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Capacitated Facility Location with Distance-Dependent Coverage under Demand Uncertainty

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Abstract

We consider a facility location problem to determine the points of dispensing medicine or supplies in a large-scale emergency. For this problem we consider capacitated facilities, a distance-dependent coverage function, and demand uncertainty. We formulate a special case of the maximal covering location problem (MCLP) with a loss function, to account for the distance-sensitive demand, and chance-constraints to address the demand uncertainty. This model decides both the locations to open and the supplies assigned to each location. We solve this problem with a locate-allocate heuristic. We illustrate the use of the model by solving a case study of locating facilities to address a large-scale emergency (an anthrax attack) in Los Angeles County.

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1 Introduction

Large-scale emergency events such as a bio-terrorist attack, or a natural calamity such as an earthquake, can strike with little or no warning. Such situations lead to a big surge in the demand for medical supplies, without which an emergency response cannot be carried out effectively. For the purpose of risk mitigation and a reduction in response time, it is important to decide locations from which medical supplies can be rapidly distributed. Decisions such as determining which of these facilities across the affected region need to be *opened* play an important role in reducing casualties. An important consideration in selecting the location of these facilities is the coverage of the demand areas. Covering models have been proven to be very useful in solving facility location problems in emergency-related scenarios [Gendreau et al. (2006) and Jia et al. (2006)]. A demand point is treated as covered only if a facility can be available to provide the certain service to the demand point within a required distance or time from a facility.

In a large-scale emergency due to an Anthrax attack in a large urban area, federal medical supplies from the Strategic National Supplies (SNS) would be delivered within 24 hours to the affected area, and it is the responsibility of the local authority to develop efficient disbursement plans. An effective disbursement plan would involve setting up points of dispersion (POD) to distribute the supplies and have the population go to the PODs. Under such circumstances, the decisions that need to be made include the locations of the facilities (or PODs) to be opened and the amount of supplies to be stored at each open facility. As can be expected, these decisions are impacted by the large degree of uncertainty associated with the location of the emergency and the number of people affected. In addition, because of unpredictable human behavior in an emergency situation, it is uncertain whether a population would go to their closest POD to obtain their medicines, if at all. The further a person is from the closest facility, the more likely he/she would be to deviate or decide that they do not want the medicine. We, therefore, assume the demand experienced by a facility from a certain demand point to be dependent on the distance between them. Since the objective is to maximize coverage, we assign the amount of supplies required to meet this objective to the facilities that are opened. Since under-stocking could lead to unnecessary casualties,

there is an incentive to increase supply. However, the speed at which people are serviced at a POD and the physical dimensions of these locations impose a restriction on the maximum rate of service. For this reason we assume facilities to be capacitated.

In this paper, we develop an optimization model to solve a capacitated facility location problem to decide which of the pre-positioned facilities need to be opened in the event of a large-scale emergency. We use chance-constraints to model the uncertainty in the demand. We use a *loss function* to denote that the demand “felt” by an open facility from a demand point is a function of the distance between them. This loss function is used to set an upper-bound on the coverage that a facility can provide to a demand point.

The rest of the paper is organized as follows. In Section 2, we provide a brief review of prior work on covering problems and stochastic facility location. In Section 3, we describe in-depth the capacitated facility location optimization model with chance-constraints that we used, and in Section 4 we present our solution algorithm to solve the optimization problem. We present results from simulation experiments which verify the performance of our model and solution algorithm in Section 5. Finally, in Section 6, we list our conclusions and future work.

2 Literature Review

Given that our objective is to design an effective response strategy to a large-scale emergency to reduce casualties, a maximal covering location problem (MCLP) [Church and ReVelle (1974)] would best suit our purpose. In the event of a large-scale emergency, a very important decision would be to locate facilities in tune with the location and intensity (that is, number of affected people) of the attack. Thus, it seems natural to use MCLP or something similar to help decide the facilities to be opened. One of the key assumptions of the MCLP is that a demand point is assumed to be fully covered if located within a distance r from the facility and not covered if it is farther than r away from the facility. However, in case of an emergency scenario, due to the gravity of the situation and the possible damage to the transportation network, it would be next to impossible to precisely predict the facility to

which the affected people might travel to get the medications, making it unrealistic to enforce the binary coverage assumptions of the MCLP. Instead, it would be closer to reality to think of the coverage offered by each facility in terms of coverage levels. In the generalized MCLP (GMCLP) as defined by Berman and Krass (2002), each demand point i has multiple sets of coverage levels, with corresponding coverage radii, that is, if a facility is located within a distance r_i from i , then the coverage level is $a_i(r_i)$. The coverage levels can be thought of as a decreasing step function of the distance between a demand point and an open facility. This work was extended by Berman et al. (2003) to the gradual covering decay model where they considered general forms of the coverage decay function. Berman et al. (2009) considered the variable radius covering problem, where the decision-maker needs to determine coverage radii for the facilities, in addition to the numbers and locations of facilities, to cover a discrete number of demand points with minimal cost. In our work, we adopt the idea of multiple coverage levels proposed by Berman and Krass (2002), albeit with slight modifications. This is explained in detail in the following section.

