In [1]:
```
from scipy.integrate import odeint
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy.optimize as optimize
```

1.1 carbon cycle without the buffer effect

In [29]:
```
# 导入化石燃料排放的数据
df=pd.read_csv('global.1751_2008.ems.txt',sep='\s+',skiprows=27,
               names=['year','total','gas','liquids','cement solids','gas production
df
```

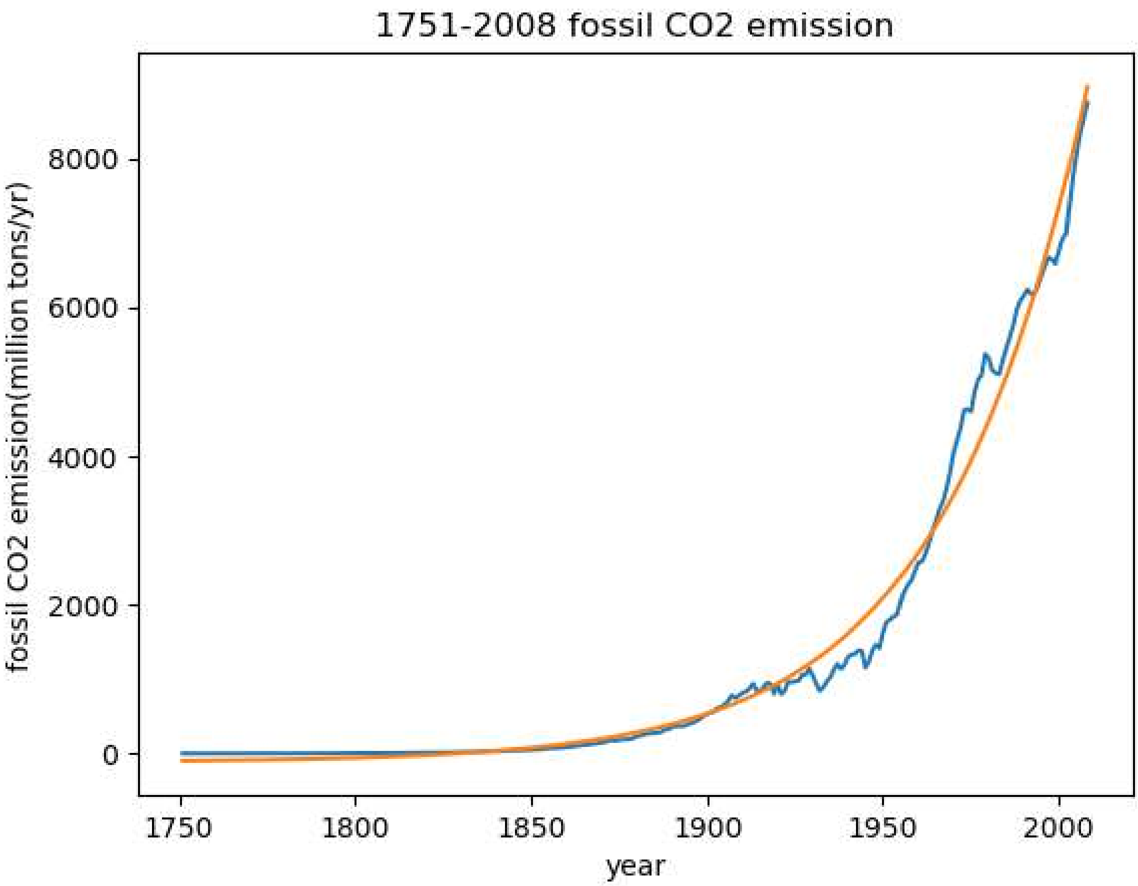
Out[29]:

	year	total	gas	liquids	cement solids	gas production	per flaring	capita
0	1751	3	0	0	3	0	0	NaN
1	1752	3	0	0	3	0	0	NaN
2	1753	3	0	0	3	0	0	NaN
3	1754	3	0	0	3	0	0	NaN
4	1755	3	0	0	3	0	0	NaN
...	...	...	...	...	...	...	...	...
253	2004	7782	1431	3027	2971	298	55	1.21
254	2005	8086	1473	3071	3162	320	61	1.24
255	2006	8350	1519	3080	3333	355	62	1.27
256	2007	8543	1551	3074	3468	382	68	1.28
257	2008	8749	1616	3095	3578	386	73	1.30

