Block Ciphers

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CUNY - Hunter College

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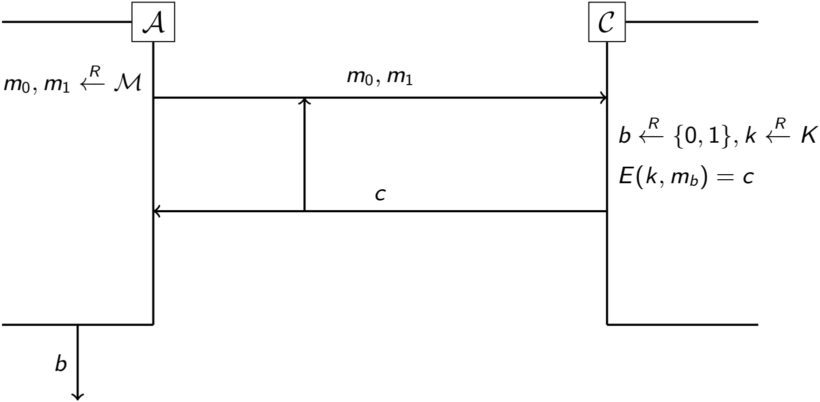
One Time Pad Security Game: Chosen Plaintext Attack


$\mathcal{A}$
$\mathcal{C}$
$m_0, m_1 \xleftarrow{R} \mathcal{M}$
m_0, m_1
$b \xleftarrow{R} \{0,1\}, k \xleftarrow{R} K$
$E(k, m_b) = c$
c
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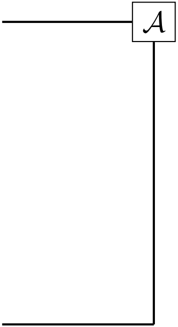

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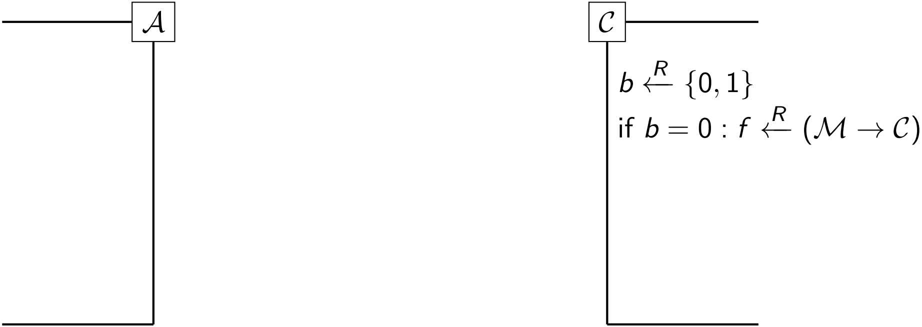

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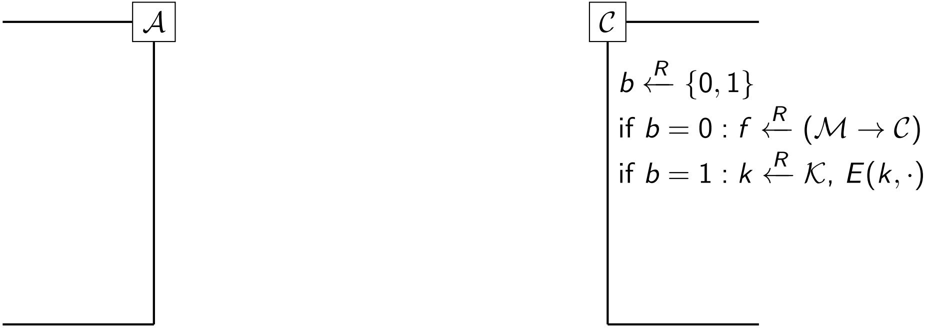

