matrices one can constructed two transfer matrices $\tau(u)$ and $\bar{\tau}(u)$, satisfying

$$[\tau(u), \tau(v)] = [\tau(u), \bar{\tau}(v)] = [\bar{\tau}(u), \bar{\tau}(v)] = 0. \quad (29)$$

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- This proves the integrability of two-loop ABJM spin chain.
[\[Minahan, Zarembo, 08\]](#)[\[Bak, Rey, 08\]](#)

Bethe roots

- Eigenstates of \mathbb{H} can be constructed using R -matrices and the states are parameterized by three set of Bethe roots,

$$u_1, \cdots, u_{K_{\mathbf{u}}}, \quad (30)$$

$$v_1, \cdots, v_{K_{\mathbf{v}}}, \quad (31)$$

$$w_1, \cdots, w_{K_{\mathbf{w}}}. \quad (32)$$

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- One selection rule for $\langle \mathcal{B}_R | \mathbf{u}, \mathbf{v}, \mathbf{w} \rangle$ being nonzero is that $K_{\mathbf{u}} = K_{\mathbf{v}} = K_{\mathbf{w}} = L$.

Bethe ansatz equations

- These Bethe roots should satisfy the following Bethe ansatz equations,

$$1 = \left(\frac{u_j + \frac{i}{2}}{u_j - \frac{i}{2}} \right)^L \prod_{\substack{k=1 \\ k \neq j}}^{K_u} S(u_j, u_k) \prod_{k=1}^{K_w} \tilde{S}(u_j, w_k), \quad (33)$$

$$1 = \prod_{\substack{k=1 \\ k \neq j}}^{K_w} S(w_j, w_k) \prod_{k=1}^{K_u} \tilde{S}(w_j, u_k) \prod_{k=1}^{K_v} \tilde{S}(w_j, v_k), \quad (34)$$

$$1 = \left(\frac{v_j + \frac{i}{2}}{v_j - \frac{i}{2}} \right)^L \prod_{\substack{k=1 \\ k \neq j}}^{K_v} S(v_j, v_k) \prod_{k=1}^{K_w} \tilde{S}(v_j, w_k), \quad (35)$$

Bethe ansatz equations

- In the previous page, the S-matrices $S(u, v)$ and $\tilde{S}(u, v)$ are given by

$$S(u, v) \equiv \frac{u - v - i}{u - v + i}, \quad \tilde{S}(u, v) \equiv \frac{u - v + \frac{i}{2}}{u - v - \frac{i}{2}}. \quad (36)$$

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- The eigenvalues of $\tau(u)$, $\bar{\tau}(u)$, \mathbb{H} on the Bethe state $|\mathbf{u}, \mathbf{v}, \mathbf{w}\rangle$ can be expressed in terms of the Bethe roots, $\mathbf{u}, \mathbf{v}, \mathbf{w}$.

IBS from WLs

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Theorem

If a four-dimensional matrix $K(u)$ satisfies the following boundary Yang-Baxter equation,

$$\begin{aligned} R_{12}(u-v)K_1(u)R_{12}(u+v)K_2(v) &= K_2(v)R_{12}(u+v) \\ K_1(u)R_{12}(u-v), \end{aligned} \quad (38)$$

the boundary state

$$|\mathcal{B}_M\rangle \equiv M^{I_1}_{J_1} M^{I_2}_{J_2} \cdots M^{I_L}_{J_L} |I_1, J_1, \cdots, I_L, J_L\rangle = (M^I_J |I, J\rangle)^{\otimes L}, \quad (39)$$

with $M = K(-1)$ is integrable in the sense explained in the next page.

A key selection rule

- When the condition of the theorem is satisfied, we have that $|\mathcal{B}_M\rangle$ satisfying the following untwisted integrable condition,

$$\tau(-u-2)|\mathcal{B}_M\rangle = \tau(u)|\mathcal{B}_M\rangle. \quad (40)$$

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- This leads to the pairing condition which states that $\langle \mathcal{B}_M | \mathbf{u}, \mathbf{v}, \mathbf{w} \rangle$ is non-zero only when the selection rule

$$\mathbf{u} = -\mathbf{v}, \quad \mathbf{w} = -\mathbf{w} \quad (41)$$

is satisfied.

IBS from WLs

- Using this theorem, we can prove that the boundary state from bosonic 1/6-BPS Wilson loop, $|\mathcal{B}_R\rangle$ is integrable.
- We just take $K(u) = R$. (Notice this R is the one appearing in the definition of $|\mathcal{B}_R\rangle$, it is not the R -matrices in the ABA approach.)
- Similarly we proved that the half-BPS and one-third BPS WLs give integrable boundary state.

Non-integrable boundary states

- For the boundary state from a generic(*) fermionic 1/6-BPS WL, we found numerically a Bethe state whose Bethe roots do not satisfy the pairing condition, but has non-zero overlap.

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Non-integrable boundary states

- For the boundary state from a generic(*) fermionic $1/6$ -BPS WL, we found numerically a Bethe state whose Bethe roots do not satisfy the pairing condition, but has non-zero overlap.
- This shows that this boundary state is not integrable.
- * By 'generic', we mean that this WL is neither half-BPS nor bosonic $1/6$ -BPS.

Overlaps

- We obtained the following formula for the normalized overlap between $|\mathcal{B}_R\rangle$ and a Bethe state,

$$\frac{|\langle \mathcal{B}_R | \mathbf{u}, \mathbf{v}, \mathbf{w} \rangle|^2}{\langle \mathbf{u}, \mathbf{v}, \mathbf{w} | \mathbf{u}, \mathbf{v}, \mathbf{w} \rangle} = \prod_{i=1}^{K_{\mathbf{w}}/2} \frac{w_i^2}{w_i^2 + 1/4} \times \frac{\det G^+}{\det G^-}. \quad (42)$$

- Here the Bethe roots satisfy the pairing condition, G^\pm are Gaudin determinants depending on $\mathbf{u}, \mathbf{v}, \mathbf{w}$.
- This result was obtained using [Gombor, Bajnok, 20][Gombor, Kristjansen, 22] and passed non-trivial checks based on numerical computations.

Summary

- By studying WL one-point function at tree level, we found that bosonic $1/6$ -BPS, half-BPS and $1/3$ -BPS WLs lead to integrable boundary states.

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Summary

- By studying WL one-point function at tree level, we found that bosonic $1/6$ -BPS, half-BPS and $1/3$ -BPS WLs lead to integrable boundary states.
- For generic fermionic $1/6$ -BPS WLs, the corresponding boundary states are not integrable.
- We computed the norm of the overlap of some integrable boundary states from WLs and the Bethe states.

Outlook

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- Correlators of BPS WLs and CPOs from localization and/or holography?

Thanks for Your Attention !