There exists a fair amount of literature on facility location models dealing with response to an emergency. One of the earliest models in this area was developed by Toregas et al. (1971) where they developed the location problem as a set covering problem with equal costs in the objective. The sets were composed of the potential facility points within a certain distance or time from each demand point. They solved this problem using linear programming techniques. Rawls and Turnquist (2006) presented a two-stage stochastic optimization model to locate facilities and assign supply to them under an emergency scenario. They develop a mixed-integer program to address uncertainty in the demands and in the capacity of the transportation network. Berman and Gavious (2007) presented competitive location models to locate facilities that contain resources required for response to a terrorist attack. They consider the worst-case scenario where the terrorist has knowledge of the location of the facilities and the State needs to take this into account. Jia et al. (2006) presented an uncapacitated version of the covering model to locate staging areas in the event of a large-scale emergency. The location of the facilities and the allocation of the demand points to the open facilities are primarily based on distance constraints. In our paper, we extend this model to a capacitated facility location model. Given the uncertainty associated with a large-scale

emergency, it is important to accurately determine the quantity of supplies that need to be stocked at each potential facility site. To address this issue, we consider the available supply at each facility to be a variable.

Facility location models aid decisions that are expensive and difficult to change. Hence, to aid decisions that are robust and valid for a long period of time, facility location models should consider the uncertainty associated with the demand, supply and distance parameters over a time horizon. In the past, researchers have utilized stochastic and robust optimization approaches to model uncertainty in facility location problems. Snyder (2006) gives an in-depth review of the work done using these two approaches. Mean-outcome models minimize the expected travel cost or maximize the expected profit. The mean-outcome models introduced by Cooper (1974), Sheppard (1974) and Mirchandani and Oudjit (1980) minimized expected costs or distance. Balachandran and Jain (1976) presented a capacitated facility location model with an objective to minimize expected cost of location, production and transportation. Berman and Odoni (1982) considered travel-times to be scenario-based, with transitions between states or scenarios occurring according to a discrete-time Markov process. The objective was to minimize travel times and facility relocation costs. Weaver and Church (1983), Mirchandani et al. (1985), Louveaux (1986) and Louveaux and Peeters (1992) presented stochastic versions of the P-median problem to choose facilities and allocate demand points. Berman and Drezner (2008) presented a P-median problem that handles uncertainty by minimizing the expected cost of serving all demand nodes in the future. Mean-variance models address the variability in performance and the decision-maker's aversion towards risk. Such models include Jucker and Carlson (1976) and Hodder and Dincer (1986). Yet another method to model uncertainty is chance-constrained programming. In this procedure, the parameters that are unknown at the time of planning are assumed to follow certain probability distributions. A chance-constraint requires the probability of a certain constraint, involving the uncertain parameter(s), holding to be sufficiently high. Carbone (1974) formulated a P-median model to minimize the distance traveled by a number of users to fixed public facilities. The uncertainty in the number of users at each demand node is handled using chance-constraints. The model seeks to minimize a threshold and ensure that the total travel distance is within the threshold with a probability α . Interested readers

are referred to Snyder (2006) and Snyder and Daskin (2004) where the authors provide an in-depth literature review of papers dealing with robust facility location.

While there exists a large amount of literature in the area of capacitated facility location, there have not been many papers on maximal covering models that use chance-constraints to deal with demand uncertainty. In an application like the one presented in this paper, where the number of people affected by a large-scale emergency and its location are unknown well in advance, facility location modeling under uncertainty is vital. Our model assigns the supply to be stored at each facility by considering it as a decision variable that depends on an unknown demand. Previous papers that have supply as a variable [Louveaux (1986) and Rawls and Turnquist (2006)] did not consider the relation between supply and a random unknown demand. Since the supply at each facility depends on a demand unknown to us apriori, we model this as a capacitated facility location problem and use chance-constraints to handle the demand uncertainty.

3 Facility Location Model for Large-Scale Emergencies

In this section, we provide a detailed explanation of the capacitated covering model with chance-constraints to handle demand uncertainty. As explained earlier, our objective is to maximize the percentage of the affected population that successfully receives the medications. That is, our goal is to maximize coverage or minimize unmet demand.

