MTHS24 – Exercise sheet 7

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Lecture material

Discussed topics:
- Correlation functions
- Statistics and the Jackknife
- Finite-volume symmetry
- GEVP

References:
- A personal favorite: Lattice methods for Quantum Chromodynamics (T. Degrand, C. DeTar), available here
- Quantum Chromodynamics on the Lattice (C. Gattringer, C. Lang), available here
- Lattice Quantum Chromodynamics: Practical Essentials (F. Knechtli, M. Günther, M. Peardon), available here

Exercices

7.1 Single-meson operators

Using the following transformation properties for spinors under charge and parity conjugation

$$\psi\left(\vec{x},t\right) \xrightarrow{\mathcal{P}} \psi\left(\vec{x},t\right)^{\mathcal{P}} = \gamma_4 \psi\left(-\vec{x},t\right) \qquad \bar{\psi}\left(\vec{x},t\right) \xrightarrow{\mathcal{P}} \bar{\psi}\left(\vec{x},t\right)^{\mathcal{P}} = \bar{\psi}\left(-\vec{x},t\right)\gamma_4,$$

$$\psi(x) \xrightarrow{\mathcal{C}} \psi(x)^{\mathcal{C}} = C^{-1}\bar{\psi}(x)^T \qquad \bar{\psi}(x) \xrightarrow{\mathcal{C}} \bar{\psi}(x)^{\mathcal{C}} = -\psi(x)^T C \qquad C\gamma_\mu C^{-1} = -\gamma_\mu^T.$$

Prove that the π^+ operator seeing in the lectures does indeed behave as expected. What about a ρ-type meson operator?

7.2 Single-meson correlation function

Let's study the functional form os single-particle correlation functions. To do so, start from the definition of the 2-pt correlator for finite T extension:

$$\left\langle O(t)O^\dagger(0)\right\rangle_T = \frac{1}{Z_T}\,\mathrm{tr}(e^{-T\widehat{H}}O(t)O^\dagger(0)), \qquad Z_T = \mathrm{tr}(e^{-TH}). \tag{1}$$

First, use the Heisenberg picture to describe the temporal evolution of the operators to arrive at

$$\left\langle O(t)O^\dagger(0)\right\rangle_T = \frac{1}{Z_T}\sum_{m,n} e^{-(T-t)E_m}e^{-tE_n}|\langle m|O|n\rangle|^2. \tag{2}$$

Now, assume that within the complete set of states there are both particles and antiparticles so that $\langle m|O|n\rangle = \langle \bar{n}|O|\bar{m}\rangle$, what formula do you get?
Ext

7.3 Bias and the Jackknife

We are gonna study now how bias arises and how to correct it with the Jackknife. First, given our original sample x, with mean $\langle x \rangle = X$, we note that, unless the function f is linear

$$\langle f(x) \rangle \neq f(X). \tag{3}$$

We call the difference between these two quantities bias. In practice, in a sampling problem X is not exactly known but approximated via the number of samples in x. We would like to propagate this information to our final observables without introducing statistical bias. First, by expanding the difference between $f(x)$ and $f(X)$ to second order, prove that the bias is proportional to the variance σ^2 of x. In the Jackknife we have

$$\widehat{x}_i \equiv \frac{1}{N-1} \sum_{k \neq i} x_k = X + \frac{1}{N-1} \sum_{k \neq i} \delta x_k \quad \delta x_k = x_k - X. \tag{4}$$

Now, using that $\widehat{f}_i = f(\widehat{x}_i)$, expand it to second order again and prove that $\langle \widehat{f}_i \rangle - f(X) = \frac{1}{2(N-1)} f''(X) \sigma^2$, the bias is now of order $\mathcal{O}(1/N)$. Is there a way you can combine both results to reduce the bias an extra order?

7.4 Character tables

- Using what we have seen in the lectures, construct the character table for C_{3v}. To do so, start by listing all classes and numbers of symmetry operations.
- Then use the Schur's lemma to obtain which irreps are represented in the table and get their characters.

7.5 Lets code!

In this practical exercise we perform correlated fits to MonteCarlo sampled two-point correctors, calculated from Wilson lines for a $L = 32$ lattice with periodic boundary conditions.

- Close the code repository, https://github.com/JointPhysicsAnalysisCenter/MTHS-Code
- Navigate to the folder `07-22-Lattice/` from within the repo
- Install `conda`, all dependencies of the project following tips on README.md
- Launch Jupyter Lab from the terminal
- In your web browser, navigate to the Jupyter Lab interface to '07-22-Lattice/fit_correlators.ipynb'
- Follow the instructions in the notebook to try and obtain a good set of correlated Jackknife fits to the correlators provided in the folder.