Fall 2023 Final Project

MAE 565

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Introduction

The RS-25 engine, renowned for its use in the Space Shuttle main engines (SSMEs) and future planned applications in NASA's Space Launch System (SLS), operates on a staged combustion cycle. This closed-cycle propulsion system employs liquid hydrogen (LH2) as fuel and liquid oxygen (LO2) as the oxidizer. The propellant feed system plays a crucial role in delivering these propellants to the main combustion chamber efficiently. In this analysis, we will focus on understanding the intricate flow paths of LH2 and LO2, their combustion in the preburner, and the subsequent injection into the main combustion chamber at 100% of the engine's rated power level (RPL). To elucidate the propellant flow paths and combustion process in the RS-25 engine, we will delve into the system's components.

Report Layout

In this project report, a detailed analysis of the RS-25 engine's propellant feed system is presented, covering various crucial components. The report begins with an exploration of fluid properties and an in-depth examination of the low-pressure fuel turbo pump (LPFTP), encompassing pump properties, temperature calculations, and power requirements. Subsequently, the focus shifts to the turbine section and the high-pressure fuel turbo pump (HPFTP) pump, providing insights into their functions and performance metrics. The report then delves into the intricacies of the high-pressure liquid hydrogen (HP LH2) flow splits and the low-pressure oxidizer turbopump (LPOTP) pump. The analysis extends to the high-pressure oxidizer turbopump (HPOTP) pump and the boost pump, offering a comprehensive understanding of their roles and operational characteristics. Further sections explore pre-burner fuel and oxidizer dynamics, turbine sections for pre-burners, main injectors, and the thrust chamber. The latter part of the report scrutinizes nozzle dimensions, thrust calculations, and efficiency considerations. This systematic organization allows for a thorough exploration of the RS-25 engine's propellant feed system, providing valuable insights into each component's functionality and contribution to the overall engine performance.

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11. Thrust Sea Level

Fluid Properties.

Looking at the RS-25 engine, there are many values known. The temperatures in the tanks, densities, specific heat C_v, specific heat C_p, γ_{H2}, and SSME vertical acceleration. This is all shown in table 1.

Fluid (Gas and Liquid) Properties			
	LO2 Temperature in Tank	90	K
	LH2 Temperature in Tank	20	K
	LO2 Density	1141	kg/m^3
	LH2 Density	70.8	kg/m^3
	GH2 Density Entering LPFTP		
	LO2 Specific heat C_v	1669	J/kg-K
	LH2 Specific heat C_v	9668	J/kg-K
	GO2 Specific heat C_p	919.1	J/kg-K
	GH2 Specific Heat C_p	14340	J/kg-K
	GH2 C_v	4672	
	GH2 Gamma value	1.483	---
	SSME vertical acceleration (T/M)	14.9	m/s^2

Table 1 Given Values

LPFTP:

Pump Inlet

Starting at the fuel pump in the low pressure fuel turbo pump, the mas flow 67.1 $\frac{kg}{s}$ and the pressure increase of 1.6 MPa the pump is know. The pressure being feed to the pump will be the pressure after $H2$ provided by the tank after the valve is mentioned to be 207 kPa and the temperature being 20 K.

Pump Outlet

The pressure exiting the pump is found by adding the 1.6 MPa pressure increase of the pump with the initial pressure entering the fuel pump as shown in equation 1.

$$P_{out} = P_{in} + \Delta P_{fuel\,pump} = 1807\ kPa \tag{1}$$

Using equation 2, can find the temperature of the fuel exiting the fuel pump using the given pumps efficiency, pressure difference of the fuel, density of the fuel, and C_v.

$$T_2 = \left(\frac{1-\eta_p}{\eta_p}\right)\left(\frac{p_{t2}-p_{t1}}{\rho_{H_2}C_v}\right) + T_1 = 21\ K \tag{2}$$

The power of the fuel pump is found by equation for the work of a pump using the efficiency of a pump, pressure difference and the density of the fluid going the pump as shown in equation 3.

$$\left|\dot{W}\right|_{actual} = \dot{m}\frac{1}{\eta_p}\frac{(p_{t2}-p_{t1})}{\rho} = 2.5\ MW \tag{3}$$

