Multi-Party Computation

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Motivating Examples

- How can we compute on encrypted data?
- How can Alice and Bob figure out who is wealthier without revealing their net worths?
- How can Signal learn if any of your contacts are using signal without Signal learning anything?
- How can two people check if their genome's share any characteristics without revealing any other information about their genomes?
- How could we share ML training data without revealing our underlying datasets?

Homework Example: Voting Problem

- Recall from HW 1, the challenge of computing the vote of the class
- This protocol had some notable security flaws (two actors could collaborate and discover someone else's vote) (someone could lie about their vote)
- However, it is a good example of multi-party computation as honest actors can collaborate to compute the collective vote

Warm Up

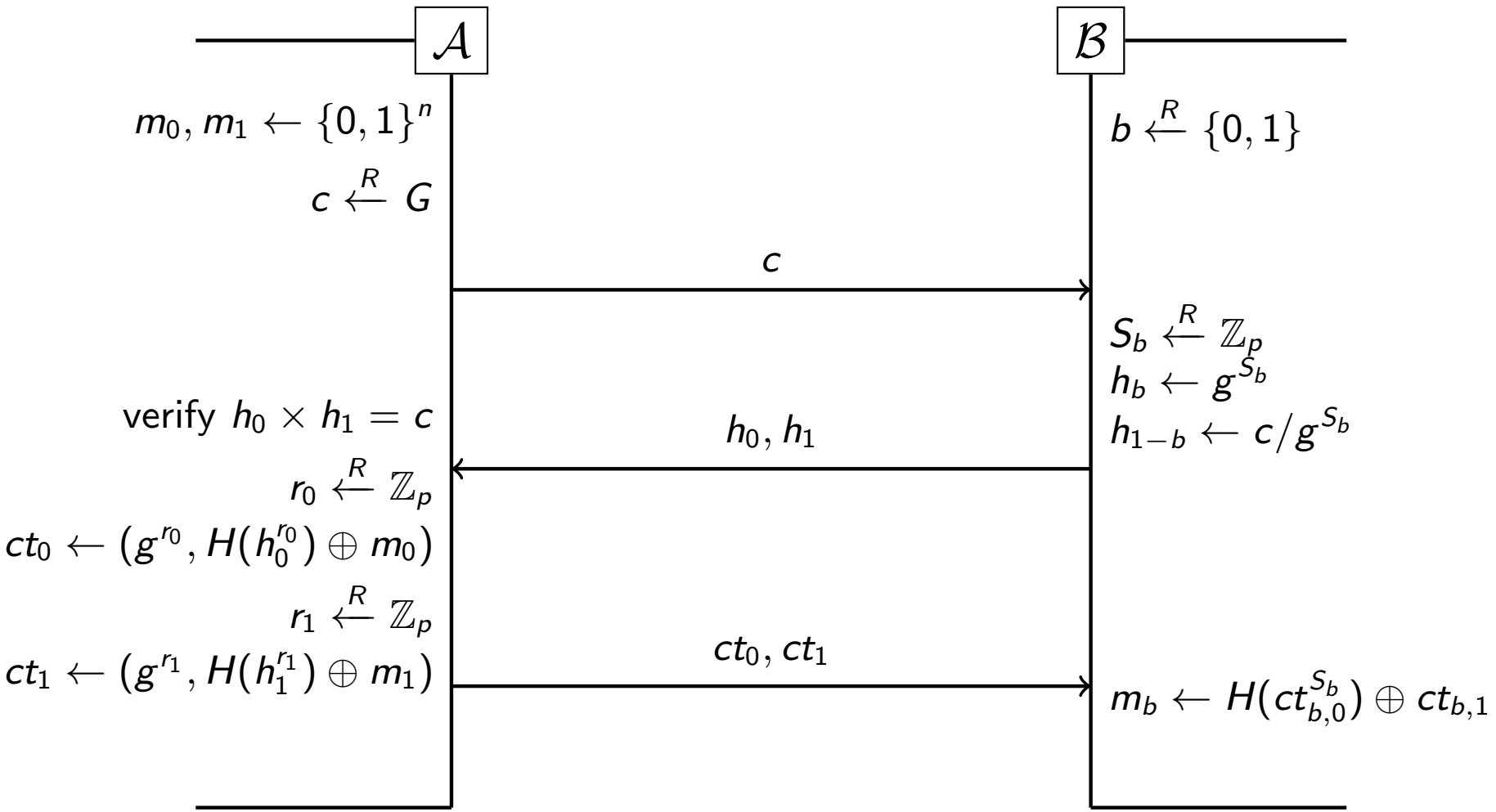
- Can we securely compute XOR?
- Can we securely compute OR?
- Can we securely compute AND?

