

Assignment 3

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Question 1

Download the relevant data

```
fredr_set_key("3f87daa0298229fc49f2d28e0752dc58")
codes<-c("GDPC1","GDPDEF","FEDFUNDS")

###Setting the base variables
M <- length(codes)                # how many variables (M)
t0 <- as.Date("1955-01-01")       # start date
t1 <- as.Date("2020-12-31")       # end date
t <- seq.Date(t0,t1,by="1 quarter") # frequency, setting the increment of the sequence
bigT <- length(t)                 # length of time series (T)

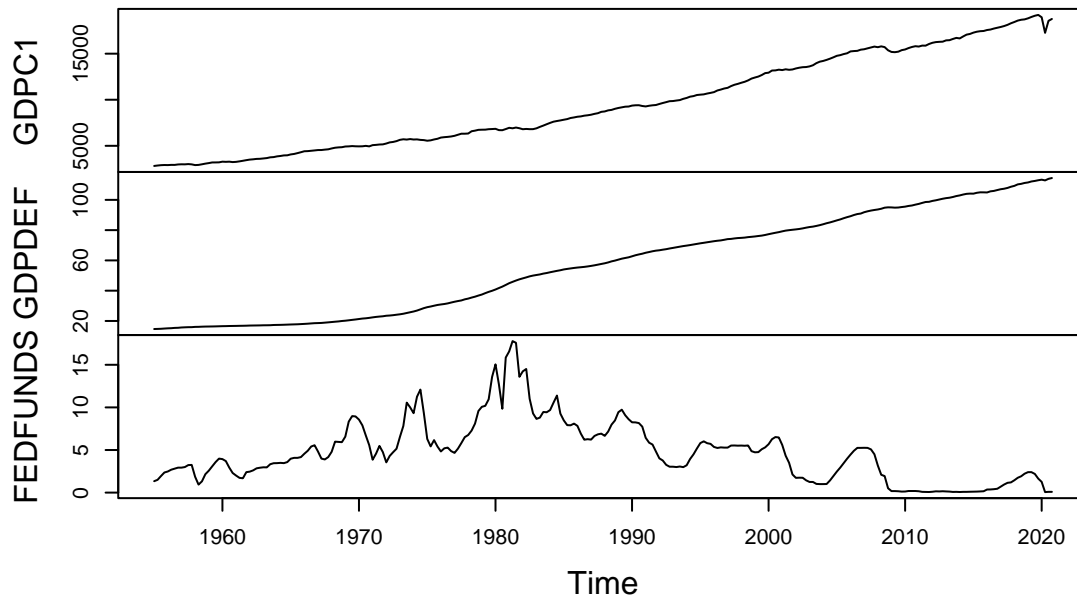
###Generate an empty matrix for the time series to be put in
xraw<-matrix(NA, bigT, M)
colnames(xraw)<-codes
rownames(xraw)<-as.character(t)

###Programming a function to import the time series and make it part of the matrix
for (ii in 1:M){
  temp<-fredr(
    series_id<-codes[ii],
    observation_start=as.Date("1955-01-01"),
    observation_end=as.Date("2020-12-31"),
    frequency="q",
    aggregation_method="avg"
  )
  xraw[(t%in%temp$date),ii]<-(temp$value)
}

###Transform the matrix into a time series
xrawtseries<-xraw<-ts(xraw, start=c(1955,1), end=c(2020,4), frequency=4)

###Plot the Macro Time Series
plot.ts(xrawtseries, main="Macro Time Series")
```

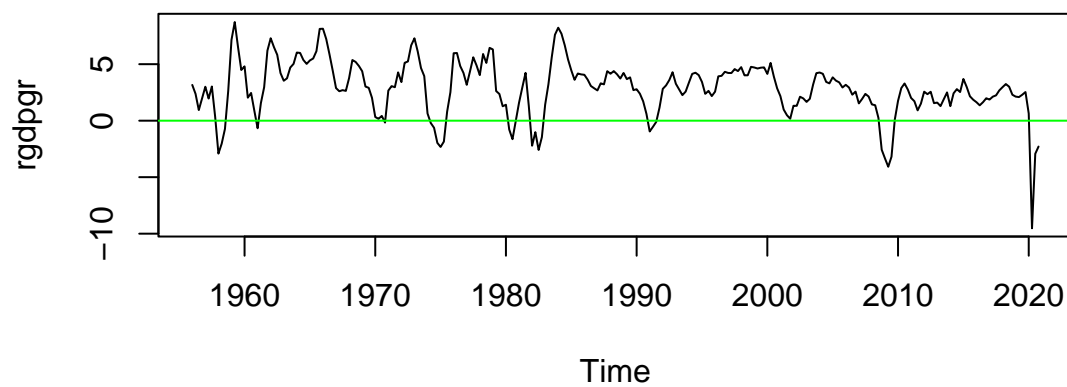
Macro Time Series



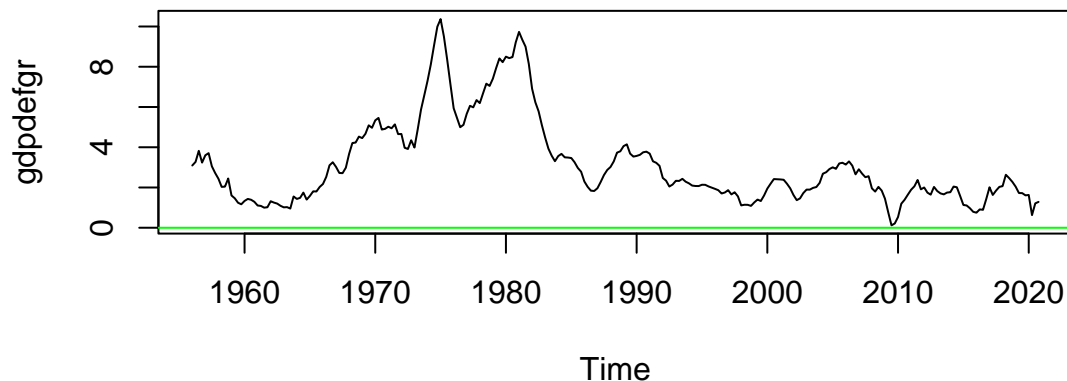
Question 2

Growth rates

```
#Real GDP  
rgdpgr<-diff(log(xrawtseries[,1]), lag=4)*100  
plot.ts(rgdpgr)  
abline(h=0, col="green")
```



```
#GDP Deflator
gdpdefgr<-diff(log(xrawtseries[,2]), lag=4)*100
plot.ts(gdpdefgr)
abline(h=0, col="green")
```