258 rows × 8 columns

In [32]:
```
# 注意到排放是随时间变化，对其进行指数拟合y=exp(a1*x+a2)+a3，得到化石燃料排放的二氧化
def fossil_emiss(x,a1,a2,a3): # 定义拟合的函数
    return np.exp(a1*x+a2)+a3

# 设置拟合的初始值
a1=0.1
a2=0.1
a3=0
p0=[a1,a2,a3]

# 调用拟合函数
para,cov=optimize.curve_fit(fossil_emiss,df['year'],df['total'],p0=p0)

# 计算拟合后的结果
y_fit=[fossil_emiss(a,*para) for a in df['year']]

# 画真实值和拟合后的图
plt.plot(df['year'],df['total'])
plt.plot(df['year'],y_fit)
plt.xlabel('year')
plt.ylabel('fossil CO2 emission(million tons/yr)')
plt.title('1751-2008 fossil CO2 emission')
```

```
plt.show()
print(para)
```




```
[ 2.43953715e-02 -3.98729502e+01 -1.12317046e+02]
```

In [33]:

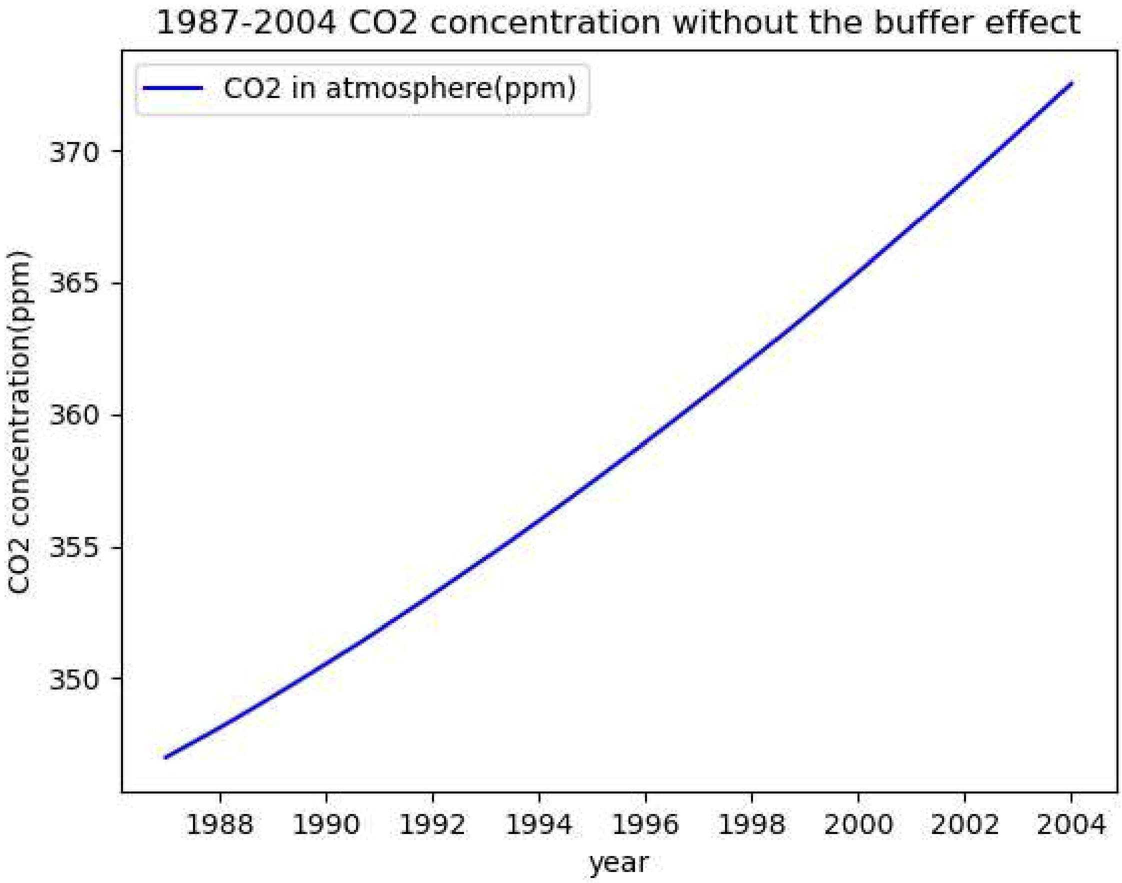
```
# 定义没有buffer效应的碳循环函数
def carbon_cycle(y,t,k12,k21):
    # a为化石燃料的二氧化碳排放
    N1,N2,a = y
    dydt=[-k12*N1+k21*N2+a,k12*N1-k21*N2,2.43953715e-02*a+2.43953715e-02*1.12317046
    return dydt
```

