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1. Referencia

Algorítmo	Parámetros	Función
sort, stable_sort	f, 1	ordena el intervalo
nth_element	f, nth, l	void ordena el n-esimo, y
		particiona el resto
fill, fill_n	f, l / n, elem	void llena [f, l) o [f,
		f+n) con elem
lower_bound, upper_bound	f, l, elem	it al primer / ultimo donde se
		puede insertar elem para que
		quede ordenada
binary_search	f, l, elem	bool esta elem en [f, l)
copy	f, l, resul	hace $resul+i=f+i \ \forall i$
find, find_if, find_first_of	f, l, elem	it encuentra i \in [f,l) tq. i $=$ elem,
	/ pred / f2, l2	$\operatorname{pred}(i), i \in [f2, l2)$
count, count_if	f, l, elem/pred	cuenta elem, pred(i)
search	f, l, f2, l2	busca $[f2,l2) \in [f,l)$
replace, replace_if	f, l, old	cambia old / pred(i) por new
	/ pred, new	
reverse	f, 1	da vuelta
partition, stable_partition	f, l, pred	pred(i) ad, !pred(i) atras
min_element, max_element	f, l, [comp]	$it \min, \max de [f,l]$
lexicographical_compare	f1,l1,f2,l2	bool con [f1,l1];[f2,l2]
next/prev_permutation	f,l	deja en [f,l) la perm sig, ant
set_intersection,	f1, l1, f2, l2, res	[res,) la op. de conj
set_difference, set_union,		
set_symmetric_difference,		
push_heap, pop_heap,	f, l, e / e /	mete/saca e en heap [f,l),
make_heap		hace un heap de [f,l)
is_heap	f,1	bool es [f,l) un heap
accumulate	f,l,i,[op]	$T = \sum \text{oper de [f,l)}$
inner_product	f1, l1, f2, i	$T = i + [f1, 11) \cdot [f2, \dots)$
partial_sum	f, l, r, [op]	$r+i = \sum /oper de [f,f+i] \forall i \in [f,l)$
builtin_ffs	unsigned int	Pos. del primer 1 desde la derecha
builtin_clz	unsigned int	Cant. de ceros desde la izquierda.
_builtin_ctz	unsigned int	Cant. de ceros desde la derecha.
_builtin_popcount	unsigned int	Cant. de 1's en x.
_builtin_parity	unsigned int	1 si x es par, 0 si es impar.
_builtin_XXXXXXII	unsigned ll	= pero para long long's.

2. Estructuras

2.1. RMQ (static)

```
1 // Dado un arreglo y una operacion asociativa idempotente:
  // get(i, j) opera sobre el rango [i, j).
   // Restriccion: LVL >= ceil(log n). Usar [ ] para llenar
   // el arreglo y luego build().
   struct RMQ {
     #define LVL 17
6
     tipo vec[LVL][1 << (LVL + 1)];
7
     tipo &operator [](int p){ return vec[0][p]; }
8
     tipo get(int i, int j){ // intervalo [i, j)
9
       int p = 31 - \_builtin\_clz(j - i);
10
       return min(vec[p][i], vec[p][j - (1 << p)]);</pre>
11
12
     void build(int n) { // O(n log n)
13
       int mp = 31 - __builtin_clz(n);
14
       forn(p, mp) forn(x, n - (1 << p))
15
         vec[p + 1][x] = min(vec[p][x], vec[p][x + (1 << p)]);
16
     }
17
18 };
```

2.2. RMQ (dynamic)

```
// Dado un arreglo y una operacion asociativa con neutro:
   // get(i, j) opera sobre el rango [i, j).
   typedef int node; // Tipo de los nodos
   #define MAXN 100000
   #define operacion(x, y) max(x, y)
   const int neutro = 0;
   struct RMQ {
7
     int sz;
8
     node t[4*MAXN];
9
     node &operator [](int p){ return t[sz + p]; }
10
     void init(int n){ // O(n lg n)
11
       sz = 1 << (32 - __builtin_clz(n));</pre>
12
       forn(i, 2*sz) t[i] = neutro;
13
14
     void updall(){//0(n)}
15
       dforn(i, sz){
16
         t[i] = operacion(t[2*i], t[2*i + 1]);
17
       }
18
```

```
19
     node get(int i, int j){ return get(i, j, 1, 0, sz); }
20
     node get(int i, int j, int n, int a, int b){ // O(\lg n)
21
       if(j <= a || i >= b) return neutro;
22
       if(i <= a && b <= j) return t[n];
23
       int c = (a + b)/2;
24
       return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n + 1, c, b));
25
26
     void set(int p, node val){ // O(lg n)
27
       for(p += sz; p > 0 && t[p] != val;){
         t[p] = val;
29
         p /= 2;
         val = operacion(t[p*2], t[p*2 + 1]);
32
    }
   } rmq;
  // Uso:
36 | cin >> n; rmq.init(n); forn(i, n) cin >> rmq[i]; rmq.updall();
                            2.3. RMQ (lazy)
1 // Dado un arreglo y una operacion asociativa con neutro:
2 // get(i, j) opera sobre el rango [i, j).
   typedef int node; // Tipo de los elementos del arreglo
   typedef int alt; // Tipo de la alteracion
   #define operacion(x, y) (x + y)
   const node neutro = 0; const alt neutro_alt = 0;
   #define MAXN 100000
   struct RMQ {
     int sz;
     node t[4*MAXN];
     alt dirty[4*MAXN];
11
     node &operator [](int p){ return t[sz + p]; }
12
     void init(int n){ // O(n lg n)
13
       sz = 1 \ll (32 - \_builtin\_clz(n));
14
       forn(i, 2*sz){
15
         t[i] = neutro;
16
         dirty[i] = neutro_alt;
17
       }
18
19
     void push(int n, int a, int b){ // Propaga el dirty a sus hijos
20
       if(dirty[n] != neutro_alt){
21
         t[n] += dirty[n]*(b - a); // Altera el nodo
22
```

```
if(n < sz){
23
           dirty[2*n] += dirty[n];
^{24}
           dirty[2*n + 1] += dirty[n];
25
26
         dirty[n] = 0;
27
28
29
     node get(int i, int j, int n, int a, int b){ // O(lg n)
30
       if(j <= a || i >= b) return neutro;
31
       push(n, a, b); // Corrige el valor antes de usarlo
32
       if(i <= a && b <= j) return t[n];
33
       int c = (a + b)/2;
34
       return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n + 1, c, b));
35
36
     node get(int i, int j){ return get(i, j, 1, 0, sz); }
37
     // Altera los valores en [i, j) con una alteración de val
38
     void alterar(alt val, int i, int j, int n, int a, int b){ // O(lg n)
39
       push(n, a, b);
40
       if(i \le a \mid | i >= b) return:
41
       if(i <= a && b <= j){
42
         dirty[n] += val;
43
         push(n, a, b);
44
         return;
45
46
       int c = (a + b)/2;
47
       alterar(val, i, j, 2*n, a, c); alterar(val, i, j, 2*n + 1, c, b);
48
       t[n] = operacion(t[2*n], t[2*n + 1]);
49
50
     void alterar(alt val, int i, int j){ alterar(val, i, j, 1, 0, sz); }
51
52 | } rmq;
                        2.4. RMQ (persistente)
   typedef int tipo;
```

```
typedef int tipo;
tipo oper(const tipo &a, const tipo &b){
    return a + b;
}
struct node {
    tipo v; node *1, *r;
    node(tipo v):v(v), l(NULL), r(NULL) {}
    node(node *1, node *r) : l(l), r(r){
    if(!1) v = r->v;
    else if(!r) v = l->v;
```

```
else v = oper(1->v, r->v);
    }
12
13
   };
   node *build (tipo *a, int tl, int tr) { // modificar para tomar tipo a
     if(tl + 1 == tr) return new node(a[tl]);
     int tm = (tl + tr) >> 1;
16
     return new node(build(a, tl, tm), build(a, tm, tr));
17
18
   node *upd(int pos, int new_val, node *t, int tl, int tr){
19
     if(tl + 1 == tr) return new node(new_val);
     int tm = (tl + tr) >> 1;
21
     if(pos < tm) return new node(upd(pos, new_val, t->1, tl, tm), t->r);
22
     else return new node(t->1, upd(pos, new_val, t->r, tm, tr));
23
24
   tipo get(int 1, int r, node *t, int tl, int tr){
25
     if(1 == tl && tr == r) return t->v;
     int tm = (tl + tr) >> 1;
     if (r \le tm) return get (l, r, t \rightarrow l, tl, tm);
     else if(l \ge tm) return get(l, r, t \ge r, tm, tr);
     return oper(get(1, tm, t->1, tl, tm), get(tm, r, t->r, tm, tr));
31 }
```

2.5. Fenwick Tree

```
1 // Para 2D: tratar cada columna como un Fenwick Tree,
  // agregando un for anidado en cada operacion.
   struct Fenwick {
     static const int sz = 1000001;
     tipo t[sz];
5
     void adjust(int p, tipo v) { // p en [1, sz), 0(\lg n)
6
       for(int i = p; i < sz; i += (i \& -i)) t[i] += v;
7
8
     tipo sum(int p){ // Suma acumulada en [1, p], O(lg n)
9
       tipo s = 0;
10
       for(int i = p; i; i -= (i \& -i)) s += t[i];
11
       return s;
12
13
     tipo sum(int a, int b){ return sum(b) - sum(a - 1); }
14
     // Obtener mayor valor con suma acumulada menor o igual que x.
15
     // Para el menor, pasar x - 1 y sumar 1 al resultado.
16
     int getind(tipo x){ // O(lg n)
17
       int idx = 0, mask = n;
18
       while(mask && idx < n) {</pre>
19
```

```
int z = idx + mask:
20
         if(x >= t[z])
^{21}
           idx = z, x = t[z];
^{22}
         mask >>= 1;
23
       }
^{24}
       return idx;
25
26
27 | };
                             2.6. Union Find
   struct UnionFind{
     vector<int> f;//the array contains the parent of each node
2
     void init(int n){f.clear(); f.insert(f.begin(), n, -1);}
3
     int comp(int x){return (f[x]=-1?x:f[x]=comp(f[x]));}//0(1)
4
     bool join(int i, int j) {
5
       bool con=comp(i)==comp(j);
6
       if(!con) f[comp(i)] = comp(j);
7
       return con;
8
     }};
9
                         2.7. Disjoint Intervals
   bool operator< (const ii &a, const ii &b) {return a.fst<b.fst;}
   //Stores intervals as [first, second]
   //in case of a collision it joins them in a single interval
   struct disjoint_intervals {
     set<ii>> segs;
5
     void insert(ii v) {//O(lgn)
       if(v.snd-v.fst==0.) return://0J0
       set<ii>>::iterator it.at:
8
       at = it = segs.lower_bound(v);
       if (at!=segs.begin() && (--at)->snd >= v.fst)
10
         v.fst = at->fst, --it;
11
       for(; it!=segs.end() && it->fst <= v.snd; segs.erase(it++))</pre>
12
         v.snd=max(v.snd, it->snd):
13
       segs.insert(v);
14
15
<sub>16</sub> };
                             2.8. RMQ (2D)
struct RMQ2D{//n filas x m columnas
     int sz;
2
```

```
RMO t[4*MAXN]:
     RMQ &operator[](int p){return t[sz/2+p];}//t[i][j]=i fila, j col
4
     void init(int n, int m){\frac{}{/0(n*m)}}
       sz = 1 \ll (32-\_builtin\_clz(n));
6
       forn(i, 2*sz) t[i].init(m); }
     void set(int i, int j, tipo val){//O(lgm.lgn)
       for(i+=sz; i>0;){
         t[i].set(j, val);
10
         i/=2;
11
         val=operacion(t[i*2][j], t[i*2+1][j]);
12
13
     tipo get(int i1, int j1, int i2, int j2){return get(i1, j1, i2, j2, 1, 0, sz
14
         ):}
     //O(lgm.lgn), rangos cerrado abierto
15
     int get(int i1, int j1, int i2, int j2, int n, int a, int b){
16
       if(i2<=a || i1>=b) return 0;
       if(i1<=a && b<=i2) return t[n].get(j1, j2);
18
       int c=(a+b)/2;
19
       return operacion(get(i1, j1, i2, j2, 2*n, a, c),
20
            get(i1, j1, i2, j2, 2*n+1, c, b));
21
     }
22
   } rmq;
   //Example to initialize a grid of M rows and N columns:
   RMQ2D rmg; rmg.init(n,m);
  forn(i, n) forn(j, m){
   int v; cin >> v; rmq.set(i, j, v);}
                               2.9. Big Int
   #define BASEXP 6
   #define BASE 1000000
   #define LMAX 1000
   struct bint{
       int 1;
5
       11 n[LMAX];
6
       bint(11 x=0){
7
           1=1;
8
           forn(i, LMAX){
9
               if (x) l=i+1;
10
               n[i]=x %BASE;
11
               x/=BASE;
12
13
           }
14
```

```
}
15
       bint(string x){
16
       l=(x.size()-1)/BASEXP+1;
17
           fill(n, n+LMAX, 0);
18
           ll r=1;
19
           forn(i, sz(x)){
20
               n[i / BASEXP] += r * (x[x.size()-1-i]-'0');
21
               r*=10; if(r==BASE)r=1;
22
           }
23
       }
24
       void out(){
25
       cout << n[l-1];
26
       dforn(i, l-1) printf("%6.61lu", n[i]);//6=BASEXP!
27
     }
28
     void invar(){
29
       fill(n+1, n+LMAX, 0);
30
       while(1>1 && !n[1-1]) 1--;
31
    }
32
33
   bint operator+(const bint&a, const bint&b){
     bint c;
35
       c.1 = max(a.1, b.1);
36
       11 q = 0;
37
       forn(i, c.1) q += a.n[i]+b.n[i], c.n[i]=q %BASE, q/=BASE;
38
       if(q) c.n[c.l++] = q;
39
       c.invar();
40
       return c;
41
^{42}
   pair<bint, bool> lresta(const bint& a, const bint& b) // c = a - b
43
44
     bint c:
45
       c.1 = max(a.1, b.1);
46
       11 a = 0:
47
       forn(i, c.1) q += a.n[i]-b.n[i], c.n[i]=(q+BASE) %BASE, q=(q+BASE)/
48
           BASE-1:
       c.invar():
49
       return make_pair(c, !q);
50
51
   bint& operator-= (bint& a, const bint& b){return a=lresta(a, b).first;}
   bint operator- (const bint&a, const bint&b){return lresta(a, b).first;}
   bool operator< (const bint&a, const bint&b){return !lresta(a, b).second
       ;}
55 bool operator <= (const bint&a, const bint&b) {return lresta(b, a).second
```

```
;}
   bool operator == (const bint&a, const bint&b) {return a <= b && b <= a;}
   bint operator*(const bint&a, ll b){
       bint c;
       11 q = 0;
       forn(i, a.l) q += a.n[i]*b, c.n[i] = q 'BASE, q/=BASE;
       c.1 = a.1;
       while(q) c.n[c.l++] = q %BASE, q/=BASE;
       c.invar();
       return c;
65
   bint operator*(const bint&a, const bint&b){
       bint c:
67
       c.l = a.l+b.l:
       fill(c.n, c.n+b.1, 0);
       forn(i, a.1){
           11 q = 0;
           forn(j, b.l) q += a.n[i]*b.n[j]+c.n[i+j], c.n[i+j] = q \text{BASE}, q
                /=BASE:
           c.n[i+b.1] = q;
73
       }
74
       c.invar();
       return c;
76
77
   pair<br/><br/>bint, 11> ldiv(const bint& a, 11 b)\{// c = a / b : rm = a \% b \}
     bint c;
79
     11 \text{ rm} = 0:
80
     dforn(i, a.1){
81
                rm = rm * BASE + a.n[i];
82
               c.n[i] = rm / b;
83
                rm % b;
84
       }
       c.1 = a.1:
86
       c.invar():
       return make_pair(c, rm);
88
89
   bint operator/(const bint&a, ll b){return ldiv(a, b).first;}
   11 operator %(const bint&a, ll b) {return ldiv(a, b).second;}
   pair<bint, bint> ldiv(const bint& a, const bint& b){
     bint c;
93
       bint rm = 0;
94
       dforn(i, a.l){
95
           if (rm.l==1 && !rm.n[0])
96
```

```
rm.n[0] = a.n[i]:
97
            else{
98
                dforn(j, rm.l) rm.n[j+1] = rm.n[j];
99
                rm.n[0] = a.n[i];
100
                rm.l++;
101
            }
102
            ll q = rm.n[b.1] * BASE + rm.n[b.1-1];
103
            ll u = q / (b.n[b.l-1] + 1);
104
            11 v = q / b.n[b.l-1] + 1;
105
            while (u < v-1){
106
                11 m = (u+v)/2;
107
                if (b*m \le rm) u = m;
108
                else v = m:
109
            }
110
            c.n[i]=u;
111
            rm-=b*u;
112
        }
113
      c.l=a.l;
114
        c.invar():
115
        return make_pair(c, rm);
116
117
    bint operator/(const bint&a, const bint&b){return ldiv(a, b).first;}
118
   bint operator%(const bint&a, const bint&b){return ldiv(a, b).second;}
```

