Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		2207181

Simpler is Better: ARIMAX-XGARCH and Markowitz Theory Based Portfolio Strategy

Summary

Gold and Bitcoin as two investment options in the market are favored by a large number of investors, but how to make a portfolio investment to make the maximum return with a limited principal is a great concern for many investors. Based on the historical price data of Gold and Bitcoin, we derive the cointegration between Gold and Bitcoin prices, and forecast the return and risk of Gold and Bitcoin, and finally abstract the investment strategy problem as an optimization problem and solve it using **Markowitz Mean-Variance theory** to maximize the return.

For problem 1, we divide the strategy solution into three steps: first, a large number of empirical studies show that Gold price and Bitcoin price affect each other due to the risk competition and cointegration relationship between them, so we build a **MIDAS-Quantile Cointegration Regression Model** to explore its cointegration; then, we use time series analysis based on historical data to predict the return and risk. The **ARIMAX model** and the improved **XGARCH model** have a simpler form and better results than the complex and ill-defined machine learning and deep learning models; finally, we abstracted the optimal strategy solution problem into an optimization problem based on **Markowitz Mean-Variance theory** and solved it, resulting in a total value of about **$167,300** held by 2021-9-11.

For problem 2, we conducted the comparison experiments with the maximum Sharpe model, the risk parity model, and the popular reinforcement learning model, such as Proximal Policy Optimization (PPO) model, and verified that our model is indeed the optimal model and that the reinforcement model performs the worst. At the same time, we also compare the return cycles and find that the model build in Problem 1 with daily return maximization as the objective is optimal.

For problem 3, which is actually a sensitivity analysis of the model to transaction costs, it is analyzed that transaction costs do have an impact on strategy and returns. Higher transaction costs tend to make returns lower.

The non-technical memo for problem 4 is listed below.

Our return maximization strategy solution model is simple but achieves better performance than many popular algorithms, with better confidence, interpretability, robustness, and scalability, illustrating that models are not necessarily more complex in terms of performance and interpretability, but often complex models not perform better because of lacking interpretability. Sometimes simpler is better.

Keywords: Portfolio Strategy, MIDAS-Conditional Quantile Cointegration Regression, ARIMAX, XGARCH, Markowitz Model

Contents

1 Introduction

1.1 Problem Background

Trading strategy is a systematic method for trading in the securities market, which may involve multiple factors such as investment style, market value, etc. At the same time, the trading strategy should be re-evaluated and adjusted regularly according to changes in market conditions or personal goals.

In recent years, with the progress of artificial intelligence and computer technology, quantitative trading is flourishing in the market. Quantitative trading is based on advanced mathematical models, and uses computer and big data technology to develop trading strategies to obtain a probability of excess returns.

Compared with Gold, Bitcoin has the properties of high yield, high volatility, exemption from regulation and tax exemption, and will have tremendous room for development in the financial field. Bitcoins, sometimes referred to as new types of Gold, can replace Gold to hedge inflation, become new hedge assets and complement Gold for hedging, which is of great interest to investors in the financial sector.

1.2 Restatement of Problems

Based on the daily price of a troy ounce of Gold and a single price in U.S.dollars from 9/11/2016 to 9/10/2021, we need to solve the following problems:

- As for an trader with 1000 cash in U.S.dollars at first, who pays 2% and 1% commission for each purchase or sale(bitcoin and Gold), how can he or she choose the best daily investment strategy to maximize his or her total assets without knowing the future data?
- How to verify that this strategy is the optimal strategy.
- Exploring the impact of commission on strategy models
- Write up models and findings as non-technical Memo.

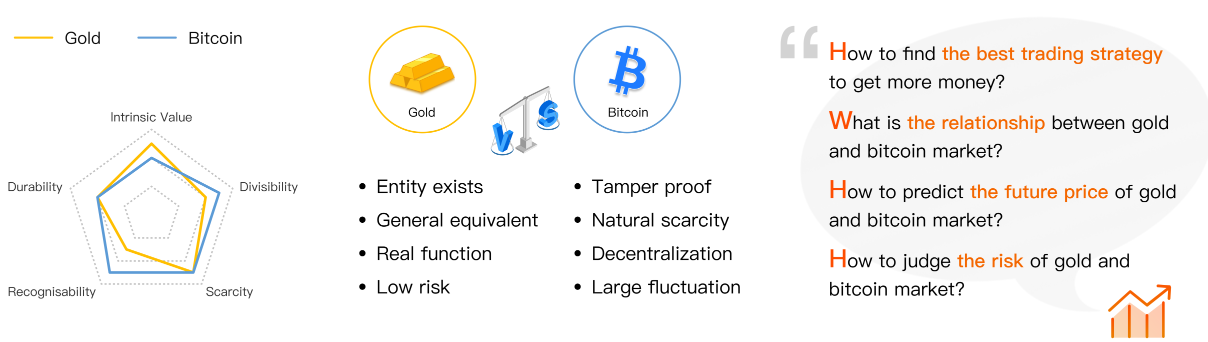


Figure 1: Introduction and Background of Problems

1.3 Analysis and Our Work

1.3.1 Problem 1

For the evaluation of the optimal strategy, we need to make some inquiry into the basic statistical properties of the data and divide our model solving steps into three based on them. First, The research of Ye Wuyi [10], Francisco jareno [3], Syed Zwick [12], sang Hoon Kang [8] and others shows that there is cointegration between the price of Gold and the price of Bitcoin. The change trend of risk in Gold market and bitcoin market is roughly opposite. There is a cointegration relationship between bitcoin's rate of return and Gold's rate of return, and Gold price can also be used as one of the factors to predict bitcoin price.Therefore, it is necessary to conduct a co - integration analysis. Second, according to the result of analysis, consider whether another series should be introduced into the model as an exogenous variable to forecast future prices or returns and volatility based on historical data, and this process can take the ARIMAX series of models and GARCH series of models that are often used in traditional econometrics, or many machine learning and deep learning models used by many scholars recently. Third, after getting the more accurate time series analysis, we build a model based on market laws and economic principles to determine the optimal investment strategy, which will be abstracted as an optimization problem.

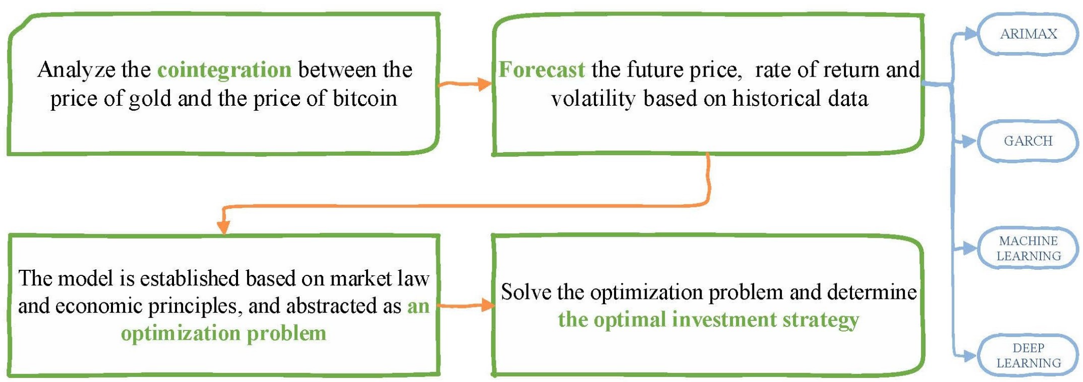


Figure 2: The workflow for problem 1

1.3.2 Problem 2

Problem2 is a complement to problem 1. The investigation of the optimal strategy can be carried out in two aspects: the first aspect is the comparison of multiple strategies, we can select the commonly used portfolio models for comparison; on the other hand, due to the difference between short-term return investment and long-term return investment strategies, we can focus on the return cycle, i.e.: whether we focus on daily, weekly or monthly returns, and come up with an answer through a comparison experiment.

1.3.3 Problem 3

The results of the model can be further explored by changing the commission parameter in the model, which is the exploration and supplementation based on the problem 1.

As we can see, the subsequent problems are based on the modeling of problem 1.

