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PROBING THE SPIN STRUCTURE OF THE PROTON USING POLARIZED
PROTON-PROTON COLLISIONS AND THE PRODUCTION OF W-BOSONS

A Dissertation submitted in partial satisfaction
of the requirements for the degree of

Doctor of Philosophy

in

Physics

by

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Acknowledgments

Advisors and Mentors are some of the most important people any scientist will encounter in their professional career. Time and again, I have heard colleagues speak of "that one inspirational" person that drove them to be their best, and knew how to "grow" a researcher.

I am very greatful to my advisor, Ken Barish, whose calm, stoic and unabated support helped guide me through my research. Ken involved me in many aspects of the research group at UCR, beyond the scientific work. He insured that I was exposed to all aspects of research in particle physics, including writing grants, reviewing literature, mentoring younger students, building detectors, running a particle accelelrator detector, and of course, data analysis. Ken has always had the uncanny ability to know "who to talk to" for nearly any problem I might have. Ken connected me with other excellent physicists, who helped me grow as a researcher, and he gave me the freedom I needed to pursue my interests, and move in the scientific directions I felt most fruitful, while helping to provide an overall direction for my academic career and research.

Beyond all this, the single most important thing Ken has done for me, is to give me a second chance in graduate school. When he accepted me into his group, I was an undoubtedly risky choice. I struggled mightily my first year in grad school. I earned poor grades, and even had to re-take a class. In fact, my performance was so poor, that my teaching responsibilities were reduced, and eventually, I lost my graduate division fellowship, which ultimately meant that I had no income, or means of supporting myself; I was effectively dismissed from graduate school. However, I was interested in the research carried out by Ken and Rich Seto's heavy ion group, so I talked to Ken, who graciously accepted me into the group, provided me with academic and financial support, and even flew me out to Brookhaven National Lab my first summer of graduate school. I finally got to dive into 'real' physics research. I think it was this vote of confidence from Ken, as well as the awesome physics happening at the PHENIX experiment which gave me the confidence to wholeheartedly devote myself to my studies and research. Without Ken's vote of confidence, I fear that my graduate career would have been over in short order.

While at Brookhaven National Lab, I encountered graduate students, post docs, research staff, and other amazing physicists who taught me an incredible amount, and showed both patience, kindness, friendship and mentorship to me. Richard Hollis was one of the first people I encountered in my research group at UCR - I have never met a more

patient person. Richard helped me get my bearings, and set me straight, during my early (and later) years of graduate school. Oleg Eyser was with our group at that time as well - although I recall that he was less than thrilled to have yet another green graduate student constantly asking questions, taking time away from his work. He still made time to teach me, and introduced me to the very complicated PHENIX software system. Oleg challenged me, and expected me to find answers for myself, and was unrelenting in that regard, which I am certain made me a better researcher.

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I have to acknowledge Joe Seele as well, in this regard, as he probably more than anyone else, set me on the path to learning to program well, and using a computer effectively - these skills, so often neglected in Particle Physics, have paid off for me, many, many times over.

W Analysis Crew

Ralf Seidl, Francesca Giordano, Sangwha Park, Daniel Jumper, Abraham Meles, Chong Kim,

Friends and Family

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Bob Beaumier, Marian Beaumier, Joe Beaumier, David Beaumier, Emily Vance, Jackie Hubbard, Alexander Anderson-Natalie, Corey Kownacki, Chris Heidt, Pat Odenthal, Behnam Darvish Sarvestani, Oleg Martynov,

Some say that it takes a village to raise a child. The same can be said of raising a graduate student up to earning a PhD. This thesis is dedicated to the multitude who have helped me become the man I am today, and to students who struggle, and their mentors who do not give up on them.

ABSTRACT OF THE DISSERTATION

PROBING THE SPIN STRUCTURE OF THE PROTON USING POLARIZED PROTON-PROTON COLLISIONS AND THE PRODUCTION OF W-BOSONS

by

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Doctor of Philosophy, Graduate Program in Physics
University of California, Riverside, August 2016
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This thesis discusses the process of extracting information about the spin structure of protons, specifically, spin contributions from the sea of quarks and anti-quarks, which are kinematically distinct from the 'valence quarks'. We have known since the 'proton-spin crisis' [1] of the late 1980s that proton spin does not entirely reside in the valence quarks, so the thrust of experimental efforts since then have been designed to determine both how to probe the proton spin structure, and how to validate models for proton spin structure. Here, I discuss one particular approach to understanding the sea-quark spin contribution, which utilizes the production of real W -bosons, and the W coupling with polarized spin structure in the proton sea, as produced from polarized proton-proton collisions. Only one of the colliding protons is longitudinally spin polarized, in this analysis, and they are collided at an energy of $500GeV$. The experimental observable used is referred to as " A_L " which is expressed mathematically as a ratio of sums and differences of various helicity combinations of singly polarized interactions between two protons, i.e. $p + p^\Rightarrow \rightarrow W \rightarrow \mu + \nu$. Once A_L has been experimentally measured, it can then be used to determine appropriate polarizations of proton sea-quarks, within a given uncertainty, if we write the cross-sections used in the calculation of A_L in terms of polarized parton distribution functions. Finally, this thesis will also include a discussion of my work experimentally determining the absolute luminosity of collisions at RHIC, which is needed as a normalization on any cross section used in the analysis. In particular, studying the cross section of the W interaction can help to validate our models for assigning a signal-to-background ratio to the $W \rightarrow \mu$ events.

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Chapter 1

Introduction

A brief note - figures used here without attribution were either: produced by me, produced in collaboration with others in my working group, or obtained by authors who labeled them for reuse without attribution. Other figures here are all fair-use.

THIS THESIS IS CURRENTLY A DRAFT. IT HAS NOT BEEN SUBMITTED AND HAS MISSING CITATIONS AND ATTRIBUTIONS.

1.1 A Brief History of the Proton

The angular momentum of the proton and neutron has been a subject of study for the last 20 years [CITATION NEEDED]. One of the challenges of particle physics is to create a framework which can accurately describe matter, as well as predict the behavior of matter at all energy scales. Protons and neutrons are baryons which make up the majority of the mass in the visible universe, yet fully understanding the origins of their properties - such as mass and spin, still eludes us. However, through the application of the scientific method over many generations of physicists, we have magnificently described this important particle, and understood much of its properties. However, one property which still defies our descriptions is its fundamental angular momentum, spin.

Our understanding of the proton has evolved and sharpened since the first experiments in deep inelastic scattering showed that the proton is not a fundamental particle [23]. Gell-Mann later planted the seeds of a theoretical framework which could in part describe some of the structure of baryons, a class of hadrons which we may naively describe

as composed of three ‘valence quarks’ [CITATION NEEDED]. We can apply well known spin-sum rules to the individual spins of the valence quarks which compose the proton in our naive valence-model to produce a correct prediction for the protons’ spin $\frac{1}{2}$. When experimenters set out to measure the contribution of these valence quarks in 1988 at the EMC experiment [1], they were flabbergasted to find that the valence quarks carry only a small fraction of the proton’s spin. Although recent papers [24] suggest that this ‘spin crisis’ is simple due to mis-attribution of spin, most literature to date has focused on understanding how to model the proton with parton distribution functions. These parton distribution functions come in many varieties, and probe different degrees of freedom within the proton, in both the case of unpolarized parton distribution functions, and polarized parton distribution functions.

1.2 Scope and Objectives of This Work

This thesis will describe the research I carried out between May of 2010 through August of 2016. I will often quote work that was carried out in active collaboration with Ralf Seidel, Francesca Giordano, Daniel Jumper, Sanghwa Park, Abraham Meles and Chong Kim. Daniel, Abraham, Ralf, Francesca, and myself all worked on the 2013 polarized proton data set taken at RHIC with PHENIX. This analysis comprises the body of work devoted to calculating A_L for the $W \rightarrow \mu$ decay. Since 2013, the five of us collaborated closely on all aspects of the work, which provided invaluable cross-checks at nearly every stage. Many of the figures in this document were produced by our collective efforts, and I will do my best to cite when possible, if one analyzer played a particularly large role in generating the data or visualization, however after several years of working together, I will certainly fail to attribute, or mis-attribute at times.

The other portion of this thesis will discuss the Vernier Analysis, which is instrumental for every single-cross-section calculation taken with RHIC data. The thrust of the Vernier Analysis is to determine the beam luminosity at PHENIX’s interaction point, so as to normalize these cross-section calculations. This is done with a series of specialized Vernier-Scans, where beams are scanned across one-another in order to measure beam geometry. The luminosity can then be calculated from first principals, and compared to the advertised machine luminosity published by RHIC’s collider-accelerator department. I be-

gan working with the Vernier Analysis under the tutelage of K. Oleg Eyser, but eventually moved to work independently on the analysis, producing an entire software framework for handling data cleaning, analysis, visualization and simulation.

Chapter 2

Historic Perspective on the Structure of Matter and Spin

2.1 The Phenomena of Spin

Spin is a fundamental quantity possessed by all elementary particles. We use the word 'spin' to describe the property, because particles which possess spin, behave as though they have some kind of intrinsic, hidden rotation, as if they were 'spinning'. The dimension of spin, therefore is angular momentum. What is somewhat bizarre about spin, is that we do not observe anything physically spinning - although there are some phenomena (such as orbital angular momenta) which can be naively thought of as a 'spinning system' (but this description escapes classical analogy, due to its quantum, probabilistic nature). The role of Spin in Physics is of foundational importance, and yet, we have not successfully produced a model which can accurately predict the spin of hadrons.

The presence of spin in relativistic particles creates the phenomena of chirality, which has huge implications for how elementary particles can generate structure in matter itself [CITATION NEEDED]. In the case of the weak interaction, the presence of spin, which creates Chiral spinors breaks the left-right symmetry of weak coupling in matter (a fact which will be exploited in this thesis to probe the spin of the proton sea).

The phenomena of spin also changes the rules for how ensembles of particles may exist in a potential. Particles with spin are fermions, and because these particles must obey Fermi-statistics, we can observe structure in matter in the universe [CITATION

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NEEDED]. Without spin, the world as we know would collapse on itself, making any kind of extended non-exotic structures which currently exist by virtue of the Pauli exclusion principal, impossible.

2.2 A Brief History of Relevant Physics

The study of Spin is really just an outgrowth of the general study of matter. Our models for matter, and the underlying structure of matter (in the modern sense), represents over a hundred years of experimental and theoretical efforts, and thousands of years of contemplating what makes up the universe.

Although indulgent on my part, I find it interesting, and humbling, to try and map out the path that humanity and science has trodden on its way to understanding the building blocks of the universe. To find the first time that humanity had murmurings that suggested our visible world is built from invisible, fundamental building blocks, we must travel back, nearly 2,500 years into the past.

