



## MTHS24 – Exercise sheet 6

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## Lecture material

### Discussed topics:

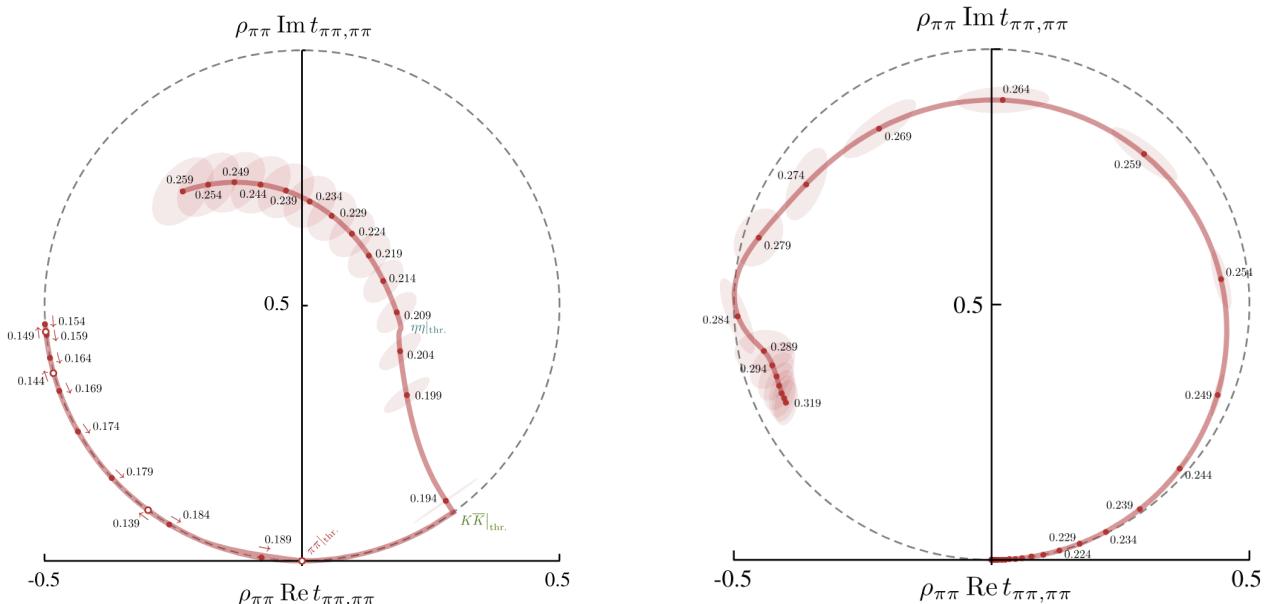
- Lippmann-Schwinger equation, Bethe-Salpeter equation, and K-matrix
- Lineshape analysis and Breit-Wigner formula
- Complex algebra, dispersion relations
- Analytic continuation and pole search
- Khury-Treiman equations

- ### References:
- A.D. Martin, T.D. Spearman, Elementary Particle Theory, [inSpire](#)
  - Review on Novel approaches in hadron spectroscopy by JPAC, [inspire](#)

## Exercises

### 6.1 Argand diagrams from lattice

The  $\pi\pi$  scattering with unphysical pion mass ( $m_\pi = 391$  MeV) for S (left) and D (right) partial waves is studied using [lattice calculations](#). Scattering amplitudes are presented on the Argand diagrams (parametric plot of energy in Real/Imaginary coordinates) as a function of energy of the system. The values are given units of  $E_{\text{cm}} \cdot t$  where  $t \cdot m_\pi = 0.06906$ .



Using information on the diagrams, answer the following questions:

- Estimate masses of  $K$  and  $\eta$  particles.

- (b) Find the elastic energy region for the S and D waves.

**Solution:** Elastic region is defined as the range of energy values for which  $\pi\pi \rightarrow \pi\pi$  process dominates. For the S-wave, the elastic region lies for the  $E_{\text{cm}}t$  range of [0.139, 0.189] (after this point, the amplitude hits the  $K\bar{K}$  threshold. Also, after this point, the curve starts going inside the unitarity circle). For the D-wave, this region exists until value of 0.229.

- (c) Locate the energy value for which the S-wave peak.

**Solution:** The S-wave peaks at the point  $E_{\text{cm}}t = 0.154$ , which is at energy  $E_{\text{cm}} = 0.872$  GeV.

- (d) Estimate the mass and decay width for the D wave resonance.

**Solution:** The D-wave resonance is observed at  $E_{\text{cm}}t = 0.284$ , this is the point where there is a kink in the argand diagram. *Note : I can locate where the resonance is. I am confused about how to proceed from there, because I can get center of mass energy from the point, and maybe equate it to the pole value. But I would expect a complex output but I cannot read it out properly from the Argand diagram.*

- (e) Sketch the amplitude phase versus energy of the system for both partial waves.

## 6.2 Escape room in the complex plane

- (a) Characterize the complex structure of functions  $\sqrt{x}$  and  $\log(-x)$  by finding the branch points, branch cuts and number of complex (Riemann) sheets in the complex plane.
- (b) Repeat (a) for a function  $f(x) = \sqrt{x} - \sqrt{x-1}$ .
- (c) Construct a complex function with two branch points at  $+i$  and  $-i$  connected by a branch cut.
- (d) Locate zeros of the function  $g(z) = \sqrt{z} + i + 1$ .
- (e) Find residue of the function  $1/g(z)$  by computing a circular integral about the complex pole.