2 Preparation of the Models

2.1 Assumptions

In order to simplify the problem, we make the following assumptions before building the model:

1. It is assumed that the trading volume of bitcoin and Gold can be any positive real number. This assumption is due to the fact that if the unitization of bitcoin and Gold is considered, the solution of the model if a discretization process and will be more complicated.

2. It is assumed that there will be no non-fee loss in the investment process.This assumption guarantees that investors can invest arbitrarily in order to obtain more returns.

2.2 Notations

The symbols in the table are for reference only. If there are symbols not marked in the table, the meaning shall be subject to the text.

Table 1: Notations

Symbol	Definition
$r(t)$	The Return Rate at t
$p(t)$	The Price at t
$r_b(t)$	The Return Rate of Bitcoin at t
$r_g(t)$	The Return Rate of Gold at t
$p_b(t)$	The Price of Bitcoin at t
$p_g(t)$	The Price of Gold at t
$Q_\tau(r\|x)$	The Quantile of r at condition x
$z_\tau(x,\theta)$	The MIDAS term
$\psi(d,\theta)$	The Almon Polynomial
μ	The Average
ϵ_t	A White Noise
α_i	The coefficient or The transaction fee
w	The Investing fee
$r^{(d)}(t)$	The d-ordered series of $r(t)$
σ^2	The Variance or Volatility
$D(w,r)$	Risk Function
$E(w,r)$	Return Function
C,B,G	Cash, Bitcoin and Gold

3 Model Establishing and Solving of Problem I

3.1 Basic descripition of the data

To more visually discover the price change pattern of two time series, we plot the price of Bitcoin and the price of Gold over time, as shown in Figure 3:

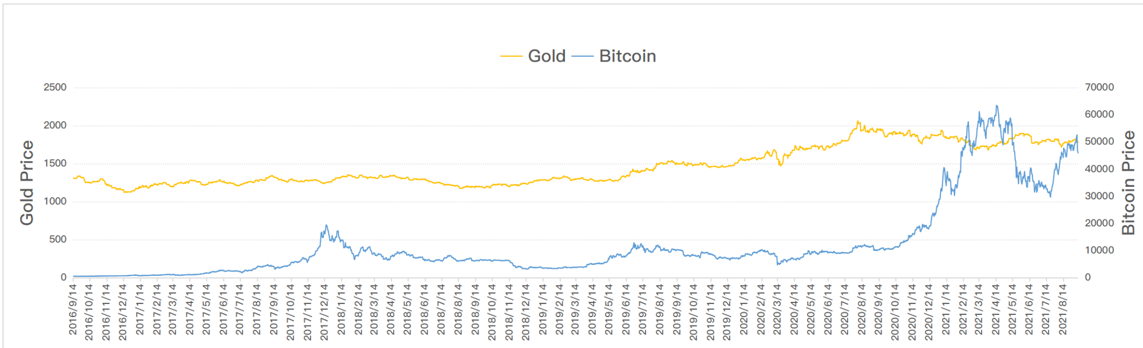


Figure 3: Original Time Series.

Also, we plot the frequency distribution histograms of Bitcoin and Gold prices, as shown in Figure 4:

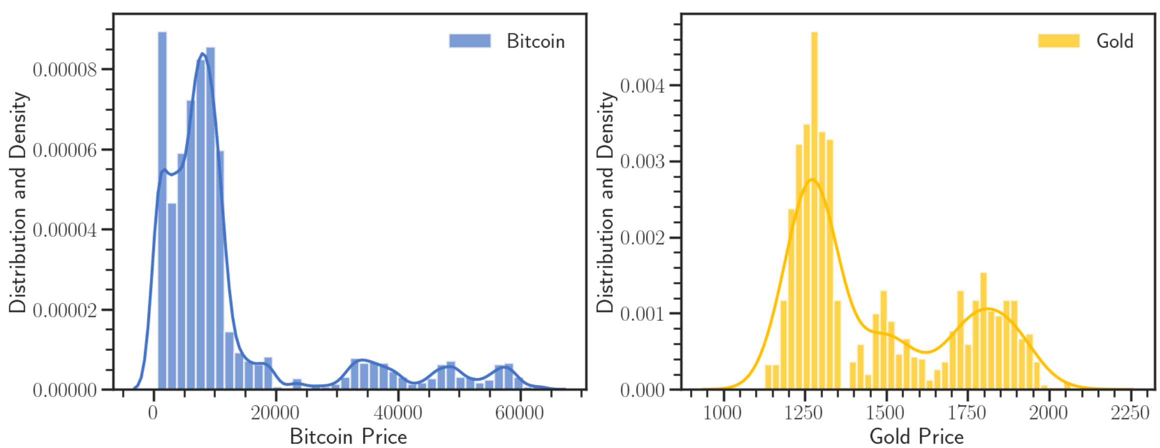


Figure 4: Distribution of two Price Series.

From Figures 3 and 4, we can find that the data have a large variation, a clear trend of growth and a fast growth rate, and are not strictly skewed(peak distributed), with multiple peaks. We believe that this is a non-stationary series, and to verify the conjecture, we conducted ADF unit root test for the two price series.The original hypothesis of the test is H0: the series is a non-stationary time series. The results show that the test statistics of Bitcoin and Gold are 0.01075 and -0.62366 respectively, and the probability values are 0.959 and 0.866 respectively, which are both much larger than 0.05, so the original hypothesis is accepted and the two series are considered as non-stationary series. If the regression method is used to model the non-stationary series, the pseudo-regression phenomenon will appear, and the predicted results will lose their economic significance. Based on this, we perform the first-order difference of the series, as shown in Figure 5.

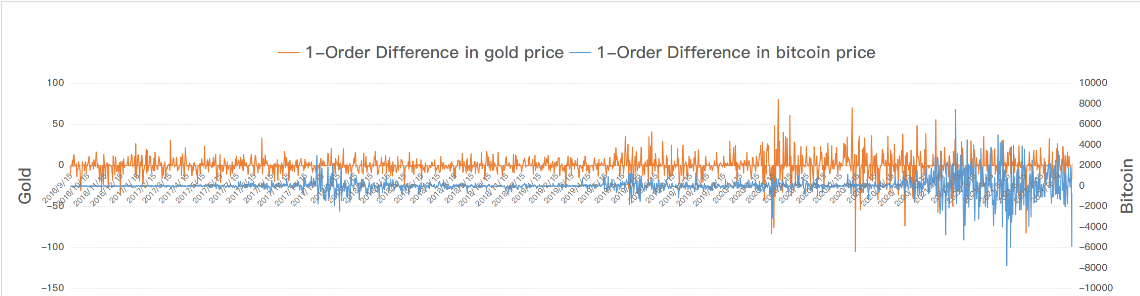


Figure 5: 1-Order Differential Time Series.

At this point we make the hypothesis H0: the difference series of the two series, and the ADP unit root test is performed again. The results show that the probability of the difference series of both Bitcoin and Gold is much less than 0.05, so the original hypothesis is rejected and the alternative hypothesis is accepted, and the difference series of both series are considered to be smooth.

In order to quantify the prices of the two stocks in further depth, we list some statistical properties of the two series in Table 2.

	mean	std	min	max	skew	kuronis
Bitcoin	12225.16	14047.60	594.08	63554.44	2.01	3.09
Gold	1463.87	249.47	1125.7	2067.15	0.65	-1.11
rt_Bitcoin	0.0024	0.042	-0.497	0.198	-0.920	13.989
rt_Gold	0.00017	0.007	-0.053	0.051	-0.452	8.944

Table 2: Basic Statistical Description of Data

According to Table 3, we can see that the Bitcoin's average price is nearly 10 times of Gold, but the std. is larger, too. This phenomenon implies that the Bitcoin can bring high return with high risk at the same time.

We also explored the autocorrelation of the series of Bitcoin and Gold. Taking different time intervals T and studying the correlation of the series p(t) with p(t-T), for both Gold and Bitcoin, the autocorrelation of the original data is very strong, until T=9, p(t-T) still has a correlation coefficient of 0.98 with the original series, which has a very strong linear correlation feature. This phenomenon verified that the two series are highly unstable, and we take the difference between them before judging the correlation, and it can be seen that after one difference the series no longer has auto-correlation. Therefore, the differenced data are more beneficial for us to deal with.