2.10. HashTables

```
//Compilar: g++ --std=c++11
  struct Hash{
2
     size_t operator()(const ii &a)const{
3
       size_t s=hash<int>()(a.fst);
4
       return hash<int>()(a.snd)+0x9e3779b9+(s<<6)+(s>>2);
5
     }
6
     size_t operator()(const vector<int> &v)const{
       size_t s=0;
8
       for(auto &e : v)
9
         s = hash<int>()(e)+0x9e3779b9+(s<<6)+(s>>2);
10
       return s;
11
     }
12
13
   unordered_set<ii, Hash> s;
  unordered_map<ii, int, Hash> m;//map<key, value, hasher>
```

2.11. Modnum

```
1 | struct mnum{
     static const tipo mod=12582917;
2
     tipo v;
3
     mnum(tipo v=0): v(v mod) {}
4
     mnum operator+(mnum b){return v+b.v;}
     mnum operator-(mnum b){return v>=b.v? v-b.v : mod-b.v+v;}
     mnum operator*(mnum b){return v*b.v;}
     mnum operator^(int n){
       if(!n) return 1;
       return n\%2? (*this)^(n/2)*(this) : (*this)^(n/2);}
10
11 | };
                           2.12.
                                 Treap para set
typedef int Key;
   typedef struct node *pnode;
   struct node{
       Kev kev;
       int prior, size;
       pnode l,r;
       node(Key key=0): key(key), prior(rand()), size(1), 1(0), r(0) {}
7
   };
8
   static int size(pnode p) { return p ? p->size : 0; }
   void push(pnode p) {
     // modificar y propagar el dirty a los hijos aca(para lazy)
11
   }
12
   // Update function and size from children's Value
   void pull(pnode p) {//recalcular valor del nodo aca (para rmq)
     p->size = 1 + size(p->1) + size(p->r);
15
16
   //junta dos arreglos
   pnode merge(pnode 1, pnode r) {
     if (!1 || !r) return 1 ? 1 : r;
19
     push(1), push(r);
20
     pnode t;
21
     if (1-\text{prior} < r-\text{prior}) 1-\text{r=merge}(1-\text{r}, r), t = 1;
22
     else r\rightarrow l=merge(1, r\rightarrow 1), t = r;
23
     pull(t);
24
     return t;
25
26
   //parte el arreglo en dos, l<key<=r
   void split(pnode t, Key key, pnode &1, pnode &r) {
28
```

if (!t) return void(1 = r = 0);

29

```
push(t);
30
       if (\text{key} \leftarrow \text{t->key}) split(\text{t->l}, \text{key}, l, \text{t->l}), r = t;
31
       else split(t->r, key, t->r, r), l = t;
32
       pull(t);
33
34
35
   void erase(pnode &t, Key key) {
36
       if (!t) return;
37
       push(t);
38
       if (key == t->key) t=merge(t->1, t->r);
39
       else if (key < t->key) erase(t->1, key);
40
       else erase(t->r, key);
41
       if(t) pull(t);
42
43
44
   ostream& operator<<(ostream &out, const pnode &t) {
     if(!t) return out;
46
       return out << t->l << t->key << ''' << t->r;
47
48
   pnode find(pnode t, Key key) {
49
       if (!t) return 0;
50
       if (key == t->key) return t;
51
       if (key < t->key) return find(t->1, key);
52
       return find(t->r, key);
53
54
   struct treap {
55
       pnode root;
56
       treap(pnode root=0): root(root) {}
57
       int size() { return ::size(root); }
58
       void insert(Key key) {
59
           pnode t1, t2; split(root, key, t1, t2);
60
           t1=::merge(t1,new node(key));
61
           root=::merge(t1,t2);
62
       }
63
       void erase(Key key1, Key key2) {
64
           pnode t1,t2,t3;
65
            split(root,key1,t1,t2);
66
            split(t2,key2, t2, t3);
67
           root=merge(t1,t3);
68
69
       void erase(Key key) {::erase(root, key);}
70
       pnode find(Key key) { return ::find(root, key); }
71
       Key &operator[](int pos){return find(pos)->key;}//ojito
72
```

```
73 };
treap merge(treap a, treap b) {return treap(merge(a.root, b.root));}
                        2.13. Treap para arreglo
   typedef struct node *pnode;
   struct node{
       Value val, mini;
       int dirty;
       int prior, size;
       pnode 1,r,parent;
       node(Value val): val(val), mini(val), dirty(0), prior(rand()), size
            (1), 1(0), r(0), parent(0) {}
   };
 8
   static int size(pnode p) { return p ? p->size : 0; }
   void push(pnode p) {//propagar dirty a los hijos(aca para lazy)
     p->val.fst+=p->dirty;
     p->mini.fst+=p->dirty;
     if(p->l) p->l->dirty+=p->dirty;
     if(p->r) p->r->dirty+=p->dirty;
     p->dirty=0;
16
   static Value mini(pnode p) { return p ? push(p), p->mini : ii(1e9, -1);
   // Update function and size from children's Value
   void pull(pnode p) {//recalcular valor del nodo aca (para rmg)
     p \rightarrow size = 1 + size(p \rightarrow 1) + size(p \rightarrow r);
     p->mini = min(min(p->val, mini(p->l)), mini(p->r));//operacion del rmq
21
     p->parent=0;
22
     if(p->1) p->1->parent=p;
23
     if(p->r) p->r->parent=p;
24
25
   //junta dos arreglos
   pnode merge(pnode 1, pnode r) {
     if (!1 || !r) return 1 ? 1 : r;
     push(1), push(r);
29
     pnode t:
30
     if (1-\text{prior} < r-\text{prior}) 1-\text{r-merge}(1-\text{r}, r), t = 1;
31
     else r\rightarrow l=merge(1, r\rightarrow 1), t = r;
32
     pull(t);
33
34
     return t;
35 }
```

```
//parte el arreglo en dos, sz(1)==tam
   void split(pnode t, int tam, pnode &1, pnode &r) {
     if (!t) return void(1 = r = 0);
38
     push(t);
39
     if (tam \le size(t->1)) split(t->1, tam, 1, t->1), r = t;
     else split(t->r, tam - 1 - size(t->l), t->r, r), l = t;
41
     pull(t);
42
43
   pnode at(pnode t, int pos) {
     if(!t) exit(1);
     push(t);
     if(pos == size(t->1)) return t;
47
     if(pos < size(t->1)) return at(t->1, pos);
     return at(t->r, pos - 1 - size(t->1));
50
   int getpos(pnode t){//inversa de at
     if(!t->parent) return size(t->1);
52
     if(t==t->parent->l) return getpos(t->parent)-size(t->r)-1;
53
     return getpos(t->parent)+size(t->l)+1;
54
55
   void split(pnode t, int i, int j, pnode &1, pnode &m, pnode &r) {
56
     split(t, i, 1, t), split(t, j-i, m, r);}
57
   Value get(pnode &p, int i, int j){//like rmq
58
     pnode 1,m,r;
59
       split(p, i, j, l, m, r);
60
       Value ret=mini(m);
61
       p=merge(1, merge(m, r));
62
       return ret:
63
64
   void print(const pnode &t) {//for debugging
     if(!t) return;
66
       push(t);
67
       print(t->1);
68
       cout << t->val.fst << ''';
69
       print(t->r);
70
71 }
```

2.14. Convex Hull Trick

```
struct Line{tipo m,h;};
tipo inter(Line a, Line b){
    tipo x=b.h-a.h, y=a.m-b.m;
return x/y+(x%?!((x>0)^(y>0)):0);//==ceil(x/y)
```

```
5 }
   struct CHT {
6
     vector<Line> c;
     bool mx;
     int pos;
9
     CHT(bool mx=0):mx(mx),pos(0){}//mx=1 si las query devuelven el max
10
     inline Line acc(int i){return c[c[0].m>c.back().m? i : sz(c)-1-i];}
11
     inline bool irre(Line x, Line y, Line z){
12
       return c[0].m>z.m? inter(y, z) <= inter(x, y)</pre>
13
                             : inter(y, z) >= inter(x, y);
14
15
     void add(tipo m, tipo h) {//0(1), los m tienen que entrar ordenados
16
           if (mx) m*=-1, h*=-1;
17
       Line l=(Line){m, h};
           if(sz(c) && m==c.back().m) { l.h=min(h, c.back().h), c.pop_back
19
                (); if(pos) pos--; }
           while(sz(c) \ge 2 \&\& irre(c[sz(c)-2], c[sz(c)-1], 1)) { c.pop_back
20
                (); if(pos) pos--; }
           c.pb(1);
21
22
     inline bool fbin(tipo x, int m) {return inter(acc(m), acc(m+1))>x;}
23
     tipo eval(tipo x){
24
       int n = sz(c);
25
       //query con x no ordenados O(lgn)
26
       int a=-1, b=n-1;
27
       while(b-a>1) { int m = (a+b)/2;
         if(fbin(x, m)) b=m;
         else a=m:
30
       }
31
       return (acc(b).m*x+acc(b).h)*(mx?-1:1);
           //query 0(1)
33
       while(pos>0 && fbin(x, pos-1)) pos--;
34
       while(pos<n-1 && !fbin(x, pos)) pos++;
35
       return (acc(pos).m*x+acc(pos).h)*(mx?-1:1);
36
    }
37
38 } ch;
                2.15. Convex Hull Trick (Dynamic)
```

```
const ll is_query = -(1LL<<62);
struct Line {
    ll m, b;
    mutable multiset<Line>::iterator it;
```

```
const Line *succ(multiset<Line>::iterator it) const:
5
       bool operator<(const Line& rhs) const {</pre>
6
           if (rhs.b != is_query) return m < rhs.m;</pre>
           const Line *s=succ(it);
           if(!s) return 0;
9
           11 x = rhs.m;
10
           return b - s->b < (s->m - m) * x;
11
       }
^{12}
13
   struct HullDynamic : public multiset<Line>{ // will maintain upper hull
       for maximum
       bool bad(iterator y) {
15
           iterator z = next(y);
16
           if (y == begin()) {
17
               if (z == end()) return 0;
18
               return y->m == z->m && y->b <= z->b;
19
           }
20
           iterator x = prev(y);
21
           if (z == end()) return y->m == x->m && y->b <= x->b;
22
           return (x-b - y-b)*(z-m - y-m) >= (y-b - z-b)*(y-m - x-m)
23
               );
24
       iterator next(iterator y){return ++y;}
25
       iterator prev(iterator y){return --y;}
26
       void insert_line(ll m, ll b) {
27
           iterator y = insert((Line) { m, b });
28
           y->it=y;
29
           if (bad(y)) { erase(y); return; }
30
           while (next(y) != end() && bad(next(y))) erase(next(y));
31
           while (y != begin() && bad(prev(y))) erase(prev(y));
32
       }
33
       ll eval(ll x) {
34
           Line 1 = *lower_bound((Line) { x, is_query });
35
           return 1.m * x + 1.b;
36
       }
37
   }h;
   const Line *Line::succ(multiset<Line>::iterator it) const{
       return (++it==h.end()? NULL : &*it);}
40
```

2.16. Gain-Cost Set

```
1 //esta estructura mantiene pairs(beneficio, costo)
2 //de tal manera que en el set quedan ordenados
```

```
3 //por beneficio Y COSTO creciente. (va borrando los que no son optimos)
   struct V{
     int gain, cost;
     bool operator<(const V &b)const{return gain<b.gain;}</pre>
   };
7
   set<V> s;
   void add(V x){
     set<V>::iterator p=s.lower_bound(x);//primer elemento mayor o igual
     if(p!=s.end() && p->cost <= x.cost) return;//ya hay uno mejor
     p=s.upper_bound(x);//primer elemento mayor
     if(p!=s.begin()){//borro todos los peores (<=beneficio y >=costo)
13
       --p;//ahora es ultimo elemento menor o igual
14
       while(p->cost >= x.cost){
15
         if(p==s.begin()){s.erase(p); break;}
         s.erase(p--);
       }
     }
19
     s.insert(x);
21
   int get(int gain){//minimo costo de obtener tal ganancia
     set<V>::iterator p=s.lower_bound((V){gain, 0});
23
     return p==s.end()? INF : p->cost;}
24
```

2.17. Set con búsqueda binaria

```
#include <ext/pb_ds/assoc_container.hpp>
  #include <ext/pb_ds/tree_policy.hpp>
  using namespace __gnu_pbds;
  typedef tree<int,null_type,less<int>,//key,mapped type, comparator
       rb_tree_tag,tree_order_statistics_node_update> set_t;
  //find_by_order(i) devuelve iterador al i-esimo elemento
7 //order_of_key(k): devuelve la pos del lower bound de k
8 //Ej: 12, 100, 505, 1000, 10000.
9 //order_of_key(10) == 0, order_of_key(100) == 1,
\frac{10}{10} //order_of_key(707) == 3, order_of_key(9999999) == 5
```

3. Algorítmos

3.1. Longest Increasing Subsecuence

```
1 //Para non-increasing, cambiar comparaciones y revisar busq binaria
2 //Given an array, paint it in the least number of colors so that each
      color turns to a non-increasing subsequence.
```

```
3 //Solution:Min number of colors=Length of the longest increasing
       subsequence
  int N, a[MAXN];//secuencia y su longitud
   ii d[MAXN+1];//d[i]=ultimo valor de la subsecuencia de tamanio i
   int p[MAXN];//padres
   vector<int> R;//respuesta
   void rec(int i){
     if(i==-1) return;
     R.push_back(a[i]);
10
     rec(p[i]);
11
12
   int lis(){//0(nlogn)}
     d[0] = ii(-INF, -1); forn(i, N) d[i+1]=ii(INF, -1);
     forn(i, N){
15
       int j = upper_bound(d, d+N+1, ii(a[i], INF))-d;
16
       if (d[j-1].first < a[i]&&a[i] < d[j].first){</pre>
17
         p[i]=d[j-1].second;
18
         d[i] = ii(a[i], i);
19
       }
20
     }
21
     R.clear();
22
     dforn(i, N+1) if(d[i].first!=INF){
23
       rec(d[i].second);//reconstruir
24
       reverse(R.begin(), R.end());
25
       return i;//longitud
26
     }
27
     return 0;
28
29 }
```

3.2. Alpha-Beta prunning

```
1 | 11 alphabeta(State &s, bool player = true, int depth = 1e9, 11 alpha = -
       INF, 11 beta = INF) { //player = true -> Maximiza
       if(s.isFinal()) return s.score;
2
     //~ if (!depth) return s.heuristic();
3
       vector<State> children;
4
       s.expand(player, children);
5
       int n = children.size();
6
       forn(i, n) {
7
           ll v = alphabeta(children[i], !player, depth-1, alpha, beta);
8
           if(!player) alpha = max(alpha, v);
9
           else beta = min(beta, v);
10
           if(beta <= alpha) break;</pre>
11
```

```
12
       return !player ? alpha : beta;}
13
                          3.3. Mo's algorithm
int n,sq;
struct Qu{//queries [1, r]
       //intervalos cerrado abiertos !!! importante!!
       int 1, r, id;
   }qs[MAXN];
   int ans[MAXN], curans;//ans[i]=ans to ith query
   bool bymos(const Qu &a, const Qu &b){
       if(a.l/sq!=b.l/sq) return a.l<b.l;</pre>
       return (a.1/sq)&1? a.r<b.r : a.r>b.r;
9
   }
10
   void mos(){
11
       forn(i, t) qs[i].id=i;
12
       sort(qs, qs+t, bymos);
13
       int cl=0, cr=0;
14
       sq=sqrt(n);
15
       curans=0;
16
       forn(i, t){ //intervalos cerrado abiertos !!! importante!!
17
           Qu &q=qs[i];
18
           while(cl>q.1) add(--cl);
19
           while(cr<q.r) add(cr++);</pre>
20
           while(cl<q.1) remove(cl++);</pre>
21
           while(cr>q.r) remove(--cr);
22
           ans[q.id]=curans;
23
       }
^{24}
25 }
```