Rather than provide a complete mathematical background to my measurement from a historic perspective, I will instead focus on the experimental historic narrative surrounding our quest to understand the structure of matter. After, I will present mathematical formalism relevant to this measurement directly - if the reader desires an exhaustive mathematical context, I invite them to read the classic tomes on Field Theory by Weinberg and reference the numerous theses written by my colleagues in theoretical physics and phenomenology.

2.2.1 Ancient Foundations

Sometime around 490 - 370 BCE lived two philosophers, Empedocles (Fig 2.1a), and Democritus (Fig 2.1b). Both men lived approximately at the same time, and made huge philosophical leaps in attempting to understand the nature of the visible world.

Democritus was part of a movement of thought which was first to make the intellectual jump that perhaps matter was not a continuum, but instead, composed of 'atomon', small, indivisible particles which when configured together, created all that is observable [CITATION NEEDED]. Empedocles was making equally important philosophical strides - in a manner complimentary to Democritus' opinion that matter must be made of atomon, Empedocles argued that matter is composed of elemental primitives [CITATION NEEDED].

Although Empedocles' 'periodic table' was only composed of Earth, Water, Fire, and Air, the idea that some unseen transmutation of elemental forces might generate ob-



(a) Empedocles [25]



(b) Democritus [26]

Figure 2.1: Two Greek philosophers, who made important philosophical contributions our understanding of matter. Empedocles (left), postulated the precursor to the elemental theory of matter [CITATION NEEDED] and Democritus (right), postulated the precursor to the atomic theory of matter.

servables in nature with quite different (but perhaps reminiscent) properties than the 'pure substances' was an important step forward. Proto-scientists were beginning to generate models which derived our complicated observations, from simpler forms.

It took centuries of cultivation, leading up to the Scientific Revolution, for the next great steps to occur, for science. Thankfully, the luminaries of the Islamic Golden Age kept the fires of inquiry burning [CITATION NEEDED].

2.2.2 The Scientific Revolution

Thanks to the mathematical foundations laid out, build, and maintained by the minds of the Islamic Golden Age, Europe was well poised to reignite the flames of scientific inquiry, during the post Renaissance Scientific Revolution [CITATION NEEDED].

This period of growth in science was unprecedented during the Scientific Revolution, thanks to the seeds of empiricism germinated during the Islamic Golden Age, fertilized by the Italian Renaissance, and helped to flourish through British Empiricism [CITATION NEEDED].



(a) Galileo [27]



(b) Newton

Figure 2.2: Giants in the age of Empiricism, Newton (left) and Galileo (right) both made foundational contributions to Physics. Galileo lived in Italy, born in 1564 and dying in 1642. Newton lived in England from 1642 until his death in 1727

2.2.2.1 Galileo Galilei

While Galileo is best known for his work in Observational Astronomy, his importance to science extends beyond this. During his years in exile for his controversial views of the heliocentric universe, he produced some of his most important scientific work in kinematics [CITATION NEEDED]. What made this work remarkable is the care that Galileo took in merging careful mathematical modeling with well designed experimentation. This methodical approach to inquiry laid the foundation for others to slowly begin to pull back the curtains obscuring physical law.

Galileo's formalization of the scientific method inexorably set science on a course to delving deep into the nature of matter, and the laws of nature.

2.2.2.2 Isaac Newton

Fittingly born in the same year as Galileo's death, Isaac Newton would carry on Galileo's legacy of rigorous mathematical modeling mixed with experimentation. Perhaps no other scientist has touched so many different aspects of physics, from theories of propagation of light, to celestial mechanics, to mathematics, and kinematics.

Newton's Principia is perhaps the most important scientific work ever published. It opened the doors of the universe in a way that nobody has since duplicated. Newton's laws of motion are still taught in school today, and although they have since been shown to be inaccurate at the smallest and largest scales, they still provide startlingly accurate predictions for the regular motion of matter.

One particularly tantalizing theory of Newton's was the corpuscular theory of light. Although not his most influential theory by far, the idea that an apparently continuous medium such as a beam of light might be made of small packets of energy (corpuscles) turned out to be partially right [CITATION NEEDED].

Newton's theories, and contributions to science are enormous, and have moved us deeper still into the underpinnings of matter. It would not be until roughly 200 years after his death, in the 19th century, that we finally can take the first steps into the world of the atomic, and sub-atomic: the world of the proton.

2.2.3 Atomic Theory

On the shoulders of giants such as Newton and Galileo, science finally came to know the tool which has been indispensable to modern particle physics: scattering. Rutherford and Thompson both carried out the most important scattering experiments in modern science, and provided us with the first hints of a hidden, quantum world, though it would not be until the 20th century that these important experiments would be fully contextualized with a theory of quantum scattering.

Scattering experiments offer a very powerful method where we one uses a well known initial state of matter (typically in the form of a beam), allows this beam to interact with an unknown configuration of matter, and measures the scattered beam. By carefully studying the kinematics of the scattered beam, we can create models which allow us to understand the structure of the target matter or describe the nature of the interaction between the beam and target.

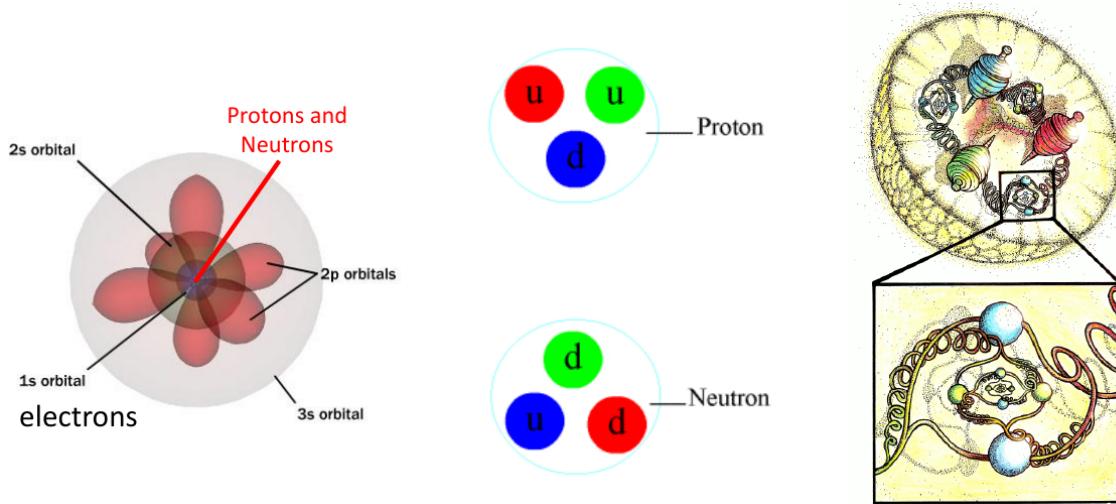


Figure 2.3: As we journey down further in scale, matter begins to look quite different. In fact, the models we use are scale dependent. Thomson 2.8, and Rutherford 2.5 began to see matter as collections of atoms (left) [2] (though not in terms of the orbital structure pictured), though it would not be until 20th century quantum mechanics that electron orbitals were discovered. Soon, nuclei were discovered to be divisible into protons and neutrons [3] (center), which in turn were discovered to be composed of a sea of quarks and gluons (right). (Right image drawn by the talented Astrid Morreale, PhD, [4])

2.2.3.1 John Dalton

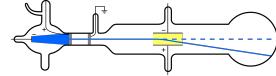
While many had postulated the existence of atoms, the first evidence based theory which suggested the existence of atoms was produced by John Dalton in the early 19th century. Dalton made an important conceptual leap to relate the existence of stoichiometric ratios in chemistry to the presence of small, individual functional units in his experiments with chemical reactions. Dalton's realization was only made possible due to his careful accounting of reactants in his experiments.

It was not until Einstein's 1905 theory on Brownian Motion was experimentally verified by Jean Perrin to place limits on the mass and size of atoms that Dalton's atomic theory was ultimately vindicated [28].

2.2.3.2 J.J. Thompson



(a) J.J. Thomson [29]



(b) Cathode Ray Tube [30]

Figure 2.4: Left: J.J. Thomson, who showed that cathode ray tubes were in fact producing the first observed subatomic particle: the electron. Right: A cartoon of Thomson's cathode ray tube setup. Electrons would be deflected by a magnetic field, sent from cathode to anode.

Thomson (Figure 2.4) would discover that atoms are not the smallest, indivisible piece of matter. In his landmark experiment, he used cathode ray scattering experiments to show that cathode rays were in fact subatomic particles. He showed these cathode rays were identical to particles given off by the photoelectric effect, and that these same particles were responsible for electric current. He had discovered the electron. And, if atoms were not the smallest piece of matter, then perhaps, atoms themselves might not be 'indivisible' as previously thought [31].

2.2.3.3 Ernest Rutherford

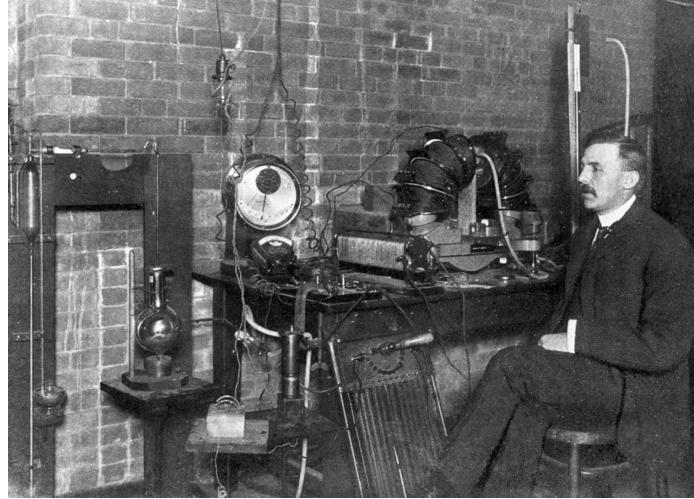


Figure 2.5: Ernest Rutherford, in his lab. [5]

Ernest Rutherford (Fig 2.5) was the first to show that atoms themselves were highly structured - and consisted of a small dense center, later called the nucleus.

Rutherford's work with radioactivity was of fundamental importance, he discovered and classified both alpha-particle radioactivity and beta-particle radioactivity. Further studies into these types of nuclear radiation would unlock the nucleus of atoms through the work of future scientists. Notably, Rutherford discovered the proton.

Rutherford's proposed planetary model for the nucleus, while technically wrong, shifted paradigms from the pudding model of atoms, to the more familiar nucleus + electron cloud model which has been spectacularly modeled and verified with the forthcoming scientists which defined the field of quantum mechanics.

Rutherford's work helped push us out of the cocoon of classical mechanics into the weird world of the quantum mechanics - scientists would soon find that the nucleus is not just a dense concentration of charge, but a probabilistic structure, with rich sub nuclear structure.