3.2 Sequence prediction models and their results

In order to make a determination of the investment strategy for the day, we need to predict the price changes of the two future series based on historical data. This is essentially a time series forecasting problem. Since investing requires a combination of return and risk, we use return and volatility to describe the level of return and risk. For the two different series we use different methods for forecasting.

Due to the large variation of Bitcoin and Gold prices, we take the logarithm and then differential them. In fact, this transformation yields exactly the logarithmic rate of return, a statistic that is also often used in return forecasting and portfolio strategies. Namely, :

$$r(t) = \ln \frac{p(t)}{p(t-1)} \tag{1}$$

3.2.1 Cointegration Analysis of Gold Price and Bitcoin Price

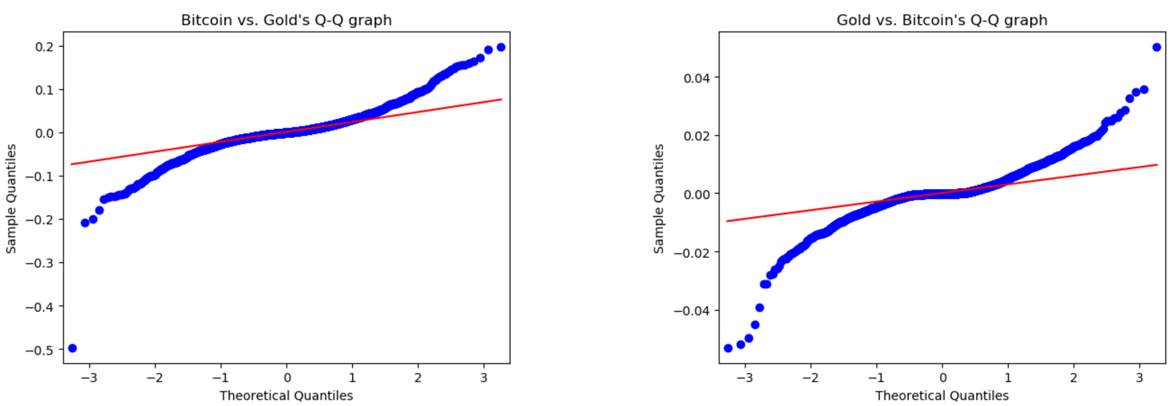
Based on previous studies, we conducted the Engel-Granger two-step cointegration test on the data. Engel-Granger Test is an approach to test if there exists cointegration between two time series. The Engle-Granger two-step procedure is:

1. By testing the smoothness of the two price variables we already know that both price series are first-order singer integer, so we can proceed to the test.

2. Using OLS to construct regression equations for the two variables, the test results are shown in Table 3. As can be seen, at a confidence level of 0.05 we can conclude that there may indeed be a preliminary cointegration relationship between Bitcoin and Gold. We also plot the Q-Q figure of the two series as shown in Figure 6.

	Variables	coef	std. err	t	P>|t|	0.025C.I.	0.975.C.I
Bitcoin	Intercept	0.0023	0.001	2.371	0.018	0.000	0.004
	Gold	0.2893	0.136	2.128	0.034	0.023	0.556
Gold	Intercept	0.0002	0.000	0.898	0.369	-0.000	0.000
	Bitcoin	0.0086	0.004	2.128	0.034	0.001	0.016

Table 3: Cointegration Test



(a) Bitcoin. (b) Gold.

Figure 6: Q-Q Plot for cointegration Test.

The data points in Figure 6 are approximately distributed near the regression line, and to verify the conjecture of smoothness, we performed the last step: the test of the residuals.

3. Test for the smoothness of the residuals: we performed the ADF test on the residual series, i.e., the Kolmogorov-Smirnov normality test and the ADF unit root test on the residuals, and the results

show that the two residuals do obey a normal distribution and are smooth series. Therefore, we can tentatively conclude that the logarithmic returns of Bitcoin and Gold are indeed fully cointegrated.

For further analysis of the cointegration of the two series, we also developed a dynamic cointegrtion analysis model based on MIDAS-Conditional Quantile Regression [10].

Since only two series of log returns of Bitcoin and Gold are available here, we establish such a quantile regression equation as:

$$Q_\tau(r_b(t)|X = r_g(t)) = a_{\tau,0}(t) + a_{\tau,1}(t)r_g(t) + a_{\tau,2}(t)z_t(r_g, \theta) \tag{2}$$

In the above equation, Q is the conditional quantile and a_0, a_1, a_2 are the coefficient functions over time, z_t is the MIDAS term:

$$z_t(x, \theta) = \sum_{d=1}^{T} \psi(d, \theta)x(t - d) \tag{3}$$

ψ is an Almon polynomial defined as :

$$\psi(d, \theta) = \frac{\exp(\theta_1 d + \theta_2 d^2)}{\sum_{d=1}^{T} \exp(\theta_1 d + \theta_2 d^2)} \tag{4}$$

where θ_1 and θ_2 are parameters.

We conducted a dynamic covariance analysis with Bitcoin and Gold as dependent variables respectively. Since Gold can be traded only 5 days per week, a conditional quantile regression was performed on the three coefficient terms with window sizes of 7 and 5, respectively, and the coefficient function variation curves over time were obtained as shown in Figure 7.

When the constant function a_0 reaches the minimum value of M, indicating that the Gold market risk reaches the maximum value of exposure. After empirical analysis, this conclusion is indeed reflected in the MIDAS-Conditional Quantile Regression Model, so we believe that there is a cointegration relationship between the two series, and there is a phenomenon of risk competition. The other series can be considered as the independent variable when making return predictions.

3.2.2 ARIMAX-based Return Forecast

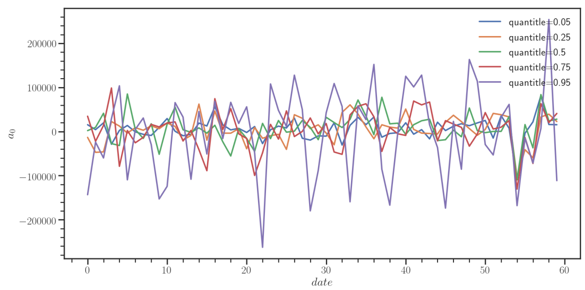
The ARIMAX model is a typical time series forecasting method that considers exogenous variables. If we use the time series ARIMA model for forecasting, we can predict the logarithmic returns for each day. The ARIMAX model is an extension of ARIMA, which contains three components, namely autoregressive(AR), difference(I) and moving average(MA) components. For each of its components, it has its recursive formula defined.

(1) Autoregressive model

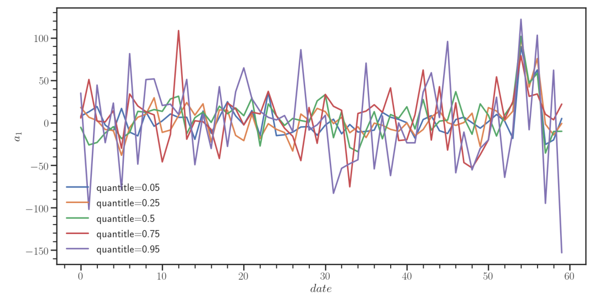
For the p-order autoregressive model, the recursive equtation is shown as:

$$y_t = \mu + \sum_{i=1}^{p} \alpha_i y_{t-i} + \epsilon_t \tag{5}$$

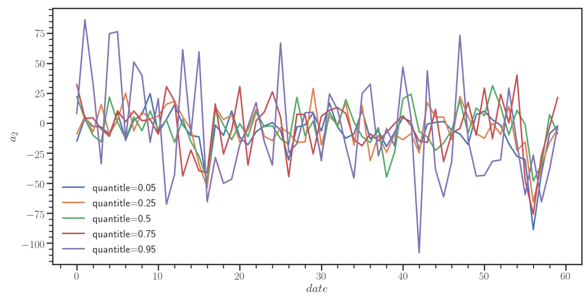
(2) Moving average model



(a) a_0.



(b) a_1.



(c) a_2.

