Let's go through the process of calculating weights using **gradient descent** for **4 samples**.

We will:

1. Define our example data.
2. Calculate the predicted price, error, and gradients.
3. Update the weights using gradient descent for each sample.
4. Repeat for multiple iterations until MSE is minimized.

Step 1: Define the Dataset and Model

Dataset

We are predicting house prices using two features:

- Area (in square feet)
- Bedrooms (number of bedrooms)

Let the dataset consist of 4 samples:

Area (x_1)	Bedrooms (x_2)	Actual Price (y)
2600	3	550000
3000	4	565000
3200	3	610000
3600	5	680000

Model

The model equation:

$$\text{Predicted Price} = w_1 \cdot \text{Area} + w_2 \cdot \text{Bedrooms} + \text{bias}$$

Initial values:

- $w_1 = 0.5, w_2 = 0.5, \text{bias} = 0$
- Learning rate (α) = 0.000000001

Step 2: Perform Gradient Descent

We'll calculate the following for each sample:

1. Predicted price
2. Error
3. Gradients for w_1, w_2, bias
4. Update weights using gradient descent.

Iteration 1

Sample 1:

- Area = 2600, Bedrooms = 3, Actual Price = 550000

1. **Predicted Price**:

$$\text{Predicted} = w_1 \cdot 2600 + w_2 \cdot 3 + \text{bias} = 0.5 \cdot 2600 + 0.5 \cdot 3 + 0 = 1301.5$$

2. **Error**:

$$\text{Error} = \text{Predicted} - \text{Actual} = 1301.5 - 550000 = -548698.5$$

3. **Gradients**:
   - For w_1:

$$\frac{\partial MSE}{\partial w_1} = 2 \cdot \text{Error} \cdot \text{Area} = 2 \cdot (-548698.5) \cdot 2600 = -2853230200$$

   - For w_2:

$$\frac{\partial MSE}{\partial w_2} = 2 \cdot \text{Error} \cdot \text{Bedrooms} = 2 \cdot (-548698.5) \cdot 3 = -3292191$$

   - For bias:

$$\frac{\partial MSE}{\partial \text{bias}} = 2 \cdot \text{Error} = 2 \cdot (-548698.5) = -1097397$$

4. **Update Weights**:
   - $w_1 = w_1 - \alpha \cdot \frac{\partial MSE}{\partial w_1} = 0.5 - 0.000000001 \cdot (-2853230200) = 0.50000285323$
   - $w_2 = w_2 - \alpha \cdot \frac{\partial MSE}{\partial w_2} = 0.5 - 0.000000001 \cdot (-3292191) = 0.50000032922$
   - $\text{bias} = \text{bias} - \alpha \cdot \frac{\partial MSE}{\partial \text{bias}} = 0 - 0.000000001 \cdot (-1097397) = 0.0010974$

Sample 2:

- Area = 3000, Bedrooms = 4, Actual Price = 565000

1. **Predicted Price**:

$$\text{Predicted} = w_1 \cdot 3000 + w_2 \cdot 4 + \text{bias} = 0.50000285323 \cdot 3000 + 0.50000032922 \cdot 4 + 0.0010974 = 1500.0198$$

2. **Error**:

$$\text{Error} = \text{Predicted} - \text{Actual} = 1500.0198 - 565000 = -563499.98$$

3. **Gradients**:
   - For w_1:

$$\frac{\partial MSE}{\partial w_1} = 2 \cdot \text{Error} \cdot \text{Area} = 2 \cdot (-563499.98) \cdot 3000 = -3380999880$$

   - For w_2:

$$\frac{\partial MSE}{\partial w_2} = 2 \cdot \text{Error} \cdot \text{Bedrooms} = 2 \cdot (-563499.98) \cdot 4 = -4507999.84$$

   - For bias:

$$\frac{\partial MSE}{\partial \text{bias}} = 2 \cdot \text{Error} = 2 \cdot (-563499.98) = -1126999.96$$

4. **Update Weights**:
   - $w_1 = 0.50000285323 - 0.000000001 \cdot (-3380999880) = 0.50000623432$
   - $w_2 = 0.50000032922 - 0.000000001 \cdot (-4507999.84) = 0.50000078002$
   - $\text{bias} = 0.0010974 - 0.000000001 \cdot (-1126999.96) = 0.0022244$

Repeat for All Samples

For each sample, repeat the steps: calculate the predicted price, compute the error, find gradients, and update the weights. After all samples, one iteration is complete. Repeat the process for multiple iterations.

Step 3: Summary of Results

After multiple iterations, the weights and bias will converge to values that minimize the MSE. This iterative process adjusts the weights to make the predicted price closer to the actual price.