In [34]:

```
# 设置初始值和参数
t=np.linspace(0, 17, 100)+1987 #时间插值
a=fossil_emiss(t,*para)

k12=105/740
k21=102/900
N1=740*1000
N2=900*1000
y0=[N1,N2,a[0]] #初始值

sol = odeint(carbon_cycle, y0, t, args=(k12,k21))/1000/740*347 #计算微分方程，并且转
```

In [35]:

```
# 画出结果图

from matplotlib.ticker import MaxNLocator
plt.plot(t, sol[:, 0], 'b', label='CO2 in atmosphere(ppm)')
plt.legend(loc='best')
plt.xlabel('year')
plt.ylabel('CO2 concentration(ppm)')
plt.gca().xaxis.set_major_locator(MaxNLocator(integer=True)) # 设置横坐标轴刻度为整
plt.title('1987-2004 CO2 concentration without the buffer effect')
plt.show()
```

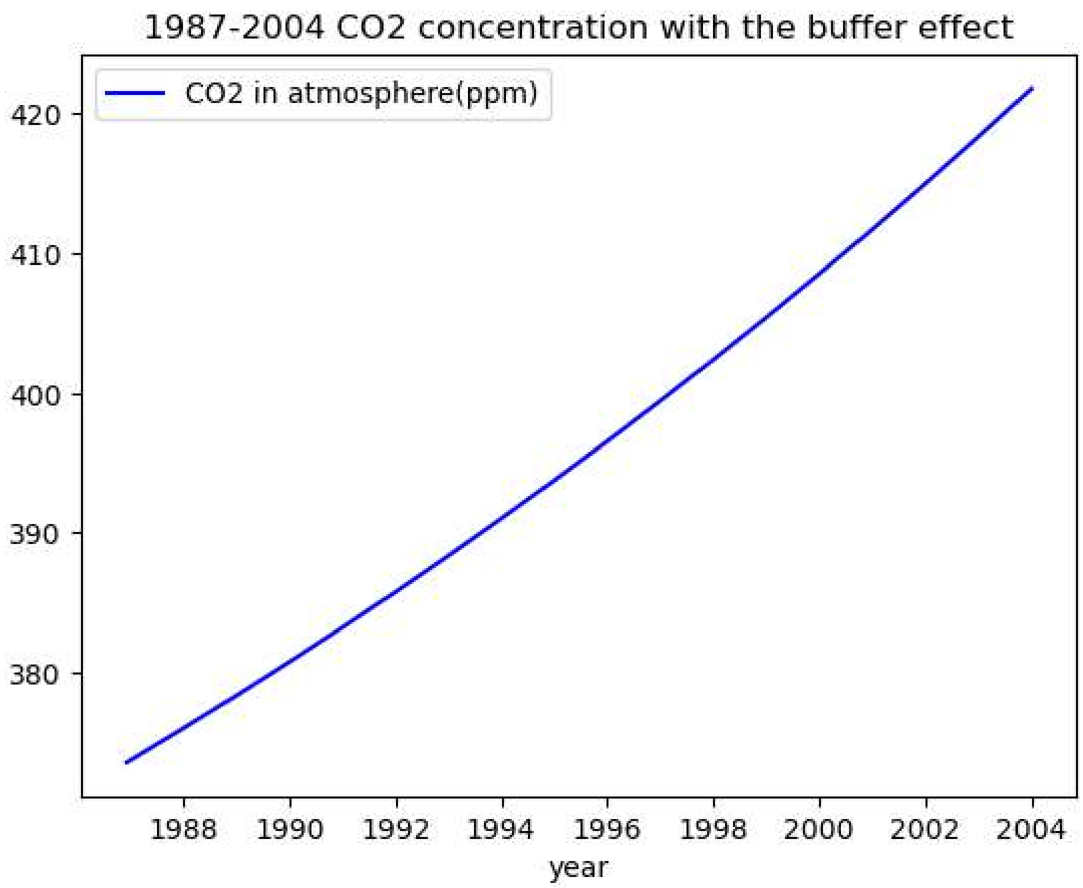


1.2 carbon cycle with the buffer effect

In [36]:
```
# 定义有buffer效应的碳循环函数
def carbon_cycle2(y,t,k12,k21,N0):
    N1,N2,a2 = y