3.1 Demand Loss Function

In the work presented in this paper, we adopt the idea of multiple coverage levels introduced by Berman and Krass (2002), albeit with slight modifications. We assume that the demand from a particular demand point i that can be serviced at a facility j is a fraction f_k of the actual demand D_i that decreases as a function of the distance between i and j . The value of f_k depends on the coverage level of j in which i is located. This assumption is made to reflect the inability of people to travel large distances (due to possible damage to infrastructure such

as roads, health conditions etc.) in an emergency scenario. For this purpose, we use a loss function that sets an upper bound on the coverage that can be offered by each open facility to all the demand points. This is different from Berman and Krass (2002) where a fixed coverage level is assumed. A second difference is the feature of multi-level coverage of each demand point. That is, a demand point can be covered by multiple facilities at various coverage levels. This insures that the number of people covered at each demand point is maximized. For instance, in Figure 1 demand point 1 is located under coverage level 1 of facility $F2$, and demand point 2 is located under coverage level 3 of $F1$ and level 3 of $F2$. This implies that the upper bound on the coverage of demand point 2 by $F1$ is $(f_3 D_2)$ and by $F2$ is $(f_3 D_2)$, where f_3 is the coverage fraction set for level 3 coverage.

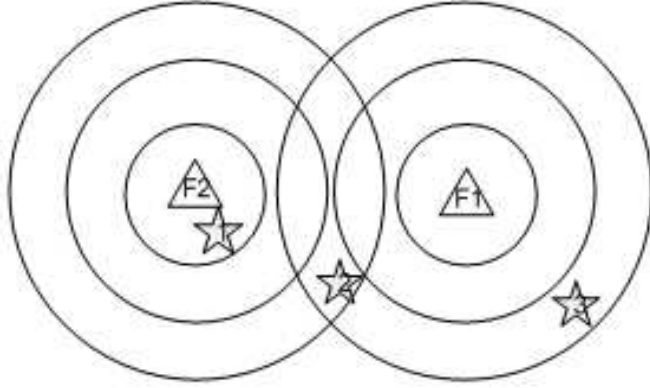


Figure 1: Demand loss function

3.2 Notation

In the model presented below, we assume that the number and the potential locations of the facilities or PODs containing the medical supplies are pre-specified. We only intend to determine which of these PODs need to be opened when a large-scale emergency occurs. We also make an assumption that the distances between every pair of facility sites and demand points is the Euclidean distance between them. Furthermore, we assume that a single demand point represents a population unique to that demand point.

We consider a set I of demand points and a set J of facility sites or PODs. We also consider

a set K levels of coverage for every open facility. We define the following parameters and decision variables.

Parameters

- D_i : demand for medical supplies from demand point i .
- N : total number of facilities that need to be opened.
- S : total supply available during an emergency.
- β : capacity of facility.
- f_k : a fraction. $f_k D_i$ defines the upper bound on the coverage that can be provided to a demand point i that lies in the k^{th} coverage level of facility j .

Decision Variables

- x_j : takes a value of 1 if facility j is open and 0 otherwise
- s_j : supply to be assigned to facility j
- t_{ij} : amount of medical supplies allocated to demand point i by facility j

3.3 Deterministic Model

Below, we first present a deterministic model for the above described problem. In this case, we assume that the demand arising from each demand point is known well in advance, and medical supplies just sufficient to meet this demand need to be stored in the facilities. In other words, a deterministic version of the problem denotes a scenario where the location and the intensity (size of the affected population) is known ahead of time.

The deterministic coverage model is given below:

$$\begin{aligned}
 \text{DM :} \quad & \text{maximize} \quad \sum_{i \in I, j \in J} t_{ij} \\
 & \sum_{j \in J} x_j = N
 \end{aligned} \tag{1}$$

$$\sum_{i \in I} t_{ij} \leq s_j \quad \forall j \in J \quad (2)$$

$$\sum_{j \in J} s_j \leq S \quad (3)$$

$$s_j \leq \beta x_j \quad \forall j \in J \quad (4)$$

$$\sum_{j | \delta_{k-1} < d_{ij} \leq \delta_k} t_{ij} \leq f_k D_i \quad \forall i \in I, k \in 1, \dots, K \quad (5)$$

$$x_i \in \{0, 1\}; s_j, t_{ij} \geq 0 \quad (6)$$

The objective of the above integer programming model is to maximize coverage. That is, minimize the number of people who are denied medications due to the shortage of supplies. The first constraint ensures that exactly N facilities are opened. Constraint (2) ensures that for each open facility j , the total supplies transported to all demand points cannot exceed the available supply at j . Constraint (3) enforces the condition that the total supply available at all the open facilities is bounded above by S . For every open facility j , constraint (4) sets the condition that the supply available at j does not exceed the facility's total capacity. Constraint (5) constitutes the loss function that defines how the effect of demand from a demand point i on an open facility j decays with an increase in distance between i and j . As per constraint (5), if a demand point i is located within the first coverage level of an open facility j , which means $k = 1$ and $\delta_0 < d_{ij} \leq \delta_1$, then $\delta_0 = 0$ and $f_1 = 1$. That is, all of D_i could potentially be covered by j , although this need not happen since i may be covered at various coverage levels by other open facilities as long as i lies within a distance δ_K from them. Constraint (5) also enforces the condition that for a demand point i that lies in the k^{th} coverage level of a facility j , where $k \neq 1$, at most a fraction f_k of the demand D_i can be covered. In addition for $k = K + 1$, wherein $\delta_K < d_{ij} \leq +\infty$, constraint (5) sets to zero the amount of supplies transported from a facility j to a demand point i that lies more than a distance δ_K away from j .