Figure 7: Visualize of Quantile Regression Formula.

For the q-th order moving average model, the recursive formula is shown as :

$$y_t = \epsilon_t + \sum_{j=1}^{q} \beta_j \epsilon_{t-j} + v \tag{6}$$

The moving average model consists of different ordered white noise series. And if we combine AR(p) and MA(q) we get ARMA(p,q) model:

$$y_t = \mu + \sum_{i=1}^{p} \alpha_i y_{t-i} + \sum_{j=1}^{q} \beta_j \epsilon_{t-j} + \epsilon_t \tag{7}$$

(3) Differential model

Sometimes ARMA(p,q) model is not fit for non-stationary series, and in this case we often differ origin in one or two times to make series stable. After d-orders differ of origin series we can apply ARMA(p,q) to new series, and we call the whole process ARIMA(p,q)

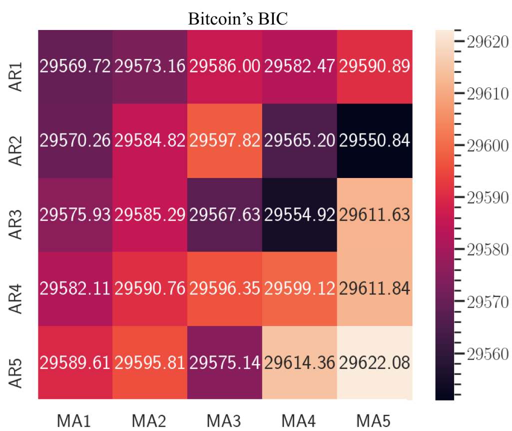
That is, the basic form of the ARIMA model can be determined from the autoregressive model order p, the difference order d and the moving average order q, which we abbreviate as ARIMA(p, d, q)

This model has some unique properties such as autocorrelation, white noise and smoothness.

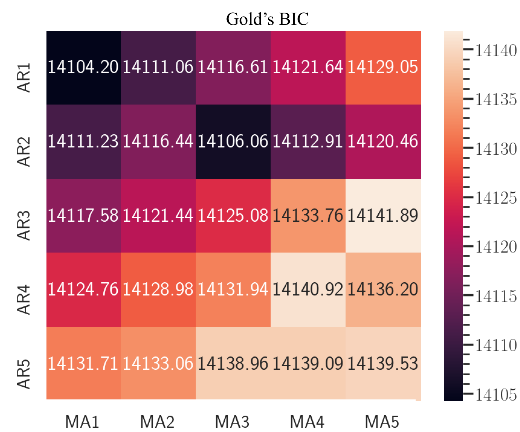
The ARIMAX model, on the other hand, introduces an exogenous variable X based on ARIMA, transforming the model form to

$$r_b(t) = \sum_{i=1}^{p} \alpha_i r_b^{(d)}(t-i) + \sum_{i=1}^{q} \beta_i \epsilon(t-i) + \gamma r_g(t) + \epsilon(t) \tag{8}$$

Using the Bayes Information Criterion(BIC) as the criteria for model parameter selection, we set both the maximum autoregressive term and the maximum moving average term order to 5. After testing, the heat map formed by the BIC values of each model with Bitcoin and Gold as dependent variables, respectively, is obtained, as shown in Figure 8:



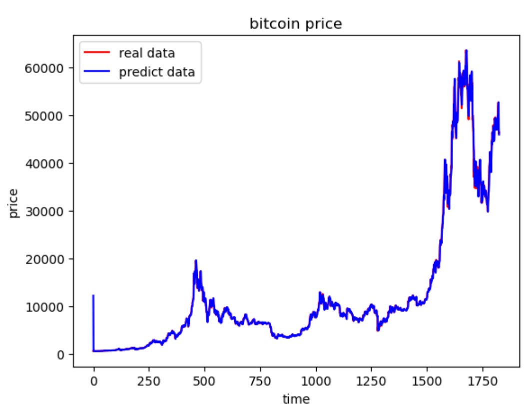
(a) Best arguments for Bitcoin.



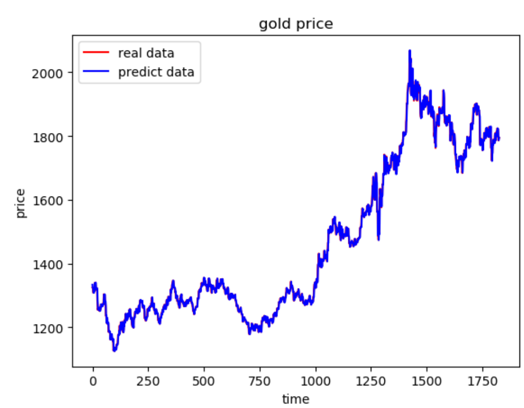
(b) Best arguments for Gold.

Figure 8: Best arguments for model based on BIC.

Since the log returns of both series are smooth series, no re-difference is required, so the difference order d can be set to 0. According to the heat map in Figure 8 the darker the color indicates the smaller BIC value, and considring the error loss of each model, we finally choose the best ARIMAX for modeling respectively. After training, the two models perform quite well in fitting the prediction of log returns, and we plot the reduction curves based on log returns, as shown in Figure 9.



(a) Fitting for Bitcoin.



(b) Fitting for Gold.

Figure 9: Fitting of ARIMAX model.

It can be seen that the model fitting results are basically consistent with the original data, showing a striking fitting effect.

For comparison with ARIMAX, we also tested the ARIMA model with the same parameter configuration, as well as the more popular machine learning models. Using linear regression as the baseline algorithm and recursive feature elimination, we obtained that the next day's logarithmic return of Bitcoin or Gold can be effectively predicted using the first four days of Bitcoin log return data and the first three days of Gold log return data. We also used the most popular deep learning methods to compare our model with ARIMAX. We used MLP, LSTM, Attention and Informer Model for comparation. The Informer model [11] has stronger ability to align long-term sequences and to handle the input and output of long-term sequences. Figure 10 is the main framework of Informer.

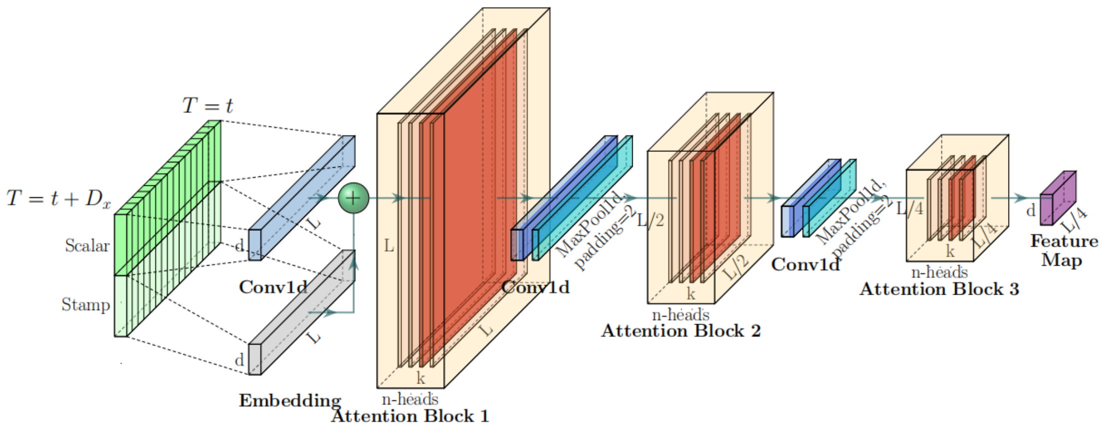
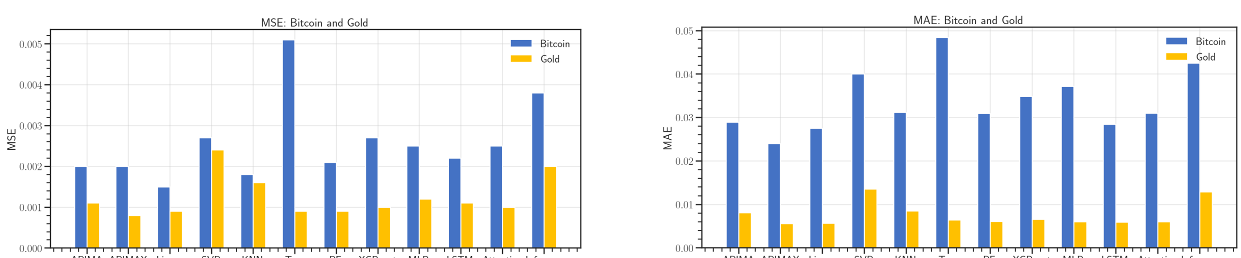


Figure 10: Main Framework of Informer.

