



MTHS24 – Exercise sheet 5

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Lecture material

Discussed topics:

- Three-body decay kinematics
- Cascade parametrization of decays
- Helicity and covariant formalism

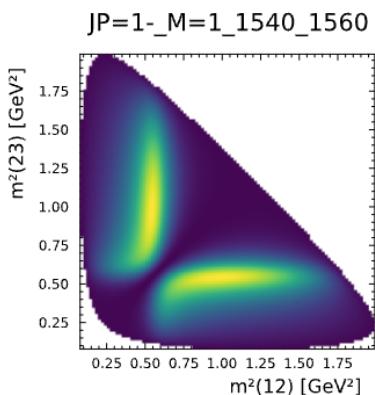
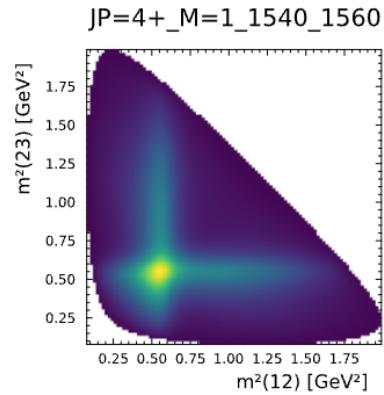
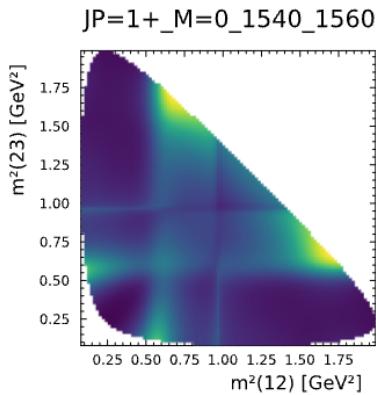
References:

- A.D. Martin, T.D. Spearman, Elementary Particle Theory, [inSpire](#)
- Eero Byckling, K. Kajantie, Particle Kinematics, [inSpire](#)