Question 3

Stationarity

```
options(width = 60)
adf.test(rgdpgr, alternative=c("stationary"))
```

```
## Warning in adf.test(rgdpgr, alternative = c("stationary")):
## p-value smaller than printed p-value
```

```
##
## Augmented Dickey-Fuller Test
##
## data: rgdpgr
## Dickey-Fuller = -5.5619, Lag order = 6, p-value =
## 0.01
## alternative hypothesis: stationary
```

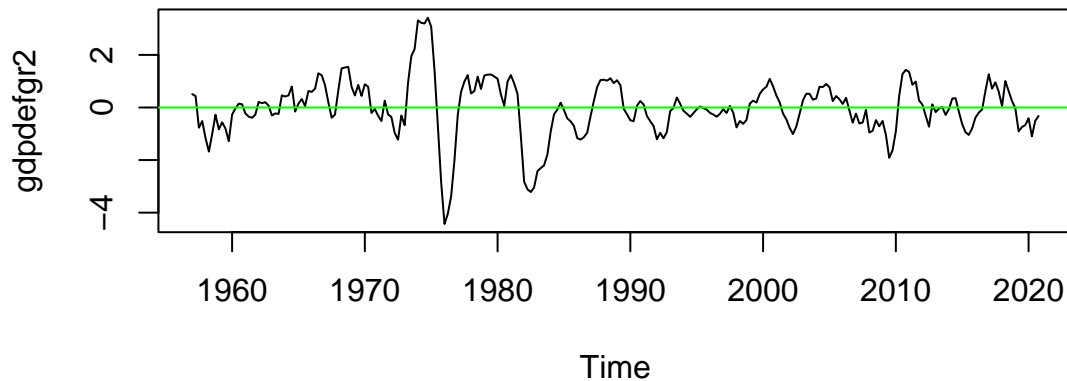
Explosiveness can be rejected for this time series, as p-value is smaller than 0.01. Furthermore, from eyeballing the time series, it seems to fluctuate around zero, with an only slightly decreasing variance.

```
adf.test(gdpdefgr, alternative=c("stationary"))
```

```
##
## Augmented Dickey-Fuller Test
##
## data: gdpdefgr
## Dickey-Fuller = -2.5778, Lag order = 6, p-value =
## 0.3325
## alternative hypothesis: stationary
```

Explosiveness cannot be rejected for this time series. This also fits the plot of the time series, as the variance in this time series shows strong outbursts, especially during the 70s and 80s. Therefore, take second differences for this time series:

```
gdpdefgr2<-diff(log(xrawtseries[,2]), lag=4, differences=2)*100
plot.ts(gdpdefgr2)
abline(h=0, col="green")
```



```
adf.test(gdpdefgr2, alternative=c("stationary"))
```

```
## Warning in adf.test(gdpdefgr2, alternative =
## c("stationary")): p-value smaller than printed p-value
```

```
##
## Augmented Dickey-Fuller Test
##
## data: gdpdefgr2
## Dickey-Fuller = -6.8963, Lag order = 6, p-value =
## 0.01
## alternative hypothesis: stationary
```

Eyeballing the plot of the time series leads to the conclusion that it is stationary, as it is varying with almost constant variance around mean (almost) zero. The `adf.test` command can reject the explosiveness of the process.

Question 4

VAR(p) and SVAR(p)

The following are the general form equations for a VAR(p) and a SVAR(p) model, respectively.

$$y_t = c + \sum_{j=1}^p A_{ij} y_{t-j} + \epsilon, \quad \epsilon \sim N(0, \Sigma)$$

$$B_0 y_t = B_0 c + \sum_{j=1}^p B_0 A_{ij} y_{t-j} + e_t, \quad e_t \sim N(0, I)$$

The problem with the error terms in “normal” VARs: they are mutually correlated. This does not allow for a clear economic interpretation. What we want to identify are structural shocks: orthogonal shocks that are not mutually correlated and have economic meaning. To identify these shocks, we need the structural representation of the VAR.

For this to work, we need to find the matrix B_0 , which multiplies the whole VAR to obtain the structural representation. However, this leads to the problem that the model contains more parameters than we can estimate from the model, implying that we need to pose $(M \times (M-1))/2$ restrictions. The matrix B_0 can be obtained using the Cholesky decomposition, which results in a lower triangular matrix that will therefore contain $(M \times (M-1))/2$ zeros above the main diagonal, which implies that there are sufficient restrictions to estimate the model.

Recursive Identification therefore means that the VAR will first be transformed using B_0 to make the error terms uncorrelated with each other before estimating the model. For this to work, we need to pin down B_0 , which is what we call identification. Recursive refers to the structure of B_0 being a lower triangular matrix. The restrictions that are imposed by B_0 not only have to include the minimum amount of restrictions for the model to be identified, but also need to have economically meaningful.

The recursive identification process is sensitive to the ordering of variables. In the Cholesky decomposition, the restrictions imply that the first variable is not contemporaneously correlated with any other variables, the second variable is only contemporaneously correlated with the first variable, and so on.