3.4 Chance-constrained Model

As mentioned previously, the demand for medical supplies from each demand point is not known apriori. Thus, we would need to plan for a range of demand values that are generated after the emergency strikes. This means that constraint (5) of **(DM)** has some degree of uncertainty around them. A simple way to handle this uncertainty would be to replace the random variable D_i with its expected value $\mathbb{E}[D_i]$. In the chance-constrained model, we assume that though we do not know the exact values of demand at the planning stage, we do know the probability distribution of demand from the demand points. We say that the probability that these constraints are not satisfied is lower than a certain confidence level $\epsilon \in (0, 1)$. Under these conditions, constraint (5) of **DM** can now be expressed as:

$$Pr \left(\sum_{j | \delta_{k-1} < d_{ij} \leq \delta_k} t_{ij} \leq f_k \delta_\epsilon^i \right) \geq 1 - \epsilon \quad \forall i \in I, k \in 1, \dots, K \quad (7)$$

In our work, we assume that the demand generated from the demand points follows a log-normal distribution with mean μ' and standard deviation σ' . We make this assumption because demand generated at a demand point cannot be negative and the log-normal distribution provides a positive support to hold this to be true. The relationship between the parameters of the log-normal and the normal distributions is defined as $\mu' = \log \mu - \frac{1}{2}\sigma'^2$, $\sigma'^2 = \log(\frac{\mu^2 + \sigma^2}{\mu^2})$. We define κ to denote the Z value of the normal distribution corresponding to the confidence level ϵ . κ is called the *safety factor*, where $\phi(\kappa) = 1 - \epsilon$. Using this relation, we linearize constraint (7) as follows:

$$\sum_{j | \delta_{k-1} < d_{ij} \leq \delta_k} t_{ij} \leq f_k e^{\mu'_i - \kappa \sigma'_i} \quad \forall i \in I, k \in 1, \dots, K \quad (8)$$

The objective function for the chance-constrained model remains unchanged from the deterministic model. We know that constraint (8) has the same structure as constraint (5), where the demand value D_i has been replaced by $e^{\mu'_i - \kappa \sigma'_i}$. While in **DM** we replace D_i by its expected value, in **CCM** we replace it with the value given by the chance-constrained expression.

4 A Locate-Allocate Heuristic

The locate-allocate heuristic was introduced by Cooper (1964), and later on, used by Larson and Brandeau (1986) to solve facility location problems. Jia et al. (2007) showed that the locate-allocate heuristic outperforms a genetic algorithm procedure and is nearly as good as a Lagrangean-relaxation heuristic for solving an uncapacitated facility location problem of locating medical supplies for a large-scale emergency. In terms of computational time, they showed that the locate-allocate heuristic is much faster than a genetic algorithm and a Lagrangean-relaxation heuristic. This was one of the main reasons why we chose to develop a locate-allocate algorithm to solve the chance-constrained model described previously. To describe the heuristic, we use the deterministic model **DM**. To solve this heuristic for **CCM**, we need to simply replace D_i with the value given by the chance-constrained expression.

The idea behind this heuristic is very simple. The first step is to choose an initial location for the N facilities to be opened. This can be done by using a simple greedy approach. In our case, we solve a simple integer program with an objective to maximize the amount of supplies transported between facilities and demand points. At this stage, we do not consider the demand loss function, but simply specify that a facility is allowed to transport supplies to demand points that lie within a radius K from it. This mixed integer program **IP₁** is as follows:

$$\begin{aligned} \mathbf{IP}_1 : \quad & \text{maximize} \quad \sum_{i \in I, j \in J} t_{ij} \\ & \sum_{j \in J} x_j = N \end{aligned} \tag{9}$$

$$\sum_{j | d_{ij} \leq \delta_K} t_{ij} \leq \frac{x_j * \sum_{j | d_{ij} \leq \delta_K} D_i}{N} \tag{10}$$

$$x_i \in \{0, 1\}; t_{ij} \geq 0 \tag{11}$$

In **IP₁** above, we initially balance the demand between all open facilities with the help of constraint (10). The second step of the heuristic is to allocate demand points to the facilities that were opened, with an objective to maximize coverage. In our case, we solve the following

linear program \mathbf{LP}_A that does this allocation of demand points to open facilities. We define a set J' to denote the set of facilities that have been opened.

