

Sumatorias

Sea la lista ordenada $\{a_1, a_2, \dots, a_n\}$ entonces:

$$\tilde{S} = a_1 + a_2 + \dots + a_n$$

$$S = \sum_{i=1}^n a_i = a_1 + a_2 + \dots + a_n$$

$$\tilde{S} = \sum_{i=5}^{10} b_i = b_5 + b_6 + b_7 + b_8 + b_9 + b_{10}$$

Sumatorias

Propiedades

S_5^{10} :

fórmulas elementales



Propiedades:

P1) Linealidad:

$$\sum_{i=1}^n a_i + k \cdot b_i = \sum_{i=1}^n a_i + k \sum_{i=1}^n b_i$$

$$\sum_{i=n}^m a_i = \sum_{i=1}^m a_i - \sum_{i=1}^{n-1} a_i$$

fórmulas elementales

$$f_1) \sum_{i=1}^n k = k + k + k + \dots + k = k \cdot n$$

$$f_2) \sum_{i=1}^n i = 1 + 2 + 3 + \dots + n-1 + n = \frac{n \cdot (n+1)}{2}$$

$$f_3) \sum_{i=1}^n i^2 = 1^2 + 2^2 + 3^2 + \dots + (n-1)^2 + n^2 = \frac{n \cdot (n+1)(2n+1)}{6}$$

$$f_4) \sum_{i=1}^n i^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n \cdot (n+1)}{2} \right]^2$$

Ejemplo: Calcular la suma de

$$\bullet \sum_{i=1}^{20} (2i+3) = 2 \sum_{i=1}^{20} i + \sum_{i=1}^{20} 3 = 2 \cdot \frac{20 \cdot (20+1)}{2} + 3 \cdot 20$$

P1 f2 f1

$$= 20 \cdot 21 + 60$$
$$= 420 + 60 = 480.$$

Ejercicios

Para calcular

Determine el valor de $\sum_{i=5}^{15} i \cdot (i - 3)$.

$$\sum_{i=5}^{15} i \cdot (i - 3) = \sum_{i=1}^{15} i \cdot (i - 3) - \sum_{i=1}^4 i \cdot (i - 3)$$

$$S = \sum_{i=1}^{15} (i^2 - 3i) - \sum_{i=1}^4 (i^2 - 3i)$$

$$S = \sum_{i=1}^{15} i^2 - 3 \cdot \sum_{i=1}^{15} i - \sum_{i=1}^4 i^2 + 3 \sum_{i=1}^4 i$$

$$S = \frac{15 \cdot (15+1) \cdot (2 \cdot 15+1)}{6} - 3 \cdot \frac{15 \cdot (15+1)}{2} - \frac{4 \cdot (4+1) \cdot (2 \cdot 4+1)}{6} + 3 \cdot \frac{4 \cdot (4+1)}{2}$$

$$S = \frac{15 \cdot 16 \cdot 31}{6} - 3 \cdot \frac{15 \cdot 16}{2} - \frac{4 \cdot 5 \cdot 9}{6} + \frac{3 \cdot 4 \cdot 5}{2}$$

$$S = 1.240 - 360 - 30 + 30$$

$$S = 880$$

Si, para todo $n \in \mathbb{N}$, $\sum_{i=1}^m a_i = 2m^2 + 3$, calcule el valor de $\sum_{i=n+1}^{2n} a_i$.

$$\begin{aligned}\sum_{i=n+1}^{2n} a_i &= \sum_{i=1}^{2n} a_i - \sum_{i=1}^n a_i \\&= \left[2 \cdot (2n)^2 + 3 \right] - \left[2n^2 + 3 \right] \\&= 2 \cdot 4n^2 + 3 - 2n^2 - 3 \\&= 8n^2 + 3 - 2n^2 - 3 \\&= 6n^2 //\end{aligned}$$