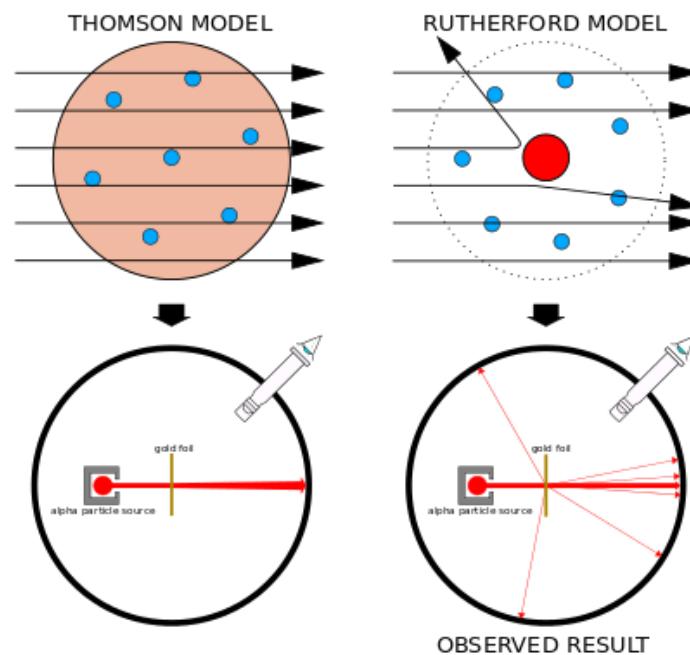


Figure 2.6: Ernest Rutherford's historic experiment, showing (top right) that atoms were composed of a small dense nucleus, in contrast to Thomson's 'pudding model' of homogeneous charge (top left). The experiment, (bottom left and right) contrast the expected results (bottom left) against the observed results (bottom right) [6].



Figure 2.7: The attendees of the Solvay Conference in Brussels, 1927 [7].

2.2.4 Early Quantum Theory

During Rutherford's time, experiments were already underway which were investigating modeling light as a wave phenomena. This was in contrast to Newton's (unverified) corpuscular theory of light. The argument whether light was wave-like or particle-like eventually lead to a classical field theory describing light, and the electromagnetic interaction, yet scientist such as Max Plank were proposing theories which required the quantization of light. **[CITATION NEEDED]**. Einstein would show that in his analysis of the photoelectric effect, that light indeed was quantized into 'corpuscles'. The nascent atomic theory of matter was also hinting at a hidden, quantized world.

At the Solvay Conference in Brussels, Figure 2.7, in 1927, we saw an unprecedented gathering of some of the most important figures in modern physics, all in one place, laying down the foundations of what would become quantum mechanics. These scientists defined the nature and rules of quantum mechanics - the weird model which accommodates a duality

of matter - both wave-like, and particle like. The notion that probing the structure of matter did not yield a simple, deterministic hierarchy of structure was revolutionary, confusing, and bizarre, and still is to this day.

It was found that not only light possesses this wave-particle duality, but also the very particles that make up atoms as well. These models were formalized by Dirac, Hilbert and Von-Neumann [CITATION NEEDED].

Though experiment tended to lead theory, regarding understanding the composition and rules of interactions in matter, in the mid 20th century, further refinements and additions to quantum mechanics gave birth to quantum field theory. While early quantum models were very successful at describing static particles trapped in static potentials - such as refining atomic theory to include predictions of observed atomic spectra, more work was needed to understand the relationship between electrical currents, light and magnetism. These concepts were all related by Maxwell [CITATION NEEDED] in the latter half of the 19th century, but did not make good predictions for systems in motion.

Dirac was first to create a model for describing the electron, its behavior in electromagnetic fields, and photon emission and absorption, under fully relativistic and quantum conditions [CITATION NEEDED]. Dirac's model was so successful, that it would become the basis for what we now call quantum electrodynamics. Much of the mathematical formalism has been reused to describe other field theories, which are the ultimate language which model and describe the structure of matter - including the insides of a proton.



(a) Paul Dirac, 1933 [32]

$$\left(\beta mc^2 + c \left(\sum_{n=1}^3 \alpha_n p_n \right) \right) \psi(x, t) = i\hbar \frac{\partial \psi(x, t)}{\partial t} \quad (2.1)$$

(b) The Dirac Equation

Figure 2.8: Paul Dirac, next to his original formulation of the Dirac Equation, describing the wave function for an electron with rest-mass m , in terms of its spacetime coordinates.

Dirac's work also began to incorporate relativistic effects in his wave equations modeling the electron, as well as crucially incorporating the spin (I.e. Dirac Spinors) of

these particles, which were important for making precise predictions for atomic spectra [CITATION NEEDED].

By this time, the proton was already known to reside in the enigmatic nucleus of atoms, however, attempts to use Quantum Electrodynamics to describe the state of the nucleus failed - it was clear that there was a very strong force, holding together the protons of a nucleus tightly - far in excess of the electromagnetic repulsion felt by the positively charged particles. There was a completely different coupling strength between this apparent strong nuclear force, and the better known electromagnetic coupling. Further complicating an understanding of the nucleus is the fact that as the length scale of probing decreases, the energies probed increase, fundamentally making the nucleus a relativistic object. Experimental physics would once again, forge ahead, in attempting to understand the inner workings of the nucleus, in the time-honored tradition of performing scattering experiments.

2.2.5 Early Particle Physics and The Eightfold Way

The hydrogen atom, and its spectra was well modeled with quantum mechanics by the end of early 20th century, however attempts to study Helium were not as successful [CITATION NEEDED]. However, in 1932, when Chadwick turned a beam of helium particles (at that time only known as α particles) on a sample of Beryllium, he observed that neutral, non-ionizing, penetrating radiation was produced [33]. Photons were ruled out as possible candidates, leading to the discovery of the neutron. Protons and neutrons were hypothesized by Heisenberg to both be the same state of a new conceptual particle, the nucleon, [34]. In the same year, Anderson discovered the positron.

By 1934, Hideki Yukawa (Fig. 2.9) had created an effective field theory for interactions of 'elementary particles' (at this time, thought to be protons and neutrons). He predicted the existence of mesons, and wrote down an effective field theory which described how protons and neutrons bind together in the nucleus [35].

Though non-relativistic quantum mechanics was mostly complete by 1934, scientists were already hard at work incorporating relativistic corrections to the theory. Experiments with cosmic rays soon revealed the existence of muons and the first observation of mesons.

Three separate paths eventually lead to the development of particle accelerators, which are to date, the best mechanism we possess in physics to probe nuclear structure. These accelerators are an outgrowth of ever more intense Rutherford-style experiments, Tandem Van-Der-Graaf generators, resonant acceleration techniques, RF linacs, and betatron accelerators [36].

By the 1950's, a cornucopia of strange new particles had been discovered, both matter and antimatter. But scientists drove forward, deeper, yearning to discover what was fundamental. By the 50's, neutrinos had been proposed, as well as Kaons, Pions, and Lambdas. Physicists were doing nuclear chemistry, in a sense, attempting to work out how quickly some particles decayed, and what decays were allowed or forbidden - science entered an age of nuclear alchemy.

"Strange" particles were discovered (K and Λ), so called because in bevatron experiments, they were produced in great quantities, but were slow to decay, unlike the faster π decay. Gell-Mann proposed that this strangeness in matter was due to a new quantum number (he called it 'strangeness'). The name stuck. [37], [38], [33]



Figure 2.9: Hideki Yukawa, the first Japanese Nobel Laureate and publisher of influential research on the theory of mesons, and other elementary particles [8].

The introduction of new conserved quantities, and the vast proliferation of particles was in full swing - the subatomic world by the 1950's was confusing, and complex. In his book "The God Particle", Leon Lederman recalled his adviser, Enrico Fermi frustratedly remarking 'Young Man, if I could remember the names of these particles, I would have been a botanist'. At this time, in the mid 1950's, the number of mesons and baryons which had been discovered were at least in the dozens, if not more.

While the use of particle accelerators were speeding us along in our search for the structure of matter, one particular invention truly revolutionized the field - the bubble chamber (Figures 2.10 and 2.11.)

The bubble chamber was essentially a large vat of supercritical fluid which could easily be caused to boil with small perturbations. This feature was exploited, by positioning a bubble chamber in a magnetic field (to cause charged tracks to bend) near the interaction

point between a particle beam and a fixed target. The bubble chamber itself was sometimes the target - since a popular liquid to use was hydrogen.



Figure 2.10: An old bubble chamber, once used at Fermilab, [9]

Invented by Donald Glaser in 1952, the bubble chamber was 'perfected' by Luis Alvarez when he helped to develop a version which could be used with liquid hydrogen. Hydrogen was desirable as a substance due to its extremely simple structure, which supplied much cleaner results than other fillings, unlike the original filler, Ether.

Soon after the advent of bubble chambers, physicists were able to macroscopically image these new, exotic particles interacting with normal matter as well as decaying - and develop novel computer techniques to analyze and catalog the massive influx of data.

The break-through came in 1961, when Gell-Mann and Nishijima leveraged recognized the underlying symmetry of the interactions taking place, and created what would be known as 'the eightfold way'. This theory created a scheme for organizing the observed baryons and mesons according to their properties in groupings called "octets". These octets



Figure 2.11: An example of the photographs taken with a Bubble Chamber, in 1973. In this picture, we see a 300 GeV proton producing particles as it travels through a hydrogen-filled bubble chamber at fermilab [10].

were in fact representations of the elements of members of the $SU(3)$ group. Another way of stating this, is that Gell-Mann had discovered the underlying structure of flavor-symmetry between the three lightest quarks - u , d , and s . This work directly led to the development of the quark model of matter, the foundation of what would become the foundation of the standard model of particles. To date, the standard model is the most successful theory describing particles, and their interactions.

Gell-Mann's quark model soon made important predictions which were later verified, notably the Ω^- , which was the ground-state particle of the spin-3/2 decuplet - discovered at Brookhaven National Laboratory (the same lab from which my research has been derived!).

Gell-Mann formalized his quark theory of matter in 1964, however, due to the unforeseen phenomena of color confinement, it would be several years before evidence of quarks composing baryons and mesons was directly obtained from deep inelastic scattering experiments.

2.2.6 Deep Inelastic Scattering and The Parton Model

Deep inelastic scattering experiments, Figure 2.12 were a natural outgrowth of Rutherford's experiment from the late 19th century. There are a few notable differences. Rutherford's scattering experiments can be modeled classically, by using a classical potential as a scattering source, and then solving as usual using an impact parameter and potential as in central force problems. Rutherford's experiments were considered generally 'elastic' because the target absorbed very little kinetic energy from the projectile, and no new particles were created from the kinetic energy of the projectile-target system.

However, in the late 20th century, scattering experiments became highly inelastic - targets would absorb a lot of kinetic energy - sometimes so much that targets would break apart and the kinetic energy of the system would create particles. When referring to scattering as 'deep inelastic', the 'deep' part refers to the process by which a scattering event occurs between a target particle and an internal, pointlike element of some complex ensemble (such as a nucleus).