Since the values are small when predicting log returns, the calculated MSE and MAE are similarly small. The MSE implies a squared term, which often appears to be a small deviation but actually has more serious consequences, i.e., the MSE is more demanding in terms of error, compared to the MAE, which is a better measure of error. A better way is to use MAE as the error measure. Using MSE and MAE as evaluation metrics, the predictive effects of the algorithm are organized in Table4 and Figure 11.



(a) MSE. (b) MAE.

Figure 11: Performances of Different Models.

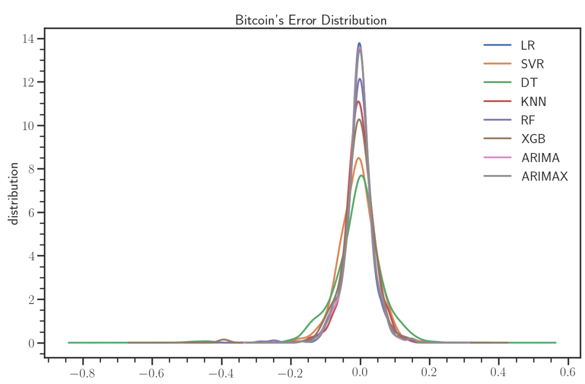
From Table4 and Figure11 we can see that the ARIMAX model has least MAE and MSE in both Bitcoin predicting and Gold predicting. An interesting phenomenon is that, the most popular deep learning approach such as LSTM and Informer are commonly shown worse performance compared

	method	MAE-Bitcoin	MSE-Bitcoin	MAE-Gold	MSE-Gold
traditional	ARIMA	0.0289	0.002	0.0081	0.0011
method	ARIMAX	**0.024**	0.002	**0.0056**	**0.0008**
	Linear	0.0275	**0.0015**	0.0057	0.0009
machine	SVR	0.04	0.0027	0.0135	0.0024
learning	KNN	0.0312	0.0018	0.0085	0.0016
approach	Tree	0.0484	0.0051	0.0064	0.0009
	RF	0.0309	0.0021	0.0061	0.0009
	XGBoost	0.0348	0.0027	0.0066	0.001
	MLP	0.0371	0.0025	0.006	0.0012
deep	LSTM	0.0284	0.0022	0.0059	0.0011
learning	Attention	0.031	0.0025	0.006	0.001
	Informer	0.0425	0.0038	0.0129	0.002

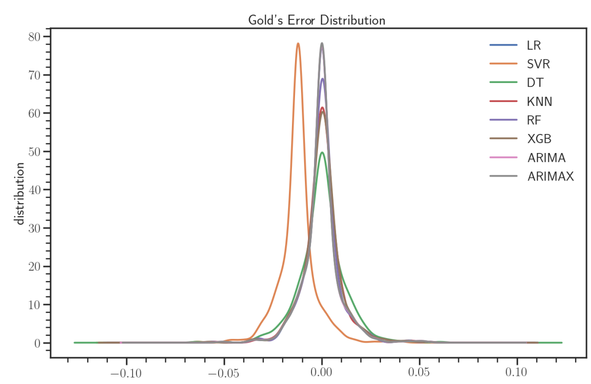
Table 4: Evaluation of Models

with ARIMAX. That implies a principle: simple methods are not necessarily bad, sometimes simpler is better.

We plot the error probability density distributions of several algorithms in Figure 12. The ARIMAX model has the least variance of error since it's distributed more centrally, so we adopt it as our predict model.



(a) Bitcoin.



(b) Gold.

Figure 12: Error Distribution of Models.

3.2.3 XGARCH-based Volatility forecasting

the GARCH model is the typical model used for volatility forecasting. The simplest ARCH(p) model is shown as:

$$\begin{cases} r(t) = \mu(t) + a(t) = \mu(t) + \sigma(t)\epsilon(t) \\ \sigma^2(t) = \alpha_0 + \alpha_1 a^2(t-1) + \alpha_2 a^2(t-2) + \cdots + \alpha_p a^2(t-p) \end{cases} \tag{9}$$

In the above equation $r(t)$ is the rate of return, which we decompose, ϵ is a random variable that obeys the standard normal distribution, μ and σ are its expected level and volatility level. The volatility is then presented in the form of variance, and the ARCH model we construct is based on the regression of returns on volatility. The parameter estimation of this model is similar to that of ARIMA in that both use extreme likelihood estimation, but a Ljung-Box test is required to test whether the ARCH property is satisfied before application. Fortunately, the probability values of the price series after the test indicate that both series can be applied to the ARCH series model.

while the GARCH(p,q) model is an improvement of it.

$$\sigma^2(t) = \alpha_0 + \sum_{i=1}^{p} \alpha_i a^2(t-i) + \sum_{j=1}^{q} \beta_j \sigma^2(t-j) \tag{10}$$

Relative to the basic ARCH model, the GARCH model introduces an autoregressive term of σ^2, which further modifies the model. Usually, we set $p, q = 1$, which is exactly the most popular GARCH(1,1) model applied.

After experiment, we find that several GARCH family models have similar effects on this volatility forecasting problem, so the more conventional GARCH model is chosen for improvement. Considering that the Gold price and Bitcoin price series have cointegration, we introduce exogenous variables to the conventional GARCH model to construct the XGARCH model for the volatility prediction problem of Bitcoin, for example, and the improved model takes the form of:

$$\sigma_b^2(t) = \alpha_0 + \sum_{i=1}^{p} \alpha_i a_b^2(t-i) + \sum_{j=1}^{q} \beta_j \sigma_b^2(t-j) + \gamma \sigma_g^2(t) \tag{11}$$

Although only one exogenous variable is introduced, the effect is still an improvement relative to the traditional GARCH model.

We predicted the volatility of Bitcoin and Gold based on our proposed XGARCH model, as shown in Figure 13:

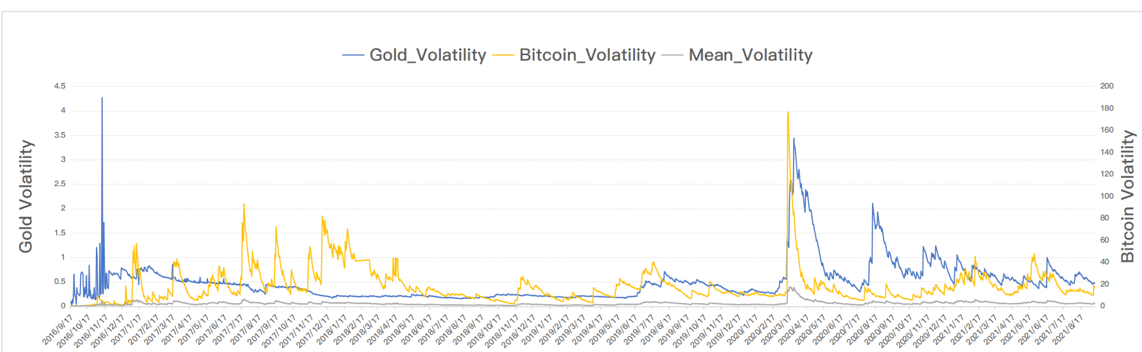


Figure 13: Volatility Prediction via XGARCH.

3.3 Markowitz Portfolio models and its result

Define w_1, w_2 as the ratio of the amount of Bitcoin and Gold purchased to the amount of cash, respectively, when w<0 means that a sale operation is performed at this point, and C, B, G as the amount of cash, Bitcoin, and Gold convertedto USD reserves, respectively. To simplify the problem, let C=1, when B and G both represent multiples of C.

