Homework V

1. Steering Vector

The channel steering vector is given by

$$e_r(\Omega) = \frac{1}{\sqrt{N_r}} \begin{pmatrix} 1 \\ e^{-j2\pi\Delta_r\Omega} \\ e^{-j2\pi\Delta_r 2\Omega} \\ \vdots \\ e^{-j2\pi\Delta_r(N_r-1)\Omega} \end{pmatrix}.$$

where Δ_r is the normalized array spacing and $\Omega = \cos\phi$ represents the direction cosine, which is related to the incidence angle. In this assignment, Δ_r is set to be $\frac{1}{2}$, which is the optimal spacing, and the incidence angles are randomly generated as $\phi \in [-\pi/2, \pi/2]$. The steering vectors are shown below:

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Signal 1: Angle = 27.65 degree
  Columns 1 through 7

   1.0000 + 0.0000i  -0.9364 - 0.3510i   0.7536 + 0.6574i  -0.4748 - 0.8801i   0.1357 + 0.9908i   0.2207 - 0.9753i  -0.5491 + 0.8358i

  Columns 8 through 14

   0.8075 - 0.5898i  -0.9632 + 0.2688i   0.9963 + 0.0864i  -0.9025 - 0.4306i   0.6940 + 0.7200i  -0.3970 - 0.9178i   0.0496 + 0.9988i

  Columns 15 through 16

   0.3042 - 0.9526i  -0.6192 + 0.7852i
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Signal 2: Angle = -69.30 degree
  Columns 1 through 7

   1.0000 + 0.0000i   0.4442 - 0.8959i  -0.6054 - 0.7959i  -0.9820 + 0.1889i  -0.2670 + 0.9637i   0.7448 + 0.6672i   0.9286 - 0.3710i

  Columns 8 through 14

   0.0801 - 0.9968i  -0.8575 - 0.5145i  -0.8419 + 0.5397i   0.1096 + 0.9940i   0.9392 + 0.3433i   0.7248 - 0.6890i  -0.2954 - 0.9554i

  Columns 15 through 16
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Signal 3: Angle = 81.05 degree
  Columns 1 through 7

   1.0000 + 0.0000i   0.8829 - 0.4695i   0.5592 - 0.8290i   0.1045 - 0.9945i  -0.3746 - 0.9272i  -0.7661 - 0.6428i  -0.9782 - 0.2079i

  Columns 8 through 14

  -0.9613 + 0.2757i  -0.7193 + 0.6947i  -0.3090 + 0.9511i   0.1737 + 0.9848i   0.6157 + 0.7880i   0.9136 + 0.4067i   0.9976 - 0.0698i

  Columns 15 through 16

   0.8480 - 0.5300i   0.5000 - 0.8661i
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Signal 4: Angle = -3.21 degree
  Columns 1 through 7

   1.0000 + 0.0000i  -1.0000 - 0.0049i   1.0000 + 0.0098i  -0.9999 - 0.0147i   0.9998 + 0.0197i  -0.9997 - 0.0246i   0.9996 + 0.0295i

  Columns 8 through 14

  -0.9994 - 0.0344i   0.9992 + 0.0393i  -0.9990 - 0.0442i   0.9988 + 0.0491i  -0.9985 - 0.0540i   0.9983 + 0.0590i  -0.9980 - 0.0639i

  Columns 15 through 16

   0.9976 + 0.0688i  -0.9973 - 0.0737i
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Signal 5: Angle = 67.05 degree
  Columns 1 through 7

   1.0000 + 0.0000i   0.3387 - 0.9409i  -0.7705 - 0.6374i  -0.8607 + 0.5091i   0.1874 + 0.9823i   0.9877 + 0.1564i   0.4817 - 0.8763i

  Columns 8 through 14

  -0.6613 - 0.7501i  -0.9298 + 0.3682i   0.0314 + 0.9995i   0.9511 + 0.3090i   0.6129 - 0.7902i  -0.5359 - 0.8443i  -0.9759 + 0.2182i

