Ruhr-University Bochum | Modern Techniques in Hadron Physics

MTHS24 – Exercise sheet 6

Morning: Mikhail Mikhasenko / Sergi Gonzalez-Solis
Afternoon: Gloria Montana, Dhruvanshu Parmar

Saturday, 20 July 2024

Lecture material

Discussed topics:

- Lippmann-Schwinger equation, Bethe-Salpeter equation, and K-matrix
- Lineshape analysis and Briet-Wigner formula
- Complex algebra, dispersion relations
- Analytic continuation and pole search
- Khury-Treiman equations

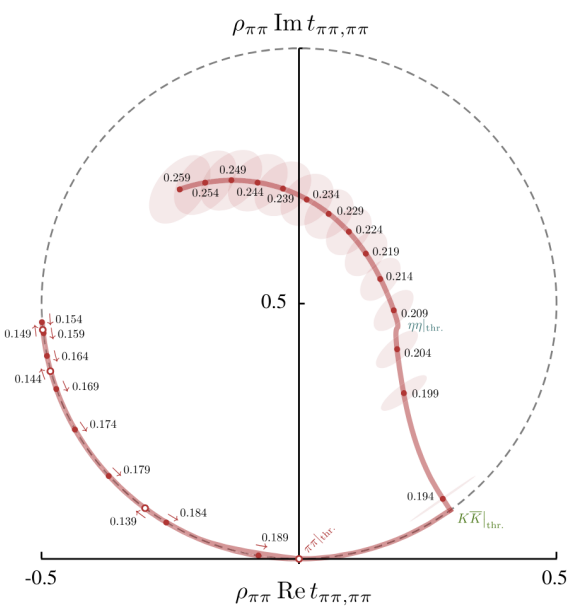
References:

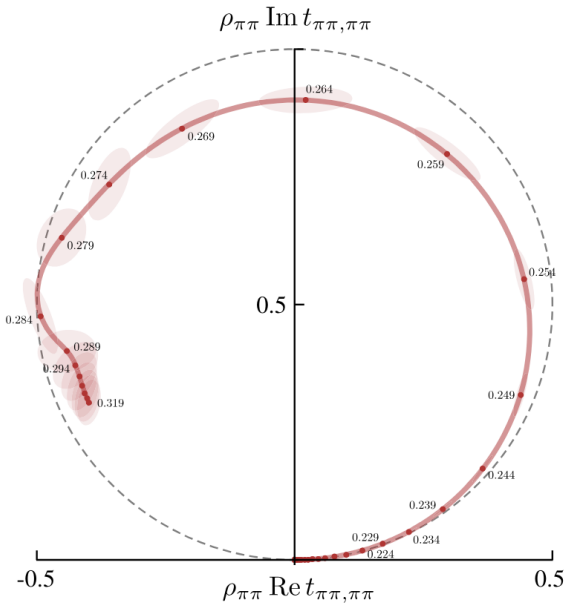
- A.D. Martin, T.D. Spearman, Elementary Particle Theory, inSpire
- Review on Novel approaches in hadron spectroscopy by JPAC, inspire

Exercices

6.1 Argand diagrams from lattice

The $\pi\pi$ scattering with unphysical pion mass ($m_\pi = 391\,\text{MeV}$) for S (left) and D (right) partial waves is studied using lattice calculations. Scattering amplitudes are presented on the Argand diagrams (parametric plot of energy in Real/Imaginary coordinates) as a function of energy of the system. The value are given units of $E_{\text{cm}} \cdot t$ where $t \cdot m_\pi = 0.06906$.





Using information on the diagrams, answer the following questions:

(a) Estimate masses of K and η particles.

(b) Find the elastic energy region for the S and D waves.

Solution: Elastic region is defined as the range of energy values for which $\pi\pi \to \pi\pi$ process dominates. For the S-wave, the elastic region lies for the $E_{\text{cm}}t$ range of [0.139,0.189] (after this point, the amplitude hits the $K\bar{K}$ threshold. Also, after this point, the curve starts going inside the unitarity circle). For the D-wave, this region exists until value of 0.229.

(c) Locate the energy value for which the S-wave peak.

Solution: The S-wave peaks at the point $E_{\text{cm}}t = 0.154$, which is at energy $E_{\text{cm}} = 0.872$ GeV.

(d) Estimate the mass and decay width for the D wave resonance.

Solution: The D-wave resonance is observed at $E_{\text{cm}}t = 0.284$, this is the point where there is a kink in the argand diagram. *Note : I can locate where the resonance is. I am confused about how to proceed from there, because I can get center of mass energy from the point, and maybe equate it to the pole value. But I would expect a complex output but I cannot read it out properly from the Argand diagram.*

(e) Sketch the amplitude phase versus energy of the system for both partial waves.

6.2 Escape room in the complex plane

(a) Characterize the complex structure of functions $\sqrt{x}$ and $\log(-x)$ by finding the branch points, branch cuts and number of complex (Riemann) sheets in the complex plane.

(b) Repeat (a) for a function $f(x) = \sqrt{x} - \sqrt{x-1}$.

(c) Construct a complex function with two branch points at +i and -i connected by a branch cut.

(d) Locate zeros of the function $g(z) = \sqrt{z} + i + 1$.

(e) Find residue of the function $1/g(z)$ by computing a circular integral about the complex pole.