In the context of Monetary Policy identification, a typical implementation is ordering the variables as output, inflation, then interest rate. This implies that output is only dependent its own structural shocks contemporaneously, inflation depends on itself and output contemporaneously and interest rate is endogenously dependent on output, inflation and interest shocks contemporaneously. This ordering has to be defended using economic intuition and cannot be verified empirically in the model.

Question 5

Frequentist VAR

```
y<-cbind(rgdpgr, gdpdefgr2, xraw[,3])

#create a subset as the first eight observations cannot be used due to differencing
yest<-y[9:nrow(y),]
colnames(yest)<-c("Outputgap", "Inflation", "FFR")
t_<-t[9:length(t)]
rownames(yest)<-as.character(t_)
```

How do you decide on the number of lags? This can be done by using the Information criteria. In R, a function is implemented that returns the calculated selection criteria. Select minimum BIC from there (results in smallest number of lags).

```
lagselection<-VARselect(yest)
lag<-lagselection$selection[3]

#Estimation procedure of the VAR(2)
var<-VAR(yest, p=lag, type=c("const"))

#Storing the coefficients of the VAR(2)
A <- as.matrix(Bcoef(var))
```

Notational Issues: Looking for the A representation of the SVAR; therefore define a 3x3 identity matrix for the A matrix with NA elements at the spots that one wants to have estimates for; set bmat=NULL in the estimation compared to the notation from task 4, A corresponds to matrix B_0 .

```
amat<-diag(3)
amat[1,1]=amat[2,2]=amat[3,3]=NA
amat[2,1]=amat[3,1]=amat[3,2]=NA
amat
```

```
##      [,1] [,2] [,3]
## [1,]   NA    0    0
## [2,]   NA   NA    0
## [3,]   NA   NA   NA
```

```
svar<-SVAR(var, Amat=amat, Bmat=NULL, lrtest = FALSE)
```

```
#calculate the impulse response functions
```

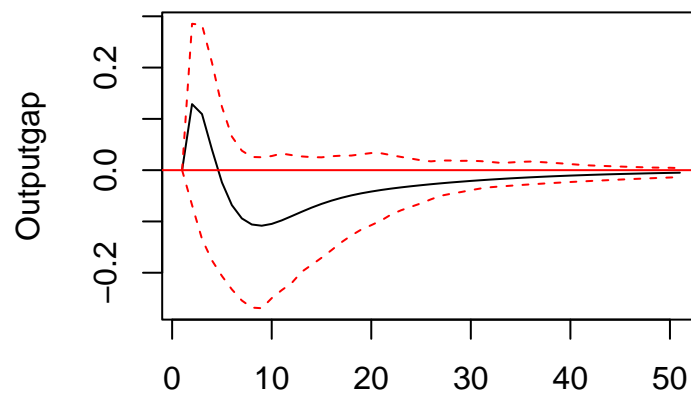
```
impulseOutputgap<-vars::irf(svar, response="Outputgap", impulse="FFR", n.ahead=50)
```

```
impulseInflation<-vars::irf(svar, response="Inflation", impulse="FFR", n.ahead=50)
```

```
impulseFFR<-vars::irf(svar, response="FFR", impulse="FFR", n.ahead=50)
```

```
plot(impulseOutputgap)
```

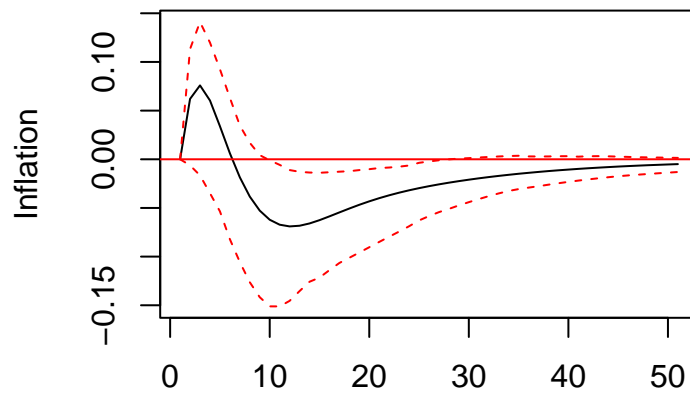
SVAR Impulse Response from FFR



95 % Bootstrap CI, 100 runs

```
plot(impulseInflation)
```

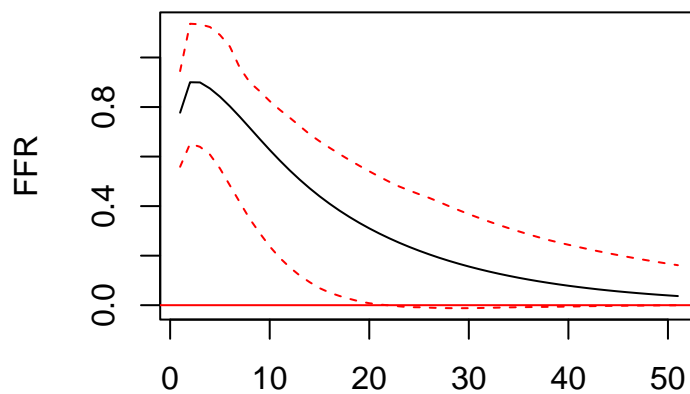
SVAR Impulse Response from FFR



95 % Bootstrap CI, 100 runs

```
plot(impulseFFR)
```

SVAR Impulse Response from FFR



95 % Bootstrap CI, 100 runs

What is the unit of measurement on each of the y-axes:

- FFR: interest rate in percentage points

- Output gap: GDP growth rate in percent
- Inflation rate: Change in inflation rate year on year in percent (?)

What is the dynamic impulse response of each of the variables to a monetary policy shock? The shock is a contractionary monetary policy shock, i.e. the interest rate is increased. This can be seen when looking at the impulse responses for the federal funds rate.