During the process of a high energy interaction between the projectile (often a beam) and the target, some kind of interaction occurs between the target and the projectile, in a way that changes the state of the projectile, and generates matter due to the high energies involved. One can observe the state of the projectile, and account for the matter which is created, and if there are laws which govern how the state of the projectile changes, or the kinds of matter that can be created, then we can run the clock backwards, reconstructing the kind of interactions that happened, to learn something about nuclear structure (or even partonic structure). In this way, one can also identify conserved quantities, which in turn suggest physical symmetries, which in turn help to build models.

One can think of interaction of a beam and target in terms of a probability of interaction - and this formalism will be discussed further in chapters related to the Vernier Analysis I worked on. Succinctly, however, one can mathematically 'separate' part of this interaction probability into a quantity called a 'cross-section', often denoted as σ for a total cross section, or $d\sigma$ for a differential cross section, or even $\frac{d\sigma}{d\Omega}$ to refer to a differential cross section scattered into a solid angle. The σ of any scattering experiment can be represented many different ways.

From a theoretical standpoint, we can represent protons (and other baryons) by selecting the relevant internal degrees of freedom we may want to study, and then devising

some rules for the internal structure. We may be interested in the momentum fraction carried by some distribution of partons, or how the relative populations of partons within a certain kinematic regime changes with distance/energy scale. For all of these cases, we use parton distribution functions, or structure functions. Concretely, these functions depend on:

A subcategory of deep inelastic scattering is 'Semi-Inclusive Deep-Inelastic Scattering'. This refers to a case where a beam (say a lepton, such as an electron) interacts inelastically with a pointlike internal structure of a target particle, and a hadron is produced (such as a π^+), which is then detected. Semi-Inclusive Deep-Inelastic scattering is then the process by which the scattered lepton and a specific hadron are measured in the final state of the interaction (but other particles that might be produced are neglected or ignored).

I said the word parton, which I have been carefully avoiding, but now the cat's out of the bag. Nuclei, as we will learn, are not elementary particles, but instead, are built up from what we assume are fundamental, elementary particles. Deep inelastic scattering experiments slowly revealed that nuclei (individual protons and neutrons) were not elementary particles, but instead, composite particles. It is natural to assume then, that the properties of protons and neutrons are not fundamental either. And in fact, the vast zoo of particles that were discovered in early inelastic scattering experiments, such as π or K or Λ (discussed briefly earlier) were not fundamental either.

In Michael Riordan's excellent 1992 summary of the discovery of quarks, Riordan lays out a very succinct and thorough history of the late 20th century experimental and theoretical works which built on Rutherford and Gell-Man's work. Riordan states that surprising 'results came from a series of electron scattering experiments...from 1967 through 1973' at MIT and SLAC, which comprised the first set of evidence produced in favor of the partonic model. As described earlier, Gell-Mann created a three-quark model to produce predictions consistent with these observations [39].

By the 1970's, collaborations between Bjorken, Feynman, and others had produced a coherent partonic model which contained quarks, and force mediating gluons. Additionally, the concept of Structure functions had been developed. Modified from Rutherford's original scattering formula, this new formula to describe the cross section of deep inelastic scattering incorporated structure functions, which separated out the momentum exchange between target and projectile (via a virtual photon), and isolated this from W_1 and

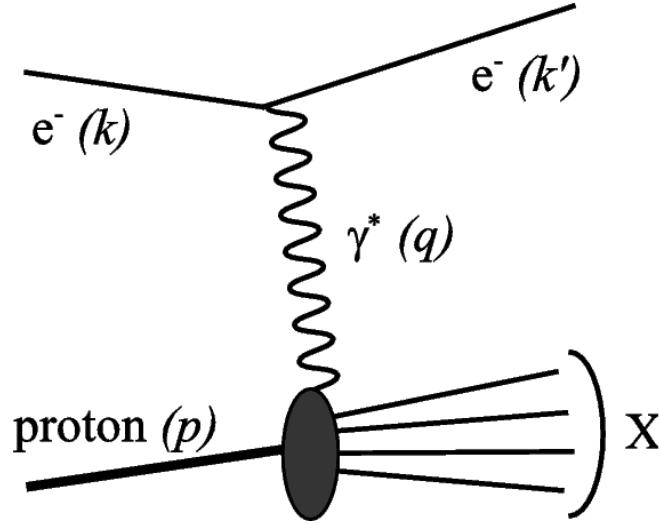


Figure 2.12: A schematic of deep inelastic scattering, where the incoming electron inelastically scatters off the proton, producing results X , via virtual photon exchange, γ^* [11]

W_2 , structure functions which were experimentally measured quantities representing the electron-proton interaction (the 'physics-y' part of the interaction).

This period of time, from 1970 - 1990 was truly the golden age of Deep Inelastic Scattering Experiments - the biggest laboratories running experiments were (and some are still running) The European Organization for Nuclear Research (CERN), The Stanford Linear Accelerator Center (SLAC), and The German Electron Synchrotron (DESY). Thousands of papers were published - some groundbreaking, such as the CERN's European Muon Collaboration experiment which showed a measurement of the spin asymmetry and determination of the proton structure function g_1 in muon-proton deep inelastic scattering [1].

The formalism of scattering theory continued to evolve during the booming period of particle physics from 1960 to present day. Though the mechanics of scattering experiments have remained essentially unchanged, vast improvements in technology in detectors, data collection and reconstruction, and beam production have evolved from Geiger and Marsden's humble beginnings to create scientific measurements of particles and their properties with exquisite and unprecedented precision. The kind of precision I'm talking about is exemplified in Brookhaven National Laboratory's E821 Muon ($g-2$) experiment - which measured the anomalous magnetic moment, $g-2$, of the muon to a precision of 7 parts in

ten million [40].

The advent of structure functions hailed an era of non-point-like baryonic matter. The mathematics of scattering formalism had to change to accomodate the underlying physical distribution of partonic matter in baryons. Deep Inelastic Scattering continued to probe various portions of these structure functions, and the structure of the standard model began to come into focus, distilled into the relatively simple mathematical structure of group theory. Concretely, the standard model is a gauge theory, which contains the internal symmetries of $SU(3) \times SU(2) \times SU(1)$, Figure 2.13. The Standard Model is said by some to be "complete" with the discovery of the Higgs Boson, yet with emergent phenomena such as proton spin, it does not provide a straightforward prediction. The model still has not included gravitation and relativistic effects fully - and probably isn't entirely correct.

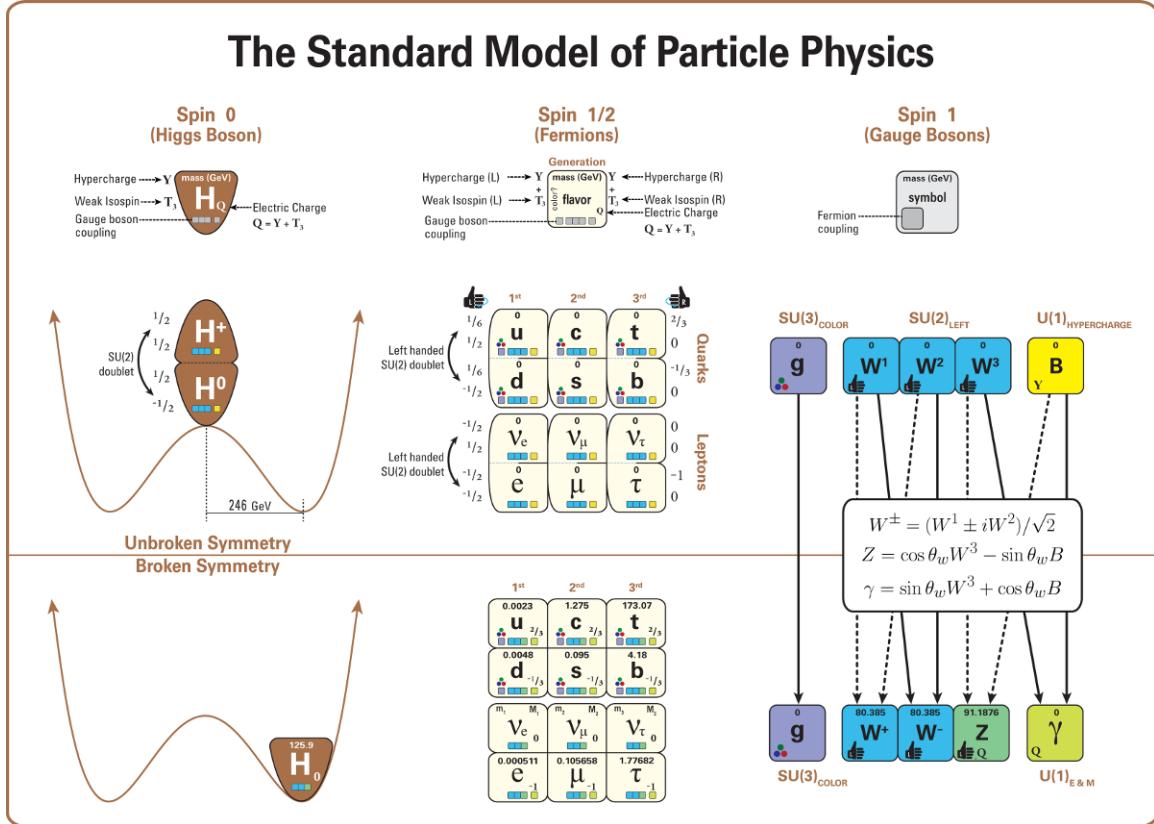


Figure 2.13: "This diagram displays the structure of the standard model (in a way that displays the key relationships and patterns more completely, and less misleadingly, than in the more familiar image based on a 4x4 square of particles). In particular, this diagram depicts all of the particles in the standard model (including their letter names, masses, spins, handedness, charges, and interactions with the gauge bosons – i.e. with the strong and electroweak forces). It also depicts the role of the Higgs boson, and the structure of electroweak symmetry breaking, indicating how the Higgs vacuum expectation value breaks electroweak symmetry, and how the properties of the remaining particles change as a consequence." [12].

2.3 World Experiments in Deep Inelastic Scattering

International Symposium on Spin Physics: http://inspirehep.net/search?ln=en&cc=Conferences&ln=en&cc=Conferences&p=Symposium+on+Spin+Physics+SPIN&action_search=Search&sf=conferencestartdate&so=d&rm=&rg=250&sc=0&of=hb

International Symposium on Deep Inelastic Scattering: http://inspirehep.net/search?ln=en&cc=Conferences&ln=en&cc=Conferences&p=Deep+Inelastic+Scattering+DIS&action_search=Search&sf=conferencestartdate&so=d&rm=&rg=250&sc=0&of=hb

The following Deep Inelastic (Inclusive and Semi-Inclusive) experiments highlighted here represent a period of science from approximately 1979 to present day, and focus mostly on efforts to understand the structure of baryons. Along the way, much of the Standard Model was discovered and codified, but I will try to keep the narrative focused on how experiments contributed to understanding both the structure of baryons in terms of parton distribution functions, and structure functions. Structure functions fall into two broad categories, within the scope of this work - cases which ignore the contribution of spin and cases which include the contribution of spin. Relevant maths will be presented largely only in the context of the experiments, which does not do a very good service to the progression of ideas on the theoretical side of physics, but hey, I'm an experimentalist. Once finished, I will present a brief overview of our best models developed for understanding the proton spin, developed in parallel with these experimental efforts, before transitioning to a discussion of RHIC, and my efforts on the subject.