$$\mathbf{LP}_A : \quad \text{maximize} \quad \sum_{i \in I, j \in J'} t_{ij}$$

$$\sum_{i \in I} t_{ij} \leq s_j \quad \forall j \in J' \quad (12)$$

$$\sum_{j \in J'} s_j \leq S \quad (13)$$

$$s_j \leq \beta \quad \forall j \in J' \quad (14)$$

$$\sum_{j \in J' | \delta_{k-1} < d_{ij} \leq \delta_k} t_{ij} \leq f_k D_i \quad \forall i \in I, k \in 1, \dots, K \quad (15)$$

$$\sum_{j \in J' | d_{ij} > \delta_K} t_{ij} = 0 \quad \forall i \in I \quad (16)$$

$$s_j, t_{ij} \geq 0 \quad (17)$$

The third step of the locate-allocate heuristic is to create clusters of demand points served by each open facility. Then, for each cluster, we try to relocate the open facility from its current site to another one such that the total travel distance between the facility and the demand points in the cluster is minimized. In our approach, we define clusters C_1, \dots, C_N to denote the N clusters formed by the N open facilities. Then, the relocate integer program can be written as follows:

$$\mathbf{IP}_R : \quad \text{minimize} \quad \sum_{C_y} \left[\Theta_{C_y, j} \sum_{i | i \in C_y} d_{ij} \right]$$

$$\sum_{j \in J} x_j = N \quad (18)$$

$$\sum_j \Theta_{C_y, j} = 1 \quad \forall y \in 1, \dots, N \quad (19)$$

$$\sum_{C_y} \Theta_{C_y, j} = x_j \quad \forall j \in J \quad (20)$$

$$x_j \in \{0, 1\} \quad \forall j \in J \quad (21)$$

Here, $\Theta_{C_y, j}$ is set to 1 if facility j is assigned to cluster C_y and 0 otherwise. M is a large constant that is used to linearize nonlinear constraints. Constraint (19) ensures that every cluster is assigned exactly one facility, and constraint (20) enforces the condition that an open facility is assigned to only one cluster, and a closed facility is assigned to none. On solving the above integer program, if a facility different from the one used in the allocate step of the heuristic is found to minimize the total travel distance for any particular cluster, then the new facility is assigned the same capacity as the previous facility. The allocate and relocate steps of the locate-allocate heuristic are repeated until no further relocation occurs. At this stage, the heuristic is terminated. In the following section, we test the performance of our heuristic and compare it with a simulated annealing heuristic presented by Berman and Drezner (2006) which also considers a distance-dependent demand.

5 Experimental Analysis

In this section, we present experiments to show how the deterministic and the chance-constrained models and the locate-allocate heuristic can be used to locate staging areas for mass distribution of the medical supplies with an objective to maximize coverage in the event of an anthrax attack on Los Angeles County. Anthrax is a deadly disease that requires vaccines and antibiotics to treat the affected persons and immunize the high-risk population. During an anthrax emergency, the strategic national stockpiles (SNS) are mobilized for local emergency medical services. We use the centroid of each census tract, representing the aggregated population in that tract as the demand points. Using this procedure, we have 1939 demand points in Los Angeles County. Under a large-scale emergency scenario, the

demand arising from each demand point could be anywhere between 0 and the aggregate population represented by it. The mean demand for the 1939 demand points were provided to us. We were also provided with approximately 200 eligible facility sites. To protect the confidentiality of the exact site locations, we use exactly 200 of these sites with a slight perturbation in their geographical coordinates. Figure 2 shows the distribution of the demand points in the County.

