$$\dot{x} = A_0 x(t) + A_1 x(t-d_1(t)) + A_2 x(t-d_1(t)-d_2(t)) \quad (1)$$

$d(t)$ 相关：$\quad d(t) = d_1(t) + d_2(t) \qquad d_3(t) = d_1(t) + d_2$

$$0 < d_1(t) < d_1 \qquad d = d_1 + d_2$$
$$0 < d_2(t) < d_2$$

$$t > t - d_1(t) > t - d_1 > t - d_2(t) - d_1 > t - d$$
$$t > t - d_2(t) > t - d_2 > t - d_1(t) - d_2 > t - d \quad (2)$$
$$t > t - d_1(t) > t - d(t) > t - d_1(t) - d_2 > t - d$$
$$t > t - d_2(t) > t - d(t) > t - d_2(t) - d_1 > t - d$$

$$V_1(t) = \xi_1^T(t) P \xi_2(t) \quad (3)$$

$$V_2(t) = \int_{t-d_1(t)}^{t} \xi_2^T(s) Q_1 \xi_2(s)\,ds + \int_{t-d(t)}^{t-d_1(t)} \xi_2^T(s) Q_2 \xi_2(s)\,ds$$
$$+ \int_{t-d_3(t)}^{t-d(t)} x^T(s) Q_3 x(s)\,ds + \int_{t-d}^{t-d_3(t)} x^T(s) Q_4 x(s)\,ds \quad (4)$$

$\dot{F}(s)$

$$V_3(t) = \int_{-d}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R \dot{x}(s)\,ds\,d\theta \quad (5)$$

$$V_1(t) = \xi_1^T(t) P \xi_1(t)$$

$$\xi_1(t) = \mathrm{col}\left\{ x(t),\ \int_{t-d_1(t)}^{t} x(s)\,ds,\ \int_{t-d(t)}^{t-d_1(t)} x(s)\,ds,\ \int_{t-d_3(t)}^{t-d(t)} x(s)\,ds,\ \int_{t-d}^{t-d_3(t)} x(s)\,ds \right\}$$

$$\dot{V}_1(t) = 2\begin{bmatrix} \xi_1(t) \end{bmatrix}^T P \begin{bmatrix} \dot{x}(t) \\ x(t) - (1-\dot{d}_1(t))\cdot x(t-d_1(t)) \\ (1-\dot{d}_1(t))\cdot x(t-d_1(t)) - (1-\dot{d}(t))\cdot x(t-d(t)) \\ (1-\dot{d}(t))\cdot x(t-d(t)) - (1-\dot{d}_1(t))\cdot x(t-d_3(t)) \\ (1-\dot{d}_1(t))\cdot x(t-d_3(t)) - x(t-d) \end{bmatrix}$$

令 $v_1(t) = \int_{t-d_1(t)}^{t} \frac{x(s)}{d_1(t)}\,ds$

$v_2(t) = \int_{t-d(t)}^{t-d_1(t)} \frac{x(s)}{d_2(t)}\,ds$ why

$v_3(t) = \int_{t-d_3(t)}^{t-d(t)} \frac{x(s)}{d_2 - d_2(t)}\,ds$

$v_4(t) = \int_{t-d}^{t-d_3(t)} \frac{x(s)}{d_1 - d_1(t)}\,ds$

$$= 2\begin{bmatrix} x(t) \\ d_1(t) v_1(t) \\ d_2(t) v_2(t) \\ (d_2 - d_2(t)) v_3(t) \\ (d_1 - d_1(t)) v_4(t) \end{bmatrix} P \begin{bmatrix} A_0 x(t) + A_1 x(t-d_1(t)) + A_2 x(t-d(t)) \\ \sim \end{bmatrix}$$

令 $e_i = [\, 0_{n\times(i-1)n}\ I_{n\times n}\ 0_{n\times(10-i)n} \,]$

$$\xi_b(t) = \mathrm{col}\{ x(t), x(t-d_1(t)), x(t-d(t)), x(t-d_3(t)), x(t-d), v_1(t), v_2(t), v_3(t), v_4(t), \dot{x}(t) \}$$