Many of the experiments to be discussed here took data over a long period of time - some experiments were in competition, while others complimented each-other.

Though there are other experiments which currently take data complimentary to RHIC, I will cover only the experiments which directly lead to RHIC's spin program, with apologies to those experiments I've left out.

EIC white paper: [13]

Discuss SLAC, CERN, DESY, and RHIC. Discuss although DESY took data with HERA from 1992 - 2007, analysis and publication of the data is still ongoing.

HepData: <http://hepdata.cedar.ac.uk/review/f2/index.shtml>

2.3.1 CERN - European Muon Collaboration: 1979-1997

Overview Paper: Highlights of the European Muon Collaboration [41]

A measurement of the spin asymmetry and determination of the structure function A_{g_1} in deep inelastic muon-proton scattering: [1].

An investigation of the spin structure of the proton in deep inelastic scattering: [42].

Although Gell-Mann's simple quark model of baryons [43] predicts the correct quantity for the spin of the proton, the work of Ashman et al (1988) [CITATION NEEDED] at the European Muon Collaboration directly measured a portion of the proton structure function g_1 and found that a rather small fraction of the proton spin comes from quarks - and most of the spin is carried by the gluons (Figure 2.14).



Figure 2.14: [FIGURE NEEDED] [CAPTION NEEDED]. Results of EMC experiment showing that the structure function g_1 , tells us a thing about proton spin.

inSPIRE: collaboration:'European Muon'

2.3.2 SLAC - E142: 1993-1994

inSPIRE: collaboration:'E142'

2.3.3 SLAC - E143: 1992-1999

inSPIRE: collaboration:'E143'

2.3.4 DESY - ZEUS: 1992-Present

inSPIRE: collaboration:'ZEUS' H1+ZEUS combined results: https://www.desy.de/h1zeus/combined_results/ ZEUS figures: http://www-zeus.desy.de/zeus_papers/zeus_papers.html

2.3.5 CERN - Spin Muon Collaboration: 1993-1998

Spin Asymmetry A_1 and structure functions g_1 [44] NLO QCD analysis of spin structure function g_1 [45] inSPIRE: collaboration:'Spin Muon'

2.3.6 SLAC - E154: 1994-1997

inSPIRE: collaboration:'E154'

2.3.7 DESY - HERMES: 1995-2007

inSPIRE: collaboration:HERMES HERMES website, publications, figures: <http://www-hermes.desy.de/>

2.3.8 SLAC - E155: 1997-2003

inSPIRE: collaboration:'E155'

2.3.9 CERN - COMPASS: 2005-Present

inSPIRE: collaboration:'COMPASS'

2.3.10 DESY - H1: 1992-Present

H1 Publications: <http://h1.desy.de/e104552/e104555/> inSPIRE: collaboration:'HERA' (HERA is the accelerator) inSPIRE: collaboration:'H1' (this is the data)

Chapter 3

Models and Associated Probes For Proton Spin Structure

3.1 Modeling the Proton Structure

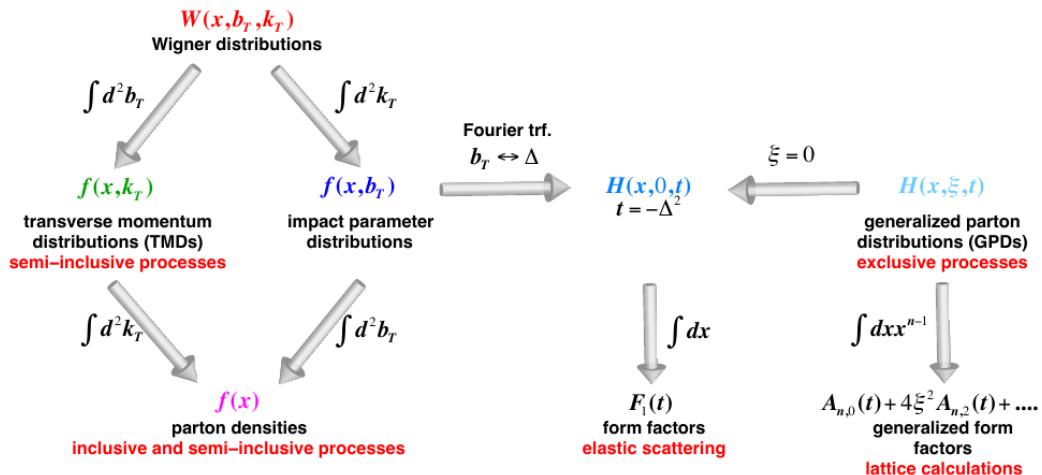


Figure 3.1: Figure from [13].

3.2 Structure Functions

Spin structure overall: [13] pp 29-31 Longitudinal spin structure: [13] pp 32-43

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \text{circle with dot}$		$h_1^\perp = \text{circle with dot} - \text{circle with dot}$ Boer-Mulders
	L		$g_{1L} = \text{circle with dot} \rightarrow - \text{circle with dot} \rightarrow$ Helicity	$h_{1L}^\perp = \text{circle with dot} \rightarrow - \text{circle with dot} \rightarrow$
	T	$f_{1T}^\perp = \text{circle with up arrow} - \text{circle with down arrow}$ Sivers	$g_{1T}^\perp = \text{circle with up arrow} - \text{circle with up arrow}$	$h_1^\perp = \text{circle with up arrow} - \text{circle with up arrow}$ Transversity $h_{1T}^\perp = \text{circle with up arrow} - \text{circle with up arrow}$

Figure 3.2: Figure from [13].

3.2.1 Parton Distribution Functions

3.2.2 Polarized Parton Distribution Functions

Discuss DSSV fits

3.2.3 Proton Spin Decomposition with the Ellis-Jeffe Sum Rule

Gauge invariant Ellis-Jeffe

$$\langle P, \frac{1}{2} | \hat{J}_z | P, \frac{1}{2} \rangle = \frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_q + J_g \quad (3.1)$$

Infinite momentum decomposition:

$$\langle P, \frac{1}{2} | \hat{J}_z | P, \frac{1}{2} \rangle = \frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_q + \Delta g + L_g \quad (3.2)$$

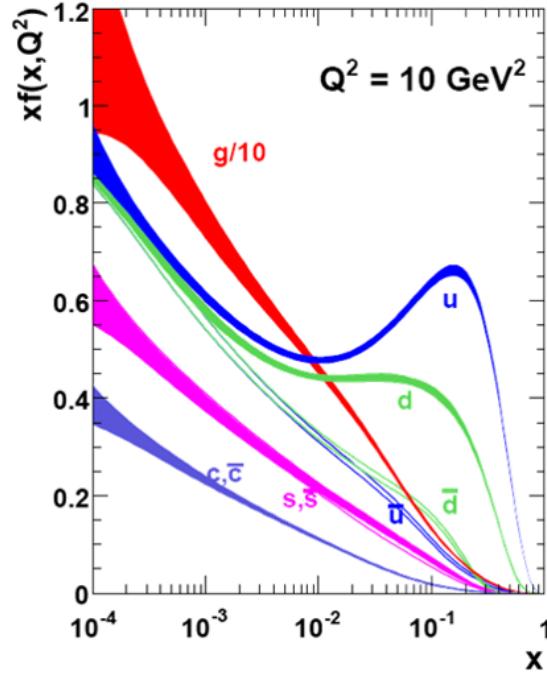


Figure 3.3: Shown: quark kinematics [CAPTION NEEDED]

Quark decomposition:

$$\Delta\Sigma = (\Delta u + \Delta \bar{u}) + (\Delta d + \Delta \bar{d}) + (\Delta s + \Delta \bar{s}) \quad (3.3)$$

3.2.4 The Spin Asymmetry: An Experimental Probe

Write in terms of the cross-section of polarized scattering.

3.3 that sweet table from Delia hasch

3.4 Experimental Probes for Proton Spin Structure

3.4.1 Physics Probes for the Proton Spin

3.4.2 W Production

The standard model tells us that W production occurs through a pure vector-axial interaction, this implies that the helicity of the parents particles - in particular $u + \bar{d} \rightarrow W^+$

and $\bar{u} + d \rightarrow W^-$ have fixed helicities, due to the relativistic final state neutrino (which is not measured, of course). To visualize the leading order of W production, with regards to the quark-sea element being probed, the leading order diagrams for the interaction are shown in Figure 3.5 [14]

Since Δq , the polarized parton distribution function can be split into contributions from valence quarks, and also sea quarks, understanding $\Delta \bar{q}$ is an important step towards understanding Δq better to better understand the total proton spin.

Though both protons in the collision are polarized, the polarization of one participant proton can be effectively ignored by summing over all polarization states for one of the two protons. With this assumption, we may construct a single spin asymmetry for colliding protons by counting difference in the number of positively and negatively polarized W's produced in collisions, scaled by the total production:

$$A_L^W = \frac{1}{P} \times \frac{N_-(W) - N_+(W)}{N_-(W) + N_+(W)} \quad (3.4)$$

This is a relatively easy experimental probe to measure (assuming that we can accurately count events which produced a W, which naturally, is nearly impossible, as we will see in Section 6.2.6).

As we saw earlier, in Section 3.2.2, we can write an asymmetry in terms of the scattering cross section for the process responsible for particle yields. These cross-sections were shown to be written in terms of polarized parton distribution functions, thus, we cut to the chase to write down the full expression of the theoretical asymmetries for this process in terms of those parton distribution functions.

The following equations all contain an implied integration over x_1 and x_2 .

