

Neural Network Parameters: **Weights, Biases, and Total Parameters**

1. Basic Terminology

Before diving into calculations, let's understand the fundamental concepts:

- **Neuron:** The basic unit in a neural network that receives inputs, applies a weighted sum, adds a bias, and passes the result through an activation function.
 - Formula: $z = w_1x_1 + w_2x_2 + \dots + w_nx_n + b$
 - w_i : Weight for input x_i
 - b : Bias for the neuron
- **Layer:** A collection of neurons operating together. Layers are categorized as:
 - **Input Layer:** Where the data enters the network (no weights/biases here).
 - **Hidden Layers:** Intermediate layers between input and output layers.
 - **Output Layer:** Produces the final prediction/output.
- **Parameters:** Trainable variables in the network.
 - **Weights:** One for each connection between neurons in consecutive layers.
 - **Biases:** One bias for each neuron (except input neurons).

2. Calculating Parameters

To calculate the **total number of parameters** in a neural network, we need to compute weights and biases layer by layer.

Weight Calculation

The number of weights is determined by the connections between neurons in two consecutive layers:

$$\text{Number of weights} = (\text{Number of neurons in previous layer}) \times (\text{Number of neurons in current layer})$$

Bias Calculation

Each neuron (except input neurons) has one bias associated with it:

$$\text{Number of biases} = (\text{Number of neurons in current layer})$$

Example Neural Network

Suppose we have the following architecture:

1. **Input Layer:** 4 neurons (x_1, x_2, x_3, x_4)
 2. **Hidden Layer 1:** 5 neurons (h_1, h_2, h_3, h_4, h_5)
 3. **Hidden Layer 2:** 3 neurons (h_6, h_7, h_8)
 4. **Output Layer:** 2 neurons (y_1, y_2)
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Step-by-Step Calculation

Layer 1 (Input → Hidden Layer 1)

- Weights:
 - Connections between 4 input neurons and 5 hidden neurons:
$$4 \times 5 = 20$$
- Biases:
 - 1 bias for each of the 5 neurons in Hidden Layer 1:
$$5$$

Layer 2 (Hidden Layer 1 → Hidden Layer 2)

- Weights:
 - Connections between 5 neurons in Hidden Layer 1 and 3 neurons in Hidden Layer 2:
$$5 \times 3 = 15$$
- Biases:
 - 1 bias for each of the 3 neurons in Hidden Layer 2:
$$3$$

Layer 3 (Hidden Layer 2 → Output Layer)

- Weights:
 - Connections between 3 neurons in Hidden Layer 2 and 2 neurons in the Output Layer:
$$3 \times 2 = 6$$
- Biases:
 - 1 bias for each of the 2 neurons in the Output Layer:

Total Parameters

Add all the weights and biases across layers:

$$\text{Total Weights} = 20 + 15 + 6 = 41$$

$$\text{Total Biases} = 5 + 3 + 2 = 10$$

$$\text{Total Parameters} = 41 + 10 = 51$$

General Formula for Total Parameters

For a neural network with L layers:

1. **Weights:**

$$\text{Total Weights} = \sum_{i=1}^L (\text{neurons in layer } i) \times (\text{neurons in layer } (i + 1))$$

2. **Biases:**

$$\text{Total Biases} = \sum_{i=1}^L (\text{neurons in layer } (i + 1))$$

3. Total Parameters:

$$\text{Total Parameters} = \text{Total Weights} + \text{Total Biases}$$

Conceptual Notes

- **Increasing Parameters:** Adding more neurons or layers increases the parameters, which makes the model more expressive but risks overfitting.
- **Bias Role:** Bias shifts the activation function, enabling the network to model data better.
- **Weight Updates:** During training, weights and biases are adjusted using optimization algorithms (e.g., Gradient Descent).