Markowitz Mean-Variance Theory [5] is widely used to solve the problem of optimal portfolio selection. This theory mainly determines the optimal investment portfolio by studying the expected return and the variance and covariance of each asset. It is also the first time that mathematical statistics methods are introduced into portfolio theory.

Based on Markowitz Mean-Variance Theory, we can establish model follow:

We define risk function and return function as $D(w,r)$ and $E(w,r)$. The risk is defined as:

$$D(w_1 r_b + w_2 r_g) = w^T \Sigma w = w^T \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix} w \tag{12}$$

there σ_1, σ_2 are volatility and σ_{12} is covariance, and we can calculate σ_{12} by $\sigma_{12} = \rho_{12}\sigma_1\sigma_2$. There $\rho_{12} = 0.645$.

For the income function, if it is considered that all Bitcoin and Gold will be cashed out on the next day, then the discounted discount minus the purchase price and transaction amount on that day is the expected income:

$$E(w_1 r_b + w_2 r_g) = (1-\alpha_1)(e^{r_b(t)} - 1)w_1 + (1-\alpha_2)(e^{r_g(t)} - 1)w_2 - (\alpha_1 w_1 + \alpha_2 w_2) \tag{13}$$

The model can be written as:

$$\begin{aligned} &\min_w D(w,r) \\ &\max_w E(w,r) \\ &s.t. \begin{cases} w_1 + w_2 \le 1 \\ -B \le w_1 \le 1 \\ -G \le w_2 \le 1 \end{cases} \end{aligned} \tag{14}$$

This is a multi-target optimization problem. To simplify this problem, we adopt τ multiplier:

$$\begin{aligned} &\min_w \tau D(w,r) - E(w,r) \\ &s.t. \begin{cases} w_1 + w_2 \le 1 \\ -B \le w_1 \le 1 \\ -G \le w_2 \le 1 \end{cases} \end{aligned} \tag{15}$$

Solving this problem can get the corresponding strategy w. After testing, it is more appropriate for us to select τ as 0.6.

4 Model Establishing and Solving of Problem II

4.1 Compared with Other Strategies

4.1.1 Maximum Sharpe Ratio Model

William Sharpe, winner of the 1990 Nobel Prize in Economics, believed that when investors build a risky investment portfolio, they should at least require the return on investment to reach the return of a risk-free investment, or more, and based on this, he proposed the Sharpe ratio [9]. The Sharpe ratio is one of the three classic indicators that can simultaneously consider both return and risk.

The purpose of the Sharpe ratio is to calculate how much excess return a portfolio will generate per unit of total risk. If the Sharpe ratio is positive, it means that the fund's return rate is higher than the volatility risk; if it is negative, it means that the fund's operational risk is greater than the return rate. The higher the Sharpe ratio, the better the portfolio.

The Sharpe ratio is a measure of both risk and return, and it is defined as:

$$Sharp(w,r) = \frac{E(w,r) - r_f}{\sqrt{D(w,r)}} \tag{16}$$

In its definition, the $E(w,r)$ is expected return and $D(w,r)$ is volatility. The r_f is the risk-free interest rate. According to the historical data of the US market, we usually take 0.04 as an estimation of r_f.

According to its definition, we can clearly see that if $D(w,r)$ is the least and $E(w,r)$ is largest, the Sharpe ratio will be maximum. So our optimization target could be maximizing Sharpe ratio. Actually, Maximum Sharpe Ratio Model has been widely applied in investing and financial area.

We establish this model in optimization as:

$$\begin{aligned} &\min_{w} -Sharp(w,r) \\ &s.t. \begin{cases} w_1 + w_2 \le 1 \\ -B \le w_1 \le 1 \\ -G \le w_2 \le 1 \end{cases} \end{aligned} \tag{17}$$

Since the Sharpe ratio is an irrational function, its optimal solution can be obtained numerically instead of to solve its symbolic solution.

4.1.2 Risk Parity Model

Risk parity [7] was proposed in 2005 by Dr. Edward Qian, Chief Investment Officer of PanAgora. Risk parity is an asset allocation philosophy that assigns equal risk weights to different assets in a portfolio. The essence of risk parity is actually to assume that the Sharpe Ratio of various assets tends to be consistent in the long run to find the maximization of the long-term Sharpe Ratio of the portfolio.

For Bitcoin and Gold, the risk contribution rate can be calculated as:

$$\begin{cases} P_1 = 1 - \frac{w_2^2 \sigma_2^2}{D(w,r)} \\ P_2 = 1 - \frac{w_1^2 \sigma_1^2}{D(w,r)} \end{cases} \tag{18}$$

To make the two sequence risks as consistent as possible, we construct a planning model like this:

$$\begin{aligned} &\min_{w} |P_1 - P_2| \\ &s.t. \begin{cases} w_1 + w_2 \le 1 \\ -B \le w_1 \le 1 \\ -G \le w_2 \le 1 \end{cases} \end{aligned} \tag{19}$$

Optimization target is to find the best w to make $|P_1 - P_2|$ less as possible.

4.1.3 Reinforcement Learning Model

The optimal dynamic adjustment of liquid asset allocation (whether or not to hold a single asset or the allocation ratio of multiple assets) can actually be regarded as a MDP (Markovian Decision Problem) problem, so reinforcement learning can be considered for this application.

Reinforcement learning can be divided into two mainstream methods, Policy method and Value method. The more popular and effective approach is to mix the two and add a modern approach to Actor Criticism. According to research by Halperin [1], etc., the application of the Value method in finance is problematic, because it is difficult to set up a very smart model in this scenario to define a reward that returns an improper return will be a very noisy signal.

Scholars such as Saffell [6], Siyu Lin [4], Oxford University [2] have conducted in-depth research on the application of reinforcement learning methods in financial markets, and proposed the application of various models in financial investment. And most of the methods can obtain better returns for a single asset without the limit of principal.

We also tried the reinforcement learning method for policy decision-making. Unlike other researchers, we tried to use the reinforcement learning model to propose an effective profit strategy for portfolio assets under the condition of capital constraints.

We used the most common baseline currently available: Proximal Policy Optimization (PPO). PPO model is an Actor-Critic architecture that performs better in complex environments. Using the data, we build a gym simulation environment. The action dimension is two consecutive variables, one is the change of Gold and the other is the change of Bitcoin. The observation dimension is nine consecutive quantities, mainly holding cash, Gold, the number of Bitcoins and market data for Gold and Bitcoins. Reward is the value of the holding asset at a specified date.

Algorithm 1 PPO, Actor-Critic Style

for i=1, 2, ..., N do **do**
  Run policy π_θ for T timesteps, collecting $\{s_t, a_t, r_t\}$
  Estimate asvantages $\widehat{A}_t = \sum_{t'>t} \gamma^{t'-t} r_{t'} - V_\phi(s_t)$
  $\theta_{old} \leftarrow \theta$
  for j=1, 2, ..., M do **do**
    $J_{PPO}(\theta) = \sum_{t=1}^{T} \frac{\pi_\theta(a_t|s_t)}{\pi(a_t|s_t)} \widehat{A}_t - \lambda KL[\pi_{old}|\pi_\theta]$
    Update θ by a gradient method w.r.t $J_{PPO}(\theta)$
  end for
  for j = 1, 2, ..., B do **do**
    $L_{BL}(\phi) = -\sum_{t=1}^{T} (\sum_{t'>t} \gamma^{t'-t} r_{t'} - V_\phi(s_t))^2$
    Update ϕ by a gradient method w.r.t $L_{BL}(\phi)$
  end for
  if $kl[\pi_{old}|\pi_\theta] > \beta_{high} KL_{target}$ then **then**
    $\lambda \leftarrow \alpha\lambda$
  else if $kl[\pi_{old}|\pi_\theta] < \beta_{high} KL_{target}$ then **then**
    $\lambda \leftarrow \alpha/\lambda$
  end if
end for

4.1.4 Experiment and Conclusion

Suppose we carry out a cash-out operation every day to convert B and G into cash before investing, and use the L-BFGS method to backward-test the above four models based on historical data and forecast data. The multiple curve of the asset relative to the initial time point can be plotted as shown in Figure 14.