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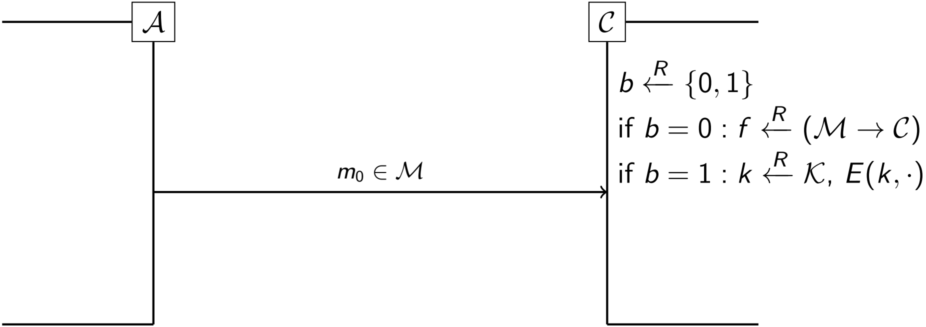

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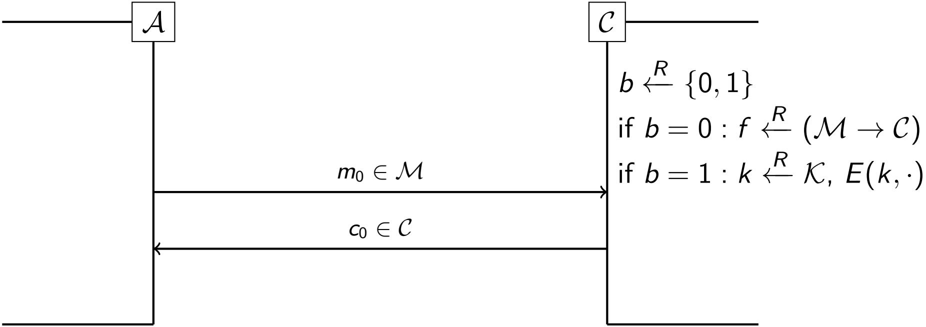

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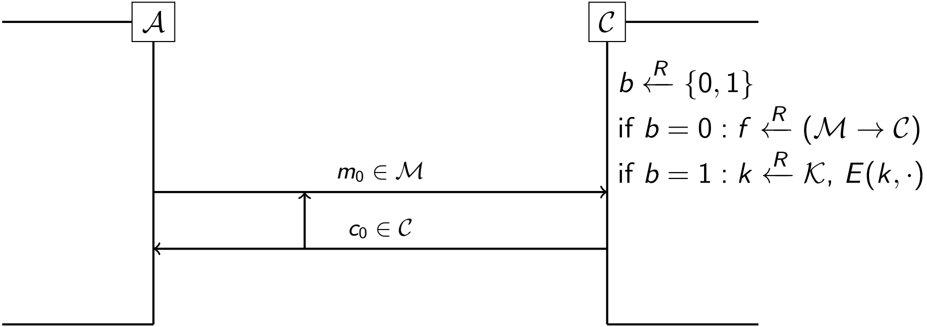
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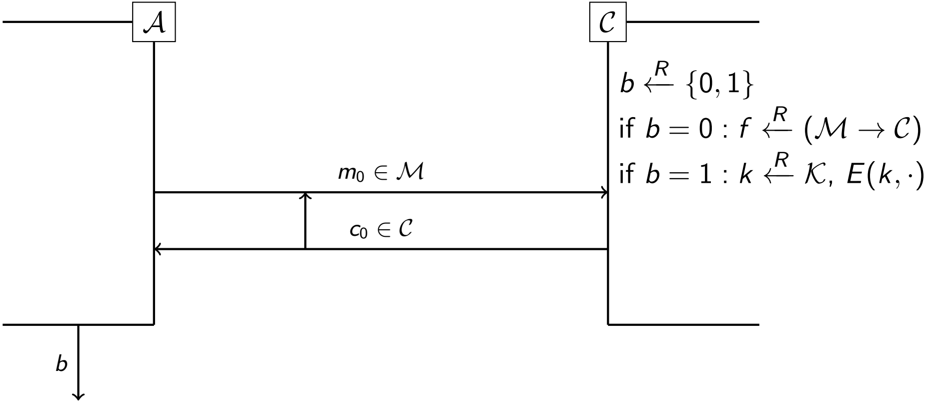