Turbine Section
Inlet

It is given that 20.3% of the mass that comes from the HPFTP goes through the thrust chamber as a heat exchanger and becomes a hot gas with a temperature of 269 K and pressure of 32.5 MPa. The mass flow can be found by simply multiplying 0.203 by the mass flux through the HPFTP, which is known to be 67.1 $\frac{kg}{s}$ as the HPFTP is being supplied by the LPFTP. Equation 4 Shows the calculations for getting the max flux of the turbo pump.

$$\dot{m}_t = (percantage\ of\ mass) * \dot{m}_{HPFTP} = 13.6\frac{kg}{s} \tag{4}$$

The pressure after fuel leaves the turbine can be found immediately by simply using the pressure ration of 1.3 in order to get the value of 25000 Kpa. The temperature is can be found by first finding the temperature ratio of the using the turbines efficiency, pressure ratio, and γ as shown in equation 5 .

$$T_2 = T_1 + (1 - \eta_T)\frac{p_{t1} - p_{t2}}{\rho c_v} \tag{5}$$

The Work being done by the turbine can be found by using equation 6, using the mass flow, C_p, and the temperature difference in the system. where the c_p of the system is found by equation 7 using the γ given and the molar weight of the gas as shown in equation 7.

$$\dot{W}_T = \dot{m}c_p(T_{t1} - T_{t2}) = 2.25\ MW \tag{6}$$

$$c_P = \frac{\gamma}{\gamma - 1} * \frac{R}{MW} \tag{7}$$

HPFTP

Pump Section

The fluid flowing to the inlet of the Pump is still a liquid. The pressure at the inlet is 1409 kPa as there is a 398 kPa pressure drop, going from the LPFTP to the HPFTP. The mas flow rate entering the pump is the 67.1 $\frac{kg}{s}$ due to conservation of mass staying the same going from the low pressure pump to the high pressure pump. There is a given pressure increase given of 41.7 MPa. The pressure leaving the pump is 43109 kPa as it is just a simple addition of the pressure increase plus the pressure entering the pump. The temperature entering the pump will just be 21 K, as it is the temperature leaving the LPFTP. The temperature leaving the HPFTP will be found by the following equation 8, with the given pump efficiency of 0.758 and the pressure difference given.

$$T_2 = \left(\frac{1 - \eta_p}{\eta_p}\right)\left(\frac{p_{t2} - p_{t1}}{\rho_{H_2} c_v}\right) + T_1 = 40.6\ K \tag{8}$$

The work required for the pump to operate under the given conditions is shown in equation 9.

$$\left|\dot{W}\right|_{actual} = \dot{m}\frac{1}{\eta_p}\frac{p_{t2} - p_{t1}}{\rho} = 52.13\ MW \tag{9}$$

HP LH2 Flow Splits

Starting off with the mass flow going to the thrust chamber cooling, it is known that 20.3% of mass flow goes from the HPFTP to the thrust chamber cooling, which means that 13.6 $\frac{kg}{s}$. The temperature entering will be 40.6 K. It is given that after exiting the thrust chamber cooling ,the pressure is 32.5 MPa and the temperature is 269 K.

Now going to the mass flow going to the expansion nozzle cooling, it is known that 42.4% of the mass flow from the HPFTP goes to the expansion nozzle cooling which is 28.5 $\frac{kg}{s}$. The temperature and pressure leaving is given and is 265 K and 35200 kPa.

37.3% of the mass flow bypasses the thrust chamber cooling and the expansion nozzle cooling, which is 25 $\frac{kg}{s}$. The mass goes into a valve and the pressure drops to 35.2 MPa and the pressure drops to 28 k and then rejoins the expansion nozzle cooling aftermath flow and together they become 53.5 $\frac{kg}{s}$ and 154 K with the pressure of 35.2 MPa. The temperature was found by using equation 10.

$$T = Y_1\left(\frac{c_{p1}}{c_p}\right)T_1 + Y_2\left(\frac{c_{p2}}{c_p}\right)T_2 \tag{10}$$