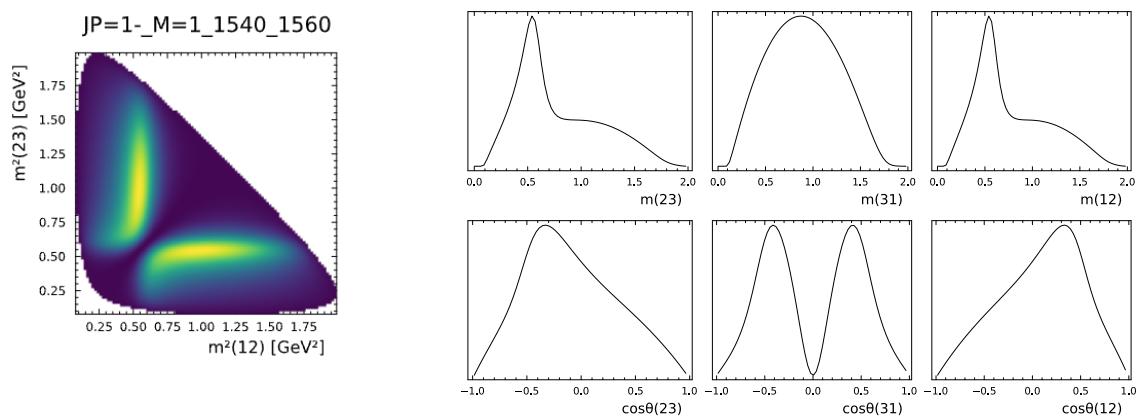
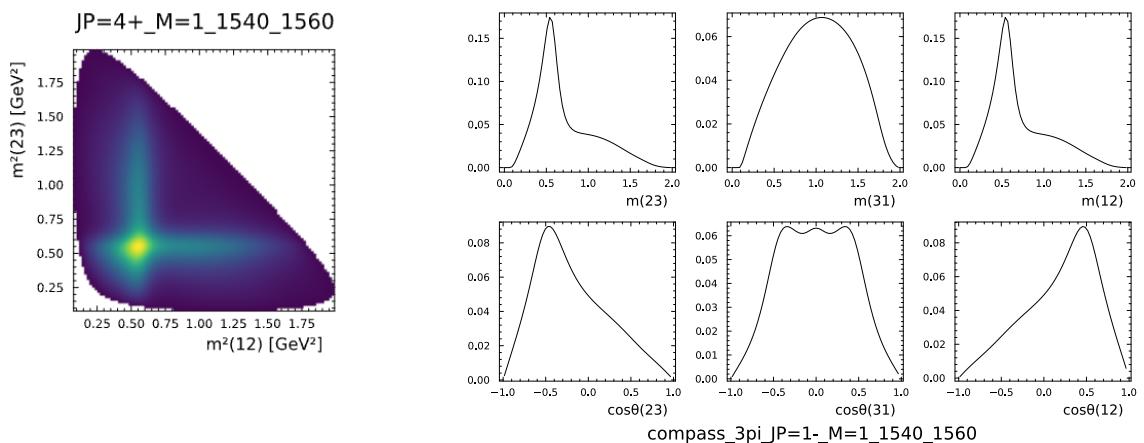
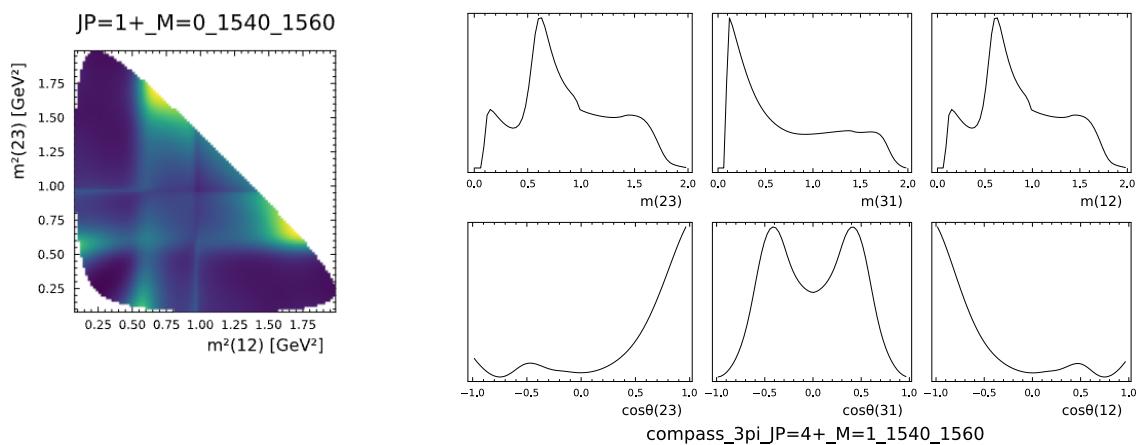
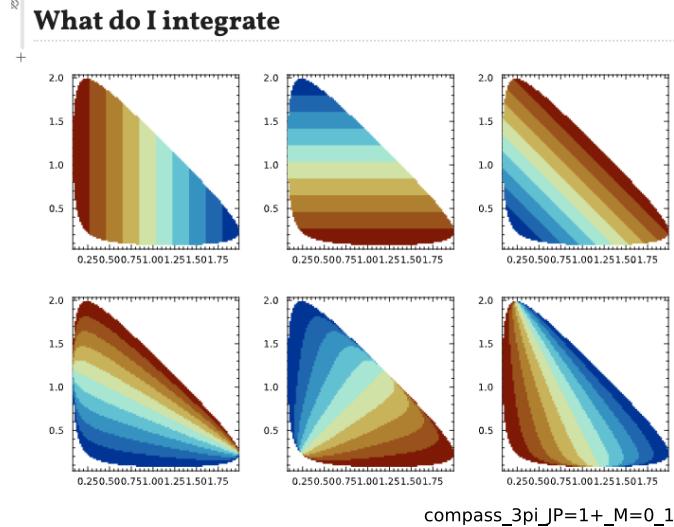
Exercises

5.1 Projections of the Dalitz Plot

For a given Dalitz plot, sketch the projections onto m_{12}^2 , m_{23}^2 , m_{31}^2 , and helicity angles for all subsystems rest frames.