  Columns 15 through 16

  -0.1253 + 0.9921i   0.8910 + 0.4539i
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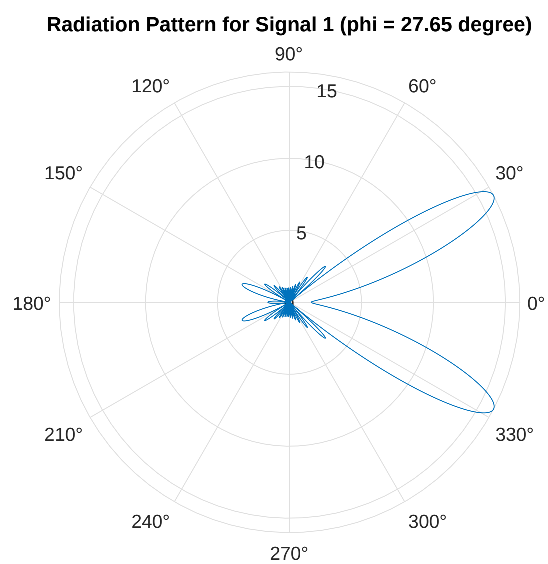
2. Radiation Pattern

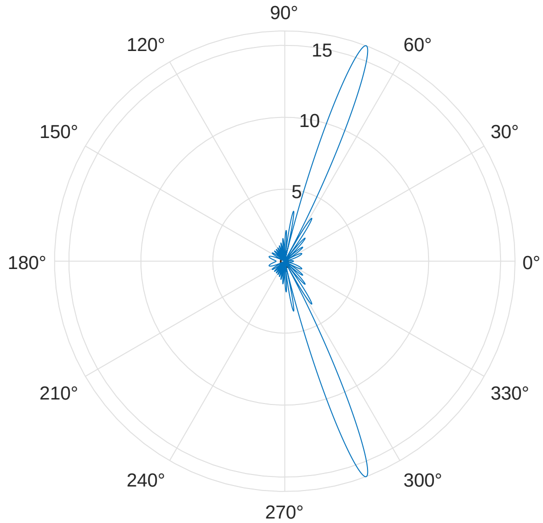
The radiation pattern is obtained by projecting the received signal onto the target steering vector. Suppose the incidence angle of the desired signal is ϕ_0, the attenuation at angle ϕ is

$$\mathrm{P}(\phi) = \left| e_r^H(\Omega) \cdot e_r(\Omega_0) \right|$$

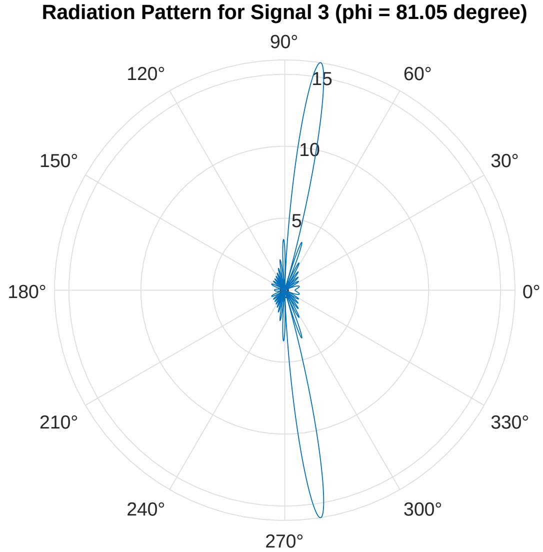
where $\Omega = \cos(\phi)$ and $\Omega_0 = \cos(\phi_0)$. The radiation patterns are shown below:

