Homework I

1. Simulation for Traffic load

Based on Erlang B formula

$$B(\rho, m) = \frac{\frac{\rho^m}{m!}}{\sum_{k=0}^{m} \frac{\rho^k}{k!}}$$

For large m, there will be overflow issue. Therefore, we take logarithm on both side to prevent overflow.

$$\log B(\rho, m) = m \log(\rho) - \sum_{k=1}^{m} \log(k) - \log\left(\sum_{k=0}^{m} \frac{\rho^k}{k!}\right)$$

Given blocking probability $B(\rho, m)$ and channel number m, we can solve the traffic load ρ. The simulation results are given as below:

m	$P_{\textbf{block}} = 1\%$	$P_{\textbf{block}} = 3\%$	$P_{\textbf{block}} = 5\%$	$P_{\textbf{block}} = 10\%$
1	0.0101	0.0309	0.0526	0.1111
2	0.1526	0.2816	0.3813	0.5954
3	0.4555	0.7151	0.8994	1.2708
4	0.8694	1.2589	1.5246	2.0454
5	1.3608	1.8752	2.2185	2.8811
6	1.9090	2.5431	2.9603	3.7584
7	2.5009	3.2497	3.7378	4.6662
8	3.1276	3.9865	4.5430	5.5971
9	3.7825	4.7479	5.3702	6.5464
10	4.4612	5.5294	6.2157	7.5106
11	5.1599	6.3280	7.0764	8.4871
12	5.8760	7.1410	7.9501	9.4740
13	6.6072	7.9667	8.8349	10.4699
14	7.3517	8.8035	9.7295	11.4735
15	8.1080	9.6500	10.6327	12.4838
16	8.8750	10.5052	11.5436	13.5001
17	9.6516	11.3683	12.4613	14.5217
18	10.4369	12.2384	13.3852	15.5480
19	11.2301	13.1150	14.3147	16.5787
20	12.0306	13.9974	15.2493	17.6132

table 1. traffic load under different number of channel ($1 \sim 20$)

m	$P_{\textbf{block}} = 1\%$	$P_{\textbf{block}} = 3\%$	$P_{\textbf{block}} = 5\%$	$P_{\textbf{block}} = 10\%$
200	179.7380	190.8859	198.5073	214.3226
201	180.7059	191.8943	199.5456	215.4278
202	181.6739	192.9028	200.5839	216.5331
203	182.6420	193.9114	201.6224	217.6383
204	183.6103	194.9201	202.6609	218.7437
205	184.5787	195.9289	203.6994	219.8490
206	185.5473	196.9378	204.7381	220.9544
207	186.5161	197.9468	205.7768	222.0598
208	187.4850	198.9559	206.8156	223.1653
209	188.4540	199.9651	207.8544	224.2708
210	189.4232	200.9744	208.8933	225.3763
211	190.3925	201.9837	209.9323	226.4818
212	191.3620	202.9932	210.9714	227.5874
213	192.3316	204.0028	212.0105	228.6931
214	193.3013	205.0124	213.0497	229.7987
215	194.2712	206.0222	214.0889	230.9044
216	195.2412	207.0320	215.1283	232.0102
217	196.2114	208.0419	216.1676	233.1159
218	197.1816	209.0519	217.2071	234.2217
219	198.1521	210.0620	218.2466	235.3275
220	199.1226	211.0722	219.2862	236.4334

table 1. traffic load under different number of channel (200 $\sim$ 220)

2. Actual traffic load

(a) No. Recall that one Erlang $\leftrightarrow$ one hour of call traffic during one gour of operatiom. Therefore, the total traffic load cannot be greater than the number of available channel.

(b) However, there are some results from (a) that do not satisfy the requirement above i.e $\rho > m$. The reason is that the actual traffic is determined by traffic load ρ and blocking probability $B(\rho, m)$, that is

$$\text{actual traffic} = \hat{\rho} = (1 - B(\rho, m))\rho$$

3. Spectral efficiency for different number of operators

Suppose there are k operator with frequency reuse factor N, then the number of channel in each cell is

$$m = \frac{600}{kN}$$

Similiar to that in (a), we solve the traffic load under different blocking probability. The results are shown below

k	$P_{\mathbf{block}} = 1\%$	$P_{\mathbf{block}} = 3\%$	$P_{\mathbf{block}} = 5\%$	$P_{\mathbf{block}} = 10\%$
1	102.9636	110.6506	115.7705	126.0823
2	46.9496	51.5697	54.5656	60.4013
3	29.0074	32.4118	34.5959	38.7874

table 3. traffic load under different number of operators

The truncking efficieny η_T is the offered traffic per channel, which is given by $\eta_T = \frac{\rho}{m}$. By translating the above traffic load into truncking efficieny, we have

k	$P_{\mathbf{block}} = 1\%$	$P_{\mathbf{block}} = 3\%$	$P_{\mathbf{block}} = 5\%$	$P_{\mathbf{block}} = 10\%$
1	0.8580	0.9221	0.9648	1.0507
2	0.7825	0.8595	0.9094	1.0067
3	0.7252	0.8103	0.8649	0.9697

table 4. trunking efficiency under different number of operators

It can be seen that no matter under which blocking probability, one operator always has the highest trunking efficiency. Hence, one user provides the highest spectral efficiency.