4. Strings

4.1. Manacher

```
int d1[MAXN];//d1[i]=long del maximo palindromo impar con centro en i
int d2[MAXN];//d2[i]=analogo pero para longitud par

//0 1 2 3 4
//a a b c c <--d1[2]=3
//a a b b <--d2[2]=2 (estan uno antes)
void manacher(){
int l=0, r=-1, n=sz(s);
forn(i, n){</pre>
```

```
int k=(i>r? 1 : min(d1[l+r-i], r-i));
                                                                                        void add(const string &s, int p=0){
9
       while(i+k<n && i-k>=0 && s[i+k]==s[i-k]) ++k;
                                                                                          if(s[p]) m[s[p]].add(s, p+1);
                                                                                   4
10
       d1[i] = k--;
                                                                                   5
11
       if(i+k > r) l=i-k, r=i+k;
                                                                                       void dfs(){
                                                                                   6
12
     }
                                                                                         //Do stuff
13
     1=0, r=-1;
                                                                                         forall(it, m)
14
     forn(i, n){
                                                                                            it->second.dfs();
                                                                                   9
15
                                                                                       }
       int k=(i>r? 0 : min(d2[1+r-i+1], r-i+1))+1;
                                                                                  10
16
       while(i+k-1 \le k = 0 \ k \le [i+k-1] == s[i-k]) k++;
                                                                                  11 };
17
       d2[i] = --k;
18
                                                                                                      4.4. Suffix Array (largo, nlogn)
       if(i+k-1 > r) l=i-k, r=i+k-1;
19
     }
20
                                                                                   1 #define MAX_N 1000
                                4.2. KMP
                                                                                     #define rBOUND(x) (x<n? r[x] : 0)
                                                                                     //sa will hold the suffixes in order.
   string T;//cadena donde buscar(where)
                                                                                     int sa[MAX_N], r[MAX_N], n;
   string P;//cadena a buscar(what)
                                                                                     string s; //input string, n=sz(s)
   int b[MAXLEN];//back table b[i] maximo borde de [0..i)
   void kmppre(){//by gabina with love
                                                                                      int f[MAX_N], tmpsa[MAX_N];
       int i = 0, j=-1; b[0]=-1;
5
                                                                                      void countingSort(int k){
       while(i<sz(P)){</pre>
6
                                                                                       zero(f);
           while(j>=0 && P[i] != P[j]) j=b[j];
                                                                                       forn(i, n) f[rBOUND(i+k)]++;
           i++, j++, b[i] = j;
8
                                                                                       int sum=0;
                                                                                  11
       }
9
                                                                                       forn(i, max(255, n)){
10
                                                                                         int t=f[i]; f[i]=sum; sum+=t;}
                                                                                  13
   void kmp(){
11
                                                                                       forn(i, n)
                                                                                  14
       int i=0, j=0;
12
                                                                                          tmpsa[f[rBOUND(sa[i]+k)]++]=sa[i];
                                                                                  15
       while(i<sz(T)){</pre>
13
                                                                                        memcpy(sa, tmpsa, sizeof(sa));
                                                                                  16
           while(j>=0 && T[i]!=P[j]) j=b[j];
14
                                                                                  17
           i++, j++;
15
                                                                                      void constructsa(){\frac{}{0} n log n)
                                                                                  18
           if(j==sz(P)) printf("Puisufounduatuindexu %duinuT\n", i-j), j=b[j
16
                                                                                       n=sz(s);
                                                                                  19
               ];
                                                                                       forn(i, n) sa[i]=i, r[i]=s[i];
                                                                                  20
       }
17
                                                                                       for(int k=1; k<n; k<<=1){
                                                                                  21
18
                                                                                         countingSort(k), countingSort(0);
                                                                                  22
19
                                                                                          int rank, tmpr[MAX_N];
                                                                                  23
   int main(){
                                                                                         tmpr[sa[0]]=rank=0;
                                                                                  24
       cout << "T=";
21
                                                                                         forr(i, 1, n)
                                                                                  25
       cin >> T;
22
                                                                                            tmpr[sa[i]] = r[sa[i-1]] \&\& r[sa[i]+k] = r[sa[i-1]+k])?
                                                                                  26
       cout << "P=";
                                                                                                rank: ++rank:
                                 4.3. Trie
                                                                                         memcpy(r, tmpr, sizeof(r));
                                                                                          if(r[sa[n-1]]==n-1) break;
                                                                                  28
1 | struct trie{
                                                                                  29
     map<char, trie> m;
                                                                                  30 }
```

13

14

```
void print(){//for debug
forn(i, n)
cout << i << 'u' <<
s.substr(sa[i], s.find( '$', sa[i])-sa[i]) << endl;}</pre>
```

4.5. String Matching With Suffix Array

```
/returns (lowerbound, upperbound) of the search
   ii stringMatching(string P){ //O(sz(P)lgn)
     int lo=0, hi=n-1, mid=lo;
     while(lo<hi){</pre>
4
       mid=(lo+hi)/2;
5
       int res=s.compare(sa[mid], sz(P), P);
6
       if(res>=0) hi=mid;
       else lo=mid+1;
8
     }
9
     if(s.compare(sa[lo], sz(P), P)!=0) return ii(-1, -1);
10
     ii ans: ans.fst=lo:
11
     lo=0, hi=n-1, mid;
12
     while(lo<hi){</pre>
13
       mid=(lo+hi)/2;
14
       int res=s.compare(sa[mid], sz(P), P);
15
       if(res>0) hi=mid:
16
       else lo=mid+1;
17
     }
18
     if(s.compare(sa[hi], sz(P), P)!=0) hi--;
19
     ans.snd=hi;
20
     return ans;
21
22 }
```

4.6. LCP (Longest Common Prefix)

```
//Calculates the LCP between consecutives suffixes in the Suffix Array.
   //LCP[i] is the length of the LCP between sa[i] and sa[i-1]
  int LCP[MAX_N], phi[MAX_N], PLCP[MAX_N];
   void computeLCP(){//0(n)}
     phi[sa[0]]=-1;
    forr(i, 1, n) phi[sa[i]]=sa[i-1];
6
     int L=0:
     forn(i, n){
8
       if(phi[i]==-1) {PLCP[i]=0; continue;}
9
       while(s[i+L]==s[phi[i]+L]) L++;
10
       PLCP[i]=L:
11
       L=max(L-1, 0);
12
```

```
15 |}
```

forn(i, n) LCP[i]=PLCP[sa[i]];

```
4.7. Corasick
1
2 | struct trie{
     map<char, trie> next;
     trie* tran[256];//transiciones del automata
     int idhoja, szhoja;//id de la hoja o 0 si no lo es
     //link lleva al sufijo mas largo, nxthoja lleva al mas largo pero que
         es hoja
     trie *padre, *link, *nxthoja;
     char pch;//caracter que conecta con padre
     trie(): tran(), idhoja(), padre(), link() {}
     void insert(const string &s, int id=1, int p=0){//id>0!!!
10
       if(p \le z(s)){
11
         trie &ch=next[s[p]];
12
         tran[(int)s[p]]=&ch;
         ch.padre=this, ch.pch=s[p];
         ch.insert(s, id, p+1);
15
16
       else idhoja=id, szhoja=sz(s);
17
18
     trie* get_link() {
19
       if(!link){
20
         if(!padre) link=this;//es la raiz
21
         else if(!padre->padre) link=padre;//hijo de la raiz
22
         else link=padre->get_link()->get_tran(pch);
23
24
       return link; }
25
     trie* get_tran(int c) {
26
       if(!tran[c]) tran[c] = !padre? this : this->get_link()->get_tran(c);
27
       return tran[c]; }
28
     trie *get_nxthoja(){
29
       if(!nxthoja) nxthoja = get_link()->idhoja? link : link->nxthoja;
30
       return nxthoja; }
31
     void print(int p){
32
       if(idhoja) cout << "foundu" << idhoja << "uuatupositionu" << p-
33
           szhoja << endl;</pre>
       if(get_nxthoja()) get_nxthoja()->print(p); }
34
     void matching(const string &s, int p=0){
```

```
}tri;
37
38
39
   int main(){
40
     tri=trie();//clear
41
     tri.insert("ho", 1);
42
     tri.insert("hoho", 2);
43
                               Suffix Automaton
  struct state {
     int len, link;
     map<char,int> next;
     state() { }
5
   const int MAXLEN = 10010;
   state st[MAXLEN*2];
   int sz, last;
   void sa init() {
    forn(i,sz) st[i].next.clear();
     sz = last = 0;
     st[0].len = 0;
     st[0].link = -1;
     ++sz:
14
15
   // Es un DAG de una sola fuente y una sola hoja
   // cantidad de endpos = cantidad de apariciones = cantidad de caminos de
        la clase al nodo terminal
  // cantidad de miembros de la clase = st[v].len-st[st[v].link].len (v>0)
        = caminos del inicio a la clase
  // El arbol de los suffix links es el suffix tree de la cadena invertida
       . La string de la arista link(v)->v son los caracteres que difieren
   void sa_extend (char c) {
     int cur = sz++;
21
     st[cur].len = st[last].len + 1;
22
     // en cur agregamos la posicion que estamos extendiendo
23
     //podria agregar tambien un identificador de las cadenas a las cuales
24
         pertenece (si hay varias)
     int p;
25
     for (p=last; p!=-1 && !st[p].next.count(c); p=st[p].link) // modificar
          esta linea para hacer separadores unicos entre varias cadenas (c
         =='$')
```

print(p); if(p<sz(s)) get_tran(s[p])->matching(s, p+1); }

```
st[p].next[c] = cur;
27
     if (p == -1)
28
       st[cur].link = 0;
29
     else {
30
       int q = st[p].next[c];
31
       if (st[p].len + 1 == st[q].len)
         st[cur].link = q;
33
       else {
34
         int clone = sz++:
         // no le ponemos la posicion actual a clone sino indirectamente
             por el link de cur
         st[clone].len = st[p].len + 1;
37
         st[clone].next = st[q].next;
         st[clone].link = st[a].link:
         for (; p!=-1 \&\& st[p].next.count(c) \&\& st[p].next[c]==q; p=st[p].
             link)
           st[p].next[c] = clone;
         st[q].link = st[cur].link = clone;
       }
    }
    last = cur;
46 }
                            4.9. Z Function
1 char s[MAXN]:
   int z[MAXN]; //z[i] = i==0 ? 0 : max k tq s[0,k) match with s[i,i+k)
   void z_function(char s[],int z[]) {
       int n = strlen(s);
4
       forn(i, n) z[i]=0;
5
       for (int i = 1, l = 0, r = 0; i < n; ++i) {
6
           if (i \le r) z[i] = min (r - i + 1, z[i - 1]);
7
           while (i + z[i] < n \&\& s[z[i]] == s[i + z[i]]) ++z[i];
8
           if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
9
       }
10
   }
11
12
  int main() {
       ios::sync_with_stdio(0);
```

5. Geometría

5.1. Punto

if(a.x < 0 && a.y <= 0)return 2;

```
struct pto{
                                                                                         if(a.x >= 0 \&\& a.y < 0)return 3;
                                                                                        assert(a.x ==0 && a.y==0);
     double x, y;
2
                                                                                  9
     pto(double x=0, double y=0):x(x),y(y){}
                                                                                         return -1;
                                                                                 10
     pto operator+(pto a){return pto(x+a.x, y+a.y);}
                                                                                      }
                                                                                 11
     pto operator-(pto a){return pto(x-a.x, y-a.y);}
                                                                                       bool cmp(const pto&p1, const pto&p2)const{
                                                                                 12
     pto operator+(double a){return pto(x+a, y+a);}
                                                                                        int c1 = cuad(p1), c2 = cuad(p2);
                                                                                 13
     pto operator*(double a){return pto(x*a, y*a);}
                                                                                        if(c1==c2) return p1.y*p2.x<p1.x*p2.y;</pre>
                                                                                 14
     pto operator/(double a){return pto(x/a, y/a);}
                                                                                             else return c1 < c2;
                                                                                 15
     //dot product, producto interno:
                                                                                      }
                                                                                 16
     double operator*(pto a){return x*a.x+y*a.y;}
                                                                                         bool operator()(const pto&p1, const pto&p2) const{
                                                                                 17
10
     //module of the cross product or vectorial product:
                                                                                        return cmp(pto(p1.x-r.x,p1.y-r.y),pto(p2.x-r.x,p2.y-r.y));
                                                                                 18
     //if a is less than 180 clockwise from b, a^b>0
                                                                                 19
     double operator^(pto a){return x*a.y-y*a.x;}
                                                                                 20 };
     //returns true if this is at the left side of line gr
                                                                                                                  5.3. Line
     bool left(pto q, pto r){return ((q-*this)^(r-*this))>0;}
     bool operator<(const pto &a) const{return x<a.x-EPS || (abs(x-a.x)<EPS
                                                                                  int sgn(ll x){return x<0? -1 : !!x;}
          && y<a.y-EPS);}
                                                                                    struct line{
   bool operator == (pto a) {return abs(x-a.x) < EPS && abs(y-a.y) < EPS;}
                                                                                      line() {}
     double norm(){return sqrt(x*x+y*y);}
18
                                                                                      double a,b,c;//Ax+By=C
     double norm_sq(){return x*x+y*y;}
19
                                                                                    //pto MUST store float coordinates!
20
                                                                                      line(double a, double b, double c):a(a),b(b),c(c){}
   double dist(pto a, pto b){return (b-a).norm();}
                                                                                      line(pto p, pto q): a(q.y-p.y), b(p.x-q.x), c(a*p.x+b*p.y) {}
   typedef pto vec;
22
                                                                                      int side(pto p){return sgn(ll(a) * p.x + ll(b) * p.y - c);}
23
                                                                                    };
                                                                                  9
   double angle(pto a, pto o, pto b){
24
                                                                                    bool parallels(line 11, line 12){return abs(l1.a*l2.b-l2.a*l1.b)<EPS;}</pre>
     pto oa=a-o, ob=b-o;
25
                                                                                    pto inter(line 11, line 12){//intersection
     return atan2(oa^ob, oa*ob);}
26
                                                                                       double det=11.a*12.b-12.a*11.b;
                                                                                 12
27
                                                                                      if(abs(det) < EPS) return pto(INF, INF); //parallels</pre>
    //rotate p by theta rads CCW w.r.t. origin (0,0)
28
                                                                                      return pto(12.b*11.c-11.b*12.c, 11.a*12.c-12.a*11.c)/det;
   pto rotate(pto p, double theta){
                                                                                 15 }
     return pto(p.x*cos(theta)-p.y*sin(theta),
30
        p.x*sin(theta)+p.y*cos(theta));
                                                                                                                5.4. Segment
31
32 }
                                                                                  1 struct segm{
                     5.2. Orden radial de puntos
                                                                                      pto s,f;
                                                                                       segm(pto s, pto f):s(s), f(f) {}
                                                                                      pto closest(pto p) {//use for dist to point
   struct Cmp{//orden total de puntos alrededor de un punto r
     pto r:
                                                                                          double 12 = dist_sq(s, f);
2
                                                                                  5
     Cmp(pto r):r(r) {}
                                                                                          if(12==0.) return s;
                                                                                  6
                                                                                         double t = ((p-s)*(f-s))/12;
     int cuad(const pto &a) const{
                                                                                  7
       if(a.x > 0 && a.y >= 0)return 0;
                                                                                          if (t<0.) return s;//not write if is a line
                                                                                  8
       if(a.x <= 0 && a.y > 0)return 1;
                                                                                          else if(t>1.)return f;//not write if is a line
                                                                                  9
6
```

