



MTHS24 – Exercise sheet 7

Morning: Vincent Mathieu / Arkaitz Rodas

Afternoon: Daniel Winney, Whytt Smith

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Lecture material

Discussed topics:

- Partial waves
- Analyticity
- Unitarity

References:

- Book 1, [inspire](#)
- Book 2, [amazon](#)
- A good review, [inspire](#)
- A good aper, [inspire](#)

Exercises

7.1 Two-body phase space

- (a) Particle A with mass M decays into two daughters with masses m_1 and m_2 . Derive the formula for the *break up momentum*, $\mathbf{p}^* = |\mathbf{p}_1| = |\mathbf{p}_2|$, in the rest frame of A . Use your result to evaluate \mathbf{p}^* for the decay $\Delta(1232) \rightarrow p\pi$.
- (b) Starting from the formula for the 2-body decay rate,

$$\Gamma_{fi} = \frac{1}{2M} \int |\mathcal{M}_{fi}|^2 \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(P - p_1 - p_2),$$

perform integrations using δ functions to obtain $d\Gamma/d\Omega$ in the centre of mass frame, $P = \begin{pmatrix} M \\ \vec{0} \end{pmatrix}$, to obtain the 2-body phase-space factor.

Hint: You may want to use the following property of the delta function:

$$\int_{-\infty}^{+\infty} g(x) \delta(f(x)) dx = \sum_{x_0} \frac{g(x_0)}{|df/dx|_{x_0}}, \text{ where } x_0 \in \{x : f(x) = 0\}.$$