$$\therefore \dot V_1(t) = 2\xi_6(t)^T \begin{bmatrix} e_1 \\ d_1(t)e_6 \\ d_2(t)e_7 \\ (d_2-d_2(t))e_8 \\ (d_1-d_1(t))e_9 \end{bmatrix}^T P \begin{bmatrix} -A_0e_1+A_1e_2+A_2e_3 \\ e_1-(1-\dot d_1(t))e_2 \\ (1-\dot d_1(t))e_2-(1-\dot d(t))e_3 \\ (1-\dot d(t))e_3-(1-\dot d_1(t))e_4 \\ (1-\dot d_1(t))e_4-e_5 \end{bmatrix} \xi_b(t)$$

$$\qquad\qquad F_1'' \qquad\qquad\qquad\qquad F_2''$$

$$= \xi_b(t)^T \left(F_1^T P F_2 + F_2^T P F_1 \right) \xi_b(t) \qquad (6)$$

$$V_2(t) = \int_{t-d_1(t)}^{t} x^T(s) Q_1 x(s)\,ds + \int_{t-d(t)}^{t-d_1(t)} x^T(s) Q_2 x(s)\,ds + \int_{t-d_3(t)}^{t-d(t)} x^T(s) Q_3 x(s)\,ds + \int_{t-d}^{t-d_3(t)} x^T(s) Q_4 x(s)\,ds$$

$$\begin{aligned}\dot V_2(t) = {} & x^T(t) Q_1 x(t) + (1-\dot d_1(t))\, x^T(t-d_1(t)) Q_1 x(t-d_1(t)) \\ & -(1-\dot d_1(t))\, x^T(t-d_1(t)) Q_2\, x(t-d_1(t)) + (1-\dot d(t))\, x^T(t-d(t)) Q_2\, x(t-d(t)) \\ & -(1-\dot d(t))\, x^T(t-d(t)) Q_3\, x(t-d(t)) + (1-\dot d_1(t))\, x^T(t-d_3(t)) Q_3\, x(t-d_3(t)) \\ & -(1-\dot d_1(t))\, x^T(t-d_3(t)) Q_4\, x(t-d_3(t)) - x^T(t-d) Q_4 x(t-d)\end{aligned}$$

$$= \begin{bmatrix} x(t) \\ x(t-d_1(t)) \\ x(t-d(t)) \\ x(t-d_3(t)) \\ x(t-d) \end{bmatrix}^T \begin{bmatrix} Q_2 & & & & \\ & (1-\dot d_1(t))(Q_1-Q_2) & & & \\ & & (1-\dot d(t))(Q_2-Q_3) & & \\ & & & (1-\dot d_1(t))(Q_3-Q_4) & \\ & & & & -Q_4 \end{bmatrix} \begin{bmatrix} \vdots \end{bmatrix}$$

$$\begin{array}{lll} & Q_2 & x(t) \\ (1-\dot d_1(t)) & (Q_1-Q_2) & x(t-d_1(t)) \\ (1-\dot d(t)) & (Q_2-Q_3) & x(t-d(t)) \\ (1-\dot d_1(t)) & (Q_3-Q_4) & x(t-d_3(t)) \\ & -Q_4 & x(t-d) \end{array}$$

$$\Rightarrow = \xi_b(t)^T \operatorname{diag}\big(e_1^T Q_2 e_1,\ e_2^T(1-\dot d_1(t))(Q_1-Q_2)e_2, \ldots$$

$$e_3^T(1-\dot d(t))(Q_2-Q_3)e_3,\ e_4^T(1-\dot d_1(t))(Q_3-Q_4)e_4,\ -e_5^T Q_4 e_5\big)\xi_b(t) \quad (7)$$

令上述对角矩阵为 Φ_1

$$\dot V_3(t) = \int_{-d}^{0} \left(\dot x(t)^T R \dot x(t) - \dot x(t+\theta)^T R \dot x(t+\theta)\right) d\theta$$

$$= d\,\dot x(t)^T R \dot x(t) - \int_{t-d}^{t} \dot x(s)^T R \dot x(s) ds$$

第二项 由(2)可分段为：

$$\dot V_{3a}(t) = \int_{t-d_1(t)}^{t} \dot x(s)^T R \dot x(s) ds + \int_{t-d}^{t-d_3(t)} \dot x(s)^T R \dot x(s) ds$$

$$\dot V_{3b}(t) = \int_{t-d(t)}^{t-d_1(t)} \dot x(s)^T R \dot x(s) ds + \int_{t-d_3(t)}^{t-d(t)} \dot x(s)^T R \dot x(s) ds$$