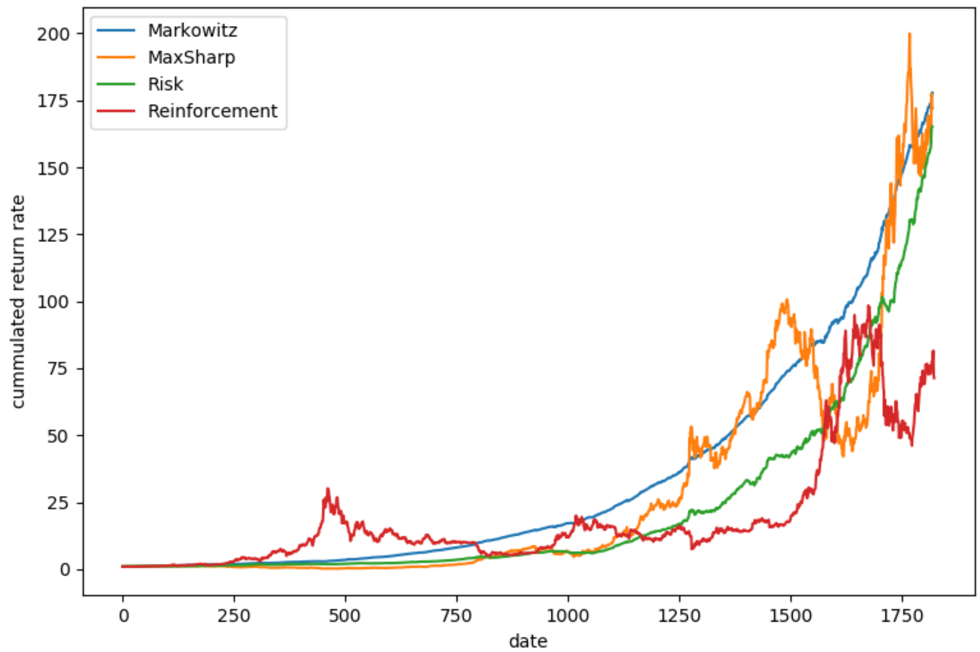


Figure 14: Comparison of Strategies.

We can clearly see that, it's stable and high-returned by adopting Markowitz Model. To better evaluate the performance of these models, we list some metrics in Table 5:

model	average return	average volatility	win rate	odds ratio	profits-loss ratio
Markowitz	34.104	19590.974	0.833	5.003	6.357
MaxSharpe	27.931	27837.734	0.608	1.555	1.281
RiskParity	21.997	18438.619	0.649	1.860	3.577
Reinforcement	18.871	7486.319	0.542	1.186	1.145

Table 5: Comparison of Strategies

Blindly pursuing the maximum Sharpe ratio often makes risks and returns not well considered separately, and it is easy to cause high-risk investments to make the average loss equally high. In fact, from the figure, we can find that the maximum Sharpe model has the highest rise and fall and profit-loss ratio, indicating that this model is quite unstable. If a special event occurs in the market, investors will likely bear huge risk of loss. An interesting phenomenon is that, Risk Parity Model doesn't consider return, but just considering risk actually makes investing return greater. A small disadvantage is that its initial growth rate is too slow. Such experimental results show that it is not suitable for short-term investment, but it shows high-yield characteristics in long-term investment.

For the popular reinforcement learning approach, its performance is almost the worse in these strategies. On one hand, it has the least average return, on the other hand, it's annual volatility is also high . This discovery also just confirms our idea: the abuse of popular machine learning or other "black box" models is not a panacea, and in many cases it may not achieve optimal results. We also find that the PPO model is more likely to purchase Bitcoins and hold them for a long time after training, probably because it is a biased strategy for pursuing higher and stable rewards. We believe that if you use a multi-agent approach to manipulate Gold and Bitcoins separately, a semi-cooperative, semi-competitive (hybrid) relationship between agents can boost the number of transactions to achieve higher returns.

4.2 Comparison of Long-term and Short-term Returns

At this time, we consider that if we do not perform daily cash-out operations, but instead cash out and liquidate all assets after a certain period, the cycle rate of return for each cash out is different from the daily rate of return, and the number of cash outs will decrease, but the rate of return may be Increase. Based on this, we tested the new model with one week and one month as the cash-out period, respectively. The back-tested yield curve of the new model is shown in Figure 12.

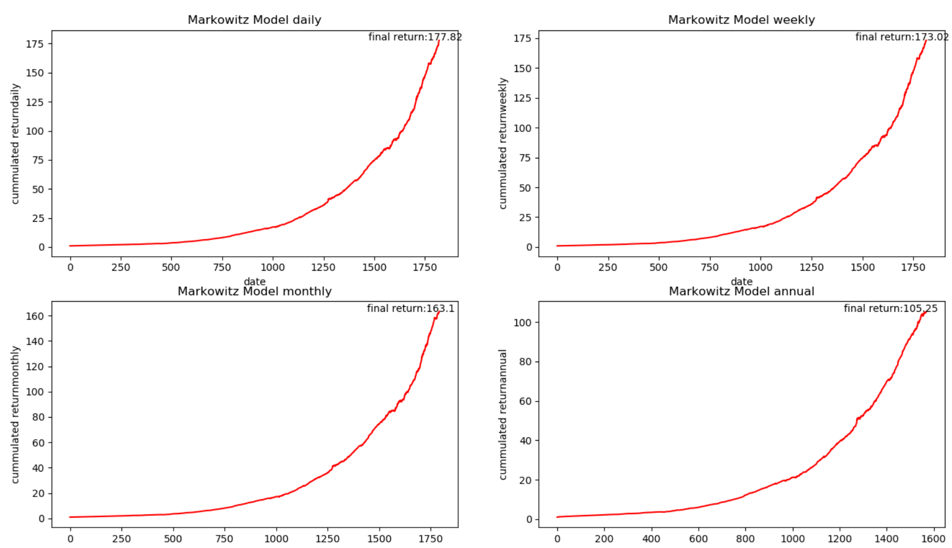


Figure 15: Comparison of different terms for investing.

We adjusted investment term and find that picking stocks according to the strategy with the highest daily return rather than a longer period can achieve better returns, which also proofs accuracy of our model.

5 Model Establishing and Solving of Problem III

For different values of α_1, α_2, taken from 0.01 to 0.03, respectively, the resulting Markowitz model curves are shown in Figure 16.

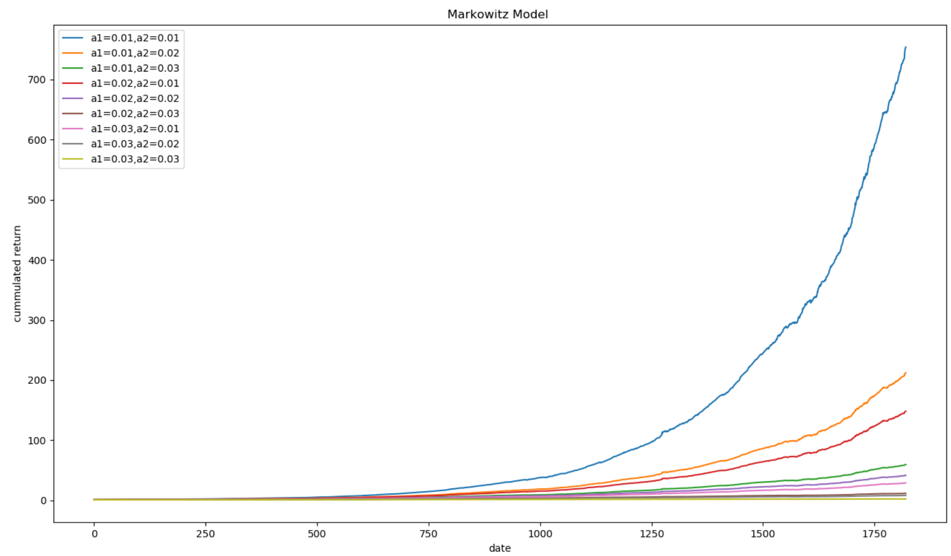


Figure 16: Comparison of different α for investing.