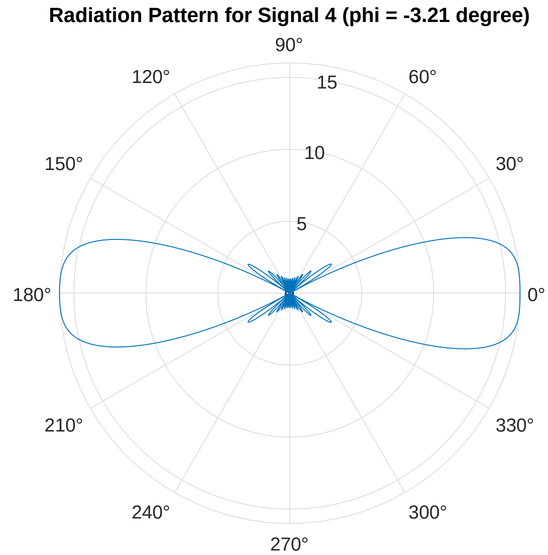


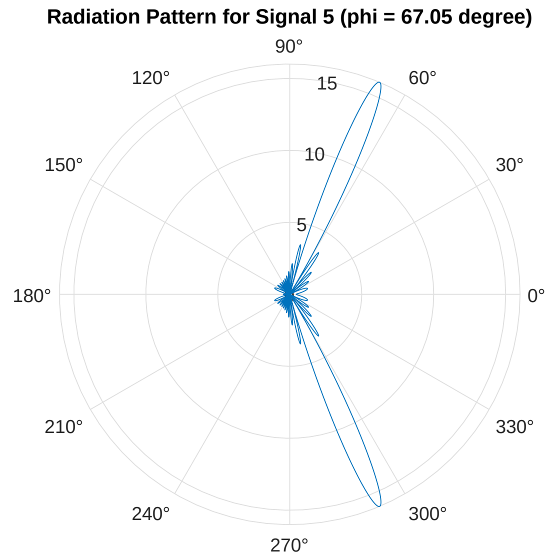
Radiation Pattern for Signal 2 (phi = -69.30 degree)



Radiation Pattern for Signal 3 (phi = 81.05 degree)







It can be observed that with receive beamforming, the main lobe is aligned with the incidence angle.

3. SIR for N = 16

Similar to that in problem 2, if the incidence angle of the desired signal is ϕ_i, then the interference power is obtained by projecting other signals onto the steering vector of desired signal.

$$\text{P}_{\text{interference}} = P \cdot \sum_{j \neq i} \left| e_r^H(\Omega_j) \cdot e_r(\Omega_i) \right|^2 .$$

Hence, the signal-to-interference power ratio (SIR) is given by

$$\text{SIR} = \frac{\left| e_r^H(\Omega_i) \cdot e_r(\Omega_i) \right|^2}{\sum_{j \neq i} \left| e_r^H(\Omega_j) \cdot e_r(\Omega_i) \right|^2}.$$

The SIRs of each signal are listed in the table below:

	Signal 1	Signal 2	Signal 3	Signal 4	Signal 5
SIR (dB)	17.7617	1.0028	13.3221	17.8966	1.2187

From the table above, it can be observed that the SIR of signals 2 and 5 is close to one. The reason is that ϕ_2 and ϕ_5 are close to each other. The steering vector of signal 2 also provides gain for signal 5, and vice versa. More precisely,

$$\left|e_r^H(\Omega_2) \cdot e_r(\Omega_2)\right|^2 \approx \left|e_r^H(\Omega_5) \cdot e_r(\Omega_2)\right|^2.$$

and

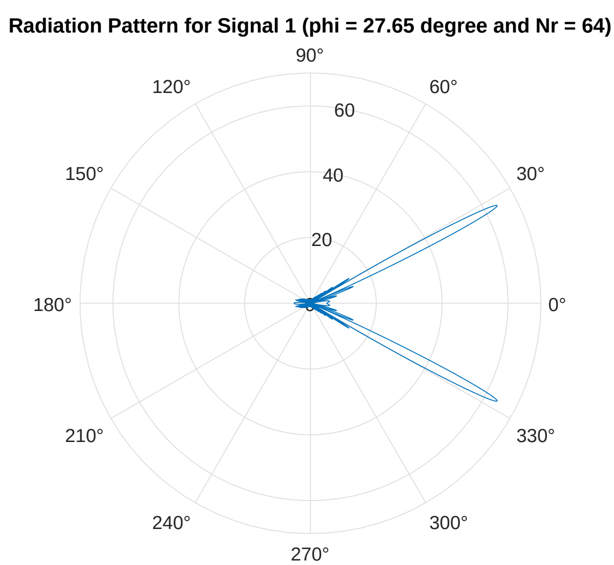
$$\left|e_r^H(\Omega_5) \cdot e_r(\Omega_5)\right|^2 \approx \left|e_r^H(\Omega_2) \cdot e_r(\Omega_5)\right|^2.$$