10

return s+((f-s)*t);

```
pto o;
11
                                                                                  7
       bool inside(pto p){return abs(dist(s, p)+dist(p, f)-dist(s, f))<EPS</pre>
                                                                                       double r;
                                                                                  8
12
                                                                                       Circle(pto x, pto y, pto z){
           ;}
                                                                                  9
                                                                                         o=inter(bisector(x, y), bisector(y, z));
   };
                                                                                  10
13
                                                                                         r=dist(o, x);
                                                                                  11
14
   pto inter(segm s1, segm s2){
                                                                                  12
     pto r=inter(line(s1.s, s1.f), line(s2.s, s2.f));
                                                                                       pair<pto, pto> ptosTang(pto p){
                                                                                  13
16
       if(s1.inside(r) && s2.inside(r)) return r;
                                                                                         pto m=(p+o)/2;
     return pto(INF, INF);
                                                                                         tipo d=dist(o, m);
18
                                                                                  15
19 }
                                                                                         tipo a=r*r/(2*d);
                                                                                         tipo h=sqrt(r*r-a*a);
                             5.5. Rectangle
                                                                                         pto m2=o+(m-o)*a/d;
                                                                                         vec per=perp(m-o)/d;
                                                                                  19
  struct rect{
                                                                                         return make_pair(m2-per*h, m2+per*h);
     //lower-left and upper-right corners
                                                                                  21
     pto lw, up;
                                                                                     }:
                                                                                  22
   };
4
                                                                                     //finds the center of the circle containing p1 and p2 with radius r
   //returns if there's an intersection and stores it in r
                                                                                     //as there may be two solutions swap p1, p2 to get the other
   bool inter(rect a, rect b, rect &r){
                                                                                     bool circle2PtsRad(pto p1, pto p2, double r, pto &c){
     r.lw=pto(max(a.lw.x, b.lw.x), max(a.lw.y, b.lw.y));
7
                                                                                             double d2=(p1-p2).norm_sq(), det=r*r/d2-0.25;
     r.up=pto(min(a.up.x, b.up.x), min(a.up.y, b.up.y));
8
                                                                                             if(det<0) return false;</pre>
                                                                                  27
   //check case when only a edge is common
                                                                                             c=(p1+p2)/2+perp(p2-p1)*sqrt(det);
    return r.lw.x<r.up.x && r.lw.y<r.up.y;</pre>
                                                                                             return true;
                                                                                  29
11 }
                                                                                     #define sqr(a) ((a)*(a))
                           5.6. Polygon Area
                                                                                  31
                                                                                     #define feq(a,b) (fabs((a)-(b))<EPS)</pre>
                                                                                     pair<tipo, tipo> ecCuad(tipo a, tipo b, tipo c){//a*x*x+b*x+c=0
   double area(vector<pto> &p){//O(sz(p))
                                                                                       tipo dx = sqrt(b*b-4.0*a*c);
     double area=0;
2
                                                                                       return make_pair((-b + dx)/(2.0*a), (-b - dx)/(2.0*a));
                                                                                  35
     forn(i, sz(p)) area+=p[i]^p[(i+1) %z(p)];
                                                                                  36
    //if points are in clockwise order then area is negative
                                                                                     pair<pto, pto> interCL(Circle c, line 1){
                                                                                  37
     return abs(area)/2;
                                                                                       bool sw=false;
                                                                                  38
6
                                                                                       if((sw=feq(0,1.b))){
                                                                                  39
   //Area ellipse = M_PI*a*b where a and b are the semi axis lengths
                                                                                       swap(1.a, 1.b);
                                                                                  40
\frac{1}{4} //Area triangle = sqrt(s*(s-a)(s-b)(s-c)) where s=(a+b+c)/2
                                                                                       swap(c.o.x, c.o.y);
                                                                                  41
                                5.7. Circle
                                                                                  42
                                                                                       pair<tipo, tipo> rc = ecCuad(
                                                                                  43
                                                                                       sqr(1.a)+sqr(1.b),
  vec perp(vec v){return vec(-v.y, v.x);}
                                                                                  44
                                                                                       2.0*1.a*1.b*c.o.y-2.0*(sqr(1.b)*c.o.x+1.c*1.a),
  line bisector(pto x, pto y){
                                                                                       sqr(1.b)*(sqr(c.o.x)+sqr(c.o.y)-sqr(c.r))+sqr(1.c)-2.0*1.c*1.b*c.o.y
    line l=line(x, y); pto m=(x+y)/2;
                                                                                  46
     return line(-1.b, 1.a, -1.b*m.x+1.a*m.y);
                                                                                  47
4
                                                                                       pair<pto, pto> p( pto(rc.first, (1.c - 1.a * rc.first) / 1.b),
  }
5
                                                                                  48
                                                                                                 pto(rc.second, (1.c - 1.a * rc.second) / 1.b) );
6 | struct Circle{
                                                                                  49
```

17 |}

4

5

6

7

9

10 11 |} forn(i, n)

pi=i:

pt.swap(shift);

```
if(sw){
50
     swap(p.first.x, p.first.y);
51
     swap(p.second.x, p.second.y);
52
53
     return p;
54
55
   pair<pto, pto> interCC(Circle c1, Circle c2){
     line 1:
     1.a = c1.o.x-c2.o.x;
58
     1.b = c1.o.y-c2.o.y;
59
     1.c = (sqr(c2.r) - sqr(c1.r) + sqr(c1.o.x) - sqr(c2.o.x) + sqr(c1.o.y)
     -sqr(c2.o.y))/2.0;
61
    return interCL(c1, 1);
62
63 }
                           5.8. Point in Poly
1 //checks if v is inside of P, using ray casting
   //works with convex and concave.
   //excludes boundaries, handle it separately using segment.inside()
   bool inPolygon(pto v, vector<pto>& P) {
     bool c = false:
    forn(i, sz(P)){
6
       int j=(i+1) \%z(P);
      if((P[j].y>v.y) != (P[i].y > v.y) &&
8
     (v.x < (P[i].x - P[j].x) * (v.y-P[j].y) / (P[i].y - P[j].y) + P[j].x))
9
         c = !c;
    }
11
     return c;
12
13 }
                  5.9. Point in Convex Poly log(n)
   void normalize(vector<pto> &pt){//delete collinear points first!
     //this makes it clockwise:
2
       if(pt[2].left(pt[0], pt[1])) reverse(pt.begin(), pt.end());
3
     int n=sz(pt), pi=0;
```

if(pt[i].x<pt[pi].x || (pt[i].x==pt[pi].x && pt[i].y<pt[pi].y))</pre>

vector<pto> shift(n);//puts pi as first point

forn(i, n) shift[i]=pt[(pi+i) %];

```
| bool inPolygon(pto p, const vector<pto> &pt){
     //call normalize first!
     if(p.left(pt[0], pt[1]) || p.left(pt[sz(pt)-1], pt[0])) return false;
14
     int a=1, b=sz(pt)-1;
     while(b-a>1){
16
       int c=(a+b)/2;
       if(!p.left(pt[0], pt[c])) a=c;
       else b=c;
19
     }
20
     return !p.left(pt[a], pt[a+1]);
22 }
                    5.10. Convex Check CHECK
| bool isConvex(vector<int> &p){//O(N), delete collinear points!
     int N=sz(p):
     if(N<3) return false:
     bool isLeft=p[0].left(p[1], p[2]);
     forr(i, 1, N)
       if(p[i].left(p[(i+1) \( \mathbb{N} \)], p[(i+2) \( \mathbb{N} \)])!=isLeft)
         return false:
7
     return true: }
                           5.11. Convex Hull
1 //stores convex hull of P in S, CCW order
   //left must return >=0 to delete collinear points!
   void CH(vector<pto>& P, vector<pto> &S){
     S.clear();
     sort(P.begin(), P.end());//first x, then y
     forn(i, sz(P)){//lower hull
6
       while(sz(S) \ge 2 \&\& S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back();
       S.pb(P[i]);
8
     }
9
     S.pop_back();
10
     int k=sz(S);
11
     dforn(i, sz(P)){//upper hull
12
       while(sz(S) \ge k+2 \&\& S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back
13
           ();
       S.pb(P[i]);
14
15
     S.pop_back();
16
```

5.12. Cut Polygon

```
1 //cuts polygon Q along the line ab
   //stores the left side (swap a, b for the right one) in P
   void cutPolygon(pto a, pto b, vector<pto> Q, vector<pto> &P){
     P.clear();
     forn(i, sz(Q)){
5
       double left1=(b-a)^(Q[i]-a), left2=(b-a)^(Q[(i+1) \%z(Q)]-a);
6
       if(left1>=0) P.pb(Q[i]);
7
       if(left1*left2<0)
8
         P.pb(inter(line(Q[i], Q[(i+1) \slash z(Q)]), line(a, b)));
9
    }
10
11 }
```

5.13. Bresenham

```
//plot a line approximation in a 2d map
   void bresenham(pto a, pto b){
    pto d=b-a; d.x=abs(d.x), d.y=abs(d.y);
    pto s(a.x<b.x? 1: -1, a.y<b.y? 1: -1);
     int err=d.x-d.y;
     while(1){
6
      m[a.x][a.y]=1;//plot
7
      if(a==b) break;
      int e2=err;
      if(e2 \ge 0) err=2*d.v, a.x+=s.x;
      if(e2 <= 0) err+= 2*d.x, a.y+= s.y;
11
    }
12
13 }
```

5.14. Rotate Matrix

```
//rotates matrix t 90 degrees clockwise
//using auxiliary matrix t2(faster)
void rotate(){
forn(x, n) forn(y, n)
t2[n-y-1][x]=t[x][y];
memcpy(t, t2, sizeof(t));
}
```

5.15. Interseccion de Circulos en n3log(n)

```
struct event {
double x; int t;
```

```
event(double xx, int tt) : x(xx), t(tt) {}
       bool operator <(const event &o) const { return x < o.x; }</pre>
4
  };
5
   typedef vector<Circle> VC;
   typedef vector<event> VE;
   int n;
   double cuenta(VE &v, double A,double B) {
       sort(v.begin(), v.end());
       double res = 0.0, lx = ((v.empty())?0.0:v[0].x);
11
       int contador = 0;
12
       forn(i,sz(v)) {
13
           //interseccion de todos (contador == n), union de todos (
14
               contador > 0)
           //conjunto de puntos cubierto por exacta k Circulos (contador ==
15
           if (contador == n) res += v[i].x - lx;
           contador += v[i].t, lx = v[i].x;
17
       }
18
       return res:
19
20
   // Primitiva de sqrt(r*r - x*x) como funcion double de una variable x.
   inline double primitiva(double x,double r) {
       if (x \ge r) return r*r*M_PI/4.0;
23
       if (x \le -r) return -r*r*M_PI/4.0;
       double raiz = sqrt(r*r-x*x);
25
       return 0.5 * (x * raiz + r*r*atan(x/raiz));
26
   }
27
   double interCircle(VC &v) {
       vector<double> p; p.reserve(v.size() * (v.size() + 2));
       forn(i,sz(v)) p.push_back(v[i].c.x + v[i].r), p.push_back(v[i].c.x
30
           - v[i].r);
       forn(i,sz(v)) forn(j,i) {
31
           Circle &a = v[i]. b = v[i]:
32
           double d = (a.c - b.c).norm();
33
           if (fabs(a.r - b.r) < d \&\& d < a.r + b.r) {
34
               double alfa = acos((sqr(a.r) + sqr(d) - sqr(b.r)) / (2.0 * d)
35
                     * a.r)):
               pto vec = (b.c - a.c) * (a.r / d):
36
               p.pb((a.c + rotate(vec, alfa)).x), p.pb((a.c + rotate(vec, -
37
                   alfa)).x);
           }
38
39
       sort(p.begin(), p.end());
40
```

4

5

```
double res = 0.0:
41
       forn(i,sz(p)-1) {
42
            const double A = p[i], B = p[i+1];
43
           VE ve; ve.reserve(2 * v.size());
44
           forn(j,sz(v)) {
45
                const Circle &c = v[j];
46
                double arco = primitiva(B-c.c.x,c.r) - primitiva(A-c.c.x,c.r
47
                double base = c.c.y * (B-A);
48
                ve.push_back(event(base + arco,-1));
49
                ve.push_back(event(base - arco, 1));
50
51
           res += cuenta(ve,A,B);
52
       }
53
       return res;
54
55 | }
```

6. Matemática

6.1. Identidades

$$\sum_{i=0}^{n} \binom{n}{i} = 2^{n}$$

$$\sum_{i=0}^{n} i \binom{n}{i} = n * 2^{n-1}$$

$$\sum_{i=m}^{n} i = \frac{n(n+1)}{2} - \frac{m(m-1)}{2} = \frac{(n+1-m)(n+m)}{2}$$

$$\sum_{i=0}^{n} i = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{n} i^{2} = \frac{n(n+1)(2n+1)}{6} = \frac{n^{3}}{3} + \frac{n^{2}}{2} + \frac{n}{6}$$

$$\sum_{i=0}^{n} i^{3} = \left(\frac{n(n+1)}{2}\right)^{2} = \frac{n^{4}}{4} + \frac{n^{3}}{2} + \frac{n^{2}}{4} = \left[\sum_{i=1}^{n} i\right]^{2}$$

$$\sum_{i=0}^{n} i^{4} = \frac{n(n+1)(2n+1)(3n^{2}+3n-1)}{30} = \frac{n^{5}}{5} + \frac{n^{4}}{2} + \frac{n^{3}}{3} - \frac{n}{30}$$

$$\sum_{i=0}^{n} i^{p} = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^{p} \frac{B_{k}}{p-k+1} \binom{p}{k} (n+1)^{p-k+1}$$

$$r = e - v + k + 1$$

Teorema de Pick: (Area, puntos interiores y puntos en el borde) $A = I + \tfrac{B}{2} - 1$

6.2. Ec. Característica

```
\begin{aligned} a_0T(n) + a_1T(n-1) + \ldots + a_kT(n-k) &= 0 \\ p(x) &= a_0x^k + a_1x^{k-1} + \ldots + a_k \\ \text{Sean } r_1, r_2, \ldots, r_q \text{ las raı́ces distintas, de mult. } m_1, m_2, \ldots, m_q \\ T(n) &= \sum_{i=1}^q \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^s \\ \text{Las constantes } c_{ij} \text{ se determinan por los casos base.} \end{aligned}
```

6.3. Combinatorio

```
forn(i, MAXN+1){//comb[i][k]=i tomados de a k
     comb[i][0]=comb[i][i]=1;
     forr(k, 1, i) comb[i][k]=(comb[i-1][k]+comb[i-1][k-1])MOD;
4
  ll lucas (ll n, ll k, int p){ //Calcula (n,k) % teniendo comb[p][p]
       precalculado.
    11 \text{ aux} = 1;
     while (n + k) aux = (aux * comb[n/p][k/p]) /p, n/=p, k/=p;
     return aux:
9 }
                    6.4. Exp. de Numeros Mod.
| 1 | 11 expmod (11 b, 11 e, 11 m)\{//0(\log b)\}
     if(!e) return 1;
    11 q= expmod(b,e/2,m); q=(q*q) m;
    return e %2? (b * q) %m : q;
5 }
          6.5. Exp. de Matrices y Fibonacci en log(n)
   #define SIZE 350
   int NN;
   double tmp[SIZE][SIZE];
   void mul(double a[SIZE][SIZE], double b[SIZE][SIZE]){ zero(tmp);
       forn(i, NN) forn(j, NN) forn(k, NN) res[i][i]+=a[i][k]*b[k][j];
       forn(i, NN) forn(j, NN) a[i][j]=res[i][j];
6
7
   void powmat(double a[SIZE][SIZE], int n, double res[SIZE][SIZE]){
       forn(i, NN) forn(j, NN) res[i][j]=(i==j);
9
       while(n){
10
           if(n&1) mul(res, a), n--;
11
           else mul(a, a), n/=2;
12
       } }
13
                6.6. Matrices y determinante O(n^3)
1 struct Mat {
       vector<vector<double> > vec;
       Mat(int n): vec(n, vector<double>(n) ) {}
3
```

