



## MTHS24 – Exercise sheet 7

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## Lecture material

### References:

- A personal favorite: Lattice methods for Quantum Chromodynamics (T. Degrand, C. DeTar), [available here](#)
- Quantum Chromodynamics on the Lattice (C. Gattringer, C. Lang), [available here](#)
- Lattice Quantum Chromodynamics: Practical Essentials (F. Knechtli, M. Günther, M. Peardon), [available here](#)

### Discussed topics:

- Correlation functions
- Statistics and the Jackknife
- Finite-volume symmetry
- GEVP

## Exercises

### 7.1 Single-meson operators

Using the following transformation properties for spinors under charge and parity conjugation

$$\begin{aligned} \psi(\vec{x}, t) &\xrightarrow{\mathcal{P}} \psi(\vec{x}, t)^{\mathcal{P}} = \gamma_4 \psi(-\vec{x}, t) & \bar{\psi}(\vec{x}, t) &\xrightarrow{\mathcal{P}} \bar{\psi}(\vec{x}, t)^{\mathcal{P}} = \bar{\psi}(-\vec{x}, t) \gamma_4, \\ \psi(x) &\xrightarrow{\mathcal{C}} \psi(x)^{\mathcal{C}} = C^{-1} \bar{\psi}(x)^T & \bar{\psi}(x) &\xrightarrow{\mathcal{C}} \bar{\psi}(x)^{\mathcal{C}} = -\psi(x)^T C & C \gamma_{\mu} C^{-1} &= -\gamma_{\mu}^T. \end{aligned}$$

Prove that the  $\pi^+$  operator seeing in the lectures does indeed behave as expected. What about a  $\rho$ -type meson operator?

### 7.2 Single-meson correlation function

Let's study the functional form of single-particle correlation functions. To do so, start from the definition of the 2-pt correlator for finite  $T$  extension:

$$\langle O(t)O^\dagger(0) \rangle_T = \frac{1}{Z_T} \text{tr}(e^{-T\hat{H}} O(t) O^\dagger(0)), \quad Z_T = \text{tr}(e^{-TH}). \quad (1)$$

First, use the Heisenberg picture to describe the temporal evolution of the operators to arrive at

$$\langle O(t)O^\dagger(0) \rangle_T = \frac{1}{Z_T} \sum_{m,n} e^{-(T-t)E_m} e^{-tE_n} |\langle m|O|n\rangle|^2. \quad (2)$$

Now, assume that within the complete set of states there are both particles and antiparticles so that  $\langle m|O|n\rangle = \langle \bar{n}|O|\bar{m}\rangle$ , what formula do you get?

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### 7.3 Bias and the Jackknife

We are gonna study now how bias arises and how to correct it with the Jackknife. First, given our original sample  $x$ , with mean  $\langle x \rangle = X$ , we note that, unless the function  $f$  is linear

$$\langle f(x) \rangle \neq f(X). \quad (3)$$

We call the difference between these two quantities bias. In practice, in a sampling problem  $X$  is not exactly known but approximated via the number of samples in  $x$ . We would like to propagate this information to our final observables without introducing statistical bias. First, by expanding the difference between  $f(x)$  and  $f(X)$  to second order, prove that the bias is proportional to the variance  $\sigma^2$  of  $x$ . In the Jackknife we have

$$\hat{x}_i \equiv \frac{1}{N-1} \sum_{k \neq i} x_k = X + \frac{1}{N-1} \sum_{k \neq i} \delta x_k \quad \delta x_k = x_k - X. \quad (4)$$

Now, using that  $\hat{f}_i = f(\hat{x}_i)$ , expand it to second order again and prove that  $\langle \hat{f}_i \rangle - f(X) = \frac{1}{2(N-1)} f''(X) \sigma^2$ , the bias is now of order  $\mathcal{O}(1/N)$ . Is there a way you can combine both results to reduce the bias an extra order?

### 7.4 Character tables

- Using what we have seen in the lectures, construct the character table for  $C_{3v}$ . To do so, start by listing all classes and numbers of symmetry operations.
- Then use the Schur's lemma to obtain which irreps are represented in the table and get their characters.

## 7.5 Lets code!

In this practical exercise we perform correlated fits to MonteCarlo sampled two-point correctors, calculated from Wilson lines for a  $L = 32$  lattice with periodic boundary conditions.

- Close the code repository

```
cd ~/Documents  
git clone https://github.com/JointPhysicsAnalysisCenter/MTHS-code MTHS-code  
cd 07-22-Lattice
```

- Downloading the folder Code/sheet\_7\_code from within the school's github repo [here](#)

**(a) Install Python in a Conda Environment with Dependencies**

Make sure you have conda installed. Easiest way for homebrew users is "brew install --cask anaconda". For linux, follow these instructions [here](#)

Create a new conda environment and install the required packages:

```
conda create --name lattice-qcd python=3.x  
conda activate lattice-qcd  
conda install numpy scipy matplotlib iminuit jupyterlab
```

**(b) Start Jupyter Lab and Open the Notebook**

**(c) Launch Jupyter Lab from the terminal:**

```
jupyter lab
```

In your web browser, navigate to the Jupyter Lab interface. Open the notebook file inside the folder you just downloaded.

**(d) Jupyter Notebook Cheat Sheet**

For basic commands and shortcuts in Jupyter Notebook, refer to the [Jupyter Notebook Cheat Sheet](#).

- Once you do so, open the .ipynb inside. Note that the first two cells just load packages and fixed variables, described in the notebook.
- Now, follow the instructions in the notebook to try and obtain a good set of correlated Jackknife fits to the correlators provided in the folder.