4. **SIR for N = 64**

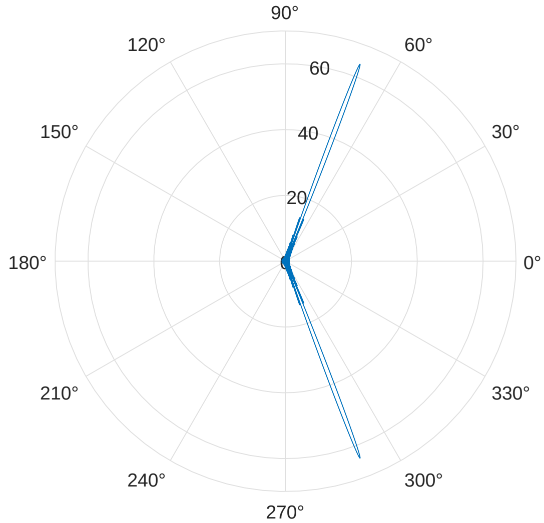
Similiar to that in (c), we may obtian the SIR for each signal under $N_r = 64$ received antenna. The SIR of each signal is listed in the table below:

	Signal 1	Signal 2	Signal 3	Signal 4	Signal 5
SIR (dB)	21.2980	16.7495	23.8789	21.1201	16.7325

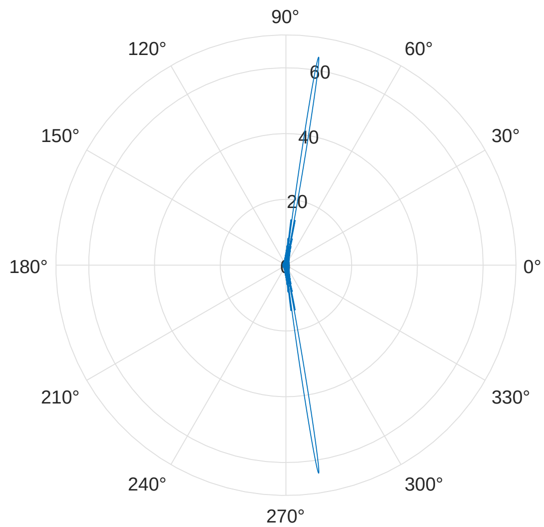
Compared to that in problem 3, it can be observed that the SIR for each signal increases as the number of receiving antennas increases. The reason can be seen from the radiation patterns



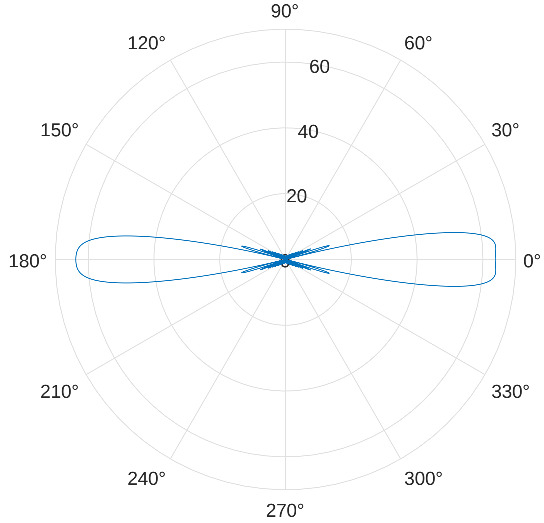
Radiation Pattern for Signal 2 (phi = -69.30 degree and Nr = 64)



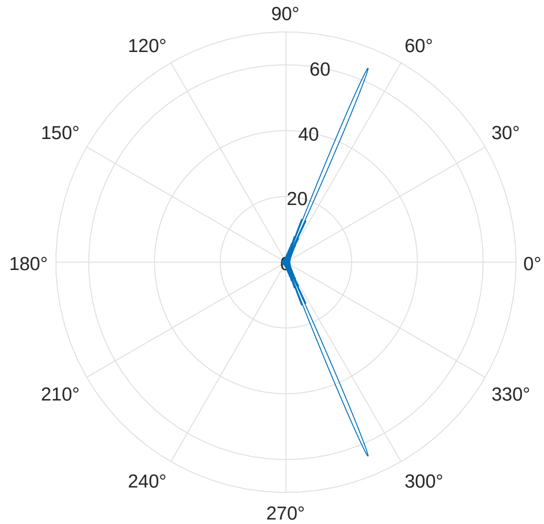
Radiation Pattern for Signal 3 (phi = 81.05 degree and Nr = 64)



Radiation Pattern for Signal 4 (phi = -3.21 degree and Nr = 64)



Radiation Pattern for Signal 5 (phi = 67.05 degree and Nr = 64)



It can be seen that as N_r increases, the beamwidth becomes narrower. Hence, the gain at angles other than the incidence angle becomes smaller, improving the angular resolvability. This also explains why the SIR of signals 2 and 5 is no longer 1.