由 Wirtinger 不等式： $\int_a^b w^T(s) R w(s) ds \geq \frac{1}{b-a} x_1^T R x_1 + \frac{3}{b-a} x_2^T R x_2$

$$x_1 = w(b) - w(a)$$
$$x_2 = w(b) + w(a) - \frac{2}{b-a}\int_a^b w(s) ds$$

$$\int_{t-d_1(t)}^{t} \dot x^T(s) R \dot x(s) ds \geq \frac{1}{d_1(t)} [x(t) - x(t-d_1(t))]^T R [x(t) - x(t-d_1(t))]$$

$$+ \frac{3}{d_1(t)} [x(t) - x(t-d_1(t))]^T R [x(t) + x(t-d_1(t)) - 2V_1(t)]$$

$$= \frac{1}{d_1(t)} \xi_b^T(t) \begin{bmatrix} e_1 - e_2 \\ e_1 + e_2 - 2e_6 \end{bmatrix}^T \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix} \begin{bmatrix} e_1 - e_2 \\ e_1 + e_2 - 2e_6 \end{bmatrix} \xi_b(t)$$

同理有 $\frac{1}{d_2(t)}\xi_b^T(t)\begin{bmatrix} e_2-e_3 \\ e_2+e_3-2e_7\end{bmatrix}^T\begin{bmatrix} R & 0 \\ 0 & 3R\end{bmatrix}\begin{bmatrix} e_2-e_3 \\ e_2+e_3-2e_7\end{bmatrix}\xi_b(t)$

$$\frac{1}{d_2-d_2(t)}\xi_b^T(t)\begin{bmatrix} e_3-e_4 \\ e_3+e_4-2e_8\end{bmatrix}^T\begin{bmatrix} R & 0 \\ 0 & 3R\end{bmatrix}\begin{bmatrix} e_3-e_4 \\ e_3+e_4-2e_8\end{bmatrix}\xi_b(t)$$

$$\frac{1}{d_1-d_1(t)}\xi_b^T(t)\begin{bmatrix} e_4-e_5 \\ e_4+e_5-2e_9\end{bmatrix}^T\begin{bmatrix} R & 0 \\ 0 & 3R\end{bmatrix}\begin{bmatrix} e_4-e_5 \\ e_4+e_5-2e_9\end{bmatrix}\xi_b(t)$$

令 $E_i = \begin{bmatrix} e_i-e_{i+1} \\ e_i+e_{i+1}-2e_{i+5}\end{bmatrix}$ $\qquad \tilde{R} = \begin{bmatrix} R & 0 \\ 0 & 3R\end{bmatrix}$

∴ $\dot{V}_{3a}(t) \geq \xi_b^T(t)\left(\frac{1}{d_1(t)}E_1^T\tilde{R}E_1 + \frac{1}{d_1-d_1(t)}E_4^T\tilde{R}E_4\right)\xi_b(t)$

$$= \xi_b^T(t)\begin{bmatrix} E_1 \\ E_4\end{bmatrix}^T\begin{bmatrix} \frac{1}{d_1(t)}\tilde{R} & 0 \\ 0 & \frac{1}{d_1-d_1(t)}\tilde{R}\end{bmatrix}\begin{bmatrix} E_1 \\ E_4\end{bmatrix}\xi_b(t)$$

$\dot{V}_{3b}(t) \geq \frac{1}{d_2(t)}E_2^T\tilde{R}E_2 + \frac{1}{d_2-d_2(t)}E_3^T\tilde{R}E_3$

$$= \xi_b^T(t)\begin{bmatrix} E_2 \\ E_3\end{bmatrix}^T\begin{bmatrix} \frac{1}{d_2(t)}\tilde{R} & \\ & \frac{1}{d_2-d_2(t)}\tilde{R}\end{bmatrix}\begin{bmatrix} E_2 \\ E_3\end{bmatrix}\xi_b(t)$$