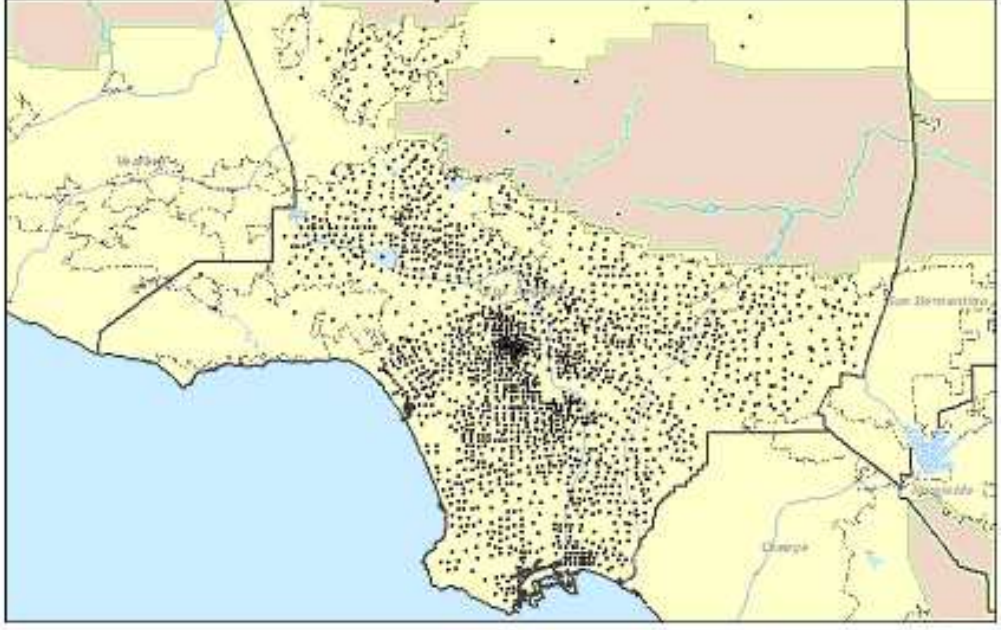


Figure 2: Locations of demand points within Los Angeles County

For the loss function, we consider three coverage levels of radii 4 mi, 8 mi and 12 mi respectively, for every open facility. Corresponding to these coverage levels, we assume $f_1 = 100\%$, $f_2 = 65\%$ and $f_3 = 30\%$. Additionally, as presented by Bravata et al. (2006), we assume the supply of vaccines in each facility (β) to be at least equal to 140,000 units per week, with a response time period of 4 weeks. This means that the facilities have a capacity of 560,000 units. In the experiments below we take the total supply S to be some fraction of the expected total demand, that is we take $S = \gamma \sum_{i \in I} \mathbb{E}[D_i]$ for some $\gamma \in (0, 1]$. We do this because the difficult situation is when there is a lack of supplies.

We coded the simulated annealing heuristic presented by Berman and Drezner (2006) in C++ and ran it using Microsoft Visual Studio 5.0 on a Pentium IV computer with 512MB

RAM. We maintained the same settings as above, and compared its performance with our locate-allocate heuristic. The simulated annealing heuristic uses an approach similar to that of Vogel’s approximation method to allocate demand points to facilities. Since their heuristic does not consider supply as a variable, we assumed the capacity of every open facility to be equal to the $\min(\frac{\gamma \sum_{i \in I} \mathbb{E}[D_i]}{N}, \beta)$. The demand experienced by a facility from a demand point is discounted as per the loss function. Facilities are sorted based on their opportunity cost for which we used distance as a measure. Demand points are allocated to facilities as long as the total demand assigned to a facility does not exceed its capacity.

The locate-allocate heuristic was coded in C++ using ILOG Concert Technology. For all the experiments provided below, CPLEX and the locate-allocate heuristic were performed on a Dell Precision 670 computer with a 3.2 GHz Intel Xeon Processor and 2 GB RAM running CPLEX 9.0. The heuristics executed rather quickly in the order of 2 minutes. To contrast, we compared the results obtained against solving the whole facility location problem with CPLEX for 2.0 CPU hours keeping the best solutions found so far. The convergence of the CPLEX solution slowed down considerably after 2.0 hours.

5.1 Deterministic Model

In this case, we use the mean demand for the 1939 demand points as the actual demand for the respective demand points. We present below the coverage that could be obtained from the deterministic model using the locate-allocate heuristic. We test the sensitivity of the algorithm for varying number of facilities to be opened N and varying service levels γ .

| γ | 100% | 90% | 80% | 70% | 50% | 30% | 20% |
|----------|-------|-------|-------|-----|-----|-----|-----|
| $N=50$ | 94.19 | 89.99 | 80 | 70 | 50 | 30 | 20 |
| $N=40$ | 89.66 | 88.32 | 80 | 70 | 50 | 30 | 20 |
| $N=30$ | 84.96 | 83.68 | 77.01 | 70 | 50 | 30 | 20 |
| $N=20$ | 74.56 | 72.26 | 72.09 | 70 | 50 | 30 | 20 |

Table 1: Coverage from the deterministic model using locate-allocate heuristic

We compare the above results with that obtained by using the simulated annealing procedure, presented by Berman and Drezner (2006), to solve the deterministic facility location problem.

The results from the simulated annealing procedure are presented below.

| γ | 100% | 90% | 80% | 70% | 50% | 30% | 20% |
|----------|-------|-------|-------|-------|-----|-----|-----|
| $N=50$ | 90.86 | 84.71 | 79.96 | 70 | 50 | 30 | 20 |
| $N=40$ | 82.45 | 80.24 | 77.58 | 69.97 | 50 | 30 | 20 |
| $N=30$ | 79.67 | 76.44 | 74.30 | 69.89 | 50 | 30 | 20 |
| $N=20$ | 71.34 | 69.91 | 68.83 | 68.67 | 50 | 30 | 20 |

Table 2: Coverage from the deterministic model using the simulated annealing heuristic

In the figure below we plot the performance of the locate-allocate (L-A) heuristic and the simulated annealing (SA) procedure for $N = 40$, and compare them to the best lower bound (LB) and upper bound (UB) of the integer program **DM** obtained using CPLEX after 2.0 CPU hours, also for $N = 40$.

