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Mathematics of Physical Models B, 10.5 ECTS
Computer lab 2

Wave equation and resonance with COMSOL Multiphysics 5.2a

Introduction

In physics, waves is a reoccurring phenomena that arises in acoustics, electromagnetics, fluid dynamics and many other fields of physics. The wave equation is a way to describe waves with a second-order linear PDE. In its simplest form, it is written as

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u, \tag{1}$$

where u is a scalar function, $u = u(\mathbf{x}, t)$[1] and c is a fixed constant with the dimension of velocity[2]. During the first part of Computer lab 2you will study eq. (1) in $1D$. In the second part the equation

$$\rho_l(x) \frac{\partial^2}{\partial t^2} v(x,t) = S \frac{\partial^2}{\partial x^2} v(x,t), \quad 0 < x < L \tag{2}$$

will be used, which describes a string of length L with a, assumed constant, tension S with an inhomogeneous linear density ρ_l that varies along the string[3].

Aim of Computer lab 2

The aim of Computer lab 2is to become accustomed with the wave equation, what the different terms describe and through your work with eqs. (1) and (2) in CMP learn how different *boundary conditions* and *initial conditions* effect the behavior of propagating waves. You will be introduced to features of CMP dealing with the *parametric solver*, *exporting data to Matlab*, the *eigenvalue solver* and *integration of coupling variables.*

Examination of Computer lab 2

The examination of Computer lab 2consist of a report (in English) which as usual should be well proof-read, all figures should be referred to, answers to questions should be in fluent text e.t.c. For each exercise, the report should contain

- Some kind of problem description.
- Relevant plots and answers to the specified questions.

This report should be sent as a *pdf* with the name *Lab#FirstnameLastname.pdf*, e.g Lab2PekkaPekkalainen.pdf, to: phni0004@student.umu.se. Always fill in mail subject as *fmm lab#*, otherwise it might be treated as spam. The report should cover:

Dimension analysis [4]

- Why c in eq.(1) should have the same dimension as velocity.
- What dimension S in eq. (2) haves.

Exercise 1

- What kind of boundary condition is specified for the PDE?
- Graphs similar to fig. 1 with a brief explanation of what they describe.

Exercise 2

- Explanation of what roles a, h_0 and α have in the PDE and the limits for a.

[1] $\mathbf{x} = x_1, x_2, ..., x_n$.
[2] Perform a dimension analysis of eq. (1) and you will see why.
[3] Since equation (2) is considered in $1D$, the dimension of ρ_l is $\frac{M}{L}$ (*mass* divided by *length*); equivalently $[\rho_l]=\frac{M}{L}$.
[4] Remember to perform the analysis with physical quantities such as length, time etc. and not units. Be consequent with your notations. Example of how to write in a convenient way: $[x] = L$ (read as "dimension of x is length").

- A graph similar to fig. 2, but with the 3:rd nonzero term included in the analytic solution. As in *Exercise 1* the graph should include a brief explanation of what it describes.

Exercise 3

- Boundary conditions for both the *fixed* and *free* end.
- Your *initial condition* for the *asymmetric pulse.*
- Informative figures which confirm what is shown in fig. 3, for both kind of ends.

Exercise 4

- Graphs like those in fig. 5 for both cases, i.e. both when pulse moves from ρ_{l_1} to ρ_{l_2} and when it moves from ρ_{l_2} to ρ_{l_1}.
- An explanation of what will happen when $\rho_{l_1}/\rho_{l_2} \to 0$. Does it make sense? Motivate.
- The effect S has on the PDE.

Exercise 5

- The reason, if any, as to why you should solve for precisely $\omega = \pi$ and $\omega = 3\pi/2$.
- Graphs of the largest amplitude for both cases, as in fig. 7.

Exercise 6

- A plot for one of the eigenmodes similar to the left graph of fig. 8.
- A plot of the angular frequency as a function of mode number similar the right graph of fig. 8. How could you have foreseen its linear behavior?
- A plot of the kinetic energy as a function of frequency, similar to fig. 9. Does the ω with maximum kinetic energy correspond to the ω derived from the eigenvalue analysis?