For W^+ and u :

$$A_L^{W^+} = \frac{u_-(x_1)\bar{d}(x_2) - u_+(x_1)\bar{d}(x_2)}{u_-(x_1)\bar{d}(x_2) - u_+(x_1)\bar{d}(x_2)} \quad (3.5)$$

For W^+ and \bar{d}

$$A_L^{W^+} = \frac{\bar{d}_-(x_1)u(x_2) - \bar{d}_+(x_1)u(x_2)}{\bar{d}_-(x_1)u(x_2) + \bar{d}_+(x_1)u(x_2)} \quad (3.6)$$

Observationally, we see a superposition of 3.5 and 3.6, which is expressed as ??:

$$A_L^{W^+} = \frac{\Delta u(x_1)\bar{d}(x_2) - \Delta \bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)} \quad (3.7)$$

For the case of W^- , we observe \bar{d} and u : For W^- and d :

$$A_L^{W^+} = \frac{d_-(x_1)\bar{u}(x_2) - d_+(x_1)\bar{u}(x_2)}{d_-(x_1)\bar{u}(x_2) - d_+(x_1)\bar{u}(x_2)} \quad (3.8)$$

For W^- and \bar{u}

$$A_L^{W^+} = \frac{\bar{u}_-(x_1)d(x_2) - \bar{u}_+(x_1)d(x_2)}{\bar{u}_-(x_1)d(x_2) + \bar{u}_+(x_1)d(x_2)} \quad (3.9)$$

Observationally, we see a superposition of 3.8 and 3.9, which is expressed as 3.10:

$$A_L^{W^-} = \frac{\Delta d(x_1)\bar{u}(x_2) - \Delta \bar{u}(x_1)d(x_2)}{d(x_1)\bar{u}(x_2) + (\bar{u})(x_1)d(x_2)} \quad (3.10)$$

Kinematics of the collision can simplify the equations even further, when at very forward or very backward rapidities [14].

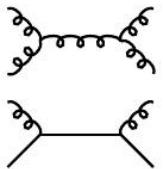
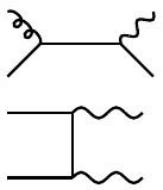
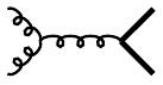
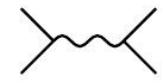
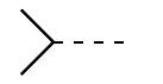
Reaction	Dom. partonic process	probes	LO Feynman diagram
$\vec{pp} \rightarrow \pi + X$ [61, 62]	$\vec{gg} \rightarrow gg$ $\vec{q}\bar{q} \rightarrow qg$	Δg	
$\vec{pp} \rightarrow \text{jet(s)} + X$ [71, 72]	$\vec{gg} \rightarrow gg$ $\vec{q}\bar{q} \rightarrow qg$	Δg	(as above)
$\vec{pp} \rightarrow \gamma + X$ $\vec{pp} \rightarrow \gamma + \text{jet} + X$ $\vec{pp} \rightarrow \gamma\gamma + X$ [67, 73, 74, 75, 76]	$\vec{q}\bar{q} \rightarrow \gamma q$ $\vec{q}\bar{q} \rightarrow \gamma q$ $\vec{q}\bar{q} \rightarrow \gamma\gamma$	Δg Δg $\Delta q, \Delta \bar{q}$	
$\vec{pp} \rightarrow DX, BX$ [77]	$\vec{gg} \rightarrow c\bar{c}, b\bar{b}$	Δg	
$\vec{pp} \rightarrow \mu^+ \mu^- X$ (Drell-Yan) [78, 79, 80]	$\vec{q}\bar{q} \rightarrow \gamma^* \rightarrow \mu^+ \mu^-$	$\Delta q, \Delta \bar{q}$	
$\vec{pp} \rightarrow (Z^0, W^\pm) X$ $\vec{pp} \rightarrow (Z^0, W^\pm) X$ [78]	$\vec{q}\bar{q} \rightarrow Z^0, \vec{q}'\bar{q}' \rightarrow W^\pm$ $\vec{q}'\bar{q}' \rightarrow W^\pm, q'\bar{q} \rightarrow W^\pm$	$\Delta q, \Delta \bar{q}$	

Figure 3.4: A summary of the various probes for longitudinally polarized protons. The "Reaction" column summarizes the reaction observed experimentally. The "Dom. partonic process" column describes the dominant process at the partonic level. The "probes" column shows which proton spin structure can be measured with the reaction. Finally, the leading order Feynman diagram for the partonic process is drawn [14].

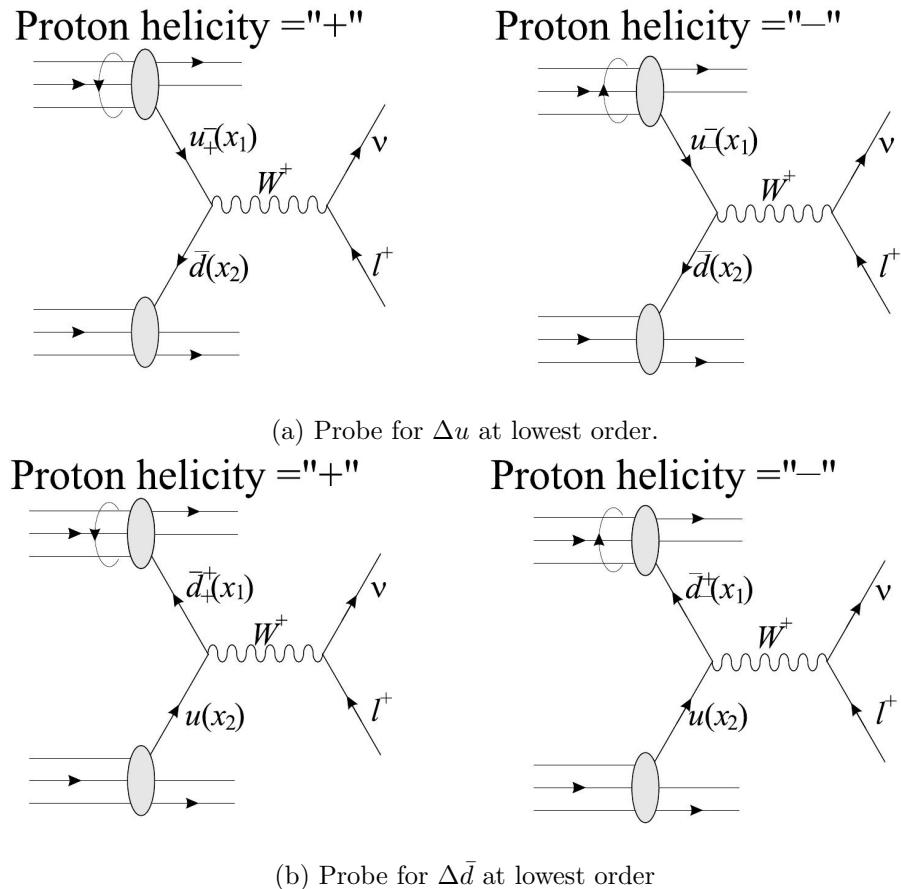


Figure 3.5: Real W^+ production as produced at PHENIX. The helicity of the initial state fixes the helicity of the partonic participants due to the relativistic final state of the neutrino + the handedness of the W boson. x_1 and x_2 are the momentum fractions of the quarks participating from the participant partons [14].

3.5 Cross Sections and Luminosity

- vernier analysis note intro, equations
- summarize the papers on Lumoninosity

Chapter 4

The Relativistic Heavy Ion Collider

4.1 Overview

While there have been many experiments which have performed deep inelastic scattering over the years, the experiments built around the Relativistic Heavy Ion Collider at Brookhaven National Laboratory are positioned to take advantage of the unique accelerator.

The Relativistic Heavy Ion Collider (RHIC) is the world's only intersecting ring particle accelerator which is capable of producing polarized proton beams. The beams are differentiated with the mnemonic "Blue" and "Yellow" beam. The Blue beam circulates clockwise when viewed from above the RHIC complex, the Yellow beam circulates counter-clockwise. As is typical for intersecting ring experiments, the beams are bunched, with bunches of ions intersecting at designated intersection points, around which experiments are built. The Blue beam is nominally used to time these collisions, such that experiments which have bunch-sensitive measurements (i.e. any experiment where bunch polarization is important) can associate the correct punch polarization with the correct collision. This will be discussed more in the section of beam-polarization ([4.3](#)).

RHIC generally separates data taking into beam 'fills' which are uniquely numbered, and for which general data characterizing the machine state is logged in various databases and online logbooks. Each fill is a unique population of beam ions, circulating around the RHIC rings. The beam ions are subdivided into discrete collections of ions called bunches. At the end of each fill, (typically 8 hours of collisions), the beam is dumped, and a new fill is generated. Experiments built around RHIC generally subdivide fills into 'runs',

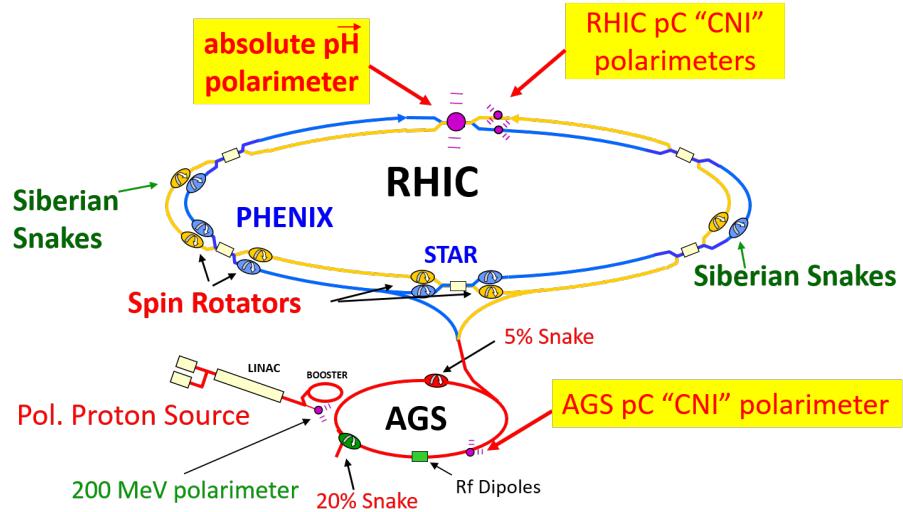
where a run is a period of time where the experiment is taking data. At PHENIX, runs are always segregated within a fill - no run will ever contain data from multiple fills.

Scientists at RHIC have come up with many ingenious ways to create and maintain beam polarization, once this is accomplished, various kinematically select probes are engineered, based on collisions observed which provide important cross-checks to DIS data as well as original discoveries and measurements of proton structure. RHIC is a unique collider in that it is quite flexible. Beams may be transversely or longitudinally polarized, a variety of ions may be used to fill the beams. To date, RHIC has collided many beam ions and species, summarized in Figure 4.2 and Figure 4.3.