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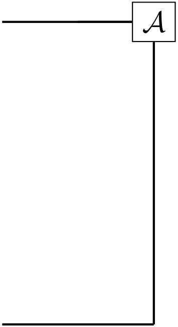

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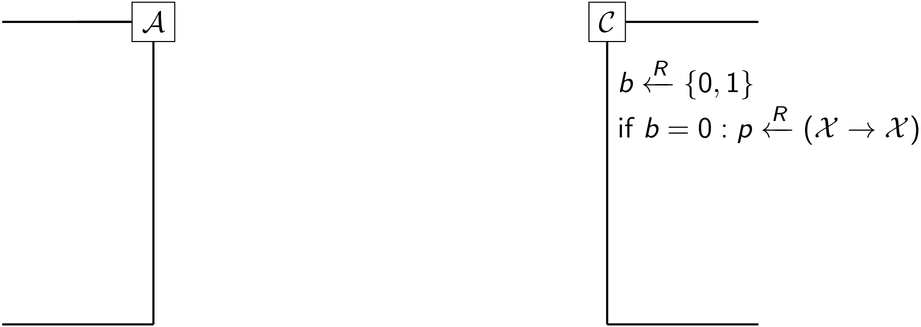

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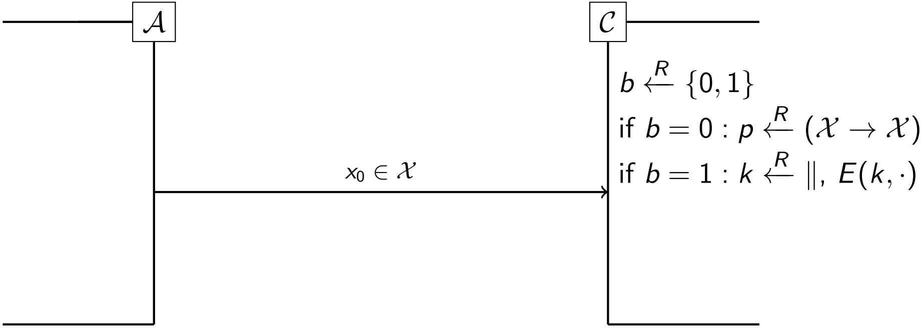

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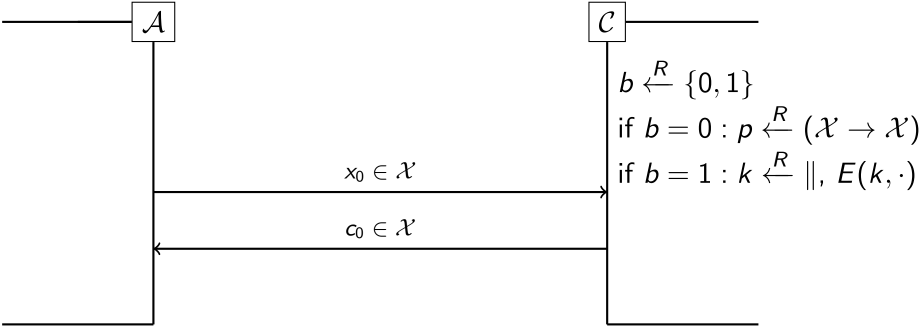