For the inflation, this would imply a decrease; however, this can only be observed in the long run; in the short run, inflation increases due to the monetary policy shock. The same holds true for the output gap (i.e. GDP growth), as this also increases in the short run before decreasing in the long run.

Increase in interest rate is persistently high for 24 periods and returns to zero over time. There is an initial increase in the interest rate after the shock, peaking after 2 periods. Initial increase in GDP growth rate for approximately 5 periods, subsequently becoming negative, then tenting back towards zero over time. Similar to the GDP growth rate, the GDP deflator initially increases, peaking at period 3, then reduces to below zero. This stays persistently below zero for all 24 periods, tending back towards zero over time.

Is this in accordance with what we would expect from theory? The responses are somewhat consistent with economic theory. Consider the basic New Keynesian model. With an contractionary monetary policy shock, interest rates stay persistently high rates over time, tending towards zero. This is consistent with our SVAR model.

For GDP growth and the inflation responses, the basic New Keynesian model does not exhibit the initial hump-shaped increase that we observe in the IRFs, but the persistent negative GDP growth and inflation rates, tending back towards zero over time, is consistent with the theoretical model.

Question 6

Robustness

Alternative to Output Gap Instead of the output gap, one can look at the Industrial Production and its change over time. Industrial Production is a key figure in economics and should also react to monetary policy shocks, as a decrease in demand and a reduction in investment due to an increase in the interest rate should certainly affect industrial production as well.

Alternative to GDP Deflator Instead of the GDP Deflator as a measure for inflation, one can look at the CPI. Measures inflation not from the production side, but from the consumption side. While it can be expected that these inflation measures diverge, it seems to be a reasonable assumption that both of the adequately depict prices.

Alternative to Federal Funds Rate Instead of the Federal Funds Rate, one can look at 3-Months Treasury Bill Secondary Markets Rates. A rise in the interest rate should lead to a rise in the Treasury Bill Market rate, as investing becomes more attractive.

```
Outputgap_Alt<-fredr(
  series_id="INDPRO",
  observation_start=as.Date("1955-01-01"),
  observation_end=as.Date("2020-12-31"),
  frequency="q",
  aggregation_method="avg"
)
Outputgap_Alt_<-ts(Outputgap_Alt$value, start=c(1955,1), end=c(2020,4), frequency=4)
Outputgap_Alt_gr<-diff(log(Outputgap_Alt_), lag=4)*100
adf.test(Outputgap_Alt_gr, alternative=c("stationary")) #stationary!
```

```
## Warning in adf.test(Outputgap_Alt_gr, alternative =
## c("stationary")): p-value smaller than printed p-value
```



```
##
## Augmented Dickey-Fuller Test
##
## data: Outputgap_Alt_gr
## Dickey-Fuller = -6.5666, Lag order = 6, p-value =
## 0.01
## alternative hypothesis: stationary

Outputgap_Alt_gr_<-Outputgap_Alt_gr[5:length(Outputgap_Alt_gr)]
yest1<-yest
yest1[,1]=Outputgap_Alt_gr_

#Selecting optimal lag length
lagselection1<-VARselect(yest1)
lag1<-lagselection1$selection[3]

#Estimation procedure of the VAR(2)
var1<-VAR(yest1, p=lag1, type=c("const"))

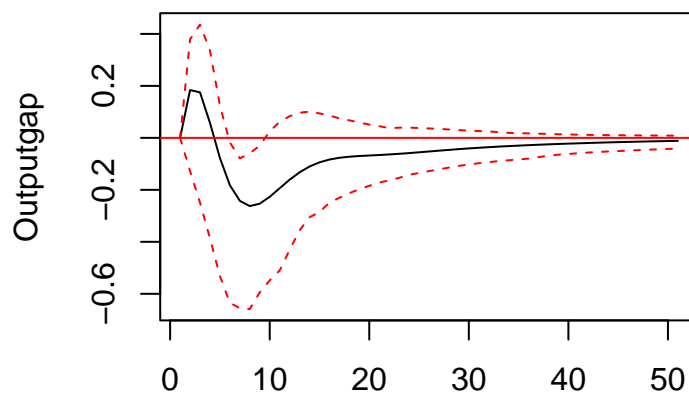
#Storing the coefficients of the VAR(2)
A1 <- as.matrix(Bcoef(var1))

#Estimating the structural VAR:
svar1<-SVAR(var1, Amat=amat, Bmat=NULL, lrtest = FALSE)

#Impulse response functions:
impulseOutputgap1<-vars::irf(svar1, response="Outputgap", impulse="FFR", n.ahead=50)
impulseInflation1<-vars::irf(svar1, response="Inflation", impulse="FFR", n.ahead=50)
impulseFFR1<-vars::irf(svar1, response="FFR", impulse="FFR", n.ahead=50)

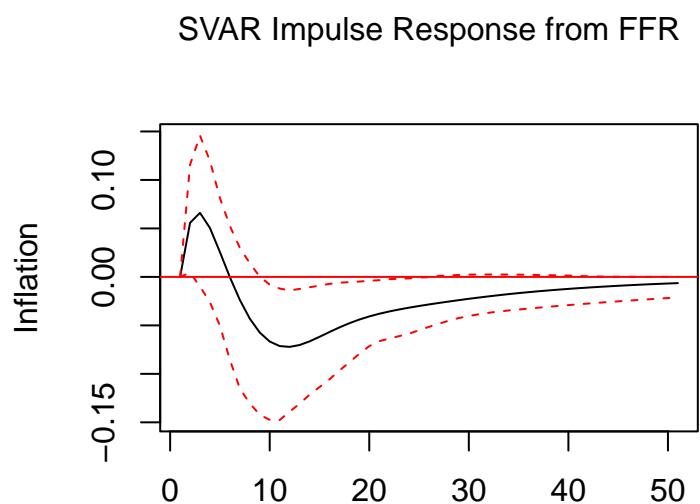
plot(impulseOutputgap1)
```