2 Oblivious Transfer

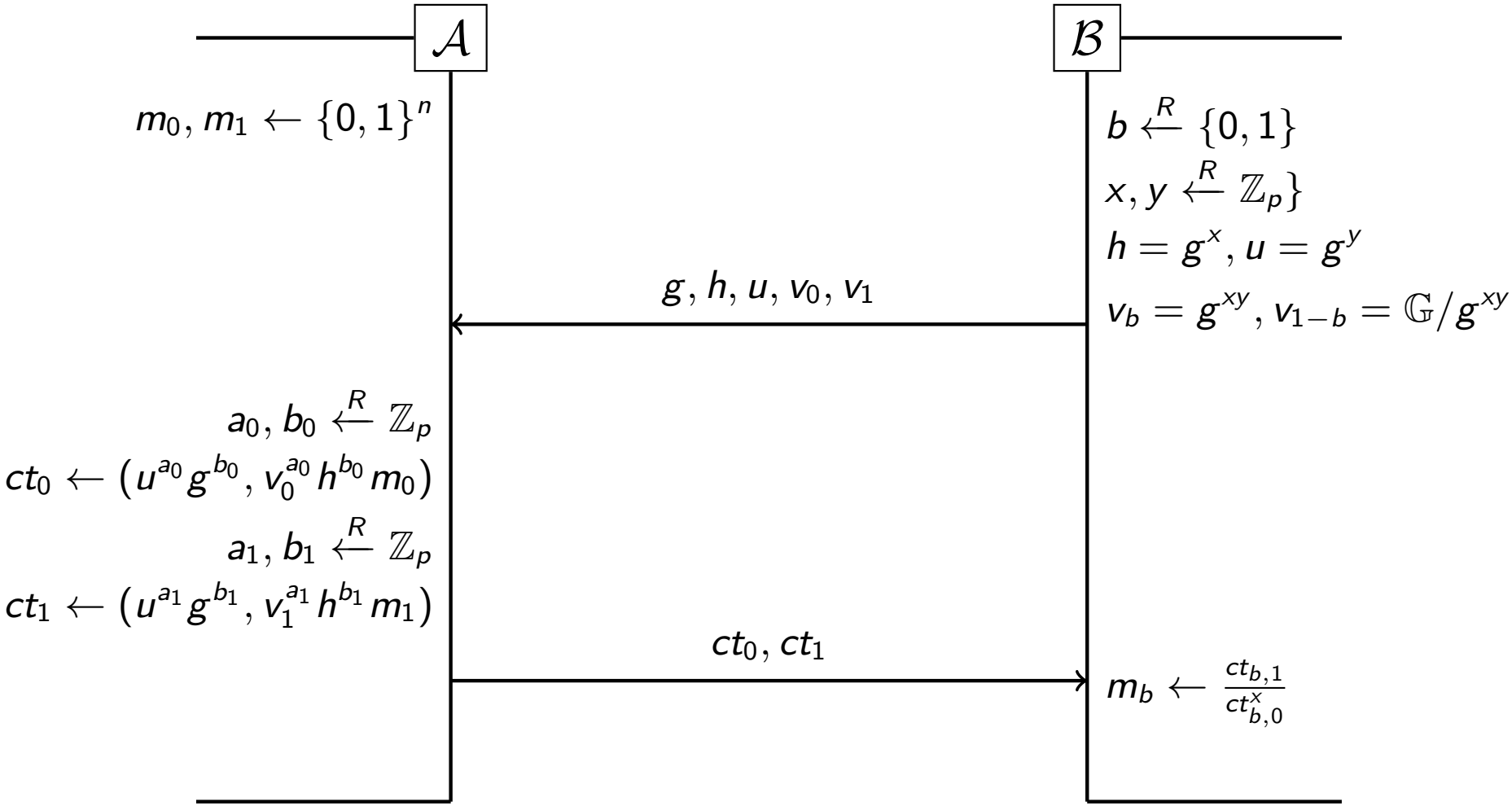
Oblivious Transfer

- The most simple building block of multi-party computation
- The idea is to establish a portocol where the sender has two messages (m_0, m_1) and the receiver learns the details of one of the messages, but not the other, and the sender doesn't know which one the receiver learned

Oblivious Transfer: Random Oracles



Oblivious Transfer: No RO



3 Shamir Secret Key Sharing

Shamir Secret Sharing: Review

- Recall our discussion of Shamir Secret sharing during the public key encryption lecture
- Essentially, it allowed us to share a secret with an arbitrary number of people and allow any subset of them to come together to unlock the secret
- I got a little excited and put it there early, now we will discuss how to actually compute it!

Shamir Secret Sharing: Mechanism

$$shares = [(-3, 0), (2, 5), (-1, -4)]$$

$$f(x) = 0 \times \frac{(x-2)(x-(-1))}{(-3-2)(-3-(-1))} + 5 \times \frac{(x-(-3))(x-(-1))}{(2-(-3))(2-(-1))} + \\ - 4 \times \frac{(x-(-3))(x-2)}{(-1-(-3))(-1-2)}$$

$$f(0) = 0 \times \frac{(0-2)(0-(-1))}{(-3-2)(-3-(-1))} + 5 \times \frac{(0-(-3))(0-(-1))}{(2-(-3))(2-(-1))} + \\ - 4 \times \frac{(0-(-3))(0-2)}{(-1-(-3))(-1-2)}$$

$$f(0) = -3$$

4 Computing on Secret Shared Data

Computing on Secret Data

- Supposed Alice, Bob, and Charlie are friends and Charlie wants to know how many marbles Alice and Bob have!
- Alice gives two random numbers to Charlie and Bob. Bob gives two random numbers to Alice and Charlie
- Alice computes $(a_{total} - s_{ac} - s_{ab}) + s_{ba}$
- Bob computes $(b_{total} - s_{bc} - s_{ba}) + s_{ab}$
- Charlie computes $s_{ac} + s_{bc}$
- They all share their results and independently compute $(a_{total} - s_{ac} - s_{ab}) + s_{ba} + (b_{total} - s_{bc} - s_{ba}) + s_{ab} + s_{ac} + s_{bc}$