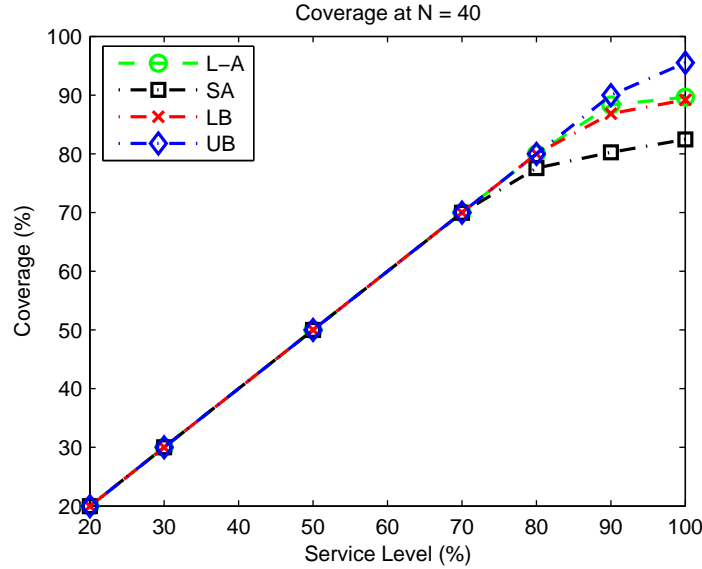


Figure 3: Comparison of locate-allocate and simulated annealing for $N = 30$

Figure 3 shows that for the settings of our problem, the locate-allocate heuristic outperforms the simulated annealing procedure in locating facilities to maximize coverage. In addition, while the coverage achieved by the locate-allocate heuristic is mostly greater than the CPLEX lower bound, the coverage achieved by simulated annealing is usually slightly below this lower bound for γ values of 100%, 90%, and 80%. This trend in both the heuristics holds true for the all the values of N tested, as shown in Tables 1 and 2 above.

5.2 Chance-constrained Model

Having verified the quality of the locate-allocate heuristic for solving the deterministic model, we now investigate the performance of the chance-constrained model and the heuristic under demand uncertainty. For all the results presented in this section, we assume that $N = 20$ and $\gamma = 0.8$. We use a log-normal distribution with the same mean values as was used in the deterministic case for generating random demand for the simulations presented in this section. The standard deviations are taken to be a certain percentage (10%, 20% etc.) of the respective mean demand values. In Table 3 below, we present the results from an experiment to study how the κ values impact the coverage provided by the chance-constrained model.

| κ | ϵ | $\sigma=10\%$ | $\sigma=20\%$ | $\sigma=40\%$ |
|----------|------------|---------------|---------------|---------------|
| 1.96 | 0.025 | 62.52 | 52.57 | 41.62 |
| 1.64 | 0.050 | 63.47 | 53.16 | 42.11 |
| 1.44 | 0.075 | 64.08 | 53.38 | 42.65 |
| 1.28 | 0.100 | 64.66 | 54.09 | 44.83 |
| 1.15 | 0.125 | 64.94 | 55.13 | 45.54 |
| 1.04 | 0.150 | 65.18 | 56.69 | 46.73 |
| 0.93 | 0.175 | 65.27 | 57.21 | 48.44 |
| 0.84 | 0.200 | 65.78 | 59.32 | 49.23 |

Table 3: Coverage from the chance-constrained model

Table 3 shows that for low values of ϵ (corresponding to high values of κ), equations (12) and (13) need to be satisfied with a high probability. Hence, $\sum_j t_{ij} \leq \delta_\epsilon$ is a tight constraint. This explains the low coverage values. As ϵ increases or κ decreases, this constraint becomes easier to satisfy, resulting in higher coverage values. As the uncertainty in the demand increases, constraints (14) and (15) become tight, thereby causing a decrease in the coverage attained.

Next, we perform two sets of simulation experiments to evaluate the quality of the locations of the open facilities and the supply stored at each open facility in terms of unmet demand through simulations. For both the simulation experiments, we fix the facility locations, that is, the x_j values for all the locations, and the s_j values for each combination of κ and σ as per Table 3. Next, we generate random demands for the 1939 demand points based on their

respective mean values for each value of σ . The performance of the corresponding facility sites and their supply under this random demand for a given κ - σ combination is recorded as the coverage obtained upon allocating the demand points, based on their random demands, to these facilities. For the first experiment, the ratio of this coverage value and the best possible coverage that could be obtained had we known this demand in advance is computed. Each recording in Table 4 below is the average of 20 such ratios.