1 Exercise 1

1.1 Problem

Solve the following PDE[5]:

$$\begin{aligned} \frac{\partial^2}{\partial t^2}u(x,t) &= 9\frac{\partial^2}{\partial x^2}u(x,t), \quad 0 < x < \pi, \quad t > 0 \\ u(0,t) &= u(\pi,t) = 0, \quad t > 0 \\ u(x,0) &= 3\sin(2x) + 12\sin(13x), \quad 0 < x < \pi \\ \frac{\partial}{\partial t}u(x,0) &= 0, \quad 0 < x < \pi \end{aligned}$$

and compare your result with the analytical solution $u_{analytic} = 3\cos(6t)\sin(2x) + 12\cos(39t)\sin(13x)$.

1.2 Procedure

Start CMP and choose $1D$ as *Space Dimension*. Select *Coefficient Form PDE* as in *Computer lab 1* and let u be the dependent variable. Since you are going to study a propagating wave select *Time Dependent* as *Study Type.*

Before you start drawing your geometry it is recommended to define any constants you might want to change during your session in *Parameters*, found when right clicking *Global Definitions.* Obviously it is way more convenient to update parameter values in one place than having to change them in many[6]. In this case you should define at least L, c, constant boundary and initial conditions as parameters. When defining anything also make sure to define its unit after your value under *Expression*[7]. This will also enable you to solve for different values of these parameters using what in CMP is called a *Parametric Sweep.* Expressions which varies in space or time you should define under *Variables* which you can find both under the node *Global Definitions* and *Definitions*[8].

[5] Problem 19 on p. 587 in *Fundamentals of Differential Equations and Boundary Value Problems* by *Nagle, Saff and Snider.*
[6] This is always recommended both in CMP and *Matlab.*
[7] As an example define the length L as L under *Name* and *pi* $[m]$ under *Expression.*
[8] The difference between *Global Definitions* and *Definitions* is described in the *Help*

When you have defined all that you want to define, you can start drawing your geometry. Remember that you have (hopefully) defined your length as a parameter! When you are pleased with your geometry you should enter the PDE, initial and boundary conditions and then mesh. Since you are working in $1D$ and solving a nice PDE you can use a very fine mesh. Before solving there are three things you will have to do:

1. Set the time range. You want to solve it for time t in $0 < t < 2$ with an appropriate time step. This is done under *Step 1: Time Dependent*.
2. The *Relative tolerance* should be decreased to 10^{-5}.
3. Decrease the *Absolute Tolerance* to 10^{-6}, you find it under *Time-Dependent Solver* located under *Solver 1*[9] under *Solver Configurations*[10].

To present your solution in a good way you need to compare your computed u with $u_{analytical}$, which should be done using a *Line Graph* showing both u and the error $u - u_{analytical}$ at a given time as in the left graph of fig. 1. It is also interesting to see how u and $u - u_{analytical}$ change over time for a specific point, as seen in right graph of fig. 1. You create this plot using *Point Graph* but first you have to define a *Cut Point*, which is found under *Data Sets*. This way, when plotting your *Point graph* you use the *Cut Point* as *Data Set*. To get points representing your data as in the picture, go to *Coloring and Style*, choose *Marker: Point* and *Positioning: in data points*.

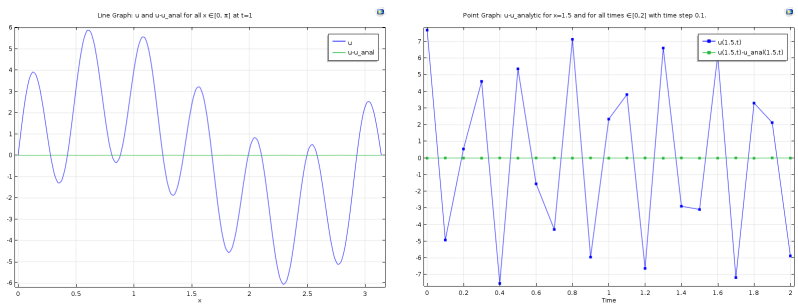
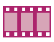


Figure 1: Both graphs shows both u and $u - u_{analytical}$. The left graph is a line graph for $t = 1$ and the right graph is a point graph for $x = 1.5$, where the points in the picture represent the actual data (i.e.: the lines do not indicate data, but clarify the chronological order of the position shifts with time).