Mat(int n, int m): vec(n, vector<double>(m)) {}

vector<double> &operator[](int f){return vec[f];}

```
const vector<double> &operator[](int f) const {return vec[f];}
6
       int size() const {return sz(vec);}
7
       Mat operator+(Mat &b) { ///this de n x m entonces b de n x m
           Mat m(sz(b), sz(b[0]));
9
           forn(i,sz(vec)) forn(j,sz(vec[0])) m[i][j] = vec[i][j] + b[i][j
10
               ];
           return m;
11
       Mat operator*(const Mat &b) { ///this de n x m entonces b de m x t
^{12}
           int n = sz(vec), m = sz(vec[0]), t = sz(b[0]);
13
           Mat mat(n,t);
14
           forn(i,n) forn(j,t) forn(k,m) mat[i][j] += vec[i][k] * b[k][j];
15
           return mat:
16
       double determinant(){//sacado de e maxx ru
17
           double det = 1:
18
           int n = sz(vec):
19
           Mat m(*this);
20
           forn(i, n){//para cada columna
21
               int k = i;
22
               forr(j, i+1, n)//busco la fila con mayor val abs
23
                    if(abs(m[j][i])>abs(m[k][i])) k = j;
24
               if(abs(m[k][i])<1e-9) return 0;
25
               m[i].swap(m[k]);//la swapeo
26
               if(i!=k) det = -det;
27
               det *= m[i][i];
28
               forr(j, i+1, n) m[i][j] /= m[i][i];
29
               //hago 0 todas las otras filas
30
               forn(j, n) if (j!= i && abs(m[j][i])>1e-9)
31
                   forr(k, i+1, n) m[j][k]-=m[i][k]*m[j][i];
32
           }
33
           return det;
34
       }
35
   };
36
37
   int n;
   int main() {
   //DETERMINANTE:
   //https://uva.onlinejudge.org/index.php?option=com_onlinejudge&Itemid=8&
       page=show_problem&problem=625
     freopen("input.in", "r", stdin);
42
       ios::sync_with_stdio(0);
43
       while(cin >> n && n){
44
           Mat m(n);
45
           forn(i, n) forn(j, n) cin >> m[i][j];
46
```

```
cout << (ll)round(m.determinant()) << endl;
cout << "*" << endl;
return 0;
}</pre>
```

6.7. Teorema Chino del Resto

$$y = \sum_{j=1}^{n} (x_j * (\prod_{i=1, i \neq j}^{n} m_i)_{m_j}^{-1} * \prod_{i=1, i \neq j}^{n} m_i)$$

6.8. Criba

```
#define MAXP 100000 //no necesariamente primo
   int criba[MAXP+1];
   void crearcriba(){
     int w[] = \{4,2,4,2,4,6,2,6\};
     for(int p=25;p<=MAXP;p+=10) criba[p]=5;</pre>
     for(int p=9;p<=MAXP;p+=6) criba[p]=3;</pre>
     for(int p=4;p<=MAXP;p+=2) criba[p]=2;</pre>
     for(int p=7,cur=0;p*p<=MAXP;p+=w[cur++&7]) if (!criba[p])</pre>
       for(int j=p*p;j<=MAXP;j+=(p<<1)) if(!criba[j]) criba[j]=p;</pre>
9
10
   vector<int> primos;
11
   void buscarprimos(){
     crearcriba();
13
     forr (i,2,MAXP+1) if (!criba[i]) primos.push_back(i);
14
15
   //^{\sim} Useful for bit trick: #define SET(i) ( criba[(i)>>5]|=1<<((i)&31) ),
         #define INDEX(i) ( (criba[i>>5]>>((i)&31))&1 ), unsigned int criba[
       MAXP/32+1;
17
18
   int main() {
     freopen("primos", "w", stdout);
     buscarprimos();
21
```

6.9. Funciones de primos

Sea $n = \prod p_i^{k_i}$, fact(n) genera un map donde a cada p_i le asocia su k_i

```
//factoriza bien numeros hasta MAXP^2
map<11,11> fact(11 n){ //0 (cant primos)
```

```
map<ll,ll> ret;
3
     forall(p, primos){
4
       while(!(n %*p)){
5
         ret[*p]++;//divisor found
6
         n/=*p;
       }
8
9
     if(n>1) ret[n]++;
10
     return ret;
11
12
    //factoriza bien numeros hasta MAXP
   map<11,11> fact2(11 n){ //0 (1g n)}
     map<ll,ll> ret;
15
     while (criba[n]){
       ret[criba[n]]++;
17
       n/=criba[n];
18
     }
19
     if(n>1) ret[n]++;
20
     return ret:
21
22
    //Usar asi: divisores(fac, divs, fac.begin()); NO ESTA ORDENADO
23
   void divisores(const map<11,11> &f, vector<11> &divs, map<11,11>::
       iterator it, ll n=1){
       if(it==f.begin()) divs.clear();
25
       if(it==f.end()) { divs.pb(n); return; }
26
       ll p=it->fst, k=it->snd; ++it;
27
       forn(_, k+1) divisores(f, divs, it, n), n*=p;
28
29
   ll sumDiv (ll n){
30
     ll rta = 1;
31
     map<ll,ll> f=fact(n);
32
     forall(it, f) {
33
     11 \text{ pot} = 1, \text{ aux} = 0;
34
     forn(i, it->snd+1) aux += pot, pot *= it->fst;
35
     rta*=aux;
36
     }
37
     return rta;
38
39
   11 eulerPhi (ll n){ // con criba: O(lg n)
40
     11 \text{ rta} = n;
41
     map<ll,ll> f=fact(n);
^{42}
     forall(it, f) rta -= rta / it->first;
43
     return rta;
44
```

```
45 }
   11 eulerPhi2 (11 n){ // 0 (sqrt n)
     11 r = n;
     forr (i,2,n+1){
      if ((11)i*i > n) break;
      if (n \% i == 0){
         while (n\% == 0) n/=i;
         r = r/i; }
    }
53
     if (n != 1) r= r/n;
     return r;
55
   }
56
57
   int main() {
     buscarprimos();
     forr (x,1, 500000){
       cout << "x_i = " << x << endl;
       cout << "Numero | de | factores | primos: | | " << numPrimeFactors(x) << endl;</pre>
       cout << "Numero, de, distintos, factores, primos:, " <<
63
            numDiffPrimeFactors(x) << endl;</pre>
       cout << "Suma_de_factores_primos:_" << sumPrimeFactors(x) << endl;</pre>
64
       cout << "Numero_de_divisores:_" << numDiv(x) << endl;</pre>
       cout << "Suma_de_divisores:_" << sumDiv(x) << endl;</pre>
       cout << "Phi de Euler: " << eulerPhi(x) << endl;</pre>
67
     }
68
     return 0;
70 }
```

6.10. Phollard's Rho (rolando)

```
13 | 11 expmod (11 b, 11 e, 11 m)\{\frac{1}{0}
                                                                                        return d==n? rho(n):d:
                                                                                 56
                                                                                    }
     if(!e) return 1;
                                                                                 57
14
     11 q= expmod(b,e/2,m); q=mulmod(q,q,m);
15
                                                                                 58
    return e %2? mulmod(b,q,m) : q;
                                                                                    map<ll,ll> prim;
16
                                                                                    void factRho (ll n){ //O (lg n)^3. un solo numero
17
                                                                                      if (n == 1) return;
18
   bool es_primo_prob (ll n, int a)
                                                                                      if (rabin(n)){
19
                                                                                        prim[n]++;
20
     if (n == a) return true;
                                                                                        return;
21
     11 s = 0, d = n-1;
22
     while (d \% 2 == 0) s++, d/=2;
                                                                                      11 factor = rho(n):
                                                                                 66
23
                                                                                      factRho(factor);
24
     11 x = expmod(a,d,n);
                                                                                      factRho(n/factor):
25
     if ((x == 1) \mid | (x+1 == n)) return true:
                                                                                 69 | }
26
27
                                                                                                                 6.11. GCD
     forn (i, s-1){
28
       x = mulmod(x, x, n);
29
                                                                                  tipo gcd(tipo a, tipo b){return a?gcd(b %a, a):b;}
       if (x == 1) return false;
30
       if (x+1 == n) return true;
                                                                                                          6.12. Extended Euclid
31
32
     return false;
                                                                                  void extendedEuclid (ll a, ll b) \{ //a * x + b * y = d \}
33
                                                                                      if (!b) { x = 1; y = 0; d = a; return;}
34
                                                                                     extendedEuclid (b, a%);
35
   bool rabin (ll n){ //devuelve true si n es primo
                                                                                      11 x1 = v;
     if (n == 1) return false;
                                                                                      11 y1 = x - (a/b) * y;
37
     const int ar[] = \{2,3,5,7,11,13,17,19,23\};
                                                                                      x = x1; y = y1;
38
     forn (j,9)
                                                                                 7 }
39
       if (!es_primo_prob(n,ar[j]))
40
                                                                                                                 6.13. LCM
         return false;
41
     return true;
42
                                                                                  tipo lcm(tipo a, tipo b){return a / gcd(a,b) * b;}
43
                                                                                                               6.14. Inversos
44
   ll rho(ll n){
45
       if( (n & 1) == 0 ) return 2:
                                                                                  1 #define MAXMOD 15485867
46
       11 x = 2, y = 2, d = 1;
                                                                                   ll inv[MAXMOD];//inv[i]*i=1 mod MOD
47
       ll c = rand() % n + 1;
                                                                                    void calc(int p){//O(p)
48
       while(d == 1){
49
                                                                                      inv[1]=1;
           x = (mulmod(x, x, n) + c) %n:
                                                                                      forr(i, 2, p) inv[i] = p-((p/i)*inv[p\%i])\%;
50
           y = (mulmod(y, y, n) + c) %n;
                                                                                  6
51
           y = (mulmod(y, y, n) + c) %n;
                                                                                    int inverso(int x){\frac{1}{0}(\log x)}
52
           if(x - y \ge 0) d = gcd(x - y, n);
                                                                                      return expmod(x, eulerphi(MOD)-2);//si mod no es primo(sacar a mano)
53
           else d = gcd(y - x, n);
                                                                                      return expmod(x, MOD-2);//si mod es primo
54
       }
55
                                                                                 10 }
```

Simpson 6.15.

```
double integral(double a, double b, int n=10000) {//O(n), n=cantdiv
     double area=0, h=(b-a)/n, fa=f(a), fb;
     forn(i, n){
3
       fb=f(a+h*(i+1));
4
       area+=fa+ 4*f(a+h*(i+0.5)) +fb, fa=fb;
5
6
    return area*h/6.;}
                             6.16. Fraction
tipo mcd(tipo a, tipo b){return a?mcd(b%a, a):b;}
  struct frac{
     tipo p,q;
3
     frac(tipo p=0, tipo q=1):p(p),q(q) {norm();}
     void norm(){
       tipo a = mcd(p,q);
       if(a) p/=a, q/=a;
       else q=1;
8
       if (q<0) q=-q, p=-p;}
9
     frac operator+(const frac& o){
10
       tipo a = mcd(q, o.q);
11
       return frac(p*(o.q/a)+o.p*(q/a), q*(o.q/a));}
12
     frac operator-(const frac& o){
13
       tipo a = mcd(q, o.q);
14
       return frac(p*(o.q/a)-o.p*(q/a), q*(o.q/a));}
15
     frac operator*(frac o){
16
       tipo a = mcd(q, o.p), b = mcd(o.q, p);
17
       return frac((p/b)*(o.p/a), (q/a)*(o.q/b));}
18
     frac operator/(frac o){
19
       tipo a = mcd(q,o.q), b = mcd(o.p,p);
20
       return frac((p/b)*(o.q/a),(q/a)*(o.p/b));}
21
     bool operator<(const frac &o) const{return p*o.q < o.p*q;}</pre>
     bool operator==(frac o){return p==o.p&kq==o.q;}
23
24 };
                            6.17. Polinomio
           int m = sz(c), n = sz(o.c):
1
           vector<tipo> res(max(m,n));
2
           forn(i, m) res[i] += c[i];
3
```

forn(i, n) res[i] += o.c[i];

return poly(res); }

4

5

```
poly operator*(const tipo cons) const {
6
       vector<tipo> res(sz(c));
7
           forn(i, sz(c)) res[i]=c[i]*cons;
8
           return poly(res); }
9
       poly operator*(const poly &o) const {
10
           int m = sz(c), n = sz(o.c);
11
           vector<tipo> res(m+n-1);
12
           forn(i, m) forn(j, n) res[i+j]+=c[i]*o.c[j];
13
           return poly(res);
14
     tipo eval(tipo v) {
15
       tipo sum = 0;
16
       dforn(i, sz(c)) sum=sum*v + c[i];
17
       return sum: }
18
       //poly contains only a vector<int> c (the coeficients)
     //the following function generates the roots of the polynomial
   //it can be easily modified to return float roots
     set<tipo> roots(){
22
       set<tipo> roots;
       tipo a0 = abs(c[0]), an = abs(c[sz(c)-1]);
       vector<tipo> ps,qs;
       forr(p,1,sqrt(a0)+1) if (a0 \%p==0) ps.pb(p),ps.pb(a0/p);
26
       forr(q,1,sqrt(an)+1) if (an \% ==0) qs.pb(q),qs.pb(an/q);
       forall(pt,ps)
28
         forall(qt,qs) if ( (*pt) % (*qt)==0 ) {
           tipo root = abs((*pt) / (*qt));
30
           if (eval(root)==0) roots.insert(root);
31
         }
32
       return roots; }
33
   };
34
   pair<poly,tipo> ruffini(const poly p, tipo r) {
     int n = sz(p.c) - 1;
36
    vector<tipo> b(n);
37
     b[n-1] = p.c[n];
38
     dforn(k,n-1) b[k] = p.c[k+1] + r*b[k+1];
     tipo resto = p.c[0] + r*b[0];
40
     poly result(b);
41
     return make_pair(result,resto);
42
43
   poly interpolate(const vector<tipo>& x,const vector<tipo>& y) {
       poly A; A.c.pb(1);
       forn(i,sz(x)) { poly aux; aux.c.pb(-x[i]), aux.c.pb(1), A = A * aux;
46
     poly S; S.c.pb(0);
```

```
forn(i,sz(x)) { poly Li;
48
       Li = ruffini(A,x[i]).fst;
49
       Li = Li * (1.0 / Li.eval(x[i])); // here put a multiple of the
50
           coefficients instead of 1.0 to avoid using double
       S = S + Li * v[i]; }
51
     return S;
52
53
54
   int main(){
55
     return 0;
57 }
```

6.18. Ec. Lineales

```
|bool resolver_ev(Mat a, Vec y, Vec &x, Mat &ev){
     int n = a.size(), m = n?a[0].size():0, rw = min(n, m);
     vector<int> p; forn(i,m) p.push_back(i);
3
     forn(i, rw) {
4
       int uc=i. uf=i:
       forr(f, i, n) forr(c, i, m) if(fabs(a[f][c])>fabs(a[uf][uc])) {uf=f;
       if (feq(a[uf][uc], 0)) { rw = i; break; }
7
       forn(j, n) swap(a[j][i], a[j][uc]);
8
       swap(a[i], a[uf]); swap(y[i], y[uf]); swap(p[i], p[uc]);
9
       tipo inv = 1 / a[i][i]; //aca divide
10
       forr(j, i+1, n) {
11
         tipo v = a[j][i] * inv;
12
        forr(k, i, m) a[j][k]-=v * a[i][k];
13
         y[j] -= v*y[i];
14
15
     } // rw = rango(a), aca la matriz esta triangulada
16
     forr(i, rw, n) if (!feq(y[i],0)) return false; // checkeo de
17
         compatibilidad
     x = \text{vector} < \text{tipo} > (m, 0);
18
     dforn(i, rw){
19
       tipo s = y[i];
20
       forr(j, i+1, rw) s -= a[i][j]*x[p[j]];
^{21}
       x[p[i]] = s / a[i][i]; //aca divide
22
23
     ev = Mat(m-rw, Vec(m, 0)); // Esta parte va SOLO si se necesita el ev
24
     forn(k, m-rw) {
25
       ev[k][p[k+rw]] = 1;
26
       dforn(i, rw){
27
```

```
tipo s = -a[i][k+rw];
forr(j, i+1, rw) s -= a[i][j]*ev[k][p[j]];
ev[k][p[i]] = s / a[i][i]; //aca divide

return true;

return true;
}
```

