Exponentially Weighted Moving Average (EWMA)

An Exponentially Weighted Moving Average (EWMA) is a method used to smooth time series data by assigning exponentially decreasing weights to past observations. This technique helps emphasize recent data points while still considering the influence of older data.

Mathematical Formulation

The formula for EWMA is:

$$S_t = \alpha X_t + (1 - \alpha) S_{t-1}$$

Where:

- S_t: Smoothed value at time t
- α: Smoothing factor ($0 < \alpha \leq 1$)
- X_t: Actual observation at time t
- S_{t-1}: Smoothed value at time $t - 1$

Properties

- The smoothing factor α determines the weight of recent versus older observations.
  - A higher α gives more weight to recent data, making the series more responsive to changes.
  - A lower α smoothens the series more but is less responsive to changes.
- For S_t, the initial value S_0 is usually set as the first observation, X_0, or the mean of the series.

Example

Data:

Let's assume the following time series data for sales over 7 days:

$$X = [50, 55, 60, 65, 70, 75, 80]$$

Smoothing Factor:

Let $\alpha = 0.3$.

Calculations:

1. Set $S_0 = X_0 = 50$.
2. Apply the formula iteratively:

   $$S_1 = \alpha X_1 + (1 - \alpha) S_0$$

   For $t = 1$:

   $$S_1 = 0.3(55) + 0.7(50) = 16.5 + 35 = 51.5$$

   For $t = 2$:

   $$S_2 = 0.3(60) + 0.7(51.5) = 18 + 36.05 = 54.05$$

   Continue similarly for all time steps:

   - $S_3 = 0.3(65) + 0.7(54.05) = 55.83$
   - $S_4 = 0.3(70) + 0.7(55.83) = 58.08$
   - $S_5 = 0.3(75) + 0.7(58.08) = 61.16$

- $S_6 = 0.3(80) + 0.7(61.16) = 65.21$

Smoothed Values:

$$S = [50, 51.5, 54.05, 55.83, 58.08, 61.16, 65.21]$$

How EWMA Helps in Smoothing

1. **Noise Reduction**: EWMA reduces the effect of random fluctuations by averaging the data with more weight given to recent points.
2. **Trend Capture**: By controlling α, EWMA captures trends in the data:
   - High α emphasizes changes quickly.
   - Low α emphasizes long-term trends.
3. **Exponential Decay**: The weights decrease exponentially as you move further back in time:

   $$\text{Weight for } X_{t-k} = \alpha(1-\alpha)^k$$

   Thus, recent points have the highest influence.

Visualization of Smoothing

If plotted, the smoothed curve S will appear less jagged than the original data X, illustrating the smoothing effect.

Applications

- **Time Series Analysis**: EWMA is widely used in stock price analysis, weather trends, and sales forecasting.
- **Control Charts**: Used in quality control to detect small shifts in process mean.
- **Signal Processing**: Helps in denoising signals.

Python Code for EWMA

python

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```
import pandas as pd import numpy as np import matplotlib.pyplot as plt # Data data = [50, 55, 60, 65, 70, 75, 80] alpha = 0.3 # Calculate EWMA ewma = pd.Series(data).ewm(alpha=alpha, adjust=False).mean() # Plot plt.plot(data, label='Original Data', marker='o') plt.plot(ewma, label='EWMA (α=0.3)', marker='o') plt.title('Exponentially Weighted Moving Average') plt.legend() plt.show()
```

This will produce a plot showing the original data and the smoothed EWMA curve.