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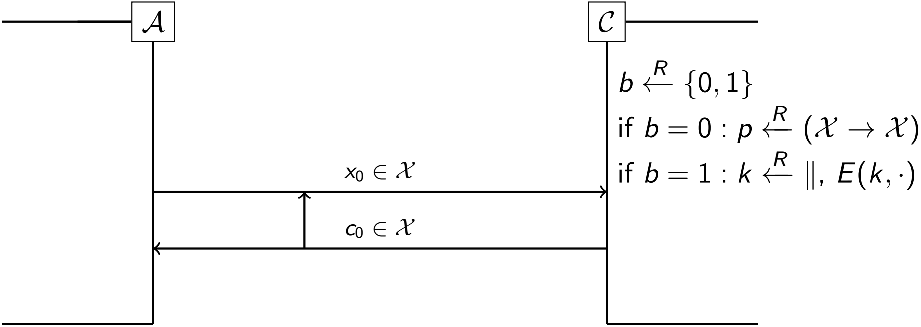

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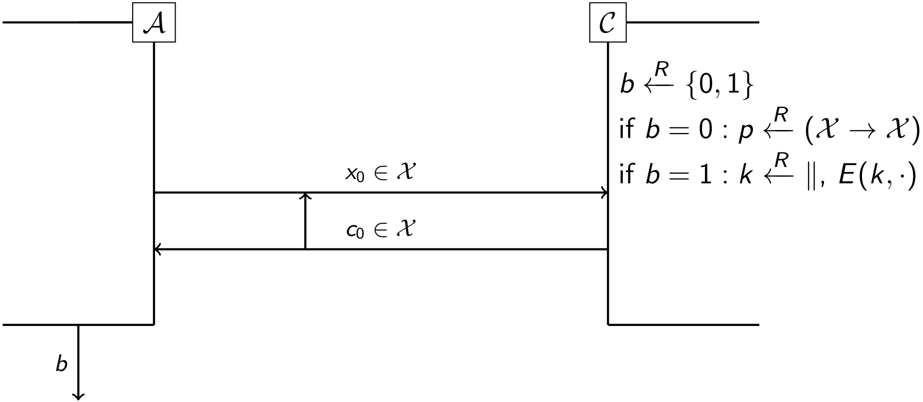

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Security Lemma

- a secure PRP is equivalent to a secure PRF

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- Shares the correctness requirement with Shannon Ciphers $D(k, E(k, m)) = m$

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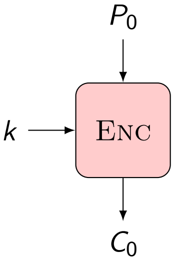

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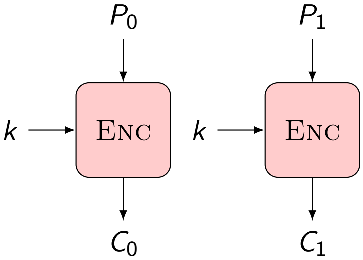

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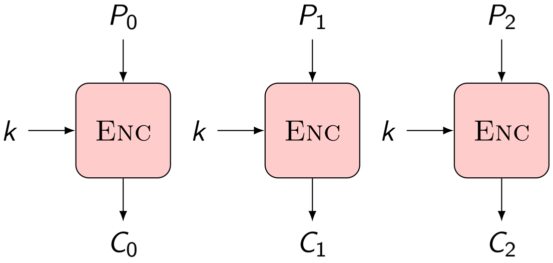

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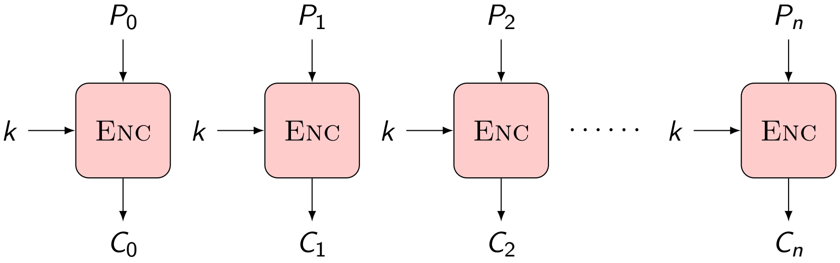