Furthermore, take a look at how the PDE behaves over time in an animation. You can create an animation by clicking  in the tool bar (the option is also available right-clicking *Export* and choosing *Animation*). *Player* allows you to see the animation directly, while *File* allows you to export the animation to a file.

2 Exercise 2

2.1 Problem

A vibrating string is described by the PDE[11]:

$$\begin{aligned}
\frac{\partial^2}{\partial t^2}u(x,t) &= \alpha^2 \frac{\partial^2}{\partial x^2}u(x,t), \quad 0 < x < L, \quad t > 0 \\
u(0,t) &= u(L,t) = 0, \quad t > 0 \\
u(x,0) &= f(x), \quad 0 < x < L \\
\frac{\partial}{\partial t}u(x,0) &= g(x), \quad 0 < x < L
\end{aligned}$$

If the string is pulled into a V-shape, with height h_0 at $x = a$, and then released at $t = 0$ the initial conditions are:

$$\begin{aligned}
f(x) &= \begin{cases} h_0 \frac{x}{a}, & 0 < x < a \\ h_0 \frac{1-x}{1-a}, & a < x < L \end{cases} \\
g(x) &= 0
\end{aligned}$$

Choosing $L = 1$, the analytical solution is $u_{analytic} = \frac{2h_0}{\pi^2 a(1-a)} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin(n\pi a) \sin(n\pi x) \cos(n\pi\alpha t)$.

[9] This might have the name *Solution 1*.

[10] The *Relative tolerance* and *Time-Dependent Solver* should have these values all throughout Computer lab 2.

[11] Problem 5 on p. 636 in *Fundamentals of Differential Equations and Boundary Value Problems* by *Nagle, Saff and Snider*.

2.2 Procedure

This exercise is very similar to the previous one, meaning the procedures are pretty much identical. There are however a few new *parameters* to define: a, h_0 and α. Start with $a = 0.5$, $h_0 = 1$ and $\alpha = 1$. It is <u>essential</u> that you understand what these parameters describe and when you have defined your PDE, vary them and compare the results using an *animation*. Try predicting which effect your changes to the parameters will have on the animation and see if you are correct! If you are having trouble defining $f(x)$, which is done under *Variables*, there is a short-hand notation for *if-statements* called *boolean operators* you can use. They look and work as follows: $x < C$ evaluates to *true* which CMP interprets as a 1 when the variable x is less than the constant C and *false* (i.e. 0) when this is not true. This, of course also work for $x > C$. These can combined as many times as you want. For example $A*(B < x)*(x < C)$ evaluates to A when $B < x < C$ and 0 for any other value of x.

When you have investigated the roles of the parameters, solve the PDE for you start values ($a = 0.5$, $h_0 = 1$ and $\alpha = 1$) and compare your solution to the analytical solution $u_{analytic}$. Writing out all the terms of $u_{analytic}$ is not possible, but how many terms should be used? Start by including the first nonzero term of $u_{analytic}$, i.e. $u_{analytic_1} = \frac{2h_0}{\pi^2 a(1-a)} \sin(\pi a)\sin(\pi x)\cos(\pi \alpha t)$, to get a picture similar to the left in fig. 2. Thereafter, include the second nonzero term[12] to see how the error decreases similarly to as in the right graph in fig. 2. Lastly, include the third nonzero term of the analytical solution - what happens to the error?

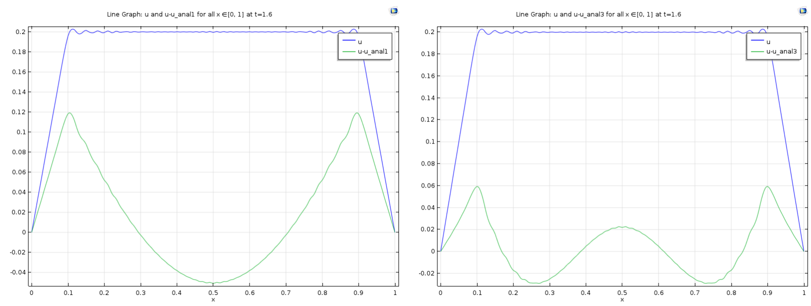


Figure 2: Both graphs show examples of how u and $u - u_{analytical}$ might look like at $t = 1.6$ (for different approximations of $u_{analytical}$). u_{anal1} in the left graph includes the 1:st nonzero term of the analytical solution, whilst u_{anal3} in the right graph includes the 2 first nonzero terms.