LPOTP

Pump

It is known that the fluid going through the inlet of the pump is a liquid with the given temperature and pressure of the tank. The pressure of the inlet is 689 kPa after the valve that connects the tank to the LPOTP and a temperature of 90 K. The mass flow being fed to the pump is 401 $\frac{kg}{s}$, pressure is 2.1 MPa and the efficiency is 0.632. The outlet pressure was found by using adding the know pressure increase across the pump to the inlet pressure from the inlet of the pump which gives the value of 2.789 MPa. The temperature of at the outlet of the pump is found with equation 12.

$$T_2 = \left(\frac{1-\eta_p}{\eta_p}\right)\left(\frac{p_{t2}-p_{t1}}{\rho_{H_2}c_v}\right) + T_1 = 91\,K \tag{12}$$

The work being done the pump is found with the use with the use of the known efficiency of the pump, pressure difference across the pump, density, and the mass flow going through the pump as shown in equation 13.

$$|\dot{W}|_{actual} = \dot{m}\frac{1}{\eta_p}\frac{(p_{t2}-p_{t1})}{\rho} = 1.168\,MW \tag{13}$$

The power being produce by turbine is the same work being used by the pump as shown in equation 13, which is 1.168 MW. The pressure that exits the turbine is 2.8 MPa, which is a value given in the document that is fed into the HPOTP, assuming there is no pressure loss going from the LPOTP to the HPOTP.

The following part is extremely important for the most of the unknown values found from throughout the spreadsheet. The temperature in entering the tube was an unknown and an initial guess was made. This value was kept throughout the spreadsheet and taken through the HPOTP and at the end matched with the output of the HPOTP and then new guesses where made until the outlet temperature of the HPOTP and the inlet temperature of the LPOTP matched. The final value was found to be around 101 K. The pressure entering the LPOTP is 29.2 MPa, which is given as it is directly connected to the exit of the HPOTP. The mass flow through the turbine was found by the manipulation of the Work formula of turbine as shown by using equation 14.

$$\dot{m} = \eta_T \frac{p_{t1} - p_{t2}}{\rho} \left(\frac{1}{\dot{W}_{actual}} \right) = 78.3 \frac{kg}{s} \tag{14}$$

The temperature exit of the pump is found by using equation 15.

$$T_2 = T_1 + (1 - \eta_T) \frac{p_{t1} - p_{t2}}{\rho c_v} = 110.9K \tag{15}$$

HPOTP

Pump

The mass flow entering the pump is a liquid at a pressure of 2.8 MPa, with a mass flow of 479 kg/s, exiting pressure of 29.6 MPa, and an efficiency of 0.681 as given by the document. The temperature of the mass flow can be found by using enthalpy conservation, since we are using mass-specific heats and the ratios between the combination of the mass from exiting from the turbine and pump from the LPOTP. The temperature can be found using equation 16 with the combination of both mass flows as mentioned.

$$T = Y_1 \left(\frac{c_{p1}}{c_p} \right) T_1 + Y_2 \left(\frac{C_{p2}}{c_p} \right) T_2 = 94\ K \tag{16}$$

Where Y_1 and Y_2 are mass fractions as shown in equation 17 and c_p is found using equation 18.

$$Y_2 = \frac{\dot{m}_1}{\dot{m}}, \quad Y_2 = \frac{\dot{m}_2}{\dot{m}} \tag{17}$$

$$c_p = Y_1 c_{p1} + Y_2 c_{p2} \tag{18}$$

The temperature exiting the pump can be found by finding by using isentropic with the efficiency equation 19.

$$T_2 = (1 - \eta_T)\frac{p_{t1} - p_{t2}}{\rho c_v} + T_1 = 101\ K \tag{19}$$

The work required for the pump to operate under the given conditions is shown in equation 20.

$$\left|\dot{W}\right|_{actual} = \dot{m}\frac{1}{\eta_p}\frac{p_{t2} - p_{t1}}{\rho} = 16.5\ MW \tag{20}$$

Boost Pump

The flow going through the boost pump Is still a liquid as it is the flow coming from the HPOTP which is still a liquid. The boost pump has a mass flow with a temperature of 101 K and pressure of 29.6 kPa. The pressure is found on the document and the temperature is retrieved from the temperature of the HPOTP exit conditions. As stated in the document, the pressure of the mass flow is then increased to 50.2 MPa with a given boost pump efficiency of 0.803. The mass flow rate in the is also given by the document as a mass flow rate of 48.3 $\frac{kg}{s}$. The exiting temperature was found with equation 21 and the power required to operate such pump is given by equation 22.