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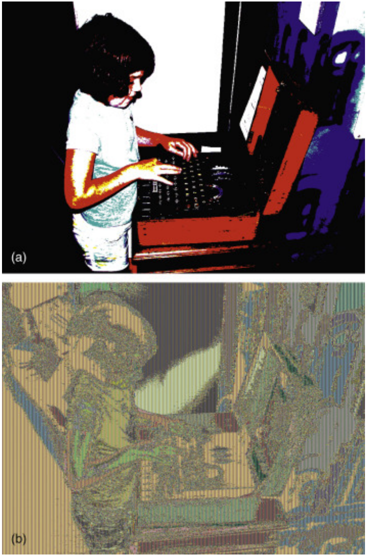
Future HW: describe an attack to break CPA given ECB

Image Encryption using ECB



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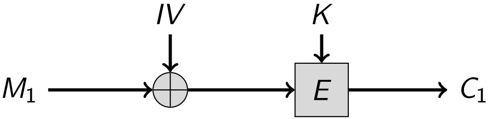

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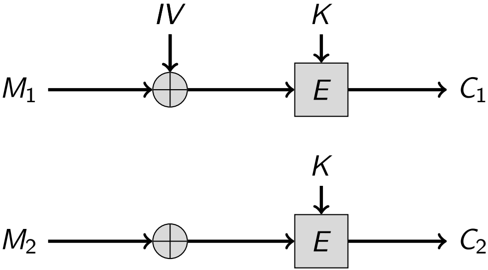

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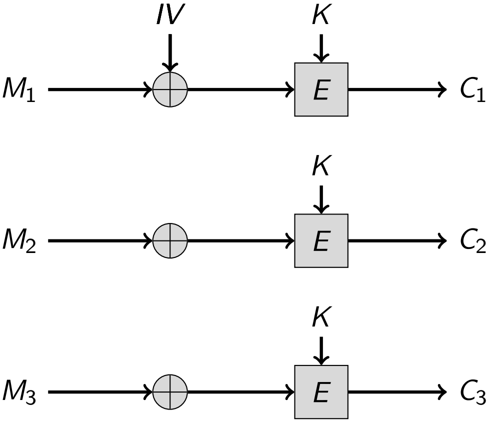

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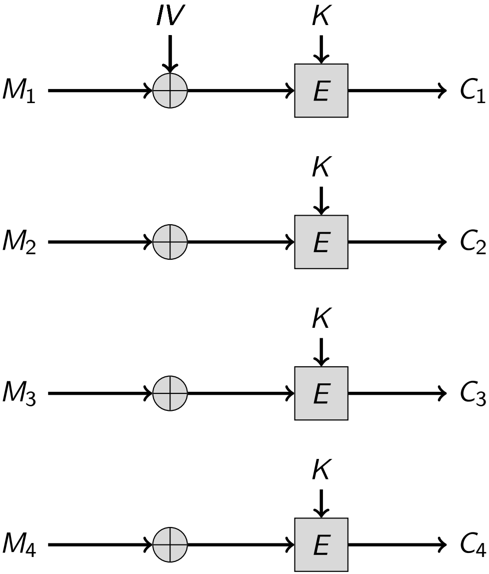

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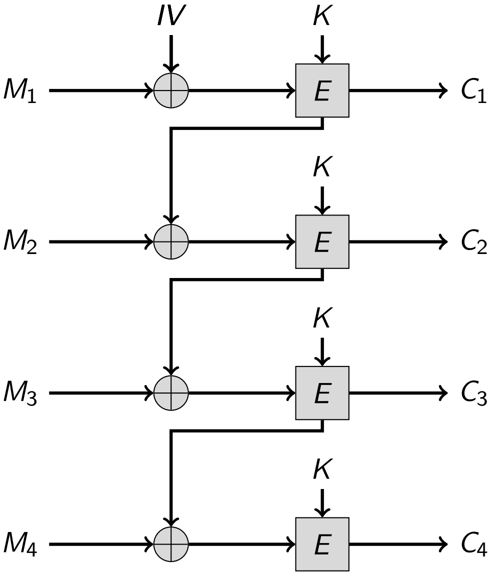

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- It is best to use a random IV every message and send it with the cipher text

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