6.19. FFT

```
1 //~ typedef complex<double> base; //menos codigo, pero mas lento
  //elegir si usar complejos de c (lento) o estos
   struct base{
       double r,i;
       base(double r=0, double i=0):r(r), i(i){}
       double real()const{return r;}
       void operator/=(const int c){r/=c, i/=c;}
7
   };
8
   base operator*(const base &a, const base &b){
       return base(a.r*b.r-a.i*b.i, a.r*b.i+a.i*b.r);}
   base operator+(const base &a, const base &b){
       return base(a.r+b.r, a.i+b.i);}
   base operator-(const base &a, const base &b){
       return base(a.r-b.r. a.i-b.i):}
   vector<int> rev; vector<base> wlen_pw;
   inline static void fft(base a[], int n, bool invert) {
       forn(i, n) if(i<rev[i]) swap(a[i], a[rev[i]]);</pre>
17
     for (int len=2; len<=n; len<<=1) {</pre>
18
       double ang = 2*M_PI/len * (invert?-1:+1);
19
       int len2 = len>>1;
20
       base wlen (cos(ang), sin(ang));
21
       wlen_pw[0] = base(1, 0);
22
           forr(i, 1, len2) wlen_pw[i] = wlen_pw[i-1] * wlen;
23
       for (int i=0; i<n; i+=len) {</pre>
24
         base t, *pu = a+i, *pv = a+i+len2, *pu_end = a+i+len2, *pw = &
25
             wlen_pw[0];
         for (; pu!=pu_end; ++pu, ++pv, ++pw)
26
           t = *pv * *pw, *pv = *pu - t,*pu = *pu + t;
27
       }
28
     if (invert) forn(i, n) a[i]/= n;}
   inline static void calc_rev(int n){//precalculo: llamar antes de fft!!
       wlen_pw.resize(n), rev.resize(n);
32
```

```
int lg=31-__builtin_clz(n);
33
       forn(i, n){
34
       rev[i] = 0:
35
           forn(k, lg) if(i&(1<<k)) rev[i]|=1<<(lg-1-k);
36
       }}
37
   inline static void multiply(const vector<int> &a, const vector<int> &b,
       vector<int> &res) {
     vector<base> fa (a.begin(), a.end()), fb (b.begin(), b.end());
       int n=1; while(n < max(sz(a), sz(b))) n <<= 1; n <<= 1;
       calc_rev(n);
     fa.resize (n), fb.resize (n);
     fft (&fa[0], n, false), fft (&fb[0], n, false);
     forn(i, n) fa[i] = fa[i] * fb[i]:
    fft (&fa[0], n, true);
     res.resize(n);
      forn(i, n) res[i] = int (fa[i].real() + 0.5); }
   void toPoly(const string &s, vector<int> &P){//convierte un numero a
       polinomio
       P.clear():
49
      dforn(i, sz(s)) P.pb(s[i]-'0');}
```

6.20. Tablas y cotas (Primos, Divisores, Factoriales, etc)

```
Factoriales
0! = 1
                  11! = 39.916.800
1! = 1
                   12! = 479.001.600 \ (\in int)
2! = 2
                   13! = 6.227.020.800
3! = 6
                  14! = 87.178.291.200
4! = 24
                   15! = 1.307.674.368.000
5! = 120
                   16! = 20.922.789.888.000
6! = 720
                  17! = 355.687.428.096.000
7! = 5.040
                  18! = 6.402.373.705.728.000
8! = 40.320
                  19! = 121.645.100.408.832.000
9! = 362.880
                  20! = 2.432.902.008.176.640.000 \ (\in \text{tint})
10! = 3.628.800 \mid 21! = 51.090.942.171.709.400.000
       \max \text{ signed tint} = 9.223.372.036.854.775.807
      max unsigned tint = 18.446.744.073.709.551.615
```

Primos

 $\begin{array}{c} 601\ 607\ 613\ 617\ 619\ 631\ 641\ 643\ 647\ 653\ 659\ 661\ 673\ 677\ 683\ 691\ 701\ 709\ 719\ 727\\ 733\ 739\ 743\ 751\ 757\ 761\ 769\ 773\ 787\ 797\ 809\ 811\ 821\ 823\ 827\ 829\ 839\ 853\ 857\ 859\\ 863\ 877\ 881\ 883\ 887\ 907\ 911\ 919\ 929\ 937\ 941\ 947\ 953\ 967\ 971\ 977\ 983\ 991\ 997\ 1009\\ 1013\ 1019\ 1021\ 1031\ 1033\ 1039\ 1049\ 1051\ 1061\ 1063\ 1069\ 1087\ 1091\ 1093\ 1097\ 1103\\ 1109\ 1117\ 1123\ 1129\ 1151\ 1153\ 1163\ 1171\ 1181\ 1187\ 1193\ 1201\ 1213\ 1217\ 1223\ 1229\\ 1231\ 1237\ 1249\ 1259\ 1277\ 1279\ 1283\ 1289\ 1291\ 1297\ 1301\ 1303\ 1307\ 1319\ 1321\ 1327\\ 1361\ 1367\ 1373\ 1381\ 1399\ 1409\ 1423\ 1427\ 1429\ 1433\ 1439\ 1447\ 1451\ 1453\ 1459\ 1471\\ 1481\ 1483\ 1487\ 1489\ 1493\ 1499\ 1511\ 1523\ 1531\ 1543\ 1549\ 1553\ 1559\ 1567\ 1571\ 1579\\ 1583\ 1597\ 1601\ 1607\ 1609\ 1613\ 1619\ 1621\ 1627\ 1637\ 1657\ 1663\ 1667\ 1669\ 1693\ 1697\\ 1699\ 1709\ 1721\ 1723\ 1733\ 1741\ 1747\ 1753\ 1759\ 1777\ 1783\ 1787\ 1789\ 1801\ 1811\ 1823\\ 1831\ 1847\ 1861\ 1867\ 1871\ 1873\ 1877\ 1879\ 1889\ 1901\ 1907\ 1913\ 1931\ 1933\ 1949\ 1951\\ 1973\ 1979\ 1987\ 1999\ 2003\ 2011\ 2017\ 2027\ 2029\ 2039\ 2053\ 2063\ 2069\ 2081\\ \end{array}$

Primos cercanos a 10^n

9941 9949 9967 9973 10007 10009 10037 10039 10061 10067 10069 10079
99961 99971 99989 99991 100003 100019 100043 100049 100057 100069
999959 999961 999979 999983 1000003 1000033 1000037 1000039
9999943 9999971 99999991 10000019 10000079 10000103 10000121
99999941 9999959 99999971 9999989 100000007 100000037 100000039 100000049
99999893 99999999 99999937 1000000007 1000000009 1000000021 1000000033

Cantidad de primos menores que 10^n

```
\pi(10^1) = 4; \pi(10^2) = 25; \pi(10^3) = 168; \pi(10^4) = 1229; \pi(10^5) = 9592

\pi(10^6) = 78.498; \pi(10^7) = 664.579; \pi(10^8) = 5.761.455; \pi(10^9) = 50.847.534

\pi(10^{10}) = 455.052,511; \pi(10^{11}) = 4.118.054.813; \pi(10^{12}) = 37.607.912.018
```

Divisores Cantidad de divisores (σ_0) para $algunos \ n/\neg \exists n' < n, \sigma_0(n') \geqslant \sigma_0(n)$

```
\sigma_0(60) = 12 \; ; \; \sigma_0(120) = 16 \; ; \; \sigma_0(180) = 18 \; ; \; \sigma_0(240) = 20 \; ; \; \sigma_0(360) = 24 \sigma_0(720) = 30 \; ; \; \sigma_0(840) = 32 \; ; \; \sigma_0(1260) = 36 \; ; \; \sigma_0(1680) = 40 \; ; \; \sigma_0(10080) = 72 \sigma_0(15120) = 80 \; ; \; \sigma_0(50400) = 108 \; ; \; \sigma_0(83160) = 128 \; ; \; \sigma_0(110880) = 144 \sigma_0(498960) = 200 \; ; \; \sigma_0(554400) = 216 \; ; \; \sigma_0(1081080) = 256 \; ; \; \sigma_0(1441440) = 288 \sigma_0(4324320) = 384 \; ; \; \sigma_0(8648640) = 448 Suma de divisores (\sigma_1) para algunos n/\neg \exists n' < n, \sigma_1(n') \geqslant \sigma_1(n) \sigma_1(96) = 252 \; ; \; \sigma_1(108) = 280 \; ; \; \sigma_1(120) = 360 \; ; \; \sigma_1(144) = 403 \; ; \; \sigma_1(168) = 480 \sigma_1(960) = 3048 \; ; \; \sigma_1(1008) = 3224 \; ; \; \sigma_1(1080) = 3600 \; ; \; \sigma_1(1200) = 3844 \sigma_1(4620) = 16128 \; ; \; \sigma_1(4680) = 16380 \; ; \; \sigma_1(5040) = 19344 \; ; \; \sigma_1(5760) = 19890 \sigma_1(8820) = 31122 \; ; \; \sigma_1(9240) = 34560 \; ; \; \sigma_1(10080) = 39312 \; ; \; \sigma_1(10920) = 40320 \sigma_1(32760) = 131040 \; ; \; \sigma_1(35280) = 137826 \; ; \; \sigma_1(36960) = 145152 \; ; \; \sigma_1(37800) = 148800 \sigma_1(60480) = 243840 \; ; \; \sigma_1(64680) = 246240 \; ; \; \sigma_1(65520) = 270816 \; ; \; \sigma_1(70560) = 280098 \sigma_1(95760) = 386880 \; ; \; \sigma_1(98280) = 403200 \; ; \; \sigma_1(100800) = 409448 \sigma_1(491400) = 2083200 \; ; \; \sigma_1(498960) = 2160576 \; ; \; \sigma_1(514080) = 2177280
```

|bool hasNegCycle(){

```
\begin{array}{l} \sigma_1(982800) = 4305280 \; ; \; \sigma_1(997920) = 4390848 \; ; \; \sigma_1(1048320) = 4464096 \\ \sigma_1(4979520) = 22189440 \; ; \; \sigma_1(4989600) = 22686048 \; ; \; \sigma_1(5045040) = 23154768 \\ \sigma_1(9896040) = 44323200 \; ; \; \sigma_1(9959040) = 44553600 \; ; \; \sigma_1(9979200) = 45732192 \end{array}
```

7. Grafos

7.1. Dijkstra

```
#define INF 1e9
   int N;
2
   #define MAX_V 250001
   vector<ii> G[MAX_V];
   //To add an edge use
   #define add(a, b, w) G[a].pb(make_pair(w, b))
   ll dijkstra(int s, int t){\frac{1}{0}(|E| \log |V|)}
     priority_queue<ii, vector<ii>, greater<ii> > Q;
     vector<ll> dist(N, INF); vector<int> dad(N, -1);
     Q.push(make_pair(0, s)); dist[s] = 0;
     while(sz(Q)){
       ii p = Q.top(); Q.pop();
12
       if(p.snd == t) break;
13
       forall(it, G[p.snd])
14
         if(dist[p.snd]+it->first < dist[it->snd]){
15
           dist[it->snd] = dist[p.snd] + it->fst;
16
           dad[it->snd] = p.snd;
17
           Q.push(make_pair(dist[it->snd], it->snd)); }
18
     }
19
     return dist[t];
20
     if(dist[t]<INF)//path generator</pre>
21
       for(int i=t; i!=-1; i=dad[i])
22
         printf("%d%c", i, (i==s?'\n':','));}
23
                           7.2. Bellman-Ford
   vector<ii> G[MAX_N];//adv. list with pairs (weight, dst)
   int dist[MAX_N];
   void bford(int src){//O(VE)
     dist[src]=0:
4
    forn(i, N-1) forn(j, N) if(dist[j]!=INF) forall(it, G[j])
       dist[it->snd]=min(dist[it->snd], dist[j]+it->fst);
6
7
```

```
forn(j, N) if(dist[j]!=INF) forall(it, G[j])
10
       if(dist[it->snd]>dist[j]+it->fst) return true;
11
     //inside if: all points reachable from it->snd will have -INF distance
12
     return false;
13
14 }
                          7.3. Floyd-Warshall
1 //G[i][j] contains weight of edge (i, j) or INF
  //G[i][i]=0
  int G[MAX_N] [MAX_N];
   void floyd(){\frac{}{0(N^3)}}
   forn(k, N) forn(i, N) if(G[i][k]!=INF) forn(j, N) if(G[k][j]!=INF)
     G[i][j]=min(G[i][j], G[i][k]+G[k][j]);
7
   }
   bool inNegCycle(int v){
     return G[v][v]<0;}
   //checks if there's a neg. cycle in path from a to b
   bool hasNegCycle(int a, int b){
    forn(i, N) if(G[a][i]!=INF && G[i][i]<0 && G[i][b]!=INF)
       return true:
13
     return false;
14
15 }
                              7.4. Kruskal
 struct Ar{int a,b,w;};
bool operator<(const Ar& a, const Ar &b){return a.w<b.w;}</pre>
   vector<Ar> E;
   ll kruskal(){
       11 cost=0:
       sort(E.begin(), E.end());//ordenar aristas de menor a mayor
6
       uf.init(n);
7
       forall(it, E){
8
           if(uf.comp(it->a)!=uf.comp(it->b)){//si no estan conectados
9
               uf.unir(it->a, it->b);//conectar
10
               cost+=it->w;
11
           }
12
       }
13
       return cost;
14
15 }
                                7.5. Prim
```

```
bool taken[MAXN];
  priority_queue<ii, vector<ii>, greater<ii> > pq;//min heap
   void process(int v){
       taken[v]=true;
       forall(e, G[v])
           if(!taken[e->second]) pq.push(*e);
6
7
8
   11 prim(){
9
       zero(taken);
10
       process(0);
11
       11 cost=0;
12
       while(sz(pq)){
13
           ii e=pq.top(); pq.pop();
14
           if(!taken[e.second]) cost+=e.first, process(e.second);
15
       }
16
       return cost;
17
18 }
```

7.6. 2-SAT + Tarjan SCC

```
//We have a vertex representing a var and other for his negation.
   //Every edge stored in G represents an implication. To add an equation
       of the form a | |b, use addor(a, b)
   //MAX=max cant var. n=cant var
   #define addor(a, b) (G[neg(a)].pb(b), G[neg(b)].pb(a))
   vector<int> G[MAX*2];
   //idx[i]=index assigned in the dfs
   //lw[i]=lowest index(closer from the root) reachable from i
   int lw[MAX*2], idx[MAX*2], qidx;
   stack<int> q;
   int qcmp, cmp[MAX*2];
   //verdad[cmp[i]]=valor de la variable i
   bool verdad[MAX*2+1];
12
13
   int neg(int x) { return x>=n? x-n : x+n;}
   void tin(int v){
     lw[v]=idx[v]=++qidx;
16
     q.push(v), cmp[v]=-2;
17
     forall(it, G[v]){
18
       if(!idx[*it] || cmp[*it]==-2){
19
         if(!idx[*it]) tjn(*it);
20
         lw[v]=min(lw[v], lw[*it]);
21
```

```
}
22
     }
23
     if(lw[v]==idx[v]){
24
       int x;
25
       do{x=q.top(); q.pop(); cmp[x]=qcmp;}while(x!=v);
26
       verdad[qcmp] = (cmp[neg(v)] < 0);</pre>
       qcmp++;
28
     }
29
30
    //remember to CLEAR G!!!
   bool satisf(){\frac{}{0}}
     memset(idx, 0, sizeof(idx)), qidx=0;
     memset(cmp, -1, sizeof(cmp)), qcmp=0;
     forn(i, n){
35
       if(!idx[i]) tjn(i);
36
       if(!idx[neg(i)]) tjn(neg(i));
37
     }
38
     forn(i, n) if(cmp[i] == cmp[neg(i)]) return false;
     return true:
40
41 }
```