Solution:



5.2 Spin Sum

Polarization vectors of a spin-one particle are given by

$$\epsilon_{\pm 1}^\mu(\theta) = \left(0, \mp \frac{\cos \theta}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, \pm \frac{\sin \theta}{\sqrt{2}}\right) \quad (1)$$

$$\epsilon_0^\mu(\theta) = \left(\frac{p}{m}, \frac{E}{m} \sin \theta, 0, \frac{E}{m} \cos \theta\right) \quad (2)$$

- (a) Evaluate the amplitude for the decay of a_1 meson ($J^P = 1^+$) to $\rho\pi$ using the covariant expression, $\mathcal{M} = \epsilon_{a_1} \cdot \epsilon_\rho$ in the following frames of reference:

- a_1 is at rest and the decay particles are aligned along the z -axis,
- ρ is at rest and a_1 and π are aligned along the z -axis, compare to (a).

Solution: The first one is $-(1, \gamma_\rho, 1)$, the second one is $-(1, \gamma_{a_1}, 1)$. Both equal since the two frames are connected by a boost with $\gamma = (m_{a_1}^2 + m_\rho^2 - m_\pi^2)/(2m_{a_1}m_\rho)$.

- (b) Compare three matrix elements in (a) configurations to the expectations from the helicity formalism,

$$A_{\lambda_{a_1} \lambda_\rho}^L = H_{\lambda_\rho, 0}^L d_{\lambda_{a_1}, \lambda_\rho}^1(\theta), \quad (3)$$

where the helicity coupling is parametrized in LS scheme reads,

$$H_{\lambda_\rho, 0}^L = \langle L, 0; 1, \lambda_\rho | 1, \lambda_\rho \rangle. \quad (4)$$

Which partial waves are allowed in the decay, and what value of L the covariant matrix \mathcal{M} element correspond to?

Solution: The S-wave corresponds to $(1, 1, 1)$, while D-wave is $(1, -2, 1)/\sqrt{10}$. Covariant formalism mixes partial waves.

5.3 Spin of a New Λ_b^{**0} State

A new Λ_b^{**0} state has been discovered decaying into $\Lambda_b^0 \pi^+ \pi^-$ with a prominent Σ_b^* resonance line on the Dalitz plot. The decay intensity distribution along the Σ_b^* band is provided in the supplementary material, which includes the helicity angle distribution. Your task is to determine the spin J of the Λ_b^{**0} state.

- (a) Write down the decay matrix element for $\Lambda_b^{**0} \rightarrow \Lambda_b^0 \pi^+ \pi^-$ using helicity formalism.
- (b) Identify the partial waves in the decay $\Sigma_b^* \rightarrow \Lambda_b^0 \pi$.
- (c) Determine the partial waves in the decay $\Lambda_b^{**0} \rightarrow \Sigma_b^* \pi$ for $J^P = \frac{1}{2}^\pm, \frac{3}{2}^\pm, \frac{5}{2}^\pm$.
- (d) Compute the unpolarized differential distribution given by:

$$\frac{dI}{d \cos \theta} = \sum_{\lambda_0, \lambda_1}^{\{-1/2, 1/2\}} \left| \langle L, 0; 3/2, \lambda_0 | J, \lambda_0 \rangle d_{\lambda_0, \lambda_1}^{3/2}(\theta) \langle 1, 0; 1/2, \lambda_1 | 3/2, \lambda_1 \rangle \right|^2$$