    bf=3.69+1.86e-2*(N1/740/1000*347)-1.8e-6*((N1/740/1000*347)**2) #buffer factor
    dydt=[-k12*N1+k21*(N0+bf*(N2-N0))+a2,
          k12*N1-k21*(N0+bf*(N2-N0)),
          2.43953715e-02*a2+2.43953715e-02*1.12317046e+02]
    return dydt
```

In [37]:
```
# 设置初始值和参数
t2=np.linspace(0, 253, 253)+1751 #时间插值，从preindustry开始积分
a2=fossil_emiss(t2,*para)

k12=105/740
k21=102/900
N0=821*1000
N1=618*1000
N2=821*1000
y0=[N1,N2,a2[0]] #初始值(工业革命前的初始值)

sol2 = odeint(carbon_cycle2, y0, t2, args=(k12,k21,N0))/740/1000*347 #计算微分方程,
```

In [39]:
```
# 画图
plt.plot(t2[235:253], sol2[235:253,0], 'b', label='CO2 in atmosphere(ppm)') # 截取19
plt.legend(loc='best')
plt.xlabel('year')
plt.gca().xaxis.set_major_locator(MaxNLocator(integer=True)) # 设置横坐标轴刻度为整
plt.title('1987-2004 CO2 concentration with the buffer effect')
plt.show()
```