SVAR Impulse Response from FFR



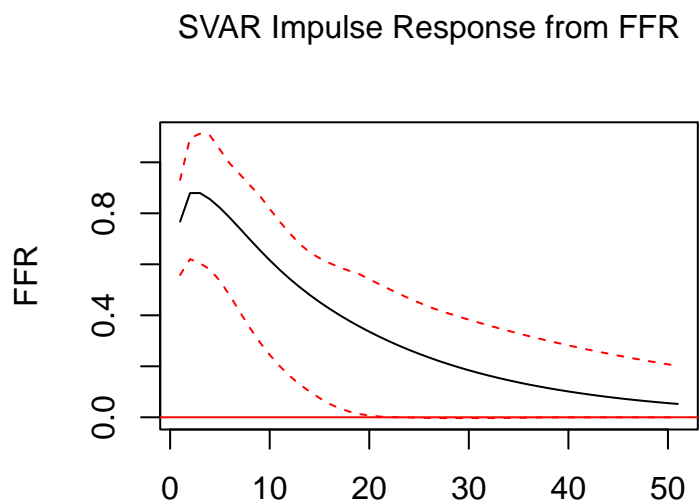
95 % Bootstrap CI, 100 runs

```
plot(impulseInflation1)
```



95 % Bootstrap CI, 100 runs

```
plot(impulseFFR1)
```



95 % Bootstrap CI, 100 runs

Results are very comparable to initial model.

```

Inflation_Alt<-GDPDEF_Alt<-fredr(
  series_id="CPIAUCSL",
  observation_start=as.Date("1955-01-01"),
  observation_end=as.Date("2020-12-31"),
  frequency="q",
  aggregation_method="avg"
)

Inflation_Alt_<-ts(Inflation_Alt$value, start=c(1955,1), end=c(2020,4), frequency=4)
Inflation_Alt_gr<-diff(log(Inflation_Alt_), lag=4)*100
adf.test(Inflation_Alt_gr, alternative=c("stationary")) #stationary

```

```

##
## Augmented Dickey-Fuller Test
##
## data: Inflation_Alt_gr
## Dickey-Fuller = -3.1916, Lag order = 6, p-value =
## 0.08972
## alternative hypothesis: stationary

```

```

Inflation_Alt_gr_<-Inflation_Alt_gr[5:length(Inflation_Alt_gr)]
yest2<-yest
yest2[,1]=Inflation_Alt_gr_

#Selecting optimal lag length
lagselection2<-VARselect(yest2)
lag2<-lagselection2$selection[3]

#Estimation procedure of the VAR(6)
var2<-VAR(yest2, p=lag2, type=c("const"))

#Storing the coefficients of the VAR(2)
A2 <- as.matrix(Bcoef(var2))

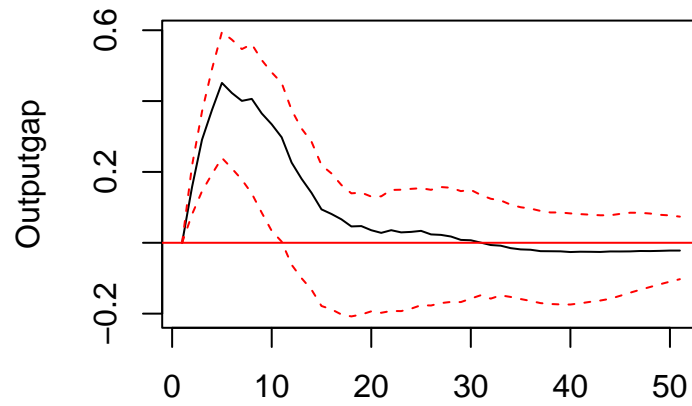
#Estimating the structural VAR:
svar2<-SVAR(var2, Amat=amat, Bmat=NULL, lrtest = FALSE)

#Impulse response functions:
impulseOutputgap2<-vars::irf(svar2, response="Outputgap", impulse="FFR", n.ahead=50)
impulseInflation2<-vars::irf(svar2, response="Inflation", impulse="FFR", n.ahead=50)
impulseFFR2<-vars::irf(svar2, response="FFR", impulse="FFR", n.ahead=50)

plot(impulseOutputgap2)