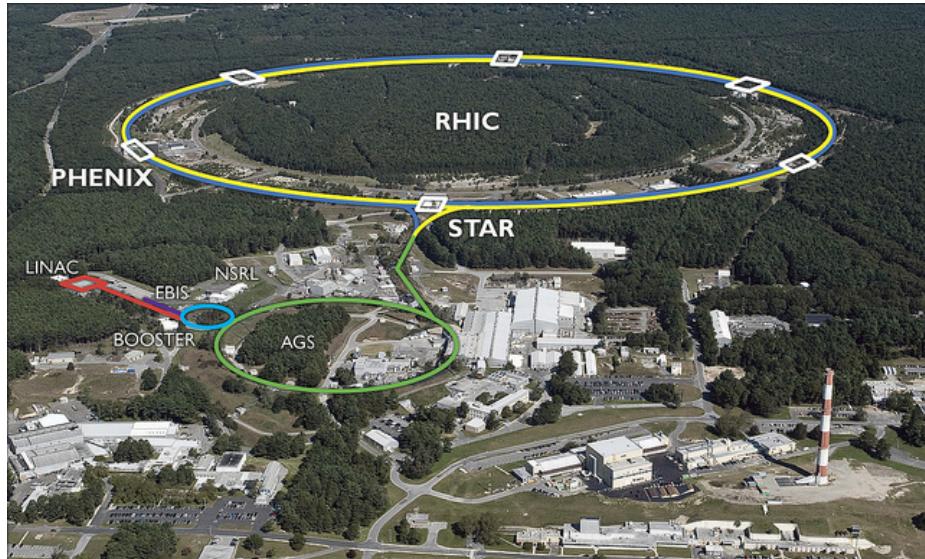
RHIC is an experiment which has been built on top of previous accelerator experiments which have been reused magnificently - a Linear Accelerator, a booster ring, and an Alternating Gradient Synchrotron, all of which now have been repurposed to create a high quality collider physics experiment. Many experiments are still set up around various egress points along the acceleration chain, which are publicised on the Brookhaven National Laboratory website www.bnl.gov

At the time of writing of this thesis (Spring of 2016), there are two experiments which are actively taking data from collisions produced by RHIC: The Pioneering High Energy Nuclear Interaction Experiment (PHENIX, Section 4.4, Figure 4.5), and the Solenoidal Tracker at RHIC (STAR, Figure 4.5). STAR and PHENIX are complimentary to each other - PHENIX has a very high precision centrally covering Electromagnetic Calorimeter, and other high precision detectors, but lacks full kinematic coverage, whereas STAR has lower precision, but has nearly full kinematic coverage around the beam intersection at its center.

RHIC's luminosity and beam polarization has been continuously improving (Figure 4.4) since RHIC was first turned on. As we will discuss later (4.5), this increased luminosity observed in 2013, was maximally leveraged with upgrades to the PHENIX detector.



(a) Diagram of RHIC Accelerator Complex, (Figure from Kiyoshi Tanida)



(b) Aerial photograph of RHIC Complex [46]

Figure 4.1: A diagram of the acceleration process of RHIC is shown in the top panel, and aerial view is on bottom. RHIC is nearly four miles in circumference and collides a variety of ions at center-of-mass energies between 62GeV and 510GeV .

RHIC operating modes and total integrated luminosity delivered to 6 experiments						
Run	species	total particle energy [GeV/nucleon]	calendar time in physics	total delivered luminosity	average store polarization, (H-jet) [*]	
Run-1 CY2000, FY2000 33.6 cryo-weeks	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$	27.9	3 shifts	$< 0.001 \mu\text{b}^{-1}$	—	
	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$	65.2	5.3 weeks	$20 \mu\text{b}^{-1}$	—	
Run-2 CY2001/02, FY2001/02 40.7 cryo-weeks	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$	100.0	15.9 weeks	$258 \mu\text{b}^{-1}$	—	
	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$ polarized p + p	9.8 100.2	2 shifts 8.3 weeks total, no continuous physics operation	$0.4 \mu\text{b}^{-1}$ 1.4 pb^{-1}	— 14%	
Run-3 CY2002/03, FY2003 30.4 cryo-weeks	d + $^{197}\text{Au}^{79+}$	100.7 + 100.0	10.2 weeks	73 nb^{-1}	—	
	polarized p + p	100.2	9.0 weeks total, no continuous physics operation	5.5 pb^{-1}	34%	
Run-4 CY2003/04, FY2004 26.7 cryo-weeks	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$	100.0	12.0 weeks	3.53 nb^{-1}	—	
	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$ polarized p + p	31.2 100.2	9 days 6.1 weeks total, no continuous physics operation	$67 \mu\text{b}^{-1}$ 7.1 pb^{-1}	— 46%	
Run-5 CY2004/05, FY2005 31.4 cryo-weeks	$^{63}\text{Cu}^{29+} + ^{63}\text{Cu}^{29+}$	100.0	7.8 weeks	42.1 nb^{-1}	—	
	$^{63}\text{Cu}^{29+} + ^{63}\text{Cu}^{29+}$ polarized p + p	31.2 11.2 100.2	12 days 5 shifts 9.4 weeks	1.5 nb^{-1} 0.02 nb^{-1} 29.5 pb^{-1}	— — 47%	
Run-6 CY2006, FY2006 21.2 cryo-weeks	polarized p + p	204.9	2 stores	0.1 pb^{-1}	30%	
	polarized p + p	100.2	13.1 weeks	88.6 pb^{-1}	55%	
	polarized p + p	31.2	12 days	1.05 pb^{-1}	50%	

Figure 4.2: Runs 1 - 3 at RHIC focused on commissioning work for experiments measuring collisions at RHIC. Work was mostly characterized by heavy-ion measurements related to understanding Quark-Gluon Plasma. The spin program began with Run 5. Table produced from data posted at the RHIC run page [15].

RHIC operating modes and total integrated luminosity delivered to 6 experiments					
Run	species	total particle energy [GeV/nucleon]	calendar time in physics	total delivered luminosity	average store polarization, (H-jet)*
Run-7 CY2006/07, FY2006 18.4 cryo-weeks	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$	100.0	12.8 weeks	7.25 nb ⁻¹	—
	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$	4.6	3 shifts total, no continuous physics operation	small	—
Run-8 CY2007/08, FY2008 19.0 cryo-weeks	d + $^{197}\text{Au}^{79+}$	100.7 + 100.0	9.0 weeks	437 nb ⁻¹	—
	polarized p + p	100.2	3.4 weeks	38.4 pb ⁻¹	44%
	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$	4.6	3 shifts	small	—
Run-9 CY2008/09, FY2009 22.0 cryo-weeks	polarized p + p	249.9	4.1 weeks	110 pb ⁻¹	34%
	polarized p + p	100.2	9.9 weeks	114 pb ⁻¹	56%
	polarized pp2pp	100.2	3.5 days	0.6 nb ⁻¹	63%
	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$	100.0	10.9 weeks	10.3 nb ⁻¹	—
	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$	31.2	2.9 weeks	544 μb ⁻¹	—
Run-10 CY2009/10, FY2010 27.1 cryo-weeks	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$	19.5	1.8 weeks	206 μb ⁻¹	—
	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$	3.85	4.6 weeks	4.23 μb ⁻¹	—
	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$	5.75	1.4 weeks	7.8 μb ⁻¹	—
	polarized p + p	249.9	9.7 weeks	166 pb ⁻¹	48%
	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$	9.8	1.4 weeks	33.2 μb ⁻¹	—
Run-11 CY2010/11, FY2011 24.4 cryo-weeks	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$	100.0	6.4 weeks	9.79 nb ⁻¹	—
	$^{197}\text{Au}^{79+} + ^{197}\text{Au}^{79+}$	13.5	8 days	63.1 μb ⁻¹	—
	polarized p + p	100.2	4.4 weeks	74.0 pb ⁻¹	59%
Run-12 CY2011/12, FY2012 22.9 cryo-weeks	polarized p + p	254.9	4.9 weeks	283 pb ⁻¹	52%
	$^{238}\text{U}^{92+} + ^{238}\text{U}^{92+}$	96.4	3.1 weeks	736 μb ⁻¹	—
	$^{63}\text{Cu}^{29+} + ^{197}\text{Au}^{79+}$	99.9 + 100.0	5.4 weeks	27.0 nb ⁻¹	—
Run-13 CY2012/13, FY2013 17.0 cryo-weeks	polarized p + p	254.9	13.3 weeks	1.04 fb ⁻¹	53%

Figure 4.3: Though RHIC is currently still running (as of May 9, 2016), I include runs here up to and including the run producing my data set (Run 13). An unprecedented 13.3 cryo-weeks of running was awarded to the W-Physics group. Table produced from data posted at the RHIC run page [15].

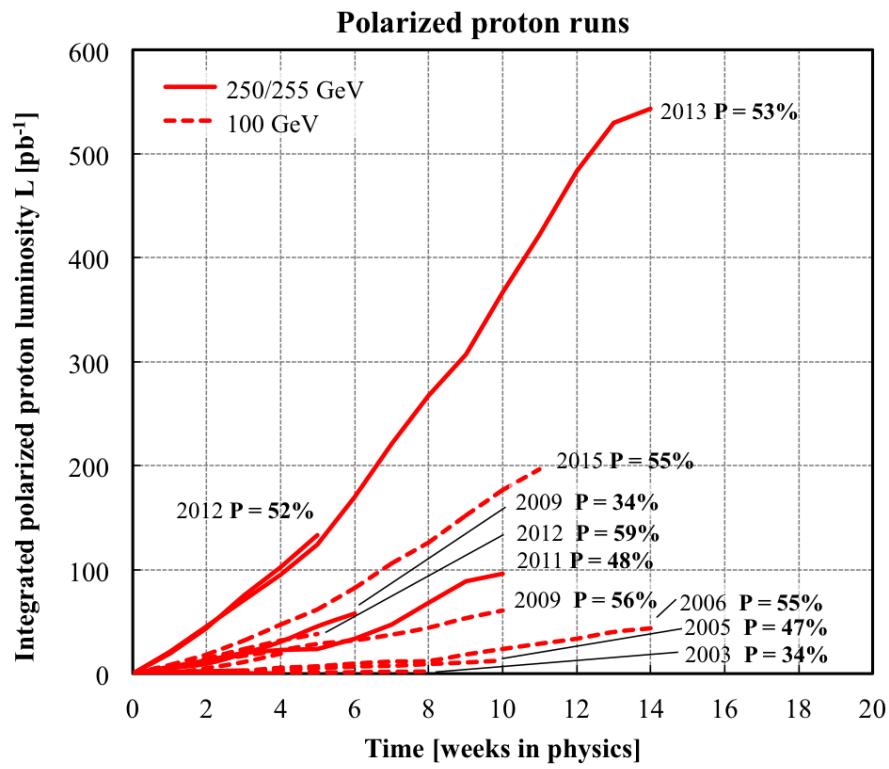


Figure 4.4: Upgrades to RHIC's electron lens have enabled massive improvements to luminosity - seen in the year 2013. The high luminosity was taken advantage of with an extra long proton+proton run. Figure obtained from [15]