1.3 reproduce Figure 2

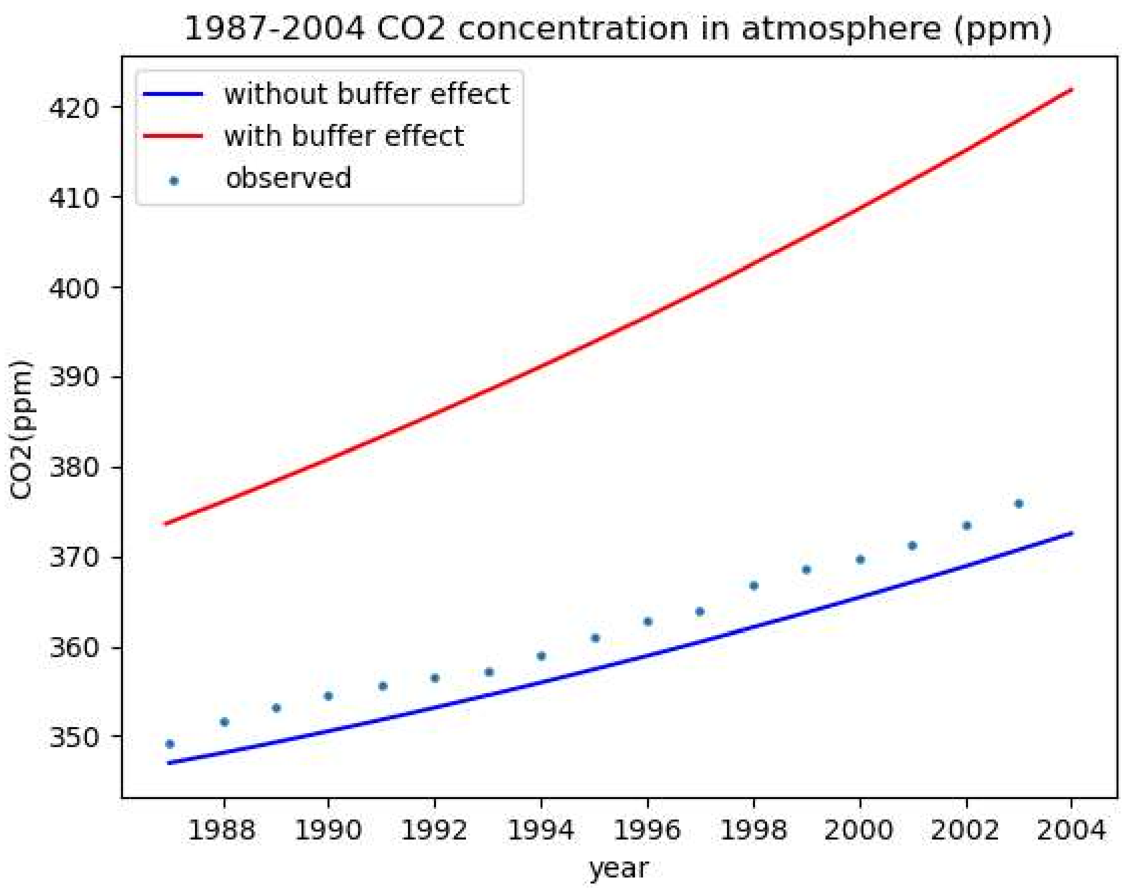
In [40]:
```
# 导入观测值数据
df2=pd.read_csv('co2_annmean_mlo.txt',sep='\s+',skiprows=57,
                names=['year','CO2','unc'])
df2
```

Out[40]:

	year	CO2	unc
0	1959	315.98	0.12
1	1960	316.91	0.12
2	1961	317.64	0.12
3	1962	318.45	0.12
4	1963	318.99	0.12
...	...	...	...
58	2017	406.76	0.12
59	2018	408.72	0.12
60	2019	411.66	0.12
61	2020	414.24	0.12
62	2021	416.45	0.12

63 rows × 3 columns

In [41]:
```
# 画图
plt.plot(t, sol[:, 0], 'b') # without buffer effect
plt.plot(t2[235:253], sol2[235:253,0], 'r') # with buffer effect
plt.scatter(df2['year'][28:45],df2['CO2'][28:45],s=5) # observed data

plt.legend(['without buffer effect','with buffer effect','observed'],loc='best')
plt.xlabel('year')
plt.ylabel('CO2(ppm)')
plt.gca().xaxis.set_major_locator(MaxNLocator(integer=True)) # 设置横坐标轴刻度为整
plt.title('1987-2004 CO2 concentration in atmosphere (ppm)')
plt.show()
# 不考虑buffer效应时比观测值低估，可能是由于人为源排放拟合时有一定的低估，因此积分之
```



seven-box model

In [42]:
```
import math
```

In [43]:
```
df3=pd.read_csv('1850-2005.txt',sep='\s+',skiprows=15,
               names=['year','global','USA','Canada','rest America','Europe','Africa
df3
```

