



MTHS24 – Exercise sheet 3

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Lecture material

Discussed topics:

- Partial waves
- Analyticity
- Unitarity

References:

- Book 1, [inspire](#)
- Book 2, [amazon](#)
- A good review, [inspire](#)
- A good paper, [inspire](#)

Exercises

3.1 Amplitude analysis

Given the Omnès function:

$$\Omega(s) = \exp \left[\frac{s}{\pi} \int_{4m_\pi^2}^\infty dz \frac{\delta(z)}{z(z-s)} \right], \quad (1)$$

(a) Consider the low-energy expansion of the pion form factor $F_\pi(s) \equiv \Omega(s)$:

$$F_\pi(s) = 1 + \frac{1}{6} \langle r_\pi^2 \rangle s + \mathcal{O}(s^2), \quad (2)$$

and deduce the sum rule:

$$\langle r_\pi^2 \rangle = \frac{6}{\pi} \int_{4m_\pi^2}^\infty dz \frac{\delta(z)}{z^2}. \quad (3)$$

(b) Assume that the phase shift $\delta(s)$ reaches $k\pi$ (k is an integer) at $s = \Lambda^2$ and stays at that value for larger s . What is the behavior of $\Omega(s)$ in the limit $|s| \rightarrow \infty$?

(c) What is the resulting function $\Omega(s)$ for an infinitely narrow resonance, i.e. consider $\delta(s) = \pi\theta(s - 4M^2)$?

3.2 Isospin

The one-pion states are of the form $|I, I_3\rangle$ with

$$|1, +1\rangle = |\pi^+\rangle, \quad |1, 0\rangle = |\pi^0\rangle, \quad |1, -1\rangle = |\pi^-\rangle. \quad (4)$$

- (a) From the three pion flavors, construct the nine different two-pion states and their decomposition into isospin states.
- (b) Invert the decomposition in (a) to get the decomposition of the isospin states into two pion states.
- (c) Do pions obey Bose statistics? What is the angular momentum (even or odd) for the states with $I = 0, 1, 2$?