7.7. Articulation Points

```
1 int N;
   vector<int> G[1000000]:
   //V[i]=node number(if visited), L[i]= lowest V[i] reachable from i
   int qV, V[1000000], L[1000000], P[1000000];
   void dfs(int v, int f){
    L[v]=V[v]=++qV;
     forall(it, G[v])
       if(!V[*it]){
         dfs(*it, v);
9
         L[v] = min(L[v], L[*it]);
         P[v] += L[*it] >= V[v];
11
       }
12
       else if(*it!=f)
13
         L[v]=min(L[v], V[*it]);
14
   }
15
   int cantart(){ //O(n)
     qV=0;
17
     zero(V), zero(P);
18
     dfs(1, 0); P[1]--;
19
     int q=0;
20
```

last = st.top(); st.pop();

e[last].comp = nbc;

36

37

```
forn(i, N) if(P[i]) q++;
                                                                                              } while (last != *ne);
                                                                                   38
                                                                                              nbc++;
22
  return q;
                                                                                   39
23 }
                                                                                              comp[u]++;
                                                                                   40
                                                                                   41
                  7.8. Comp. Biconexas y Puentes
                                                                                            b[u] = min(b[u], b[v]);
                                                                                   42
                                                                                   43
struct edge {
                                                                                   44
                                                                                            st.push(*ne);
     int u,v, comp;
2
                                                                                            b[u] = min(b[u], d[v]);
     bool bridge;
                                                                                   47
4
                                                                                       }
   vector<edge> e;
                                                                                   48
                                                                                   49 }
   void addEdge(int u, int v) {
     G[u].pb(sz(e)), G[v].pb(sz(e));
7
     e.pb((edge){u,v,-1,false});
8
9
                                                                                    const int MAXN=100001;
    //d[i]=id de la dfs
                                                                                      const int LOGN=20;
   //b[i]=lowest id reachable from i
   int d[MAXN], b[MAXN], t;
                                                                                      //L[v] holds the level of v
   int nbc;//cant componentes
                                                                                      int N, f[MAXN][LOGN], L[MAXN];
   int comp[MAXN];//comp[i]=cant comp biconexas a la cual pertenece i
                                                                                      //call before build:
   void initDfs(int n) {
     zero(G), zero(comp);
16
                                                                                        f[v][0]=fa, L[v]=lvl;
     e.clear();
17
     forn(i,n) d[i]=-1;
18
     nbc = t = 0:
19
20
   stack<int> st:
                                                                                      int climb(int a, int d){\frac{1}{0}}
   void dfs(int u, int pe) \{//0(n + m)\}
22
                                                                                        if(!d) return a;
     b[u] = d[u] = t++;
23
     comp[u] = (pe != -1);
24
                                                                                          return a;}
     forall(ne, G[u]) if (*ne != pe){
25
                                                                                      int lca(int a, int b){\frac{1}{0}}
       int v = e[*ne].u ^ e[*ne].v ^ u;
26
                                                                                        if(L[a]<L[b]) swap(a, b);
       if (d[v] == -1) {
27
                                                                                        a=climb(a, L[a]-L[b]);
                                                                                   19
         st.push(*ne);
28
                                                                                        if(a==b) return a;
         dfs(v,*ne);
29
                                                                                   21
         if (b[v] > d[u]){
30
                                                                                        return f[a][0]; }
                                                                                   22
           e[*ne].bridge = true; // bridge
31
         }
32
         if (b[v] >= d[u]) \{ // art \}
33
           int last:
34
           do {
35
```

```
else if (d[v] < d[u]) \{ // back edge
                         7.9. \text{ LCA} + \text{Climb}
  //f[v][k] holds the 2^k father of v
 | void dfs(int v, int fa=-1, int lvl=0){//generate required data
    forall(it, G[v])if(*it!=fa) dfs(*it, v, lvl+1); }
  void build(){//f[i][0] must be filled previously, O(nlgn)
   forn(k, LOGN-1) forn(i, N) f[i][k+1]=f[f[i][k]][k];}
  #define lg(x) (31-_builtin_clz(x))//=floor(log2(x))
    dforn(i, lg(L[a])+1) if(1<<i<=d) a=f[a][i], d-=1<<i;
    dforn(i, lg(L[a])+1) if(f[a][i]!=f[b][i]) a=f[a][i], b=f[b][i];
  int dist(int a, int b) {//returns distance between nodes
    return L[a]+L[b]-2*L[lca(a, b)];}
                7.10. Heavy Light Decomposition
int treesz[MAXN];//cantidad de nodos en el subarbol del nodo v
```

int dad[MAXN];//dad[v]=padre del nodo v

int szt[MAXN];

```
3 | void dfs1(int v, int p=-1){//pre-dfs
                                                                                  void calcsz(int v, int p) {
     dad[v]=p;
                                                                                       szt[v] = 1:
4
                                                                                      forall(it,G[v]) if (*it!=p && !taken[*it])
     treesz[v]=1;
5
     forall(it, G[v]) if(*it!=p){
                                                                                         calcsz(*it,v), szt[v]+=szt[*it];
       dfs1(*it, v);
                                                                                     }
                                                                                  11
       treesz[v]+=treesz[*it];
                                                                                     void centroid(int v=0, int f=-1, int lvl=0, int tam=-1) {//O(nlogn)
8
                                                                                       if(tam==-1) calcsz(v, -1), tam=szt[v];
                                                                                 13
9
                                                                                      forall(it, G[v]) if(!taken[*it] && szt[*it]>=tam/2)
10
                                                                                 14
                                                                                        {szt[v]=0; centroid(*it, f, lvl, tam); return;}
   //PONER Q EN O !!!!!
                                                                                 15
   int pos[MAXN], q://pos[v]=posicion del nodo v en el recorrido de la dfs
                                                                                       taken[v]=true;
   //Las cadenas aparecen continuas en el recorrido!
                                                                                      padre[v]=f;
                                                                                 17
                                                                                      forall(it, G[v]) if(!taken[*it])
   int cantcad:
                                                                                 18
                                                                                         centroid(*it, v, lvl+1, -1);
   int homecad[MAXN]://dada una cadena devuelve su nodo inicial
                                                                                  19
   int cad[MAXN];//cad[v]=cadena a la que pertenece el nodo
                                                                                 20 }
   void heavylight(int v, int cur=-1){
                                                                                                             7.12. Euler Cycle
     if(cur==-1) homecad[cur=cantcad++]=v;
     pos[v]=q++;
19
     cad[v]=cur;
                                                                                  int n,m,ars[MAXE], eq;
20
     int mx=-1:
                                                                                  vector<int> G[MAXN];//fill G,n,m,ars,eq
21
     forn(i, sz(G[v])) if(G[v][i]!=dad[v])
                                                                                    list<int> path;
22
       if(mx==-1 || treesz[G[v][mx]]<treesz[G[v][i]]) mx=i;</pre>
                                                                                     int used[MAXN];
23
     if(mx!=-1) heavylight(G[v][mx], cur);
                                                                                     bool usede[MAXE];
24
     forn(i, sz(G[v])) if(i!=mx && G[v][i]!=dad[v])
                                                                                     queue<list<int>::iterator> q;
25
       heavylight(G[v][i], -1);
                                                                                     int get(int v){
26
                                                                                       while(used[v]\leq z(G[v]) && usede[G[v][used[v]]]) used[v]++;
27
   //ejemplo de obtener el maximo numero en el camino entre dos nodos
                                                                                       return used[v]:
   //RTA: max(query(low, u), query(low, v)), con low=lca(u, v)
                                                                                  10
    //esta funcion va trepando por las cadenas
                                                                                     void explore(int v, int r, list<int>::iterator it){
   int query(int an, int v){//O(logn)
                                                                                      int ar=G[v][get(v)]; int u=v^ars[ar];
31
                                                                                 12
     //si estan en la misma cadena:
                                                                                       usede[ar]=true;
32
                                                                                 13
     if(cad[an] == cad[v]) return rmq.get(pos[an], pos[v]+1);
                                                                                      list<int>::iterator it2=path.insert(it, u);
33
                                                                                 14
     return max(query(an, dad[homecad[cad[v]]]),
                                                                                       if(u!=r) explore(u, r, it2);
                                                                                 15
            rmq.get(pos[homecad[cad[v]]], pos[v]+1));
                                                                                      if(get(v)<sz(G[v])) q.push(it);</pre>
35
                                                                                 16
36 | }
                                                                                 17
                                                                                     void euler(){
                    7.11. Centroid Decomposition
                                                                                       zero(used), zero(usede);
                                                                                 19
                                                                                      path.clear();
                                                                                 20
1 | int n;
                                                                                       q=queue<list<int>::iterator>();
                                                                                 21
  vector<int> G[MAXN]:
                                                                                       path.push_back(0); q.push(path.begin());
                                                                                 22
  |bool taken[MAXN];//poner todos en FALSE al principio!!
                                                                                       while(sz(q)){
                                                                                 23
   int padre[MAXN];//padre de cada nodo en el centroid tree
                                                                                        list<int>::iterator it=q.front(); q.pop();
                                                                                 24
                                                                                         if(used[*it] < sz(G[*it])) explore(*it, *it, it);</pre>
                                                                                 25
```

26

```
reverse(path.begin(), path.end());
27
28
   void addEdge(int u, int v){
29
     G[u].pb(eq), G[v].pb(eq);
30
     ars[eq++]=u^v;
31
32 }
                         7.13. Diametro árbol
  vector<int> G[MAXN]; int n,m,p[MAXN],d[MAXN],d2[MAXN];
   int bfs(int r, int *d) {
     queue<int> q;
     d[r]=0; q.push(r);
     int v:
5
     while(sz(q)) { v=q.front(); q.pop();
6
       forall(it,G[v]) if (d[*it]==-1)
7
         d[*it]=d[v]+1, p[*it]=v, q.push(*it);
8
9
     return v://ultimo nodo visitado
10
11
   vector<int> diams; vector<ii> centros;
12
   void diametros(){
13
     memset(d,-1,sizeof(d));
14
     memset(d2,-1,sizeof(d2));
15
     diams.clear(), centros.clear();
16
     forn(i, n) if(d[i]==-1){
17
       int v,c;
18
       c=v=bfs(bfs(i, d2), d);
19
       forn(_,d[v]/2) c=p[c];
20
       diams.pb(d[v]);
21
       if(d[v]&1) centros.pb(ii(c, p[c]));
22
       else centros.pb(ii(c, c));
23
     }
^{24}
^{25}
26
   int main() {
27
     freopen("in", "r", stdin);
28
     while(cin >> n >> m){
29
       forn(i,m) { int a,b; cin >> a >> b; a--, b--;
30
         G[a].pb(b);
31
         G[b].pb(a);
32
```

7.14. Chu-liu

```
void visit(graph &h, int v, int s, int r,
     vector<int> &no, vector< vector<int> > &comp,
     vector<int> &prev, vector< vector<int> > &next, vector<weight> &mcost,
     vector<int> &mark, weight &cost, bool &found) {
     if (mark[v]) {
       vector<int> temp = no;
       found = true;
       do {
         cost += mcost[v];
         v = prev[v];
         if (v != s) {
11
           while (comp[v].size() > 0) {
12
             no[comp[v].back()] = s;
13
             comp[s].push_back(comp[v].back());
             comp[v].pop_back();
15
           }
         }
17
       } while (v != s);
       forall(j,comp[s]) if (*j != r) forall(e,h[*j])
         if (no[e->src] != s) e->w -= mcost[ temp[*i] ];
20
     }
21
     mark[v] = true;
     forall(i,next[v]) if (no[*i] != no[v] && prev[no[*i]] == v)
23
       if (!mark[no[*i]] || *i == s)
24
         visit(h, *i, s, r, no, comp, prev, next, mcost, mark, cost, found)
25
26
   weight minimumSpanningArborescence(const graph &g, int r) {
       const int n=sz(g);
     graph h(n);
29
     forn(u,n) forall(e,g[u]) h[e->dst].pb(*e);
30
     vector<int> no(n);
31
     vector<vector<int> > comp(n);
32
     forn(u, n) comp[u].pb(no[u] = u);
     for (weight cost = 0; ;) {
34
       vector<int> prev(n, -1);
35
       vector<weight> mcost(n, INF);
36
       forn(j,n) if (j != r) forall(e,h[j])
37
         if (no[e->src] != no[j])
38
           if (e->w < mcost[ no[i] ])</pre>
39
             mcost[no[j]] = e->w, prev[no[j]] = no[e->src];
40
       vector< vector<int> > next(n);
41
       forn(u,n) if (prev[u] >= 0)
42
```

```
next[ prev[u] ].push_back(u);
43
       bool stop = true;
44
       vector<int> mark(n);
45
       forn(u,n) if (u != r && !mark[u] && !comp[u].empty()) {
46
         bool found = false;
47
         visit(h, u, u, r, no, comp, prev, next, mcost, mark, cost, found);
48
         if (found) stop = false;
49
       }
50
       if (stop) {
51
         forn(u,n) if (prev[u] >= 0) cost += mcost[u];
52
         return cost;
53
       }
54
    }
55
  |}
56
```

7.15. Hungarian

```
1 //Dado un grafo bipartito completo con costos no negativos, encuentra el
        matching perfecto de minimo costo.
tipo cost[N][N], lx[N], ly[N], slack[N]; //llenar: cost=matriz de
       advacencia
  int n, max_match, xy[N], yx[N], slackx[N], prev2[N]; //n=cantidad de nodos
   bool S[N], T[N]; //sets S and T in algorithm
   void add_to_tree(int x, int prevx) {
     S[x] = true, prev2[x] = prevx;
     forn(y, n) if (lx[x] + ly[y] - cost[x][y] < slack[y] - EPS)
       slack[y] = lx[x] + ly[y] - cost[x][y], slackx[y] = x;
8
9
   void update_labels(){
10
     tipo delta = INF;
11
     forn (y, n) if (!T[y]) delta = min(delta, slack[y]);
12
     form (x, n) if (S[x]) lx[x] -= delta;
13
     forn (y, n) if (T[y]) ly[y] += delta; else slack[y] -= delta;
14
15
   void init_labels(){
16
     zero(lx), zero(ly);
17
     form (x,n) form(y,n) lx[x] = max(lx[x], cost[x][y]);
18
19
   void augment() {
20
     if (max_match == n) return;
21
     int x, y, root, q[N], wr = 0, rd = 0;
22
     memset(S, false, sizeof(S)), memset(T, false, sizeof(T));
23
     memset(prev2, -1, sizeof(prev2));
```

```
forn (x, n) if (xy[x] == -1){
       q[wr++] = root = x, prev2[x] = -2;
26
       S[x] = true; break; }
27
     forn (y, n) slack[y] = lx[root] + ly[y] - cost[root][y], slackx[y] =
28
         root;
     while (true){
       while (rd < wr){
30
         x = q[rd++];
31
         for (y = 0; y < n; y++) if (cost[x][y] == lx[x] + ly[y] && !T[y]){
           if (yx[y] == -1) break; T[y] = true;
           q[wr++] = yx[y], add_to_tree(yx[y], x); }
34
         if (y < n) break; }
       if (y < n) break;
36
       update_labels(), wr = rd = 0;
       for (y = 0; y < n; y++) if (!T[y] && slack[y] == 0){
         if (yx[y] == -1)\{x = slackx[y]; break;\}
         else{
40
           T[y] = true;
           if (!S[yx[y]]) q[wr++] = yx[y], add_to_tree(yx[y], slackx[y]);
42
         }}
       if (y < n) break; }
     if (v < n){
       max_match++;
46
       for (int cx = x, cy = y, ty; cx != -2; cx = prev2[cx], cy = ty)
47
         ty = xy[cx], yx[cy] = cx, xy[cx] = cy;
48
       augment(); }
49
   }
50
   tipo hungarian(){
     tipo ret = 0; max_match = 0, memset(xy, -1, sizeof(xy));
     memset(yx, -1, sizeof(yx)), init_labels(), augment(); //steps 1-3
    forn (x,n) ret += cost[x][xy[x]]; return ret;
54
55 }
```