[12] Since a=0.5, what is the 2:nd term of $u_{analytic}$?

3 Exercise 3

3.1 Problem

Consider a pulse propagating along a string and arriving at a wall. Your task is to, by making use of a PDE as in *Exercise 2*, compute a solution for a propagating asymmetric pulse both towards a *fixed* and *free end*. Using the results from this computation you should be able to confirm what is shown in fig. 3.

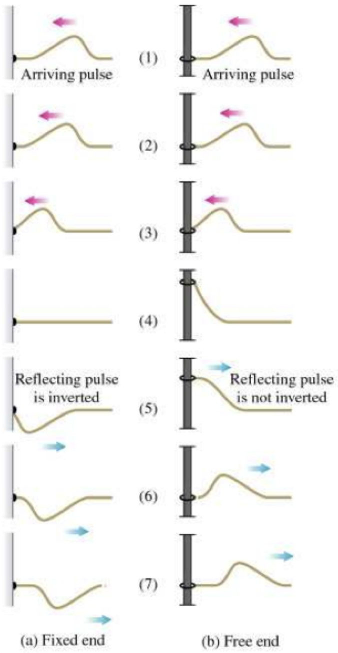


Figure 3: A schematic sketch over time for a pulse propagating towards both a fixed and free end.

3.2 Procedure

As in *Exercise 2* this one only introduce a minimum of new aspects to the previously solved problems. The hints you may need is that you have to narrow the region where $f(x) \neq 0$ and make the pulse asymmetric. The created *asymmetric initial pulse* will travel in both directions - focus on one of these propagating pulses. If you are still unsure on how to proceed another hint is to keep $g(x) = 0$. Finally, the pulse may consist of straight lines (and must be continuous), hence a triangular pulse is approved[13].

4 Exercise 4

4.1 Problem

Similar effects as in *Exercise 3* also occurs when a pulse is propagating towards a medium with different density. Consider eq. (2) for the case showed in fig. 4 where a string with density $\rho_l(x)$, consisting of two different materials, is stretched between two fixed walls. The density to the left, ρ_{l_1}, is less then the the density to the right, ρ_{l_2}, and the total length of the string is L.

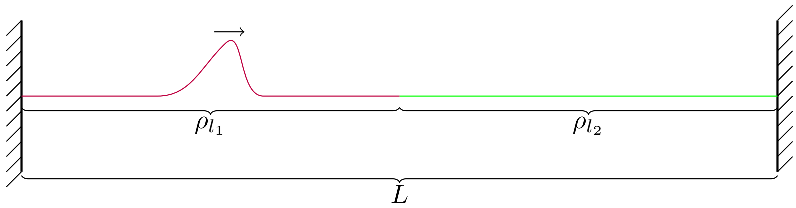


Figure 4: A inhomogeneous string of length L with two different densities ρ_{l_1} and ρ_{l_2} is stretched between to fixed walls.

[13] Remember to, as always, keep the function for your pulse as general as possible. It will make your life much easier if you can move the initial position of the pulse from one place to another just by changing the values of a few parameters or so.

Your task is to investigate what will happen if the pulse starts where $\rho_l = \rho_{l_1}$ and travels towards the right and vice versa[14]. Also, investigate what will happen if ρ_{l_2} starts to increase, i.e. the the fraction ${}^{\rho_{l_1}}/_{\rho_{l_2}} \to 0$.

4.2 Procedure

Start as in the previous exercises and create your initial pulse. The linear density $\rho_l(x)$ can be defined with the help of *boolean operators* as before, i.e. think of $\rho_l(x)$ as

$$\rho_l(x) = \begin{cases} \rho_{l_1}, & 0 < x < a \\ \rho_{l_2}, & a < x < L \end{cases} .$$

Now, by defining ρ_l under *Variables*, you should be able to create graphs resembling those in fig. 5.