```

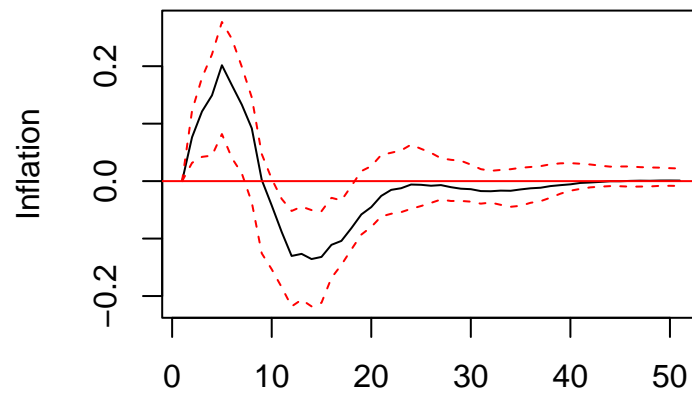
SVAR Impulse Response from FFR



95 % Bootstrap CI, 100 runs

```
plot(impulseInflation2)
```

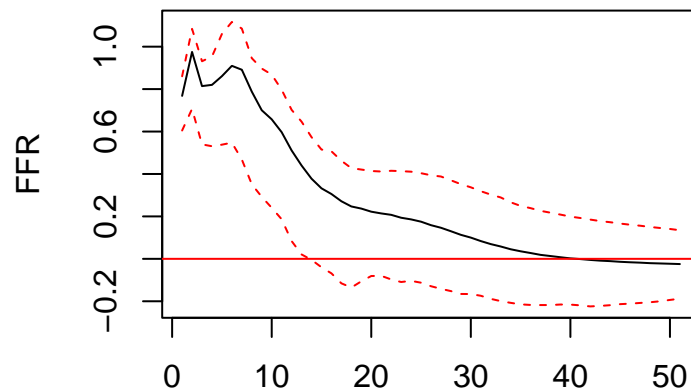
SVAR Impulse Response from FFR



95 % Bootstrap CI, 100 runs

```
plot(impulseFFR2)
```

SVAR Impulse Response from FFR



95 % Bootstrap CI, 100 runs

Appears to be very comparable to previous results.

```
FFR_Alt<-fredr(
  series_id="TB3MS",
  observation_start=as.Date("1955-01-01"),
  observation_end=as.Date("2020-12-31"),
  frequency="q",
  aggregation_method="avg"
)

FFR_Alt_<-ts(FFR_Alt$value, start=c(1955,1), end=c(2020,4), frequency=4)

#Create an updated matrix with the exchanged data:
FFR_Alt__<-FFR_Alt_[9:length(FFR_Alt_)]
yest3<-yest
yest3[,1]=FFR_Alt__

#Selecting optimal lag length
lagselection3<-VARselect(yest3)
lag3<-lagselection3$selection[3]

#Estimation procedure of the VAR(2)
var3<-VAR(yest3, p=lag3, type=c("const"))

#Storing the coefficients of the VAR(2)
A3 <- as.matrix(Bcoef(var3))

#Estimating the structural VAR:
svar3<-SVAR(var3, Amat=amat, Bmat=NULL, lrtest = FALSE)

#Impulse response functions:
```

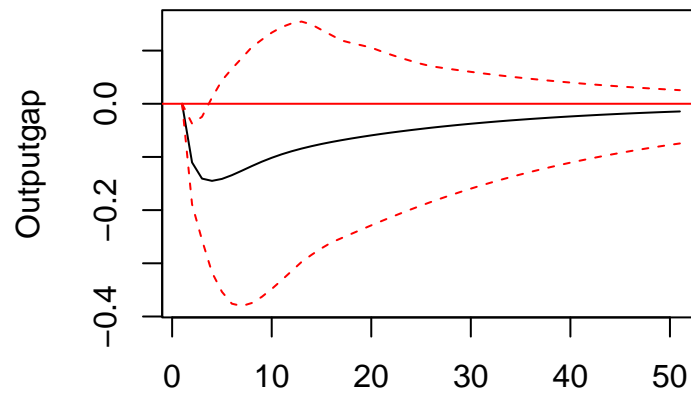
```

impulseOutputgap3<-vars::irf(svar3, response="Outputgap", impulse="FFR", n.ahead=50)
impulseInflation3<-vars::irf(svar3, response="Inflation", impulse="FFR", n.ahead=50)
impulseFFR3<-vars::irf(svar3, response="FFR", impulse="FFR", n.ahead=50)

plot(impulseOutputgap3)

```

SVAR Impulse Response from FFR



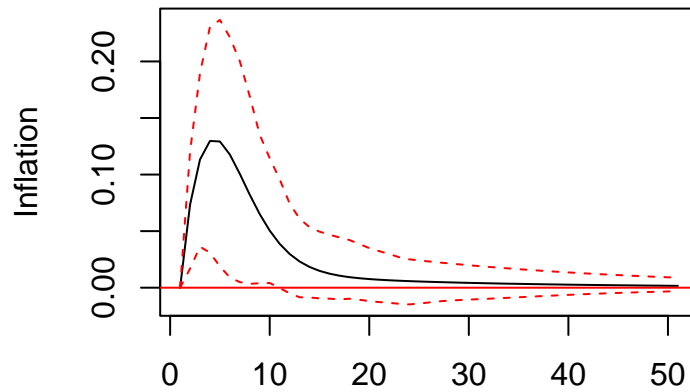
95 % Bootstrap CI, 100 runs

```

plot(impulseInflation3)