$$T_2 = (1 - \eta_T)\frac{p_{t1} - p_{t2}}{\rho c_v} + T_1 = 103\ K \tag{21}$$

$$\left|\dot{W}\right|_{actual} = \dot{m}\frac{1}{\eta_p}\frac{p_{t2} - p_{t1}}{\rho} = 1.1\ MW \tag{22}$$

Pre-burner Fuel

Fuel

The mass flow from the expansion nozzle cooling and the mass that went through the valve, both totaling at 53.5 $\frac{kg}{s}$ with be split and headed to the pre-burners. The 68% of the mass flow will head into the left-hand side where the LPFTP is positioned, for a total of 36.4 $\frac{kg}{s}$. While the 32% of the fuel goes to the right-hand side, right above the HPOTP, which is 36.4 $\frac{kg}{s}$ going to the pre-burner. Both are at a temperature of 154 K and pressure of 35.2 MPa as previously found.

Oxidizer

To find the mass flow going into the pre-burner, it is known that the pre-burner need an O/F ratio of 0.97. The mass flow of the oxidizer is found using equation 22.

$$O_2 = 0.97H = 35.3\frac{kg}{s} \tag{22}$$

The temperature of the oxidizer mass flow section is found and traced back from the boost pump exit conditions which shows that the temperature is 103K, with a pressure of 50.2 MPa. By combining the mas flow rate of the oxidizer and the fuel, a total of 71.6 $\frac{kg}{s}$ is shown to be entering the pre-burner. Table 2 shows product from the pre-burners which were all provided by the document.

Preburner product gas Y_H2	0.446	---
Preburner product gas Y_H2O	0.554	---
Preburner product gas temperature	1117	K
Preburner product gas pressure	35500	kPa
Preburner product gas gamma value	1.35	---
Preburner product gas C_p value	8088	J/kg-K
Preburner product gas MW value	3.97	g/mol

Table 2 HPFTP Pre-burners Product conditions

Turbine Section

Many of the conditions of the hot gas product from the HPFTP pre-burner is given, meaning that many of the conditions of the gas entering the HPFTP are known. In the document, the turbine inlet temperature is 1117 K, pressure is 35.5 MPa, $\frac{p_{in}}{p_{out}} = 1.52$, and turbine efficiency of 0.77. The power of turbine if easily known as it is the same as the power needed to power the HPFTP pump. The exit pressure can be found as the pressure ratio is known, so a the pressure in divided by the pressure ratio given will give the pressure out 23.355 kPa. The temperature was found with equation 23 as shown below.

$$T_2 = (1 - \eta_T)\frac{p_{t1} - p_{t2}}{\rho c_v} + T_1 = 1121\ K \tag{23}$$

Pre-burner Oxidizer

Fuel

The High-Pressure Oxidizer Turbopump's (HPOTP) preburner operates an (O/F) mass ratio of 0.668. During the combustion process, the preburner receives a flow of 11.3 kg/s of liquid oxygen (LO2) and 17.1 kg/s of gaseous hydrogen (GH2). Table 2 shows product from the pre-burners which were all provided by the document. The temperature of the LO2 is 103 K and pressure 50.2 MPa which was found in the exit states of the boost pump. The preburner max flux would just be the addition of the known GH2 and the LO2 values, which sum up to 28.4 $\frac{kg}{s}$. The rest of the values in table 3 where retrieved from the document.

Preburner product gas Y_H2	0.446	---
Preburner product gas Y_H2O	0.554	---
Preburner product gas temperature	1117	K
Preburner product gas pressure	35500	kPa
Preburner product gas gamma value	1.35	---
Preburner product gas C_p value	8088	J/kg-K
Preburner product gas MW value	3.97	g/mol

