



## MTHS24 – Exercise sheet 10

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## Lecture material

### References:

- Original Lüscher paper (stable particles): [inspire](#) - Lüscher (1985)
- Original Lüscher paper (scattering): [inspire](#) - Lüscher (1986)
- Non-rest frames: [inspire](#) - Rummukainen & Gottlieb (1995)
- Field theory approach: [inspire](#) - Kim, Sharpe, & Sachrajda (2005)
- Coupled-channels: [inspire](#) - Hansen & Sharpe (2012)
- Arbitrary number of channels and spin: [inspire](#) - Briceno (2014)

### Discussed topics:

- Two-particles quantization condition
- Finite-volume spectrum
- Interacting energy levels from resonances

## Exercises

### 10.1 Two Particles in a Box

In this problem, we explore the discrete spectrum of two particles in a finite cubic volume and its relation to their infinite volume scattering amplitude.

- (a) Enumerate the different momenta (in units of  $2\pi/L$ ) allowed for  $\mathbf{n}^2 \in \{0, 1, 2, 3, 4, 5, 6, 8, 9\}$ .
- (b) Take the limit of  $E/m$  as  $mL \rightarrow \infty$ .
- (c) Plot the non-interacting spectrum in terms of  $E^*/m$  in the rest frame  $\mathbf{n}_P = [000]$  as a function of  $mL$ .
- (d) Repeat (c) for the frames  $\mathbf{n}_P = [001], [011], [111], [002]$ .

### 10.2 Finite-Volume Function

This problem focuses on the finite-volume function  $F(E, \mathbf{P}, L)$ , which characterizes finite-volume distortions in an interacting system.

- (a) Derive  $F(E, \mathbf{P}, L)$  using the all-orders approach discussed in the lectures. Simplify the result for numerical computation.
- (b) Determine the dimensions of  $F$ .
- (c) For a system at total momentum  $\mathbf{P} = 0$ , plot  $F$  as a function of  $E^*/m, (Lq^*/2\pi)$  for fixed  $mL = 4, 5$ , and 6.

- (d) Repeat (c) for moving frame systems,  $\mathbf{n}_P = [001], [011], [111], [002]$ .

### 10.3 Connecting the Finite-Volume Function to the Spectrum

In this problem we explore how to determine the spectrum of two non-interacting particles by solving  $F^{-1} = 0$ .

- (a) Find the spectrum of two non-interacting particles in their rest frame by solving  $F^{-1} = 0$  for fixed  $mL = 4, 5$ , and  $6$ .
- (b) Repeat for moving frame systems,  $\mathbf{n}_P = [001], [011], [111], [002]$ .

### 10.4 Lüscher Quantization Condition

Here, we study the poles of the correlation matrix for interacting particles using the Lüscher quantization condition.

- (a) Show that the imaginary parts of  $\mathcal{M}^{-1}$  and  $F$  cancel.
- (b) Using the Breit-Wigner and Effective Range parameterizations, investigate the spectrum of an interacting two-particle system.