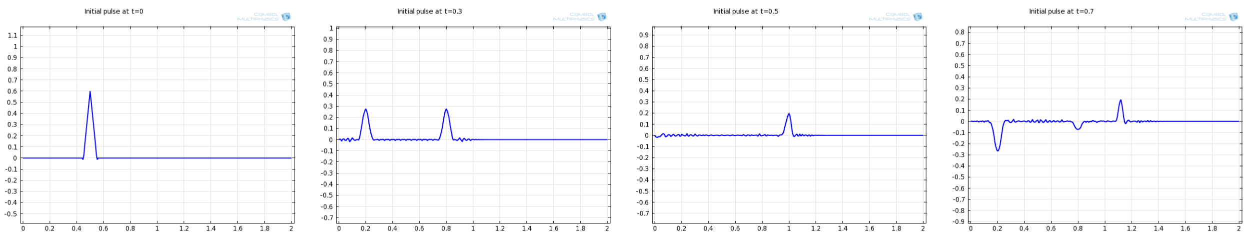


Figure 5: The pulse at $t = 0$, $t = 0.3$, $t = 0.5$ and $t = 0.7$, respectively.

When investigating the effects of a much larger ρ_{l_2} than ρ_{l_1} you use the *Parametric Sweep*[15] which you find when right clicking *Study*. Choose your predefined ρ_{l_2} as the parameter to *sweep* over in the range $\rho_{l_1} < \rho_{l_2} < n \cdot \rho_{l_1}$ where $n < 20$. You may use a quite large *step-size* as long as you are able to describe what effect the larger steps has.

5 Exercise 5

5.1 Problem

Consider a string attached to a mechanical vertical oscillator in its left end, as seen in fig. 6.

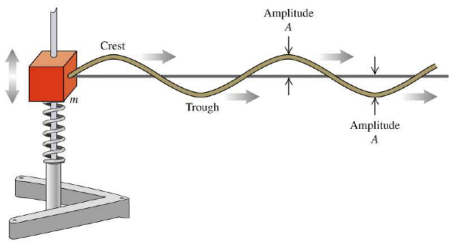


Figure 6: A string oscillating from one end.

First assume that the string's other end is attached to a fixed point[16]. This can be modelled according to the equation

$$\begin{aligned} \frac{\partial^2}{\partial t^2}u(x,t) &= \frac{\partial^2}{\partial x^2}u(x,t), \quad 0 < x < L, \quad t > 0 \\ u(0,t) &= \frac{\cos \omega t}{10}, \quad t > 0 \\ u(L,t) &= 0, \quad t > 0 \\ u(x,0) &= 0, \quad 0 < x < L \\ \frac{\partial}{\partial t}u(x,0) &= 0, \quad 0 < x < L. \end{aligned} \tag{3}$$

Solve this PDE with $L = 1$ (i.e. 10 times the amplitude for the mechanical oscillator) for two cases, $\omega = \pi$ and $\omega = {}^{3\pi}/_{2}$ over several oscillation periods $T = {}^{2\pi}/_{\omega}$. Here your goal is to explore the effect of resonance, therefore you should consider the largest amplitude in both cases. When solving, use $t = 0 : 0.1 : 9$.

[14] The pulse starts where $\rho_l = \rho_{l_2}$ and travels towards the left.

[15] This is a very useful tool, which enables you to solve over parameters. You can find more information about it in the *Help*. You might also consider solving over S to investigate what effect S might have.

[16] This is not seen in the figure.

5.2 Procedure

Start by defining $L = 1$ and $\omega = \pi$ under *Parameters*. The *left boundary condition* of your model, which you should define under *Variables*, should be $u_{left} = 0.1 \cdot \cos(\omega t)$. Now draw your geometry and enter the PDE. When setting your boundary conditions notice that they are both *Dirichlet*, and accordingly you must add two *Dirichlet Boundary Condition*, on the left side where $r = u_{left}$ and the right where $r = 0$. Then mesh and define the *time range* before computing.

You can now visualize how u changes over time using *Line Graph* and *Animation*. Since you are supposed to consider the largest amplitudes of both $\omega = \pi$ and $\omega = {}^{3\pi}/_{2}$ you could look at the *Line Graphs* for every *time step* and compare the amplitudes. This, however, would be a waste of time[17]. You should let *Matlab* do such tedious tasks for you. By right clicking *Export* you will find the option to export *Data*. Let *solution 1* be your *Data set*, choose *Select via: Stored output times* and have u as *Expression*, then export it as *omega1.txt*. When you have exported for $\omega = \pi$, change to $\omega = {}^{3\pi}/_{2}$, recompute and export it as *omega2.txt*. Now you have all the data from both cases stored in *txt-files*. Using the *Matlab script* in appendix A you will be able to create a graph looking like fig. 7, showing maximum amplitude for both cases.