Table 3 HPOTP Pre-burners Product conditions

Turbine Section

The HPOTP turbine power would just be the power required for the boost pump and the main pump in the HPOTP. When added together, the power would amount to 17.6 MW. The gas pressure of the product is 836K and the pressure is 34.4 MPa, which are provided in the document as the product exiting the preburner, is entering the turbine. The pressure exiting was found to be 23.355 MPa, as the pressure exiting both turbines have to be the same. By dividing the pressure going in by the pressure going out, the resulting $\frac{p_{in}}{p_{out}} = 1.47$. The max flux would just be 28.4 $\frac{kg}{s}$ which is just the max flux that is passing from the preburner to the turbine. The temperature exiting the turbine was found by using equation 24 and the efficiency was found by using equation 25.

$$T_2 = (1 - \eta_T)\frac{p_{t1} - p_{t2}}{\rho c_v} + T_1 = 768\ K \tag{24}$$

$$\eta_T = \frac{1 - \frac{T_{t2}}{T_{t1}}}{1 - \left(\frac{p_{t2}}{p_{t1}}\right)^{\frac{\gamma-1}{\gamma}}} = 0.823 \tag{25}$$

Main Injectors

Table 4 shows the main states of the GH2-rich product, retrieved from the HPFTP and the LPFTP, which were either already calculated or given in the document provided.

GH2-rich product mass flow rate from HPFTP supplied to injectors	71.6	kg/s
GH2-rich product temperature from HPFTP supplied to injectors	1117	K
GH2-rich product C_p from HPFTP supplied to injectors	8088	J/kg-K
GH2-rich product MW from HPFTP supplied to injectors	3.97	g/mol
GH2-rich product pressure from HPFTP supplied to injectors	35500	kPa
GH2-rich product Y_H2 from HPFTP supplied to injectors	0.446	---
GH2-rich product Y_H2O from HPFTP supplied to injectors	0.554	---
GH2 Mass flow rate from LPFTP turbine supplied to injectors	13.6	kg/s
GH2 Temperature from LPFTP turbine supplied to injectors	257.2	K
GH2 Pressure from LPFTP turbine supplied to injectors	25000	kPa
GH2 C_p from LPFTP turbine supplied to injectors	14340	J/kg-K
GH2 MW from LPFTP turbine supplied to injectors	2.02	g/mol

Table 4 GH2-rich product from the HPFTP and the LPFTP

The combined GH2-rch mass flow rate would just be the addition of the two masses which are found at the exit of the turbines for the HPOTP and the HPFTP. Which when added will come out to 85.3 $\frac{kg}{s}$. The combine temperature can be found using mass fractions and temperature fraction as shown in equation 26. While the Combined GH2-rich pressure from the resulting combination can be found by using equation 27.

$$T = Y_1\left(\frac{c_{p1}}{c_p}\right)T_1 + Y_2\left(\frac{C_{p2}}{c_p}\right)T_2 = 900.2\ K \tag{26}$$

$$p = X_1 p_1 + X_2 p_2 = 32.644\ MPa \tag{27}$$

Where X_1 and X_2 and the mole fraction as shown in equation 28.

$$X_1 = \frac{\dot{n}_1}{\dot{n}} \quad , \quad X_2 = \frac{\dot{n}_1}{\dot{n}} \tag{28}$$

The mass ratio of H2 in the GH2 rich product was found by adding the masses of H2 in each flow together and dividing it by the total mass, which gave us 0.535. The same process was used for the ratio of H20, which resulted in a mass ratio of 0.465. Next , the combination of the c_p was found with the help of equation 29, as well as the molar weight with equation 30.

$$c_p = Y_1 c_{p1} + Y_2 c_{p2} = 9087 \frac{J}{kg - K} \tag{29}$$

$$MW = X_1 MW_1 + X_2 MW_2 = 3.44 \frac{g}{mol} \tag{30}$$

Next the states O2 that is being provided from the HPOTP is given in table 5. These states have already been solved for, from finding the exiting conditions from the turbine.

GH2-rich product mass flow rate from HPOTP supplied to injectors	28.4	kg/s
GH2-rich product temperature from HPOTP supplied to injectors	836	K
GH2-rich product pressure from HPFTP supplied to injectors	34400	kPa
GH2-rich product Y_H2 from HPOTP supplied to injectors	0.549	
GH2-rich product Y_H2O from HPOTP supplied to injectors	0.451	
GH2-rich product C_p from HPOTP supplied to injectors	9073	J/kg-K
GH2-rich product MW from HPOTP supplied to injectors	3.36	g/mol

Table 5 GH2-Rich Product from HTOTP

To find the total GH2 mass flow rate, a simple addition of each source is needed which gives 113.7 $\frac{kg}{s}$.

