



## MTHS24 – Exercise sheet 5

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## Lecture material

### Discussed topics:

- Three-body decay kinematics
- Cascade parametrization of decays
- Helicity and covariant formalism

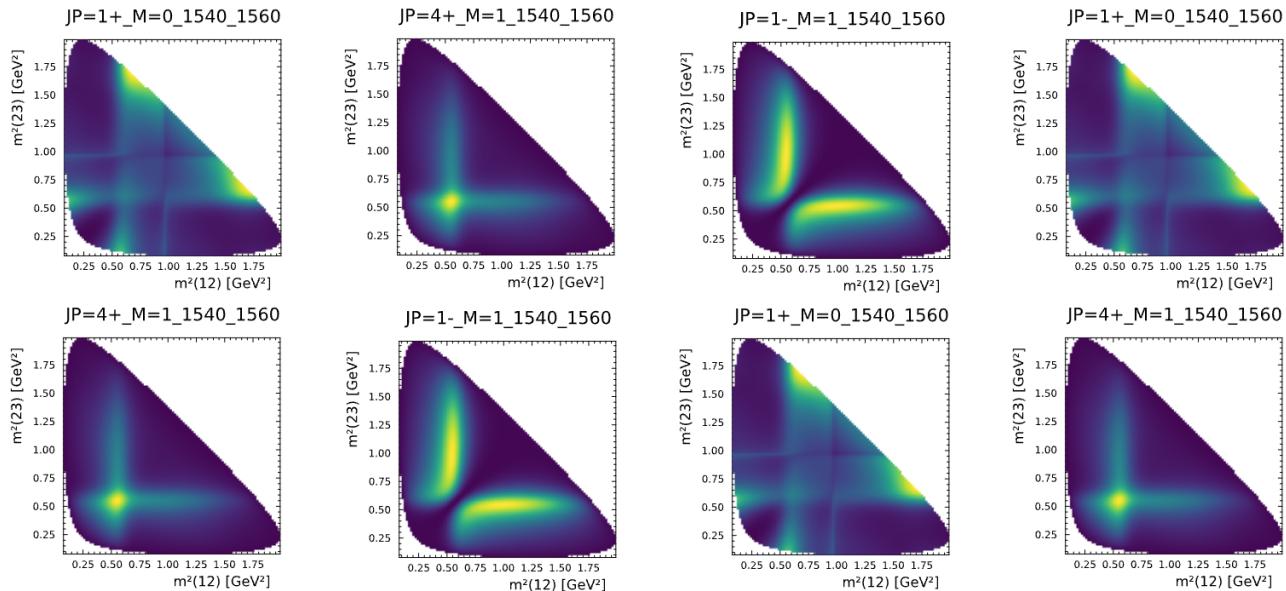
### References:

- A.D. Martin, T.D. Spearman, Elementary Particle Theory, [inSpire](#)
- Eero Byckling, K. Kajantie, Particle Kinematics, [inSpire](#)

## Exercises

### 5.1 Projections of the Dalitz Plot

For a given Dalitz plot, sketch the projections onto  $m_{12}^2$ ,  $m_{23}^2$ ,  $m_{31}^2$ , and helicity angles for all subsystems rest frames.



### 5.2 Spin Sum

Polarization vectors of a spin-one particle are given by

$$\epsilon_{\pm 1}^{\mu}(\theta) = \left( 0, \mp \frac{\cos \theta}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, \pm \frac{\sin \theta}{\sqrt{2}} \right) \quad (1)$$

$$\epsilon_0^{\mu}(\theta) = \left( \frac{p}{m}, \frac{E}{m} \sin \theta, 0, \frac{E}{m} \cos \theta \right) \quad (2)$$

- (a) Evaluate the amplitude for the decay of  $a_1$  meson ( $J^P = 1^+$ ) to  $\rho\pi$  using the covariant expression,  $\mathcal{M} = \epsilon_{a_1} \cdot \epsilon_\rho$  in the following frames of reference:

- $a_1$  is at rest and the decay particles are aligned along the  $z$ -axis,
- $\rho$  is at rest and  $a_1$  and  $\pi$  are aligned along the  $z$ -axis, compare to (a).

**Solution:** The first one is  $-(1, \gamma_\rho, 1)$ , the second one is  $-(1, \gamma_{a_1}, 1)$ . Both equal since the two frames are connected by a boost with  $\gamma = (m_{a_1}^2 + m_\rho^2 - m_\pi^2)/(2m_{a_1}m_\rho)$ .

- (b) Compare three matrix elements in (a) configurations to the expectations from the helicity formalism,

$$A_{\lambda_{a_1} \lambda_\rho}^L = H_{\lambda_\rho, 0}^L d_{\lambda_{a_1}, \lambda_\rho}^1(\theta), \quad (3)$$

where the helicity coupling is parametrized in LS scheme reads,

$$H_{\lambda_\rho, 0}^L = \langle L, 0; 1, \lambda_\rho | 1, \lambda_\rho \rangle. \quad (4)$$

Which partial waves are allowed in the decay, and what value of  $L$  the covariant matrix  $\mathcal{M}$  element correspond to?

**Solution:** The S-wave corresponds to  $(1, 1, 1)$ , while D-wave is  $(1, -2, 1)/\sqrt{10}$ . Covariant formalism mixes partial waves.

### 5.3 Spin of a New $\Lambda_b^{**0}$ State

A new  $\Lambda_b^{**0}$  state has been discovered decaying into  $\Lambda_b^0 \pi^+ \pi^-$  with a prominent  $\Sigma_b^*$  resonance line on the Dalitz plot. The decay intensity distribution along the  $\Sigma_b^*$  band is provided in the supplementary material, which includes the helicity angle distribution. Your task is to determine the spin  $J$  of the  $\Lambda_b^{**0}$  state.

- Write down the decay matrix element for  $\Lambda_b^{**0} \rightarrow \Lambda_b^0 \pi^+ \pi^-$  using helicity formalism.
- Identify the partial waves in the decay  $\Sigma_b^* \rightarrow \Lambda_b^0 \pi$ .
- Determine the partial waves in the decay  $\Lambda_b^{**0} \rightarrow \Sigma_b^* \pi$  for  $J^P = \frac{1}{2}^\pm, \frac{3}{2}^\pm, \frac{5}{2}^\pm$ .
- Compute the unpolarized differential distribution given by:

$$\frac{dI}{d \cos \theta} = \sum_{\lambda_0, \lambda_1}^{\{-1/2, 1/2\}} \left| \langle L, 0; 3/2, \lambda_0 | J, \lambda_0 \rangle d_{\lambda_0, \lambda_1}^{3/2}(\theta) \langle 1, 0; 1/2, \lambda_1 | 3/2, \lambda_1 \rangle \right|^2$$