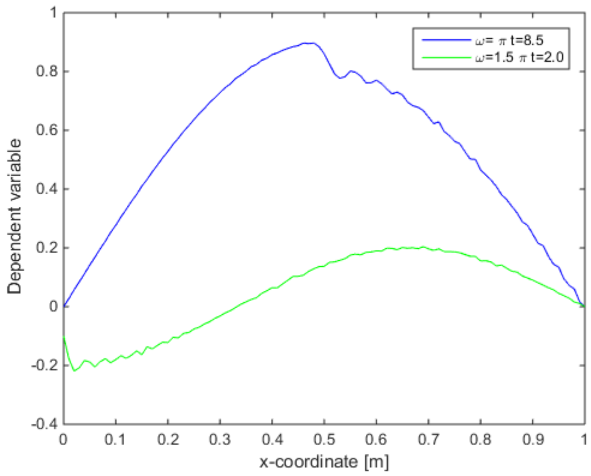


Figure 7: Maximum amplitude of u for both $\omega = \pi$ and $\omega = {}^{3\pi}/_{2}$. For $\omega = \pi$ maximum is achieved at $t = 8.5$ whilst for $\omega = {}^{3\pi}/_{2}$ at $t = 2.0$.

You can compare the *Matlab-plot* in fig. 7 with the *Line Graphs* in CMP at given t in order to confirm that you have derived what you wanted.

6 Exercise 6

6.1 Problem

You are interested in *resonant frequencies* and the *corresponding resonant modes*[18]. A straightforward way of investigating this is to consider eq. (2) under the following conditions:

$$\begin{aligned} v(0,t) &= a\cos\omega t, \quad t > 0 \\ v(L,t) &= 0, \quad t > 0 \\ v(x,0) &= 0, \quad 0 < x < L \\ \frac{\partial}{\partial t}v(x,0) &= 0, \quad 0 < x < L, \end{aligned} \tag{4}$$

then solving it for many different values of ω for sufficiently long times, noting when a frequency is resonant and the shape of its corresponding mode as done in *Exercise 5*. As you have hopefully figured out this is not the fastest way of solving this particular problem, as it would be much faster if the *initial transient behaviour* could be avoided. To do this, assume that the system has reached a *final state*, i.e. that it started long before $t = 0$. The final state can be described by

$$v(x,t) = u(x)\cos\omega t. \tag{5}$$

[17]Some might even say stupid.
[18]That is the motion/geometry of the string at the resonances.

Inserting eq. (5) into eq. (2) yields

$$
\begin{aligned}
-S\frac{\partial^2}{\partial x^2}u(x) - \rho_l(x)\omega^2 u(x) = 0,\ \ 0 < x < L \\
u(0) = a \\
u(L) = 0.
\end{aligned} \tag{6}
$$

Considering the case of constant density, i.e. $\rho_l(x) = \rho_{l_0}$, the general solution of eq. (6) is given by

$$u(x) = A\cos\left(x\omega\sqrt{\frac{\rho_{l_0}}{S}}\right) + B\sin\left(x\omega\sqrt{\frac{\rho_{l_0}}{S}}\right)$$

Using the boundary conditions from eq. (6) to find A and B, the equation

$$u(x) = a\cos\left(x\omega\sqrt{\frac{\rho_{l_0}}{S}}\right) - \frac{a\cos\left(x\omega\sqrt{\frac{\rho_{l_0}}{S}}\right)}{\sin\left(L\omega\sqrt{\frac{\rho_{l_0}}{S}}\right)}\sin\left(x\omega\sqrt{\frac{\rho_{l_0}}{S}}\right) \tag{7}$$

is derived. The resonant frequencies withhold the condition

$$\sin\left(L\omega\sqrt{\frac{\rho_{l_0}}{S}}\right) = 0$$

since that yields division by 0 in eq. (7) and $u(x)$, and thereby also $v(x,t)$, then becomes infinite. This requires

$$L\omega\sqrt{\frac{\rho_{l_0}}{S}} = n\pi,\ \ n = 1, 2, 3... \tag{8}$$

These frequencies can also be found by solving the *eigenvalue problem*

$$
\begin{aligned}
-S\frac{\partial^2}{\partial x^2}u(x) = \rho_l(x)\lambda u(x),\ \ 0 < x < L,\ \ \lambda = \omega^2 \\
u(0) = 0 \\
u(L) = 0.
\end{aligned} \tag{9}
$$