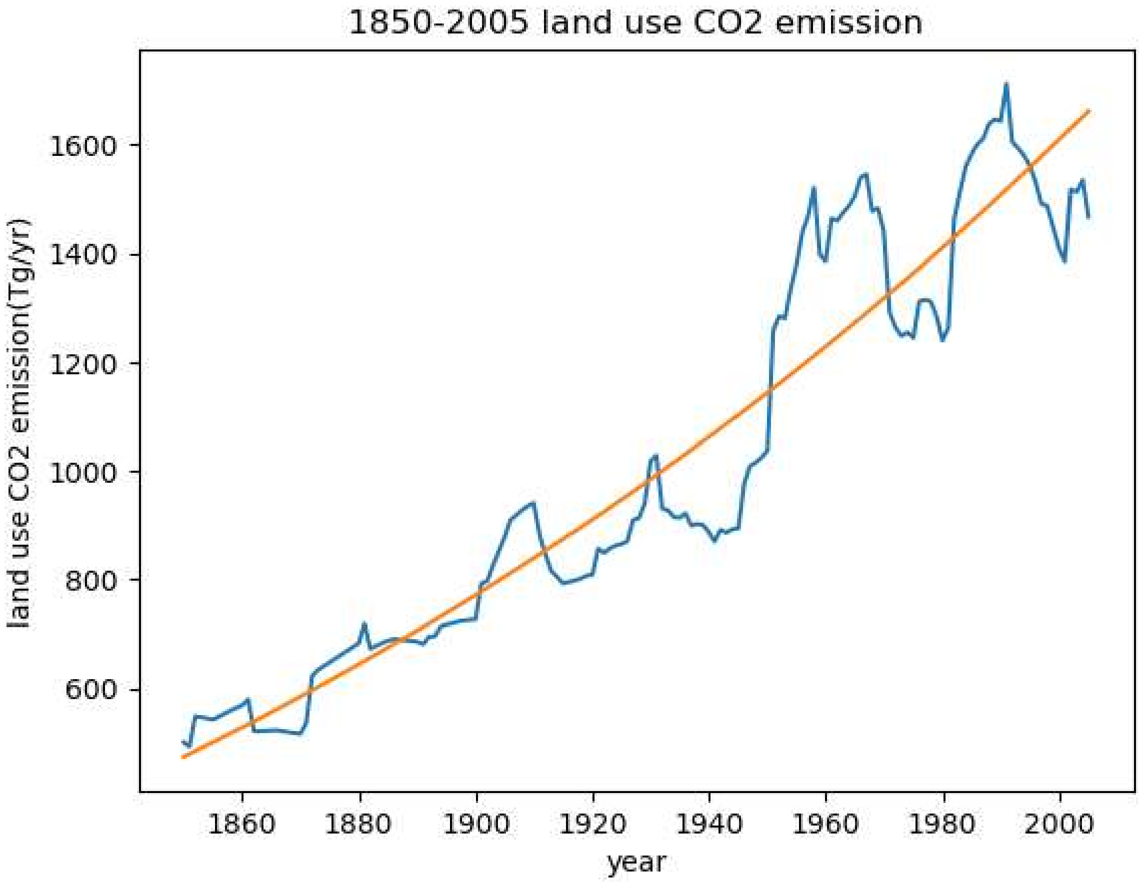
Out[43]:

	year	global	USA	Canada	rest America	Europe	Africa	Asia	Former Southest Region	China	South+	Paci
0	1850	500.6	164.1	5.5	23.5	55.0	4.0	-1.3	58.6	101.8	87.3	
1	1851	492.7	165.7	5.4	23.2	55.0	4.0	-1.1	58.6	93.1	86.9	
2	1852	548.5	230.7	5.3	22.9	55.0	4.0	-1.0	58.9	83.8	86.9	
3	1853	546.8	238.5	5.3	22.6	55.0	4.0	-1.1	59.2	74.2	87.0	
4	1854	544.8	246.2	5.3	22.4	54.9	4.0	-1.0	59.6	64.3	87.1	
...	...	...	...	...	...	...	...	...	...	...	...	
151	2001	1385.4	-31.9	17.6	643.2	-18.1	23.2	261.7	20.1	-12.9	478.5	
152	2002	1517.7	-31.9	17.6	625.5	-18.1	23.2	258.5	20.1	-12.9	631.7	
153	2003	1513.2	-31.9	17.6	616.5	-18.1	23.2	225.5	20.1	-12.9	669.3	
154	2004	1534.9	-31.9	17.6	609.4	-18.1	23.2	225.8	20.1	-12.9	697.8	
155	2005	1467.3	-31.9	17.6	606.4	-18.1	23.2	239.2	20.1	-12.9	619.7	

