Integral exemplarisch für eine Diagonal und eine Nichtdiagonalkomponente

$$(I_i)_{11} = \rho_i \int\limits_{x_i-r/2}^{x_i+r/2} \int\limits_{y_i-r/2}^{y_i+r/2} \int\limits_{z_i-r/2}^{z_i+r/2} (y-y_0)^2 + (z-z_0)^2 \, \mathrm{d}z \, \mathrm{d}y \, \mathrm{d}x$$

$$= \rho_i \int\limits_{y_i-r/2}^{y_i+r/2} \int\limits_{z_i-r/2}^{z_i+r/2} \left[((y-y_0)^2 + (z-z_0)^2)z \right]_{z_i-r/2}^{z_i+r/2} \, \mathrm{d}z \, \mathrm{d}y$$

$$= \rho_i r \int\limits_{y_i-r/2}^{y_i+r/2} \int\limits_{z_i-r/2}^{z_i+r/2} (y-y_0)^2 + (z-z_0)^2 \, \mathrm{d}z \, \mathrm{d}y$$

$$= \rho_i r^2 \left(\int\limits_{y_i-r/2}^{y_i+r/2} (y-y_0)^2 \, \mathrm{d}y + \int\limits_{z_i-r/2}^{z_i+r/2} (z-z_0)^2 \, \mathrm{d}z \right)$$

$$= \rho_i r^2 \left[\frac{(y-y_0)^3}{3} \right]_{y_i-r/2}^{y_i+r/2} + r^2 \left[\frac{(z-z_0)^3}{3} \right]_{z_i-r/2}^{z_i+r/2}$$

$$= \rho_i r^3 (y_0^2 - y_0(2y_i) + z_0^2 - z_0(2z_i) + (r^2/6 + y_i^2 + z_i^2))$$

$$= y_0^2(m_i) - y_0(2y_i m_i) + z_0^2(m_i) - z_0(2z_i m_i) + (r^2/6 + y_i^2 + z_i^2)m_i$$

$$(I_i)_{12} = \rho_i \int\limits_{x_i-r/2}^{x_i+r/2} \int\limits_{y_i-r/2}^{y_i+r/2} \int\limits_{z_i-r/2}^{z_i+r/2} -(x-x_0)(y-y_0) \, \mathrm{d}z \, \mathrm{d}y \, \mathrm{d}x$$

$$= \rho_i r \int\limits_{x_i-r/2}^{x_i+r/2} \int\limits_{y_i-r/2}^{y_i+r/2} -(x-x_0)(y-y_0) \, \mathrm{d}y \, \mathrm{d}x$$

$$= \rho_i r \int\limits_{x_i-r/2}^{x_i+r/2} -(x-x_0) \left[\frac{(y-y_0)^2}{2} \right]_{y_i-r/2}^{y_i+r/2} \, \mathrm{d}x$$

$$= -\rho_i r \left[\frac{(x-x_0)^2}{2} \right]_{x_i-r/2}^{x_i+r/2} \left[\frac{(y-y_0)^2}{2} \right]_{y_i-r/2}^{y_i+r/2}$$

$$= -\rho_i r^3 (x_i - x_0)(y_i - y_0)$$

$$= -x_0 y_0(m_i) + x_0(y_i m_i) + y_0(x_i m_i) - x_i y_i m_i$$

Ergebniss:

$$I = x_0^2 \sum_i m_i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y_0^2 \sum_i m_i \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + z_0^2 \sum_i m_i \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$- x_0 y_0 \sum_i m_i \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - x_0 z_0 \sum_i m_i \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} - y_0 z_0 \sum_i m_i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$+ x_0 \sum_i m_i \begin{pmatrix} 0 & y_i & z_i \\ y_i & -2x_i & 0 \\ z_i & 0 & -2x_i \end{pmatrix} + y_0 \sum_i m_i \begin{pmatrix} -2y_i & x_i & 0 \\ x_i & 0 & z_i \\ 0 & z_i & -2y_i \end{pmatrix} + z_0 \sum_i m_i \begin{pmatrix} -2z_i & 0 & x_i \\ 0 & -2z_i & y_i \\ x_i & y_i & 0 \end{pmatrix}$$

$$+ \sum_i m_i \begin{pmatrix} y_i^2 + z_i^2 + r^2/6 & -x_i y_i & -x_i z_i \\ -x_i y_i & x_i^2 + z_i^2 + r^2/6 & -y_i z_i \\ -x_i z_i & -y_i z_i & x_i^2 + y_i^2 + r^2/6 \end{pmatrix}$$

Inkrementell aktuallisiert werden also:

$$\sum_i m_i; \ \sum_i m_i x_i; \ \sum_i m_i y_i; \ \sum_i m_i z_i; \ \sum_i m_i x_i y_i; \ \sum_i m_i x_i z_i; \ \sum_i m_i y_i z_i;$$
$$\sum_i m_i (y_i^2 + z_i^2 + r^2/6); \ \sum_i m_i (x_i^2 + z_i^2 + r^2/6); \ \sum_i m_i (x_i^2 + y_i^2 + r^2/6)$$