We see that when $\alpha_1 = \alpha_2 = 0.01$ the final return could reach nearly 700 times of origin after 5 years. It's interesting that (0.02,0.01) is higher than (0.01,0.03). This is might because the Bitcoin's price is much more higher than Gold's which leads to Bitcoin's transaction fees are higher than Gold's and less people are willing to buy Bitcoin and don't need to take higher risk at the same time.

Since the curve in the figure16 is more discriminative, we believe that the model is sensitive to transaction fees.

6 Sensitivity Analysis

In Problem III we discussed about the sensitivity to α of our model, and we can also discuss the sensitivity to τ. In our model, our optimization target is $\tau D(w,r) - E(w,r)$, we make τ in [0.1,0.5,1,5,10,20] to explore its sensitivity. τ is used to describe how we combine risk and return and their weight , theoretically it will infect the strategy to more focus on risk or return. But we amazingly find that the model is very weakly sensitive to the τ multiplier, and the model starts to show some differences when the τ is small. When the τ is larger, the model results are not significantly different.

7 Strengths and Weaknesses

We discussed about our model and think our model has strengths and weaknesses below:

7.1 Strengths

- More convincing. Our model is based on the 5 years' price of Bitcoin and Gold via time series analysis, this model considers co-integration and correct origin model to make it performs better. We compared our model to many other approaches and get better performance. So our model is more convincing.
- More simplified. Compared with the more popular but complex machine learning and even deep learning models, we have innovated on the basis of traditional time series analysis, which is simpler in form, but achieves better results than machine learning. This makes our model easier to transfer to other econometric and time series problems.
- More universal. This method takes into account exogenous variables and co-integration effects, and can actually be used as a general model for multi-series forecasting. It is not only applied in the field of Bitcoin and Gold, but can also be transferred to the portfolio investment of stocks, and even non-economics. It has a certain generalization and migration.
- More robust. In Sensitivity Analysis we discussed how the change of arguments infect the return, and we find only α will affect this. This means this model has great robustness in some sense.

7.2 Weaknesses

- We did not consider whether Gold and Bitcoin purchases can be purchased in arbitrary quantities. That is to say, if there is a minimum unit for the purchase amount of Gold and Bitcoin, then our optimization solution process will be a discrete optimization process, which is more complicated.
- The results predicted by the prediction model can only represent one prediction level, and there is still a certain deviation from the actual level. It cannot be guaranteed to be in a profitable state, but it is guaranteed to make as many times as possible.

8 Conclusion

For Problem 1 *We established three models, namely: a dynamic cointegration analysis model based on MIDAS-conditional quantile regression to analyze the cointegration of Bitcoin and Gold sequences; a sequence prediction model based on ARIMAX-XGARCH to predict The return and volatility of the two; the return optimization model based on Markowitz's theory selects the optimal strategy. In this question, we compared a variety of sequence prediction algorithms and found that the innovative optimization method in traditional time series theory has better performance than methods such as machine learning, which confirms our principle: "simple method does not necessarily perform poorly". The idea achieves the precise planning of the strategy. The cumulative return over the final five-year period can reach 167 times the original assets.*

For Problem 2 *We compared our model with the maximum Sharpe, risk parity and reinforcement learning models, and believed that our model was the best model in the comprehensive consideration of return and risk. At the same time, we also compared the impact of different revenue cycles on the strategy, and the experimental results also confirmed that our strategy is indeed the optimal strategy.*

For Problem 3 *The sensitivity of different transaction fees to the model was explored. Combined with the subsequent sensitivity analysis, it was found that the proportion of the fee amount does affect the model results.*

For Problem 4 *The non-technical Memo for Problem 4 is above.*

Overall, we have built a simple but effective forecasting-planning model that enables precise quantification of investment strategies. The model has several advantages, it is easy to generalize and transfer, and it also confirms an idea: "A method with a simple form is not necessarily a poor performer, sometimes simpler is better".

Memo

TO: Traders
From: Team 2207181
SUBJECT: Methods to maximize the profit
DATE: February, 21, 2022

As two high-return investment options, portfolio investment strategies for Bitcoin and Gold have been a topic of considerable interest to investors. To help investors better plan their investment strategies, we have created a portfolio model to help investors better plan their investment strategies on a daily basis.

Our work Our work consists of the following components: To get the optimal strategy, we decompose the problem into three steps: first, by establishing a MIDAS-Conditional Quantile Regression Model, we verified that the price of gold and the price of bitcoin are co-integrated, the market risks of the two affect each other; second, we achieved accurate prediction of benefits and risks by building the ARIMAX-XGARCH model, and achieved better results than complex machine learning and deep learning models; finally, the optimization model established based on Markowitz Mean-Variance theory solves the daily optimal investment strategy. According to our strategy, your total assets will reach $167,300 by 2021-9-11. To verify the optimal strategy, we compare various models and our model is optimal in terms of both return and risk; we also compare the impact of different investment cycles on the model to further verify the accuracy of the model. To investigate the impact of fees on returns, we try to solve model again with different transaction fees.

Conclusion After our modeling process, we found the following pattern

1. The Gold and Bitcoin markets are covariant and unified, with risk fluctuations in one market affecting the other.
2. Targeting the expected return on daily cash-outs will achieve higher returns than strategies with longer expected return cycles.
3. The lower the fees, the higher the ultimate return on investment.
4. Not the more complex the model the better the results. Sometimes simpler is better.

Recommendation Based on the above conclusions, we would like to make the following recommendations.

1. The amount of money invested should be determined based on the predicted price of volatility and returns of Gold and Bitcoin. Not the more you invest in Bitcoin the better. However, investing according to their combined risk and return profile will yield better returns.
2. Because of the covariance between the Gold and Bitcoin, you should move more assets to the Bitcoin when the Gold market starts to experience high volatility; the same goes for the Bitcoin market when it experiences high volatility.
3. When the fees are similar, the option with lower Gold fees is preferred. This is because moving more assets to Gold allows you to take on less risk and more reward.

We will be very honored if our model is adopted.
Yours Sincerely:
Team 2207181

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Appendices

Appendix1: Core Codes for ARIMX

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import statsmodels.api as sm
import pyflux as pf
import numpy as np
from sklearn.metrics importmean_absolute_error,mean_squared_error,r2_score

data=pd.read_csv("data.csv")
datadf=data[['Bitcoin','Gold']]
datadf=np.log(datadf).diff().dropna()
trainnum = np.int(datadf.shape[0]*0.7)
traidata = datadf.iloc[0:trainnum,:]
testdata = datadf.iloc[trainnum:datadf.shape[0],:]
model = pf.ARIMAX(data=traidata,formula="Gold~Bitcoin",ar=3,ma=2,integ=0)
model_1 = model.fit("MLE")

C02pre = model.predict(h=testdata.shape[0], oos_data=testdata)
datadf.Bitcoin.plot(figsize=(15,5),c="b",label="Bitcoin")
C02pre.Bitcoin.plot(c = "r",label="Prediction")
plt.xlabel("Time")
plt.ylabel("Bitcoin")
plt.show()
```

Appendix2: Core Codes for PPO's learning

```
def learn(self, obs_batch, actions_batch, value_preds_batch, return_batch,
    old_action_log_probs_batch, adv_targ):
        values = self.model.value(obs_batch)
        mean, log_std = self.model.policy(obs_batch)
        dist = Normal(mean, log_std.exp())
        action_log_probs = dist.log_prob(actions_batch)
        dist_entropy = dist.entropy().sum(axis=-1).mean()
        ratio = paddle.exp(action_log_probs - old_action_log_probs_batch)
        surr1 = ratio * adv_targ
        surr2 = paddle.clip(ratio, 1.0 - self.clip_param,
                            1.0 + self.clip_param) * adv_targ
        action_loss = -paddle.minimum(surr1, surr2).mean()
        value_loss = 0.5 * (return_batch - values).pow(2).mean()
        (value_loss * self.value_loss_coef + action_loss - dist_entropy *
            self.entropy_coef).backward()
        self.optimizer.step()
        self.optimizer.clear_grad()
        return value_loss.numpy(), action_loss.numpy(), dist_entropy.numpy()
```