```

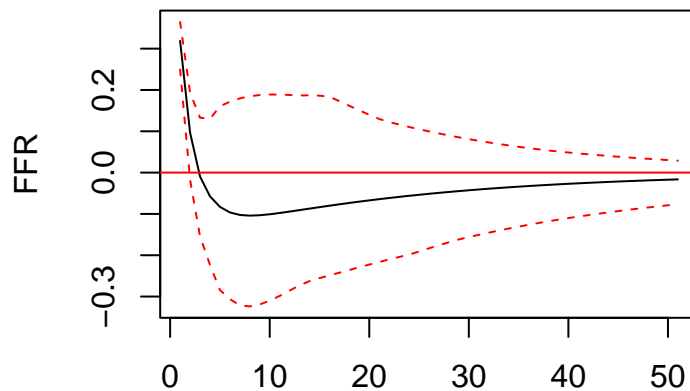
SVAR Impulse Response from FFR



95 % Bootstrap CI, 100 runs

```
plot(impulseFFR3)
```

SVAR Impulse Response from FFR



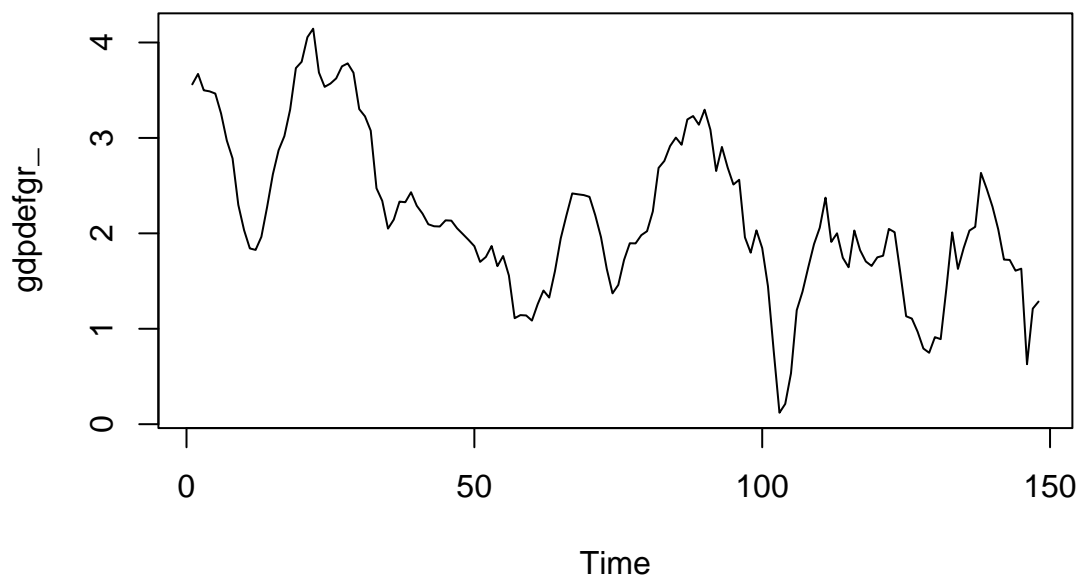
95 % Bootstrap CI, 100 runs

Very poor performance in terms of inflation as even in the long run no decrease in the inflation (i.e. contraction of the economy). Also, the impulse response of the 3 Months Treasury Bill looks rather unorthodox, as the shock is not at all persistent.

Splitting the time series and only redo the analysis for the constrained sample:

A suitable time point seems to be somewhere around the 1980s. During this time, the time series of the GDP deflator seems to be becoming stationary, which would enable us to avoid taking second differences and solely work with first differences. This assumption will be tested in the following using data from 1985 Q1 onwards:

```
gdpdef<-xrawtseries[113:264,2]
gdpdefgr_<-diff(log(gdpdef), lag=4)*100
plot.ts(gdpdefgr_)
```



```
adf.test(gdpdefgr_, alternative=c("stationary")) #stationary
```

```
## Warning in adf.test(gdpdefgr_, alternative =  
## c("stationary")): p-value smaller than printed p-value
```

```
##  
## Augmented Dickey-Fuller Test  
##  
## data: gdpdefgr_  
## Dickey-Fuller = -4.0648, Lag order = 5, p-value =  
## 0.01  
## alternative hypothesis: stationary
```

```
#Set up a new matrix for this subperiod: 1985 Q1 : 2020 Q4
```

```
yest4<-yest[109:256,]  
yest4[,2]<-gdpdefgr_
```

```
#Estimation same as before:
```

```
lagselection4<-VARselect(yest4)
```



```

lag4<-lagselection3$selection[3]

#Estimation procedure of the VAR(2)
var4<-VAR(yest4, p=lag4, type=c("const"))

#Storing the coefficients of the VAR(2)
A4 <- as.matrix(Bcoef(var4))

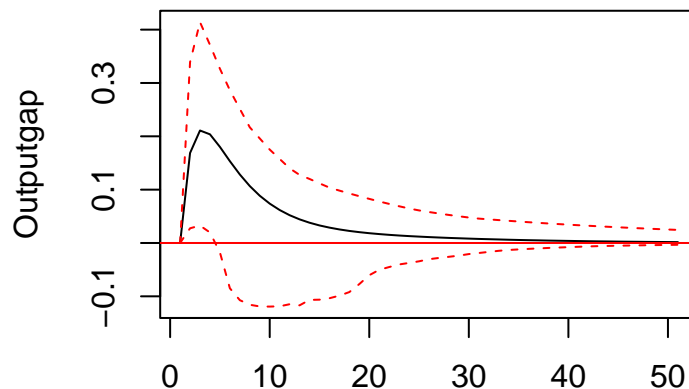
#Estimating the structural VAR:
svar4<-SVAR(var4, Amat=amat, Bmat=NULL, lrtest = FALSE)

#Impulse response functions:
impulseOutputgap4<-vars::irf(svar4, response="Outputgap", impulse="FFR", n.ahead=50)
impulseInflation4<-vars::irf(svar4, response="Inflation", impulse="FFR", n.ahead=50)
impulseFFR4<-vars::irf(svar4, response="FFR", impulse="FFR", n.ahead=50)

plot(impulseOutputgap4)

```

SVAR Impulse Response from FFR



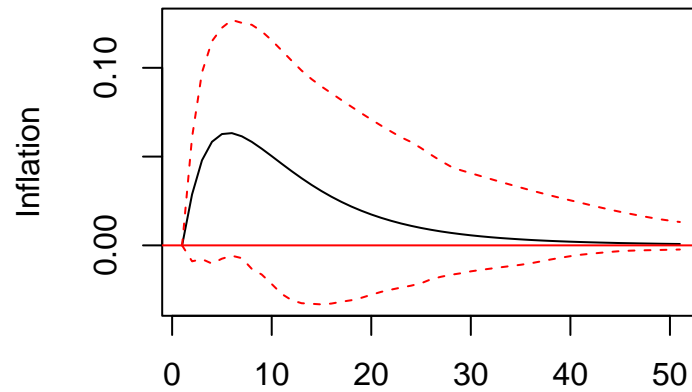
95 % Bootstrap CI, 100 runs

```

plot(impulseInflation4)

```

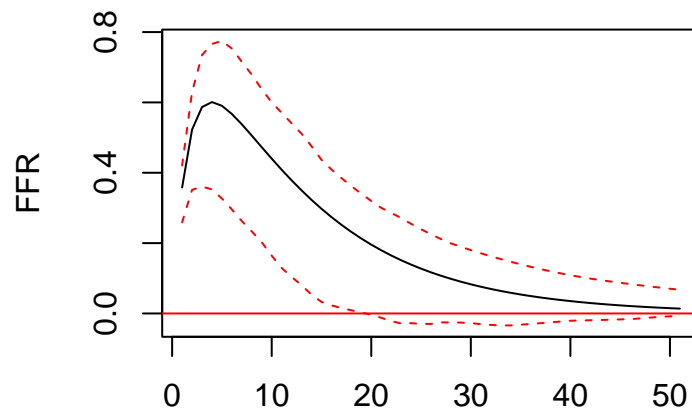
SVAR Impulse Response from FFR



95 % Bootstrap CI, 100 runs

```
plot(impulseFFR4)
```

SVAR Impulse Response from FFR



95 % Bootstrap CI, 100 runs

Very poor performance of the whole system in terms of explaining the monetary policy shock; no negative impact on inflation, same holds true for the output gap.