7.16. Dynamic Conectivity

```
struct UnionFind {
   int n, comp;
   vector<int> pre,si,c;

UnionFind(int n=0):n(n), comp(n), pre(n), si(n, 1) {
     forn(i,n) pre[i] = i; }
   int find(int u){return u==pre[u]?u:find(pre[u]);}
   bool merge(int u, int v) {
     if((u=find(u))==(v=find(v))) return false;
```

```
if(si[u]<si[v]) swap(u, v);</pre>
9
           si[u]+=si[v], pre[v]=u, comp--, c.pb(v);
10
           return true;
11
       }
12
       int snap(){return sz(c);}
13
       void rollback(int snap){
14
           while(sz(c)>snap){
15
               int v = c.back(); c.pop_back();
16
               si[pre[v]] -= si[v], pre[v] = v, comp++;
17
18
       }
19
20
   enum {ADD,DEL,QUERY};
   struct Query {int type,u,v;};
   struct DynCon {
       vector<Query> q;
24
       UnionFind dsu;
25
       vector<int> match,res;
26
       map<ii,int> last;//se puede no usar cuando hay identificador para
27
           cada arista (mejora poco)
       DynCon(int n=0):dsu(n){}
28
       void add(int u, int v) {
29
           if(u>v) swap(u,v);
30
           q.pb((Query){ADD, u, v}), match.pb(-1);
31
           last[ii(u,v)] = sz(q)-1;
32
       }
33
       void remove(int u, int v) {
34
           if(u>v) swap(u,v);
35
           q.pb((Query){DEL, u, v});
36
           int prev = last[ii(u,v)];
37
           match[prev] = sz(q)-1;
38
           match.pb(prev);
39
       }
40
       void query() {//podria pasarle un puntero donde guardar la respuesta
41
           q.pb((Query){QUERY, -1, -1}), match.pb(-1);}
42
       void process() {
43
           forn(i,sz(q)) if (q[i].type == ADD && match[i] == -1) match[i] =
44
                sz(a):
           go(0,sz(q));
45
46
       void go(int 1, int r) {
47
           if(1+1==r){
48
               if (q[1].type == QUERY)//Aqui responder la query usando el
49
```

```
dsu!
                    res.pb(dsu.comp);//aqui query=cantidad de componentes
50
                        conexas
                return;
51
           }
52
           int s=dsu.snap(), m = (l+r) / 2;
53
           forr(i,m,r) if(match[i]!=-1 && match[i]<1) dsu.merge(q[i].u, q[i</pre>
54
                ].v);
           go(1,m);
           dsu.rollback(s);
           s = dsu.snap();
57
           forr(i,1,m) if(match[i]!=-1 && match[i]>=r) dsu.merge(q[i].u, q[
58
                il.v):
           go(m,r);
59
           dsu.rollback(s);
60
61
62 }dc;
```

8. Flujo

8.1. Dinic

```
1
  const int MAX = 300:
3 // Corte minimo: vertices con dist[v]>=0 (del lado de src) VS. dist[v
       ]==-1 (del lado del dst)
4 // Para el caso de la red de Bipartite Matching (Sean V1 y V2 los
       conjuntos mas proximos a src y dst respectivamente):
5 // Reconstruir matching: para todo v1 en V1 ver las aristas a vertices
       de V2 con it->f>0, es arista del Matching
6 // Min Vertex Cover: vertices de V1 con dist[v] == -1 + vertices de V2 con
        dist[v]>0
7 // Max Independent Set: tomar los vertices NO tomados por el Min Vertex
s // Max Clique: construir la red de G complemento (debe ser bipartito!) y
        encontrar un Max Independet Set
9 // Min Edge Cover: tomar las aristas del matching + para todo vertices
       no cubierto hasta el momento, tomar cualquier arista de el
int nodes, src, dst;
int dist[MAX], q[MAX], work[MAX];
   struct Edge {
13
       int to, rev;
       11 f, cap;
14
```

```
Edge(int to, int rev, 11 f, 11 cap) : to(to), rev(rev), f(f), cap(
15
                                                                                      56
            cap) {}
   };
                                                                                      57
16
   vector<Edge> G[MAX];
   void addEdge(int s, int t, ll cap){
       G[s].pb(Edge(t, sz(G[t]), 0, cap)), G[t].pb(Edge(s, sz(G[s])-1, 0,
19
            0));}
   bool dinic_bfs(){
20
       fill(dist, dist+nodes, -1), dist[src]=0;
21
       int qt=0; q[qt++]=src;
22
       for(int qh=0; qh<qt; qh++){</pre>
23
            int u = q[qh];
24
           forall(e, G[u]){
25
                int v=e->to:
26
                if(dist[v]<0 && e->f < e->cap)
27
                    dist[v]=dist[u]+1, q[qt++]=v;
28
                                                                                      10
           }
29
                                                                                      11
       }
30
                                                                                      12
       return dist[dst]>=0;
31
                                                                                      13
32
                                                                                      14
   ll dinic_dfs(int u, ll f){
33
                                                                                      15
       if(u==dst) return f;
34
                                                                                      16
       for(int &i=work[u]; i<sz(G[u]); i++){</pre>
35
                                                                                      17
            Edge &e = G[u][i];
36
                                                                                      18
            if(e.cap<=e.f) continue;</pre>
37
                                                                                      19
            int v=e.to;
38
                                                                                      20
            if(dist[v]==dist[u]+1){
39
                                                                                      21
                    11 df=dinic_dfs(v, min(f, e.cap-e.f));
40
                                                                                      22
                    if(df>0){
41
                                                                                      23
                             e.f+=df, G[v][e.rev].f-= df;
42
                                                                                           }
                                                                                      24
                             return df; }
43
                                                                                      25 }
           }
44
       }
45
       return 0;
46
47
   ll maxFlow(int _src, int _dst){
48
       src=_src, dst=_dst;
49
       ll result=0:
50
       while(dinic_bfs()){
51
           fill(work, work+nodes, 0);
52
            while(ll delta=dinic_dfs(src,INF))
53
                result+=delta;
54
       }
55
                                                                                       9 int f, p[MAX_V];
```

```
// todos los nodos con dist[v]!=-1 vs los que tienen dist[v]==-1
          forman el min-cut
      return result; }
                              8.2. Konig
1 // asume que el dinic YA ESTA tirado
  // asume que nodes-1 y nodes-2 son la fuente y destino
  int match[maxnodes]; // match[v]=u si u-v esta en el matching, -1 si v
      no esta matcheado
  int s[maxnodes]; // numero de la bfs del koning
  queue<int> kq;
  // s[e] \%2==1 o si e esta en V1 y s[e]==-1-> lo agarras
  void koning() {//O(n)
    forn(v,nodes-2) s[v] = match[v] = -1;
    forn(v,nodes-2) forall(it,g[v]) if (it->to < nodes-2 && it->f>0)
      { match[v]=it->to; match[it->to]=v;}
    forn(v,nodes-2) if (match[v]==-1) {s[v]=0;kq.push(v);}
    while(!kq.empty()) {
      int e = kq.front(); kq.pop();
      if (s[e] %2==1) {
        s[match[e]] = s[e]+1:
        kq.push(match[e]);
      } else {
        forall(it,g[e]) if (it->to < nodes-2 && s[it->to]==-1) {
          s[it->to] = s[e]+1;
          kq.push(it->to);
                       8.3. Edmonds Karp's
  #define MAX_V 1000
  #define INF 1e9
  //special nodes
  #define SRC 0
  #define SNK 1
6 map<int, int> G[MAX_V];//limpiar esto
7 //To add an edge use
 #define add(a, b, w) G[a][b]=w
```

void push(int a, int b) {

```
void augment(int v, int minE){
                                                                                        int amt = min(excess[a], ll(G[a][b]));
                                                                                        if(height[a] <= height[b] || amt == 0) return;</pre>
     if(v==SRC) f=minE;
                                                                                  17
11
     else if(p[v]!=-1){
                                                                                        G[a][b]-=amt, G[b][a]+=amt;
12
                                                                                  18
       augment(p[v], min(minE, G[p[v]][v]));
                                                                                        excess[b] += amt, excess[a] -= amt;
13
                                                                                  19
       G[p[v]][v]-=f, G[v][p[v]]+=f;
                                                                                        enqueue(b);
14
                                                                                  20
15
                                                                                  21
                                                                                      void gap(int k) {
16
                                                                                  22
   11 maxflow(){//0(VE^2)
                                                                                        forn(v, N){
     11 Mf=0;
                                                                                          if (height[v] < k) continue;</pre>
18
                                                                                          count[height[v]]--;
     do{
19
                                                                                          height[v] = max(height[v], N+1);
       f=0:
20
                                                                                  26
       char used[MAX_V]; queue<int> q; q.push(SRC);
                                                                                          count[height[v]]++;
21
                                                                                  27
       zero(used), memset(p, -1, sizeof(p));
                                                                                          enqueue(v);
22
                                                                                  28
       while(sz(q)){
                                                                                       }
23
                                                                                   29
         int u=q.front(); q.pop();
                                                                                   30
24
         if(u==SNK) break;
                                                                                      void relabel(int v) {
25
         forall(it, G[u])
                                                                                        count[height[v]]--;
26
           if(it->snd>0 && !used[it->fst])
                                                                                        height[v] = 2*N;
27
                                                                                  33
             used[it->fst]=true, q.push(it->fst), p[it->fst]=u;
                                                                                        forall(it, G[v])
                                                                                  34
28
       }
                                                                                          if(it->snd)
29
       augment(SNK, INF);
                                                                                            height[v] = min(height[v], height[it->fst] + 1);
                                                                                  36
30
                                                                                        count[height[v]]++;
       Mf+=f;
31
                                                                                        enqueue(v);
     }while(f);
32
                                                                                   38
     return Mf;
                                                                                   39
33
34 }
                                                                                      ll maxflow() {//O(V^3)
                                                                                   40
                                                                                        zero(height), zero(active), zero(count), zero(excess);
                       8.4. Push-Relabel O(N3)
                                                                                        count[0] = N-1;
                                                                                  42
                                                                                        count[N] = 1;
                                                                                  43
                                                                                        height[SRC] = N;
   #define MAX_V 1000
                                                                                  44
   int N;//valid nodes are [0...N-1]
                                                                                        active[SRC] = active[SNK] = true;
                                                                                        forall(it, G[SRC]){
   #define INF 1e9
                                                                                  46
                                                                                          excess[SRC] += it->snd;
   //special nodes
                                                                                  47
                                                                                          push(SRC, it->fst);
   #define SRC 0
                                                                                  48
                                                                                       }
   #define SNK 1
                                                                                  49
                                                                                        while(sz(Q)) {
   map<int, int> G[MAX_V];
                                                                                  50
                                                                                          int v = Q.front(); Q.pop();
                                                                                  51
   //To add an edge use
                                                                                          active[v]=false;
   #define add(a, b, w) G[a][b]=w
                                                                                  52
                                                                                        forall(it, G[v]) push(v, it->fst);
   ll excess[MAX V]:
                                                                                  53
                                                                                        if(excess[v] > 0)
   int height[MAX_V], active[MAX_V], count[2*MAX_V+1];
                                                                                  54
                                                                                          count[height[v]] == 1? gap(height[v]):relabel(v);
   queue<int> Q;
                                                                                  55
12
                                                                                        }
   void enqueue(int v) {
                                                                                  56
                                                                                        ll mf=0;
     if (!active[v] && excess[v] > 0) active[v]=true, Q.push(v); }
                                                                                  57
```

forall(it, G[SRC]) mf+=G[it->fst][SRC];

```
return mf;
60 }
                              Min-cost Max-flow
                        8.5.
   const int MAXN=10000;
   typedef 11 tf;
   typedef 11 tc;
   const tf INFFLUJO = 1e14;
   const tc INFCOSTO = 1e14;
   struct edge {
     int u, v;
     tf cap, flow;
     tc cost;
     tf rem() { return cap - flow; }
10
11
   int nodes; //numero de nodos
12
   vector<int> G[MAXN]; // limpiar!
   vector<edge> e; // limpiar!
   void addEdge(int u, int v, tf cap, tc cost) {
     G[u].pb(sz(e)); e.pb((edge)\{u,v,cap,0,cost\});
     G[v].pb(sz(e)); e.pb((edge)\{v,u,0,0,-cost\});
17
18
   tc dist[MAXN], mnCost;
19
   int pre[MAXN];
   tf cap[MAXN], mxFlow;
   bool in_queue[MAXN];
   void flow(int s, int t) {
23
     zero(in_queue);
24
     mxFlow=mnCost=0;
25
     while(1){
26
       fill(dist, dist+nodes, INFCOSTO); dist[s] = 0;
27
       memset(pre, -1, sizeof(pre)); pre[s]=0;
28
       zero(cap); cap[s] = INFFLUJO;
29
       queue<int> q; q.push(s); in_queue[s]=1;
30
       while(sz(q)){
31
         int u=q.front(); q.pop(); in_queue[u]=0;
32
         for(auto it:G[u]) {
33
           edge &E = e[it];
34
           if(E.rem() && dist[E.v] > dist[u] + E.cost + 1e-9){ // ojo EPS
35
             dist[E.v]=dist[u]+E.cost;
36
             pre[E.v] = it;
37
```

cap[E.v] = min(cap[u], E.rem());

38

```
if(!in_queue[E.v]) q.push(E.v), in_queue[E.v]=1;
39
40
         }
41
       }
^{42}
       if (pre[t] == -1) break;
43
       mxFlow +=cap[t];
44
       mnCost +=cap[t]*dist[t];
45
       for (int v = t; v != s; v = e[pre[v]].u) {
         e[pre[v]].flow += cap[t];
         e[pre[v]^1].flow -= cap[t];
49
    }
50
51 }
```

9. Plantilla

```
1 #include <bits/stdc++.h>
   using namespace std;
   #define forr(i, a, b) for(int i = (a); i < (int) (b); i++)
   #define forn(i, n) forr(i, 0, n)
   #define forall(it, v) for(auto it = v.begin(); it != v.end(); ++it)
   #define dforn(i, n) for(int i = ((int) n) - 1; i \ge 0; i--)
   #define db(v) cerr << #v << " = " << v << endl
   #define pb push_back
   typedef long long 11;
   const int MAXN = -1;
11
12 | int main() {
13
     return 0;
14
15 }
```

10. Ayudamemoria

Cant. decimales

```
#include <iomanip>
cout << setprecision(2) << fixed;</pre>
```

Rellenar con espacios(para justificar)

```
#include <iomanip>
cout << setfill(''') << setw(3) << 2 << endl;</pre>
```

Aleatorios

```
#define RAND(a, b) (rand()%(b-a+1)+a)
rand(time(NULL));
random_shuffle(A, A + n);
```

Comparación de Doubles

```
const double EPS = 1e-9;
x == y <=> fabs(x-y) < EPS
x > y <=> x > y + EPS
x > y <=> x > y - EPS
```

Limites

```
i #include inits>
i numeric_limits<T>
i ::max()
i ::min()
::epsilon()
```

Muahaha

```
#include <signal.h>
void divzero(int p){
while(true);}

void segm(int p){
exit(0);}

//in main
signal(SIGFPE, divzero);
signal(SIGSEGV, segm);
```

Mejorar velocidad

```
i lios::sync_with_stdio(false);
```

Mejorar velocidad 2

```
//Solo para enteros positivos
inline void Scanf(int& a){
   char c = 0;
   while(c<33) c = getc(stdin);
   a = 0;
   while(c>33) a = a*10 + c - '0', c = getc(stdin);
}
```

Expandir pila

```
#include <sys/resource.h>
2 rlimit rl;
getrlimit(RLIMIT_STACK, &rl);
4 | rl.rlim_cur=1024L*1024L*256L;//256mb
5 setrlimit(RLIMIT_STACK, &rl);
                               C++11
1 g++ --std=c++1
                          Leer del teclado
freopen("/dev/tty", "a", stdin);
                        Iterar subconjunto
for(int sbm=bm; sbm; sbm=(sbm-1)&bm)
                              File setup
1 // tambien se pueden usar comas: {a, x, m, 1}
touch {a..l}.in; tee {a..l}.cpp < template.cpp
                           Releer String
string s; int n;
getline(cin, s);
  stringstream leer(s);
  while(leer >> n){
    // do something ...
6 }
```