The combine temperature can be found using mass fractions and temperature fraction as shown in equation 31. While the Combined GH2-rich pressure from the resulting combination can be found by using equation 32.

$$T = Y_1 \left(\frac{c_{p1}}{c_p}\right) T_1 + Y_2 \left(\frac{C_{p2}}{c_p}\right) T_2 = 884\, K \tag{31}$$

$$p = X_1 p_1 + X_2 p_2 = 33.09\, MPa \tag{32}$$

The mass ratio of H2 in the GH2 rich product was found by adding the masses of H2 in each flow together and dividing it by the total mass, which gave us 0.538. The same process was used for the ratio of H20, which resulted in a mass ratio of 0.462. Next , the combination of the c_p was found with the help of equation 33, as well as the molar weight with equation 34.

$$c_p = Y_1 c_{p1} + Y_2 c_{p2} = 9087 \frac{J}{kg - K} \tag{33}$$

$$MW = X_1 MW_1 + X_2 MW_2 = 3.44 \frac{g}{mol} \tag{34}$$

The pressure drop across the injector for the GH2 was given to be 16.2 MPa which makes the pressure drop at the exit of the injectors to be 16.89 MPa. The mass flow rate from the HPOTP supply to the injectors can be found to be 353 $\frac{kg}{s}$. Which was found to by subtracting the 48.3$\frac{kg}{s}$ from 401 $\frac{kg}{s}$. Which is basically just subtracting away the mass flowrate going into the boost pump from the main flow. The temperature of the LO2 entering the injectors from the HTOTP is just 101 K and the pressure is 29.6 MPa, which are just the exit conditions of the HPOTP. The pressure drops of the LO2 across the injects was given by the document to be 9 MPa, making the pressure at the exit of the injectors to be 20.6 MPa.

Thrust Chamber

The overall all O/F mass ratio is found by dividing the O2 being supplied by the remaining H2 that did not already get converted to H20. When this is calculated, the O/F value of 5.77 is given. The main combustion chamber is given to be 20.6 MPa. The mass flow rate of the combined GH2 rich preburner flow that enters the main combustion chamber is 466.4. this is done by just adding all the masses entering the main injectors. The H2 and H20 percent masses are just pulled from the already calculated values in the main injector section as well as their temperature.

The O/F mixture fraction entering is just the original Oxidizer mas flux going in, divided by the original fuel going into the system. The O2/H2 mass flux entering is essentially the same thing, but we do not include the already mixed in H2O by product. Table 6 shows the values of the combustion product gas values that was provided in the document.

Combustion product gas temperature in combustion chamber	3760	K
Combustion product gas pressure in combustion chamber	20.6	MPa
Combustion product gas Cp value in combustion chamber	8571	J/kg-K
Combustion product gas gamma value in combustion chamber	1.13	---
Combustion product gas MW in combustion chamber	16.5	g/mol

Table 6 Combustion Product Gas Values

The combustion product gas mass flux exiting the chamber would be the mass as the flux that is entering, which is 466.4 kg/s

Dimensions

Combustion chamber diameter	45.1	cm
Combustion chamber cross-sectional area A_C	0.160	m^2
Throat diameter	26.2	cm
Throat area A*	0.054	m^2
Ratio of A_C over A*	2.96	---

Table 7 Dimensions

The following areas with the given diameter dimensions were found by using the equation for the area of a circle, as shown in equation 35.

$$A = \pi r^2 \tag{35}$$

Expansion Nozzle

Nozzle exit diameter	2.304
Nozzle exit area A_e	4.17
Nozzle A_e/A*	77.3
Nozzle isentropic efficiency	0.97

Table 8 Nozzle dimensions

The nozzle diameters are given and equation 35 was used to find the areas of the diameters and simple division was used of the found area of the exit to the area of the throat to find the values in table 8.

The combustion product gas gamma value and the MW entering the nozzle are 1.13 and 16.5 g/mol respectively. These values were provided in the document.

