Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers

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Abstract

ADMM [1]

1 Literature review

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2 Different software frameworks

2.1 Apache Hadoop

lrjfzjiozjf

2.2 Mesos

lshfcizufhoizauh

2.3 Spark

zdfjzpofjz

3 Alternating Direction Method of Multipliers (ADMM)

3.1 Dual Ascent

Let consider the following convex optimization problem:

$$\begin{array}{ll} \underset{x}{\text{minimize}} & f(x) \\ \text{subject to} & Ax = b, \end{array} \tag{1}$$

with variables $x \in \mathbb{R}^n$, where $A \in \mathbb{R}^{m \times n}$ and $f : \mathbb{R}^n \to \mathbb{R}$ is convex. The Lagrangian of problem 1 is:

$$L(x, \nu) = f(x) + \nu^T (b - Ax)$$

and the dual function is:

$$g(\nu) = \min_x L(x, \nu) = -f^*(-A^T \nu) - b^T \nu$$

where ν is the dual variable, and f^* is the convex conjugate of f. The dual is given by:

$$\underset{\nu}{\text{maximize}} \qquad g(\nu)$$

with variable $\nu \in \mathbb{R}^m$. Assuming that the strong duality holds, the optimal valules of the primal and the dual are the same. Then the following relation holds:

$$x^* \quad = \quad \underset{x}{\operatorname{argmin}} \; L(x, \nu),$$

where x^* and y^* are the two optimal arguments for the primal and the dual problem.

The dual ascent method solves the dual problem using gradient ascent. Assuming that g is differentiable, we can evaluate $\nabla g(\nu)$ with $\nabla g(\nu) = Ax^+ - b$, where $x^+ = \operatorname{argmin}_x L(x, \nu)$. The dual ascent method is given by this iteration:

$$\begin{aligned} x^{k+1} &:= \underset{x}{\operatorname{argmin}} \; L(x, \nu^k) \\ \nu^{k+1} &:= \nu^k + \alpha^k (Ax^{k+1} - b), \end{aligned}$$

where $\alpha^k > 0$ is a step size, and the superscript is the iteration counter.

3.2 Method of Multipliers

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3.3 ADMM

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References

[1] S. Boyd, N. PARIKH, E. CHU, B PELEATO, and J ECKSTEIN. Distributed optimization and statistical learning via the alternating direction method of multipliers. *Foundation and Trends in Machine Learning*, pages 1–122, 2010.