由改进的矩阵不等式： $\qquad R_1, R_2$ 对称， S_1, S_2 任意

$$\begin{bmatrix} \frac{1}{\alpha}R_1 & 0 \\ 0 & \frac{1}{1-\alpha}R_2\end{bmatrix} \geq \begin{bmatrix} R_1+(1-\alpha)(R_1-S_2R_2^{-1}S_2^T) & (1-\alpha)S_1+\alpha S_2 \\ * & R_2+\alpha(R_2-S_1^TR_1^{-1}S_1)\end{bmatrix}$$

$$\dot{V}_{3a}(t)+\dot{V}_{3b}(t) \geq \xi_b^T(t)\left\{\frac{1}{d_1}\Phi_{6a}+\frac{1}{d_2}\Phi_{6b}\right\}\xi_b(t)$$

$$\tilde{\Phi}_{6a} = \begin{bmatrix} E_1 \\ E_4 \end{bmatrix}^T \begin{bmatrix} \tilde{R} + \frac{d_1 - d_1(t)}{d_1} T_1 & \frac{d_1 - d_1(t)}{d_1} X_1 + \frac{d_1(t)}{d_1} X_2 \\ * & \tilde{R} + \frac{d_1(t)}{d_1} T_2 \end{bmatrix} \begin{bmatrix} E_1 \\ E_4 \end{bmatrix}$$

$$\tilde{\Phi}_{6b} = \begin{bmatrix} E_2 \\ E_3 \end{bmatrix}^T \begin{bmatrix} \tilde{R} + \frac{d_2 - d_2(t)}{d_2} T_3 & \frac{d_2 - d_2(t)}{d_2} Y_1 + \frac{d_2(t)}{d_2} Y_2 \\ * & \tilde{R} + \frac{d_2(t)}{d_2} T_4 \end{bmatrix} \begin{bmatrix} E_2 \\ E_3 \end{bmatrix}$$

$$T_1 = \tilde{R} - X_2 \tilde{R}^{-1} X_2^T$$

$$T_2 = \tilde{R} - X_1^T \tilde{R}^{-1} X_1$$

$$T_3 = \tilde{R} - Y_2 \tilde{R}^{-1} Y_2^T$$

$$T_4 = \tilde{R} - Y_1^T \tilde{R}^{-1} Y_1$$

$\dot{V}_3$ 第一项可化为:

$$d \cdot \underbrace{(A_0 e_1 + A_1 e_2 + A_2 e_3)}_{e_s}^T R (\quad)$$

综上 $\dot{V} = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t)$

$$= \xi_b^T(t)\left(F_1^T P F_2 + F_2^T P F_1 + \Phi_1 + d e_s^T R e_s - \frac{1}{d_1}\tilde{\Phi}_{6a} - \frac{1}{d_2}\tilde{\Phi}_{6b}\right)\xi_b(t)$$

$N_1, N_2, N_3, N_4, N_5, N_6$

为对称矩阵

$$2\left[N_1 \dot{x}(t) + N_2 x(t) + N_3 x(t-d_1(t)) + N_4 x(t-d(t)) + N_5 x(t-d_2(t)) + N_6 x(t-d)\right]^T \left[\dot{x}(t) - (A+BK)x(t) - A_1 x(t-d_1(t)) - A_2 x(t-d_1(t)-d_2(t))\right] = 0$$

$$\Rightarrow \quad \xi_6^T(t)\, \Phi_2\, \xi_6(t)$$

$$\Phi_2 = \begin{bmatrix} -(N_2+N_2^T)(A+BK) & -N_2^T A_1 & -N_2^T A_2 & & N_2^T \\ -N_3^T(A+BK) & -(N_3+N_3^T)A_1 & -N_3^T A_2 & & N_3^T \\ -N_4^T(A+BK) & -N_4^T A_1 & -(N_4+N_4^T)A_2 & 0 & N_4^T \\ -N_5^T(A+BK) & -N_5^T A_1 & -N_5^T A_2 & & N_5^T \\ -N_6^T(A+BK) & -N_6^T A_1 & -N_6^T A_2 & & N_6^T \\ 0 & 0 & 0 & & 0 \\ -N_1^T(A+BK) & -N_1^T A_1 & -N_1^T A_2 & & N_1+N_1^T \end{bmatrix}$$

令 $V = N_1 BK$

$N_2 = a_2 N_1$

$N_3 = a_3 N_1$

$N_4 = a_4 N_1$

$N_5 = a_5 N_1$

$N_6 = a_6 N_1$

故

$$\Phi_2 = \begin{bmatrix} -N_2\cdot A - N_2^T\cdot A - 2\alpha_2 V & \\ -N_3^T\cdot A - \alpha_3 V & \\ -N_4^T\cdot A - \alpha_4 V & \\ -N_5^T\cdot A - \alpha_5 V & * \\ -N_6^T\cdot A - \alpha_6 V & \\ -N_1^T\cdot A - 0\, V & \end{bmatrix}$$

$$\dot{V} = \xi_b^T(t)\left(F_1^T P F_2 + F_2^T P F_1 + \Phi_1 + \Phi_2 + d\cdot e_5^T R e_5 - \frac{1}{d_1}\tilde{\Phi}_{6a} - \frac{1}{d_2}\check{\Phi}_{6b}\right)\xi_b(t)$$