In order to find the Mach at the exit of the nozzle the following equation bust be used and an iterative method must be used in order to find the value of the Unknown value of the Mach, which was found to be 4.055 using the equation 36. The Total-to-Static pressure ratio was found using equation 37, which was found to be 0.00135. The temperature could be found by doing an isentropic relation as shown in equitation 38 as shown below giving the value of 1757.4 K. The velocity at the exit could be found by using equation 39, which was found to be 4056 m/s.

$$\frac{A_e}{A^*} = \frac{1}{M_e}\left(\left(\frac{2}{\gamma+1}\right)\left(1+\frac{\gamma-1}{2}M_e^2\right)\right)^{\frac{1\gamma+1}{2\gamma-1}} * \left(1+\left(1-\frac{1}{\eta_N}\right)\frac{\gamma-1}{2}M_e^2\right)^{-\frac{\gamma}{\gamma-1}} \tag{36}$$

$$\frac{p_e}{p_{t2}} = \left(1-\frac{1}{\eta_N}\left(\frac{(\gamma-1)M_e^2}{2+(\gamma-1)M_e^2}\right)\right)^{\frac{\gamma}{\gamma-1}} \tag{37}$$

$$T_e = \left(\frac{p_e}{p_{t2}}\right)^{\frac{\gamma-1}{\gamma}} * T_t \tag{38}$$

$$M_e * \sqrt{\gamma R T_e} \tag{39}$$

The following shifting equilibriums were given show in table 9 and the velocity was found using equation 39.

Shifting Equilibrium	Combustion product gas gamma value exiting nozzle	1.256	---
Shifting Equilibrium	Combustion product gas MW exiting nozzle	12.62	g/mol
Shifting Equilibrium	M_e from non-isentropic nozzle flow w/ gamma exiting nozzle	4.22	---
Shifting Equilibrium	p_e from non-isentropic nozzle flow w/ gamma exiting nozzle	24.0	kPa
Shifting Equilibrium	T_e from non-isentropic nozzle flow w/ gamma exiting nozzle	1848	K
Shifting Equilibrium	V_e from non-isentropic nozzle flow w/ gamma exiting nozzle	5218	m/s

Table 9 Shifting Equilibrium Values

Thrust

The resulting jet thrust was found using the first part of the rocket equation 40.

$$T_J = \dot{m}_p V_e \tag{40}$$

The resulting pressure thrust was found by using the second half of the thrust equation 41.

$$(p_e - p_\infty)A_e \tag{41}$$

By combining the resulting jet thrust and the pressure thrust, we are given the rocket equation and thrust able to find the nominal thrust as shown in equation 42.

$$T = \dot{m}_p V_e + (p_e - p_\infty)A_e \tag{42}$$

The 0.8% nozzle lost is taken away and a the thrust goes from 2113 kN to 2096 kN. The actual thrust coefficient is then found by using equation 43. While the ideal thrust coefficient is solved for using equation 44.

$$c_{tactual} = \frac{T}{p_t A^*} \tag{43}$$

$$(C_T)_{Ideal} = \gamma \left(\left(\frac{2}{\gamma - 1}\right) \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}} \left(1 - \left(\frac{p_e}{p_t}\right)^{\frac{\gamma-1}{\gamma}}\right) \right)^{\frac{1}{2}} \tag{44}$$

The specific impulse was found by using equation 45.

$$I_{sp} = \frac{T}{\dot{m}_p g_0} \qquad 45$$

The process is the same for finding the values in a vacuum, the only difference being that in stead of p_∞, the pressure will be zero and we would need to use equation 46 to find the ideal thrust coefficient in a vacuum.

$$(C_T)_{Ideal} = \gamma\left(\left(\frac{2}{\gamma - 1}\right)\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}\right)^{\frac{1}{2}}$$

The efficiency of nozzle of coefficient of thrust, will just be the actual coefficient over the ideal coefficient, as was found for both cases.

Conclusion

The report provides a comprehensive analysis of a rocket propulsion system, focusing on the propellant feed system. It covers fluid properties, details of fuel and oxidizer pumps, pre-burner dynamics, and key components such as injectors and thrust chambers. The study delves into nozzle dimensions, expansion characteristics, and thrust calculations for both sea level and vacuum conditions. The organized structure of the report facilitates a clear understanding of the complex system, offering insights into the performance and efficiency of each component.