Eq. (9) is a homogeneous PDE, where both endpoints are assumed to be fixed[19] and the trivial solution $u(x) = 0$ always exist. Note, however that there are also solutions as:

$$u(x) = Asin\left(x\omega\sqrt{\frac{\rho_l}{S}}\right),\ \ \omega = \sqrt{\lambda},$$

so if $u(x) = 0$, that is $\sin\left(L\omega\sqrt{\rho_{l_0}/S}\right) = 0$, eq. (8) is satisfied with A as an arbitrary amplitude.

Your task is first to consider eq. (9) with $L = \rho_l = S = 1$. Plot both the *eigenmodes* and *angular frequency* w as functions of mode number. Secondly you should evaluate eq. (6) for a range around a specific ω with $L = \rho_l = S = 1$ and $a = 0.1$. To find out which ω in your range that is most resonant, consider the integral

$$W_{kin} = \frac{1}{4}\int_0^L \rho_l(x)\left[u(x)\right]^2 dx, \tag{10}$$

where W_{kin} is the kinetic energy. **The most resonant frequency corresponds to the highest kinetic energy**. Compare your results, i.e. the w derived from the eigenvalue analysis and the w with the highest kinetic energy in your range.

6.2 Procedure

Start a new session. Again choose $1D$ as *Space Dimension*, then *Coefficient Form PDE*, and let u be the dependent variable. Now choose *Eigenvalue* as study type. Define L and S under *Parameters* and ρ_l under *Variables*. Draw your geometry, specify the PDE according to eq. (9) and its boundary conditions. Before you compute, under *Step 1: Eigenvalue* (which you find under *Study 1*) choose 14 as *Desired number of eigenvalues*, and remember to mesh.

An example of a line graph of u for a specific eigenvalue $\lambda_4 \approx 157.9$ is shown in the left plot of fig. 8. The

[19] $v(0,t) = v(L,t) = 0$.

right graph of fig. 8 shows how the angular frequency $\omega = \sqrt{\lambda}$ depends on mode number n. To create similar graphs, create a *Line Graph* and choose the λ_n you want to look at. Then create a *Point Graph* (in a *1D Plot Group*), choose *Solution 1* as your *Data Set*, make sure *Eigenvalue selection* is *All* and that *All boundaries* are selected. Plot $\sqrt{\lambda}$ versus *Solution number*[20]. Under *Coloring and Style*, choose *Line*: *None*, *Marker: Point* and *Positioning: in data points*.

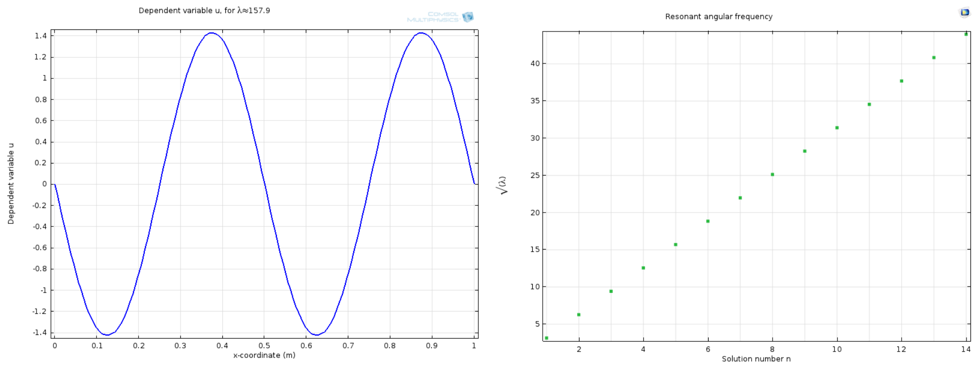


Figure 8: Left graph shows the dependent variable u for the forth eigenvalue $\lambda_4 \approx 157.9$. Right graph shows the angular frequency $\omega = \sqrt{\lambda}$ versus the corresponding mode number n.