156 rows × 12 columns

In [44]:

```
# 对land use进行指数拟合y=exp(a1*x+a2)+a3，得到land use排放的二氧化碳随时间变化的曲线
def land_use(x,a1,a2,a3): # 定义拟合的函数
    return np.exp(a1*x+a2)+a3

# 设置拟合的初始值
a1=0.1
a2=1
a3=500
p0=[a1,a2,a3]

# 调用拟合函数
para2,cov=optimize.curve_fit(land_use,df3['year'],df3['global'],p0=p0,maxfev = 1000(

# 计算拟合后的结果
y_fit2=[land_use(a,*para2) for a in df3['year']]

# 画真实值和拟合后的图
plt.plot(df3['year'],df3['global'])
plt.plot(df3['year'],y_fit2)
plt.xlabel('year')
plt.ylabel('land use CO2 emission(Tg/yr)')
plt.title('1850-2005 land use CO2 emission')

plt.show()
print(para2)
```




```
[ 4.47292783e-03 -1.19483115e+00 -7.14450749e+02]
```

In [45]:

```
# 定义seven-box model
def carbon_cycle3(y,t,k12,k21,k23,k24,k32,k34,k43,k45,k51,k67,k71,N0,b):# b是fertili
    N1,N2,N3,N4,N5,N6,N7,a3,lu = y
    bf=3.69+1.86e-2*(N1/740/1000*347)-1.8e-6*((N1/740/1000*347)**2) #buffer factor
    f=62000*(1+b*math.log(N1/618/1000)) # net primary productivity
    dydt=[-k12*N1+k21*(N0+bf*(N2-N0))+a3-f+lu+k51*N5+k71*N7,
          k12*N1-k21*(N0+bf*(N2-N0))-k23*N2+k32*N3-k24*N2,
          k23*N2-k32*N3-k34*N3+k43*N4,
          k34*N3-k43*N4+k24*N2-k45*N4,
          k45*N4-k51*N5,
          f-k67*N6-2*lu,
          k67*N6-k71*N7+lu,
          2.43953715e-02*a3+2.43953715e-02*1.12317046e+02,# fossil排放对时间求导
         4.47292783e-03*lu+4.47292783e-03*7.14450749e+02] # land use对时间求导
    return dydt
```

In [46]:

```
# 设置初始值和参数
t3=np.linspace(0, 253, 253)+1751 #时间插值, 从preindustry开始积分
a3=fossil_emiss(t3,*para)
lu=land_use(t3,*para2)

# 设置参数
k12=60/615
k21=60/842
k23=9/842
k24=43/842
k32=52/9744
k34=162/9744
k43=205/26280
k45=0.2/26280
k51=0.2/90000000
k67=62/731
k71=62/1328
```

```
N0=842*1000
b=0.38

# 设置初始值
N1=615*1000
N2=842*1000
N3=9744*1000
N4=26280*1000
N5=90000000*1000
N6=731*1000
N7=1238*1000
y0=[N1,N2,N3,N4,N5,N6,N7,a3[0],lu[0]] #初始值(工业革命前的初始值)

sol3 = odeint(carbon_cycle3, y0, t3, args=(k12,k21,k23,k24,k32,k34,k43,k45,k51,k67,k
```

In [47]:

```
# b=0.50的情形
b=0.50
sol4 = odeint(carbon_cycle3, y0, t3, args=(k12,k21,k23,k24,k32,k34,k43,k45,k51,k67,k
```

In [48]:

```
# ice core的数据只有1010-1960的数据，因此1959之后的数据用df2，将两个dataframe连接
df4=pd.read_csv('lawdome.combined.dat.txt',sep='\s+',skiprows=273,names=['year','CO2
df5=pd.concat([df4.iloc[0:189,0:2],df2[['year','CO2']]])
df5
```

Out[48]:

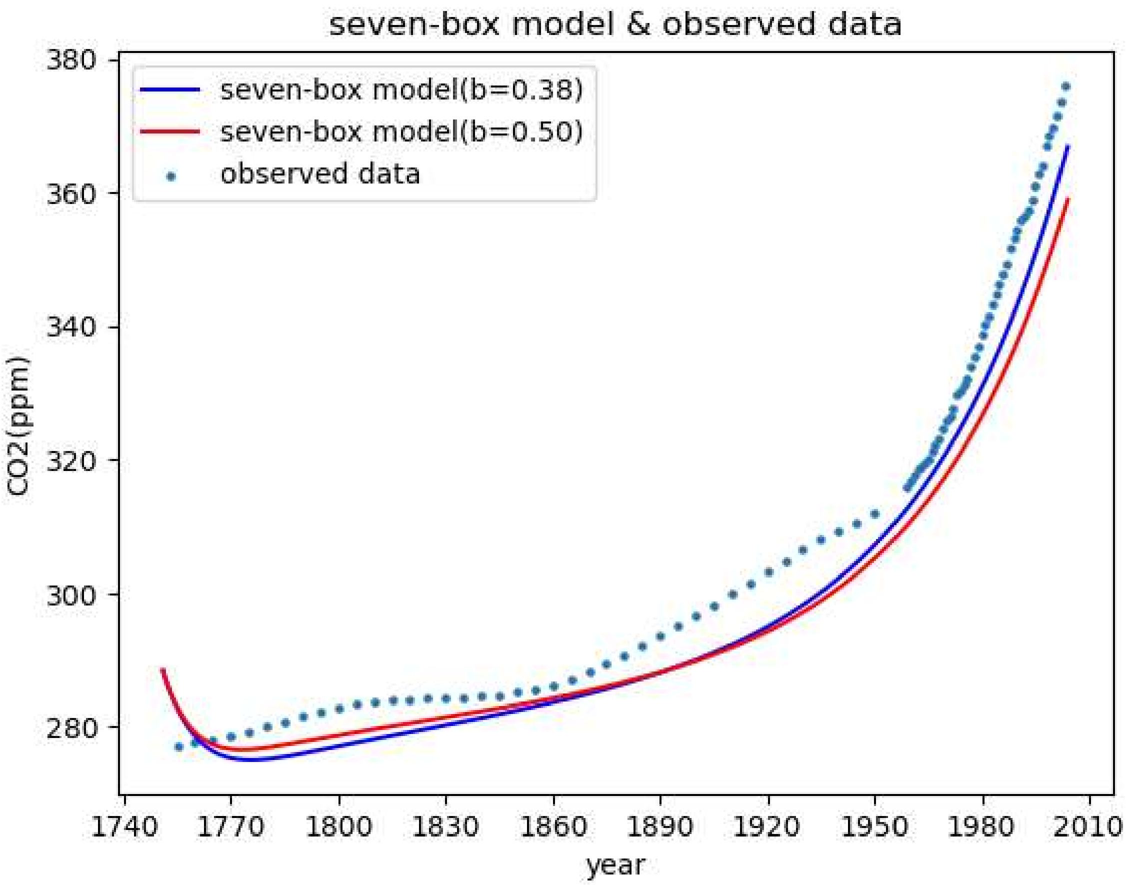
	year	CO2
0	1010	279.50
1	1015	279.60
2	1020	279.70
3	1025	279.80
4	1030	279.90
...	...	...
58	2017	406.76
59	2018	408.72
60	2019	411.66
61	2020	414.24
62	2021	416.45

252 rows × 2 columns

In [49]:

```
# 画图
plt.plot(t3, sol3[:,0], 'b') # b=0.38
plt.plot(t3, sol4[:,0], 'r') # b=0.50
plt.scatter(df5['year'][149:234],df5['CO2'][149:234],s=5) # ovserved data
plt.legend(['seven-box model(b=0.38)','seven-box model(b=0.50)','observed data'],loc
plt.xlabel('year')
plt.ylabel('CO2(ppm)')
plt.gca().xaxis.set_major_locator(MaxNLocator(integer=True)) # 设置横坐标轴刻度为整
plt.title('seven-box model & observed data')
plt.show()
```

```
C:\Users\17978\AppData\Local\Temp\ipykernel_23668\2301373622.py:4: FutureWarning: The behavior of `series[i:j]` with an integer-dtype index is deprecated. In a future version, this will be treated as *label-based* indexing, consistent with e.g. `series[i]` lookups. To retain the old behavior, use `series.iloc[i:j]`. To get the future behavior, use `series.loc[i:j]`.
  plt.scatter(df5['year'][149:234],df5['CO2'][149:234],s=5) # ovserved data
```



In [ ]: