



## MTHS24 – Exercise sheet 11

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Afternoon:



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## Lecture material

### References:

#### Discussed topics:

- Functional methods
- Dynamical Chiral Symmetry Breaking
- Spectra of conventional and exotic hadrons
- (optional: g-2, form factors,...)

- Eichmann et al., “Baryons as relativistic three-quark bound states,” PPNP **91** (2016), 1-100  
[arXiv:1606.09602 \[hep-ph\]](https://arxiv.org/abs/1606.09602).
- Eichmann et al. “Four-Quark States from Functional Methods,” FBS **61** (2020) no.4, 38  
[arXiv:2008.10240 \[hep-ph\]](https://arxiv.org/abs/2008.10240).

## Exercises

### 11.1 Diquarks

Write down spin, color and flavour wave functions for a scalar and an axialvector diquark built from

- (a) two light quarks (what is the resulting isospin ?)
- (b) two strange, charm or bottom quarks
- (c) a heavy-(not-so-heavy) combination such as bc, bs or cs.

*Hint: carefully think about symmetries...*

**Solution: I.) Scalar diquarks:**

Spin S=0  $\rightarrow \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)$  and antisymmetric

Color: From Young-Tableaux we find  $3 \otimes 3 = 6 \oplus \bar{3}$  and  $\bar{3}$  is antisymmetric, while 6 is symmetric.

Thus we need an antisymmetric flavour wave function together with  $\bar{3}$ -color and a symmetric flavour wave function together with 6-color.

We obtain for  $\bar{3}$ -color:

- (a)  $\frac{1}{\sqrt{2}} (ud - du)$  and we have  $I=0$ .
- (b) not possible
- (c)  $\frac{1}{\sqrt{2}} (bc - cb)$  and analogously for the others.

We obtain for 6-color:

- (a)  $\{\frac{1}{\sqrt{2}} (ud + du), uu, dd\}$  and we have  $I=1$ .
- (b)  $ss, cc, bb$
- (c)  $\frac{1}{\sqrt{2}} (bc + cb)$  and analogously for the others.

**II.) Axialvector diquarks:**

Spin S=1  $\rightarrow \{\frac{1}{\sqrt{2}} (\uparrow\downarrow + \downarrow\uparrow), \uparrow\uparrow, \downarrow\downarrow\}$  and symmetric

Color: Same as above. But now we need a symmetric flavour wave function together with  $\bar{3}$ -color and an antisymmetric flavour wave function together with 6-color.

Thus, flavour/color combinations are interchanged as compared to scalar diquark.

## 11.2 Four-quark states

Now think about a four-quark state with two heavy quarks and two light anti-quarks in the two flavour combinations  $bb\bar{q}\bar{q}$  and  $bc\bar{q}\bar{q}$ . Suppose, the quarks and antiquarks are arranged in scalar (S) and axialvector (A) diquarks. Which diquark combinations are possible for the following quantum numbers?

- (a)  $I(J) = 0(1)$

**Solution:**  $J = 1 \rightarrow$  we need at least one axialvector diquark, i.e. only combinations  $AA, SA, AS$  are possible.

color  $3 \otimes \bar{3}$ :

$I = 0 \rightarrow$  light diquark needs to be S  $\rightarrow$  heavy diquark needs to be A and indeed, this is possible

color  $6 \otimes \bar{6}$ :

$I = 0 \rightarrow$  light diquark needs to be A  $\rightarrow$  heavy diquark needs to be S and indeed, this is possible

- (b)  $I(J) = 1(1)$

**Solution:**  $J = 1 \rightarrow$  we need at least one axialvector diquark, i.e. only combinations  $AA, SA, AS$  are possible.

color  $3 \otimes \bar{3}$ :

$I = 1 \rightarrow$  light diquark needs to be A. Heavy diquark also needs to be A (S is not possible).

color  $6 \otimes \bar{6}$ :

$I = 1 \rightarrow$  light diquark needs to be S  $\rightarrow$  heavy diquark needs to be A, but this is not possible.

- (c)  $I(J) = 0(0)$

**Solution:**  $J = 0 \rightarrow$  we need either  $SS$  or  $AA$  (from rules of adding angular momenta).

color  $3 \otimes \bar{3}$ :

$I = 0 \rightarrow$  light diquark needs to be S  $\rightarrow$  heavy diquark also needs to be S, but this is not possible.

color  $6 \otimes \bar{6}$ :

$I = 0 \rightarrow$  light diquark needs to be A  $\rightarrow$  heavy diquark also needs to be A, but this is not possible.

*Hint: again carefully think about symmetries...*