When considering eq. (6), define $\omega = 12$ and $a = 0.1$ under *Parameters*[21]. Then add a *Stationary* study by right clicking the *root node* and selecting *Add Study*. Notice that a new study called *Study 2* appears in the *Model Builder*. When defining your PDE first change *Show equation assuming:* from *Study 1, Eigenvalue* to *Study 2, Stationary*. Remember also to update the boundary condition according to eq. (6)[22].

Add a *Parametric Sweep* by right clicking *Study 2*. Choose ω as *Parameter name* and create a range to *sweep* over, considering the value of some λ_n derived earlier. As an example, choosing $\lambda_4 \approx 157.9$ makes $\omega_4 \approx \sqrt{\lambda_4} \approx 12.6$. This would mean that a suitable range to *sweep* over could be $\omega = 12 : 0.1 : 13$.[23] Add a *line integral* under *Derived values* and add eq. (10) as *Expression*[24]. Choose *Solution 2* as *Data Set* and select the whole boundary, then evaluate the integral by pressing =. Two columns will appear under the *Graphics* window, one for ω and one for W_{kin}. To plot these two against each other press and you will get a graph similar to fig. 9.

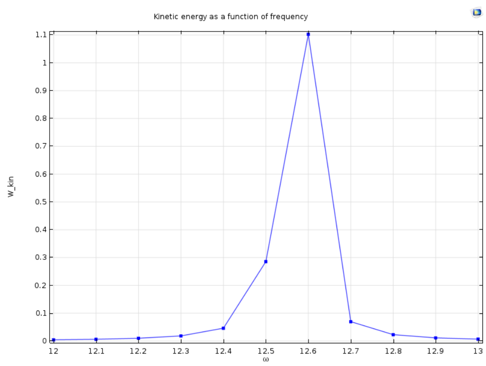


Figure 9: The kinetic energy W_{kin} as a function of frequency ω.

[20] The notations in COMSOL are often quite intuitive - writing $\sqrt{\lambda}$ as sqrt(lambda) does the job.

[21] Since you are going to perform a *Parametric Sweep* on ω, the relevant thing is that ω is *defined* in the beginning, and not what it is defined *as*. The values of ω used for computations will be the ones determined in *Parametric Sweep*. Hence, in this case you could define ω as pretty much any value under *Parameters*.

[22] If you, later on, switch between the studies - remember to always update the PDE and boundary conditions. If you get unexpected errors e.g. "mass matrix is zero", that could indicate that this has not been done correctly.

[23] In CMP written as range(12, 0.1, 13).

[24] That is 1/4*pl*u^2

A Matlab script

```
function find_max_FMM
% This program will find the maximum amplitude of u for both omega=pi
% and omega= 1.5*pi from data derived via CMP in Computerlab 2 in
% the course "Mathematics of Physical Models B, 10.5 ECTS" given at
% UmU.

%% Loading data
% u_omega#(:,1) defines the geometry, u_omega1(:,2:end) is the
% computed data.
u_omega1 = load('omega1.txt');
u_omega2 = load('omega2.txt');

%% Finding maximum
s = size(u_omega1);
max_omega1(1) = 0;
max_omega2(1) = 0;
Max_omega1 = max_omega1(1);
Max_omega2 = max_omega2(1);
for n = 2 : s(2)
    max_omega1(n) = max(abs(u_omega1(:,n)));
    max_omega2(n) = max(abs(u_omega2(:,n)));

    if max_omega1(n)>Max_omega1
        Max_omega1 = max_omega1(n);
        MAX_n1 = n;
    end

    if max_omega2(n)>Max_omega2
        Max_omega2 = max_omega2(n);
        MAX_n2 = n;
    end
end

%% Ploting the result
plot(u_omega1(:,1),u_omega1(:,MAX_n1),u_omega2(:,1),u_omega2(:,MAX_n2))
title('Maximum amplitude for \omega= \pi and \omega=1.5 \pi', ...
    'fontsize', 11, 'fontweight', 'bold');
xlabel('x-coordinate [m]');
ylabel('Dependent variable');
legend(['\omega= \pi ', sprintf('t=%.1f',0.1*(MAX_n1-2))],['\omega=1.5 \pi ',...
    sprintf('t=%.1f',0.1*(MAX_n2-2))])
```