令 $\tilde{\Psi}_1(d_1(t), d_2(t), \dot{d}_1(t), \dot{d}_2(t)) =$

$$F_1^T P F_2 + F_2^T P F_1 + \Phi_1 + \Phi_2 + d\cdot e_5^T R e_5 - \frac{1}{d_1}\tilde{\Phi}_{6a} - \frac{1}{d_2}\check{\Phi}_{6b}$$

$\tilde{\Phi}_{6a}$. $\tilde{\Phi}_{6b}$ 中 存在 R^{-1}

$$\tilde{\Phi}_{6a} = \begin{cases} \begin{bmatrix} \bar{E}_1 \\ \bar{E}_4 \end{bmatrix}^T \begin{bmatrix} 2\tilde{R} - X_2\tilde{R}^{-1}X_2^T & X_1 \\ * & \tilde{R} \end{bmatrix} \begin{bmatrix} \bar{E}_1 \\ \bar{E}_4 \end{bmatrix} & \text{if } d_1(t) = 0 \\ \begin{bmatrix} \bar{E}_1 \\ \bar{E}_4 \end{bmatrix}^T \begin{bmatrix} \tilde{R} & X_2 \\ * & 2\tilde{R} - X_1^T\tilde{R}^{-1}X_1 \end{bmatrix} \begin{bmatrix} \bar{E}_1 \\ \bar{E}_4 \end{bmatrix} & \text{if } d_1(t) = d_1 \end{cases}$$

$$\tilde{\Phi}_{6b} = \begin{cases} \begin{bmatrix} \bar{E}_2 \\ \bar{E}_3 \end{bmatrix}^T \begin{bmatrix} 2\tilde{R} - Y_2\tilde{R}^{-1}Y_2^T & Y_1 \\ * & \tilde{R} \end{bmatrix} \begin{bmatrix} \bar{E}_2 \\ \bar{E}_3 \end{bmatrix} & \text{if } d_2(t) = 0 \\ \begin{bmatrix} \bar{E}_2 \\ \bar{E}_3 \end{bmatrix}^T \begin{bmatrix} \tilde{R} & Y_2 \\ * & 2\tilde{R} - Y_1^T\tilde{R}^{-1}Y_1 \end{bmatrix} \begin{bmatrix} \bar{E}_2 \\ \bar{E}_3 \end{bmatrix} & \text{if } d_2(t) = d_2 \end{cases}$$

舒尔补

$\Rightarrow \tilde{\Psi}_1(d_1(t), d_2(t), \dot{d}_1(t), \dot{d}_2(t)) < 0$

$$\Rightarrow \begin{bmatrix} \Psi_1|_{d_1(t)=0, d_2(t)=0} & E_1^T X_2 & E_2^T Y_2 \\ * & -d_1\tilde{R} & 0 \\ * & * & -d_2\tilde{R} \end{bmatrix} < 0 \quad (8)$$

$$\begin{bmatrix} \Psi_1|_{d_1(t)=0, d_2(t)=d_2} & E_1^T X_2 & E_3^T Y_1^T \\ * & -d_1\tilde{R} & 0 \\ * & * & -d_2\tilde{R} \end{bmatrix} < 0 \quad (9)$$

$$\begin{bmatrix} \Psi_1|_{d_1(t)=d_1, d_2(t)=0} & E_4^T X_1^T & E_2^T Y_2 \\ * & -d_1\tilde{R} & 0 \\ * & * & -d_2\tilde{R} \end{bmatrix} < 0 \quad (10)$$

$$\begin{bmatrix} \Psi_1|_{d_1(t)=d_1, d_2(t)=d_2} & E_4^T X_1^T & E_3^T Y_1^T \\ * & -d_1\tilde{R} & 0 \\ * & * & -d_2\tilde{R} \end{bmatrix} < 0 \quad (11)$$

LMI: $P, Q_{1,2,3,4}, R_{1,2} < 0$

$(8-11) < 0$

$K = B^{-1} N_1^{-1} V$