| κ | ϵ | $\sigma=10\%$ | $\sigma=20\%$ | $\sigma=40\%$ |
|----------|------------|---------------|---------------|---------------|
| 1.96 | 0.025 | 0.8253 | 0.7386 | 0.6868 |
| 1.64 | 0.050 | 0.8352 | 0.7558 | 0.7078 |
| 1.44 | 0.075 | 0.8391 | 0.7734 | 0.7312 |
| 1.28 | 0.100 | 0.8502 | 0.7815 | 0.7457 |
| 1.15 | 0.125 | 0.8539 | 0.8040 | 0.7492 |
| 1.04 | 0.150 | 0.8606 | 0.8121 | 0.7598 |
| 0.93 | 0.175 | 0.8626 | 0.8200 | 0.7704 |
| 0.84 | 0.200 | 0.8646 | 0.8317 | 0.7844 |

Table 4: Comparison of the coverage from the **CCM** under a random demand to the best possible coverage had we known this demand in advance

In Table 4, the denominator values remain constant for each column. Similar to Table 3, the numerator values increase with a decrease in the κ value and decrease with an increase in uncertainty, represented by the standard deviation. Table 4 shows that in the worst case, the locations chosen by the locate-allocate heuristic cover approximately 69% of the demand that could be covered had we known this random demand well in advance. For the best-case scenario, this value increases to approximately 86%.

For the second simulation experiment, the numerator is computed as it is done for the values in Table 4 above. For the denominator, we consider the sites, the x_j and the s_j values, used by the deterministic model, presented in Table 1 with $N = 20$ and $\gamma = 0.8$. With these settings, we allocate demand points to the open facilities with the random demands same as what were considered for the numerator. Each recording in Table 5 is averaged over 20 such ratios.

Table 5 shows the merit of using the chance-constrained model to locate facilities, determine their capacities and allocate demand points to the open facilities. It also shows the relative

| κ | ϵ | $\sigma=10\%$ | $\sigma=20\%$ | $\sigma=40\%$ |
|----------|------------|---------------|---------------|---------------|
| 1.96 | 0.025 | 0.9799 | 0.9820 | 1.0152 |
| 1.64 | 0.050 | 0.9917 | 1.0048 | 1.0727 |
| 1.44 | 0.075 | 0.9962 | 1.0282 | 1.1082 |
| 1.28 | 0.100 | 1.0094 | 1.0390 | 1.1301 |
| 1.15 | 0.125 | 1.0138 | 1.0689 | 1.1353 |
| 1.04 | 0.150 | 1.0218 | 1.0797 | 1.1514 |
| 0.93 | 0.175 | 1.0242 | 1.0902 | 1.1675 |
| 0.84 | 0.200 | 1.0265 | 1.1058 | 1.1887 |

Table 5: Comparison of the performance of the **CCM** to the **DM** in response to a random demand

gain in the coverage, achieved by using the chance-constrained model over the deterministic model, increases with a decrease in the safety factor and an increase in the standard deviation. The first is due to an increase in the numerator values as κ decreases, and the second is due to a decrease in the denominator values as demand uncertainty increases. Under the best-case scenario, we see nearly a 20% gain in coverage by resorting to the locations given by the chance-constrained model.

6 Conclusions

In this study, we consider the problem of locating points of disbursement (POD) for medicines in response to a large scale emergency. To address the tremendous magnitude and low frequency of large-scale emergencies we obtain a solution that maximizes the number of people serviced under such uncertain and limited resources/time conditions.

In the problem we consider here, first, the facilities are capacitated by the service rate of a POD. Second, the demand satisfied depends on the distance to the facility. This is because in a large-scale emergency scenario, the number of people turning up at a particular POD decreases as their distance to that POD increases. Thirdly, given the unpredictability as to when and where such an emergency scenario could occur and how many people would be affected, there is significant uncertainty in demand values.

We model the problem as a capacitated facility location problem with a distance dependent

demand with an objective to maximize coverage. The number of facilities to be opened is dependent upon the type of emergency scenario, and the number of people affected. In our model, we assume that the demand that can be serviced at a facility from a particular demand point is a fraction of the demand that decreases as a function of the distance between the demand point and the facility. This assumption is made to reflect the inability of people to travel large distances (due to possible damage to infrastructure such as roads, health conditions etc.) in an emergency scenario. For this purpose, we use a loss function. We use chance constraints for the uncertainty in demand arising in the various demand points.

A locate-allocate heuristic is developed to efficiently solve this problem. We run the heuristic for various combinations of open facilities, N and service factors, γ . We investigate the performance of the heuristic for both the deterministic and the chance-constrained models. We present results to show the quality of the locations in providing coverage by selecting the staging areas in the event of an emergency. Our results showed that the heuristic performs well under these conditions of uncertain demand, and that the **CCM** performs significantly better than the **DM** under demand uncertainty.

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