



MTHS24 – Exercise sheet 11

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Afternoon:



Friday, 26 July 2024

Lecture material

References:

- Eichmann et al., “Baryons as relativistic three-quark bound states,” PPNP **91** (2016), 1-100
[arXiv:1606.09602 \[hep-ph\]](https://arxiv.org/abs/1606.09602).
- Eichmann et al. “Four-Quark States from Functional Methods,” FBS **61** (2020) no.4, 38
[arXiv:2008.10240 \[hep-ph\]](https://arxiv.org/abs/2008.10240).
- Michele Maggiore, “Modern Introduction to Quantum Field Theory”, Oxford University Press

Exercises

11.1 Diquarks

Write down spin, color and flavour wave functions for a scalar and an axialvector diquark built from

- (a) two light quarks (what is the resulting isospin ?)
- (b) two strange, charm or bottom quarks
- (c) a heavy-(not-so-heavy) combination such as bc, bs or cs.

Hint: carefully think about symmetries...

11.2 Four-quark states

Now think about a four-quark state with two heavy quarks and two light anti-quarks in the two flavour combinations $bb\bar{q}\bar{q}$ and $bc\bar{q}\bar{q}$. Suppose, the quarks and antiquarks are arranged in scalar (S) and axialvector (A) diquarks. Which diquark combinations are possible for the following quantum numbers?

- (a) $I(J) = 0(1)$
- (b) $I(J) = 1(1)$
- (c) $I(J) = 0(0)$

Hint: again carefully think about symmetries...

11.3 Pion Bethe-Salpeter Equation

The Bethe-Salpeter vertex function Γ_π of a pion can be expressed most generally by

$$\Gamma_\pi(P, p) = \sum_{i=1}^4 f_i(P, p) T_i \quad (1)$$

with tensors $T_1 = \gamma_5 \mathbb{1}$, $T_2 = \gamma_5 \hat{\mathcal{P}}$, $T_3 = \gamma_5 \not{p}$ and $T_4 = \gamma_5 [\hat{\mathcal{P}}, \not{p}]$ in Dirac-space. Here we use the normalised total momentum \hat{P} of the pion and the orthogonalised relative momentum p between quark and antiquark, i.e. $\hat{P} \cdot \hat{P} = 1$ and $p \cdot P = 0$.

The Pauli-Lubanski vector (see e.g. Maggiore, chapter 2.7) can be used to determine the spin and angular momentum quantum numbers of these tensors in the rest frame of the pion. Its square can be separated in a part referring to angular momentum and a part referring to spin:

$$L^2 = 2p^\alpha \frac{\partial}{\partial p^\alpha} + \left(p_T^\alpha p_T^\beta - p_T^2 T_p^{\alpha\beta} \right) \frac{\partial}{\partial p^\alpha} \frac{\partial}{\partial p^\beta} \quad (2)$$

$$[S^2]_{i,j}^{k,l} = \frac{3}{2} \mathbb{1}_{i,j} \otimes \mathbb{1}_{k,l} - \frac{1}{2} \left(\gamma_T^\mu \gamma_5 \hat{\mathcal{P}} \right)_{i,j} \otimes \left(\hat{\mathcal{P}} \gamma_5 \gamma_T^\mu \right)_{k,l} \quad (3)$$

Here, $\gamma_T^\mu = \gamma^\mu - \hat{P}^\mu \hat{\mathcal{P}}$, $p_T^\alpha = p^\alpha - \hat{P}^\alpha p \cdot P$ and $T_p^{\alpha\beta} = \delta^{\alpha\beta} - p^\alpha p^\beta / p^2$.

(a) Show that T_1 and T_2 are s-waves (eigenvalue 0 of L^2), whereas T_3 and T_4 are p-waves (eigenvalue 1 of L^2).

(b) Show that T_1 has eigenvalue 0 wrt. S^2 . (The same is true for T_2 .)

Hint: $\gamma_T^\mu \gamma_T^\mu = 3$

(c) Show that T_3 has eigenvalue 1 wrt. S^2 . (The same is true for T_4 , but this calculation is rather lengthy...)

Hint: Use $p \cdot P = 0$ and $\not{p} \gamma_T^\mu = -\gamma_T^\mu \not{p} + 2p^\mu$