Bonus Question

Bayesian VARs

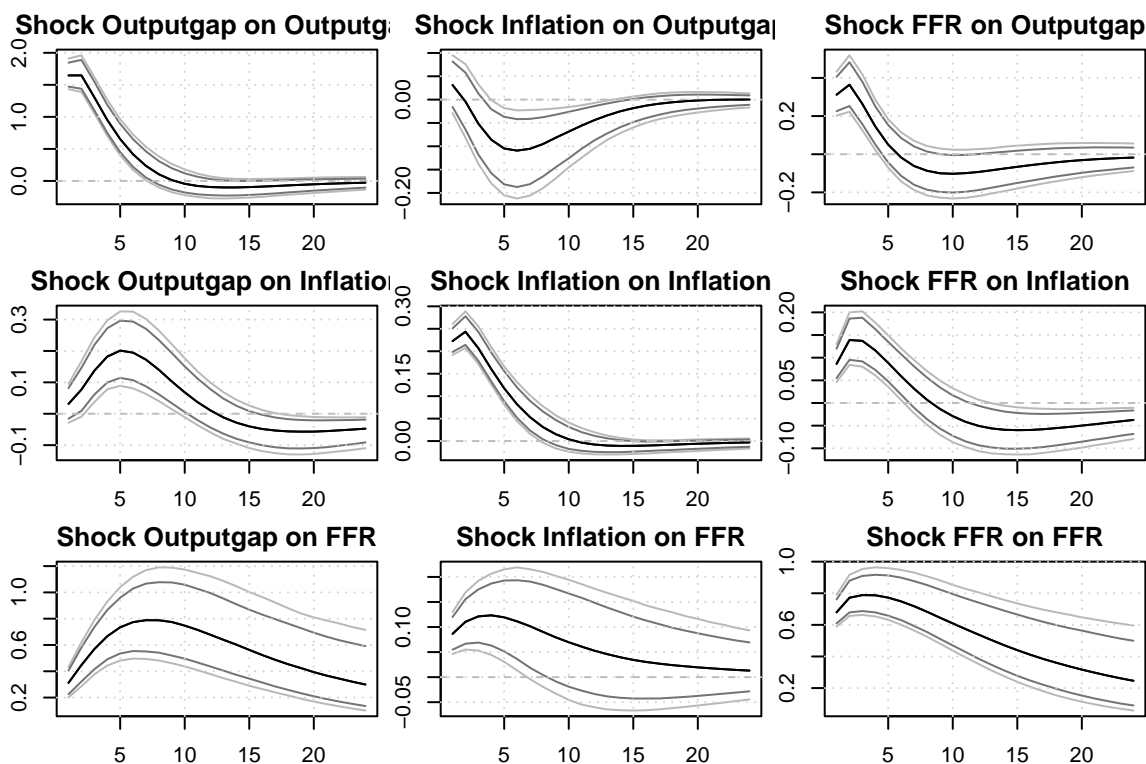
```
#For perfect reproducibility, it is necessary to set the seed:  
set.seed(123456789)
```

```
#Setting an appropriate prior  
prior<-bv_priors()
```

```
#Estimate the Bayesian VAR  
x<-BVAR::bvar(yest, lags=lag)
```

```
## Optimisation concluded.  
## Posterior marginal likelihood: -963.408  
## Hyperparameters: lambda = 0.27253  
## |  
## Finished MCMC after 4.58 secs.
```

```
#calculate the impulse response functions  
setting<-bv_irf(horizon=24, identification = FALSE)  
irf(x)<-BVAR::irf(x, setting, conf_bands=c(0.05, 0.1), n_thin=1L)  
plot(BVAR::irf(x))
```



Which prior did you use? The Minnesota prior, with nothing adjusted, everything governed/optimised by the algorithm. Using this prior regularises the data. This helps to remove the underestimation of persistence common in frequentist approaches.

What are the differences compared to the frequentist VAR? In estimation, we use the concept of belief updating; the prior is updated using the dataset to obtain the posterior. Contentwise, the puzzle that was part of the frequentist VAR also appears in the impulse Response functions; even though there is a monetary policy shock which raises the interest rate, Inflation and Outputgap first increase before decreasing.

Frequentist VAR has a small sample bias, where the persistence of parameter estimates is systematically underestimated. Frequentist VAR also faces the curse of dimensionality - the proliferation of parameters when new variables are added. Bayesian VAR approaches address these limitations through regularization and the choice of relevant priors, thus limiting the variance of parameter estimates. The interpretation of uncertainty bands is also different in both approaches - see below.

How do you interpret the uncertainty bands? The uncertainty bands result from the estimation uncertainty introduced using the Bayesian VAR. Between the bands, the credible set of impulse responses can be found. In Bayesian statistics, parameters are not considered to be fixed, but random variables. So, the uncertainty bands can be interpreted as the distribution of the parameter of interest. This reflects the fundamental epistemological uncertainty in the world.

In frequentist statistics, we think of the parameter as a fixed and unknown value, and the uncertainty band is a set of values within which the ‘true’ value should lie with a given certainty.