4.1.1 Experimental Apparatus

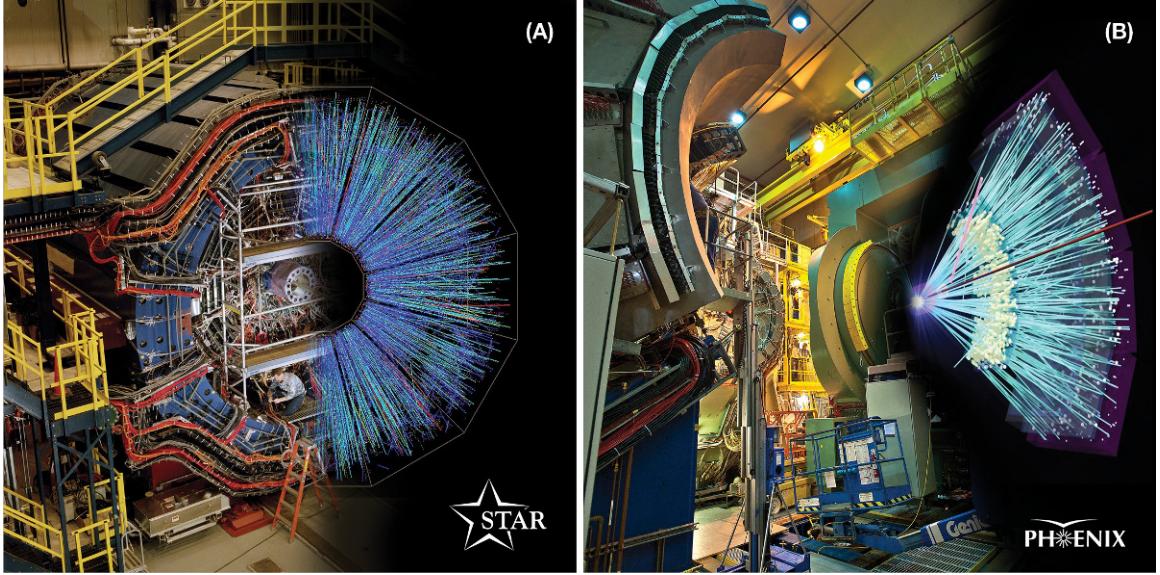


Figure 4.5: STAR (a) and PHENIX (b) with cutaways showing the event display for a heavy-ion collision as reconstructed by the detectors' electromagnetic calorimeters [16].

RHIC accelerates ions in a multi-stage process, summarized in Figure 4.1. The first stop is the **Electron Beam Ion Source**, built on top of a 200MeV linear accelerator (Linac). Once ions are injected into the Linac, they travel to the *Booster Synchrotron*. At this stage, ions are accelerated with pulsed RF fields. Once the beam of ions has been accelerated to nearly the speed of light, they are fed into the **Alternating Gradient Synchrotron** or AGS. At this time, ions are traveling at about $0.37 c$. By the time the ions leave the AGS, they are moving at $0.997 c$. Once the ions are ready, they are transferred to the **AGS-to-RHIC Line**, where a switching magnet pumps bunches of ions into either the counterclockwise circulating ring of RHIC, or the clockwise circulating ring of RHIC. The ions are 'spun-up' here to maximum speed, and are accelerated around the RHIC complex - each beam-ion travels nearly 2.4 miles in microseconds, for the duration of a physics-fill [47].

When the RHIC rings are filled with ions, the ions are bunched into rotating electromagnetic potentials called 'buckets'. There are 360 beam-buckets in total, but typically only a fraction are filled with ions. For this analysis, we took data with beams with 110 filled buckets. The sequence of beam buckets from one bunch to the next is referred to as a

'bunch' - and are rather long - Figure 4.6. A detailed presentation of beam dynamics with regards to luminosity will be presented in chapter 7.

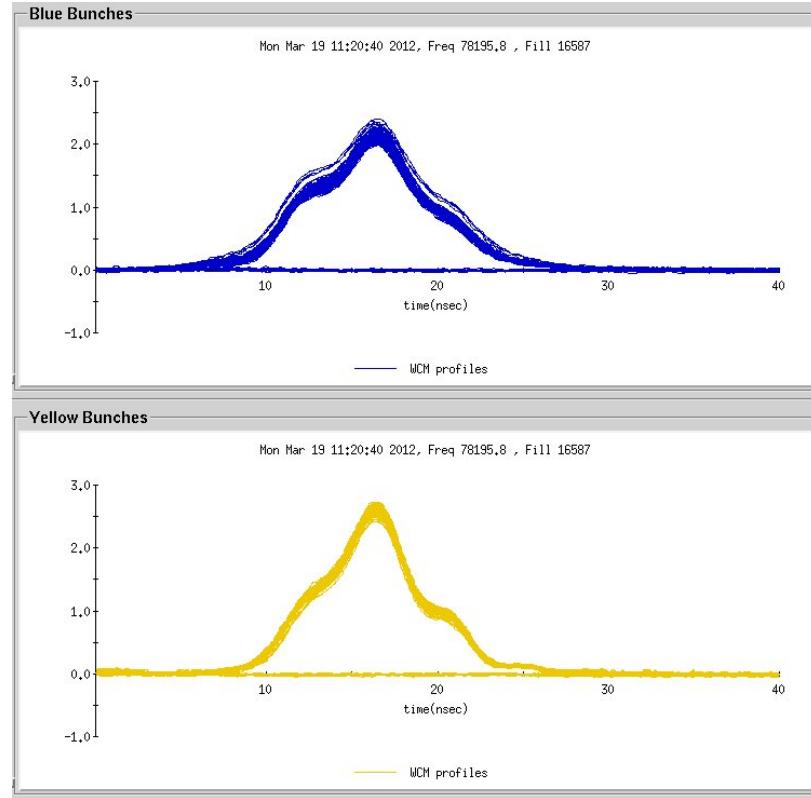


Figure 4.6: Plot courtesy of Angelika Drees, of RHIC's Collider-Accelerator department. The blue beam (blue) and yellow beam (yellow) are overlaid over a 40 nanosecond time period. Even with bunches crossing a fixed point over 40 nanoseconds, this still corresponds to an overall bunch length of about 12 meters. Conversely, the bunch width is quite narrow - with Gaussian geometry, it is between 150 millimeters and 300 millimeters depending on the beam energy. Understanding the beam bunch geometry is a crucial component to understanding total the total luminosity delivered by RHIC to PHENIX.

4.2 Production of Polarized Proton Beams

The production of polarized beams is crucial to the physics of this measurement - without polarized beams, no spin structure analysis can be done at RHIC. This is due to the fact that the helicity state of the protons in the initial state of any proton proton collision can be connected to the final observed states in a way which provides information about the spin structure function, as was discussed in section 3.1.

The production of polarized beams is a multistage process, and involves several experimental components. The importance of polarizing the beams is fully realized once polarized beams are collided at relatively high center of mass energies - where the beams behave less like polarized proton beams, but more like polarized beams of quarks and gluons [48]. Beam polarization is achieved incrementally - with polarization starting as soon as the booster and AGS stage of the acceleration process 4.1.

The RHIC Configuration Manual [18] provides a wealth of information, accelerator physics, diagrams, equations, and descriptions of the extremely precise and comprehensive approach to creating polarized proton beams and injecting them into RHIC. This work was crucial to this section of my thesis, and is recommended reading for anyone who wishes to know 'all the details' of how RHIC handles polarized beams.

4.2.1 Polarized Injection

RHIC uses an optically pumped polarized ion source (OPPIS), Figure 4.7 to produce a polarized ion source greatly in excess of RHIC's design intensity. This is used to our advantage, as the emittance of the beam can be lowered to create a highly collimated beam for physics use.

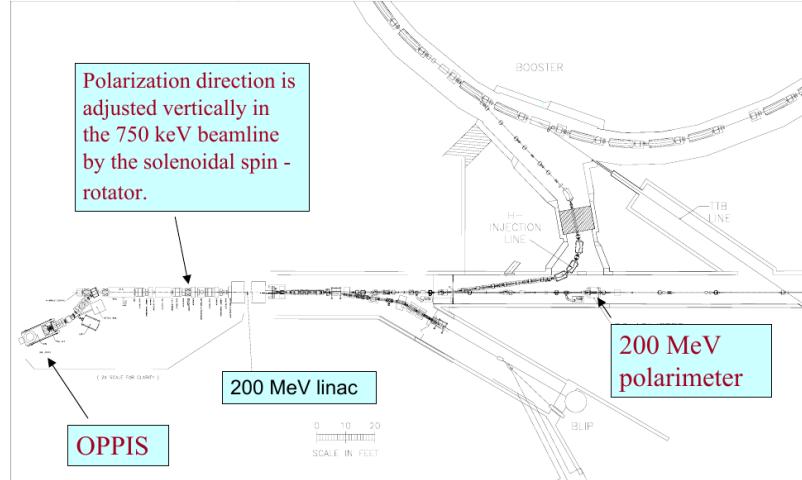


Figure 4.7: RHIC's optically pumped polarized ion source. Produces 0.5-1.0 mA current of polarized H^- ions. The optical pumping is pulsed at 400 μs , [17]

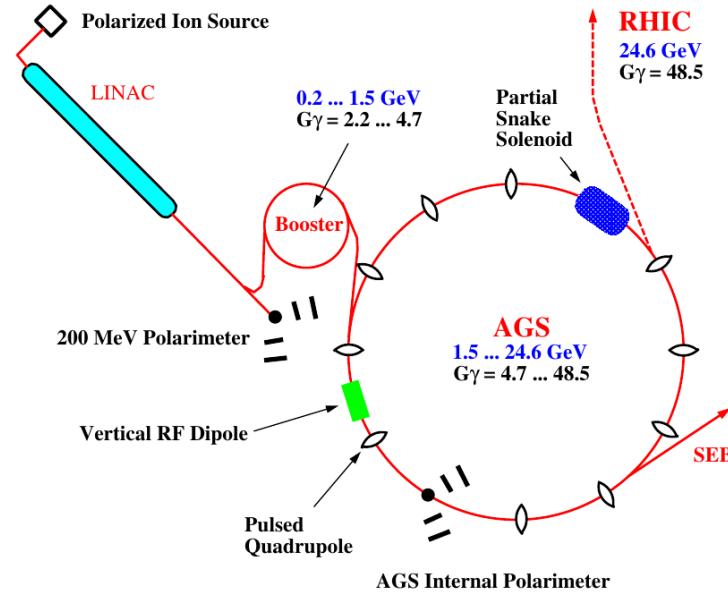
4.2.2 AGS to RHIC Transfer Line

Once ions have been optically pumped, we have a direct-current beam at approximately 80% polarization. The pumping is accomplished using Rubidium vapor. The polarized ions are then moved into the booster from the Linac, where some polarization is lost to spin precession, intrinsic to accelerating charged ions in a circular path. However, polarization is maintained, for the most part, by matching the precession resonance to the orbiting frequency of the booster ring. The Siberian snakes and spin rotators at this stage serve to incrementally flip the ion spin such that the natural depolarization works to re-polarize the orbiting ions, every full-turn. The full details of this procedure are well described in [18].

After the ions are sufficiently polarized and filled in the AGS, they are moved into the AGS to RHIC Transfer line, Figure 4.9. The beam is focused and fed through a switching magnet - which must be timed with great precision in order to fill the blue and yellow beams with the appropriate polarization patterns. In fact, the precision is so great, that the earth's curvature must be taken into account over this relatively short injection line - the entry point and exit point are bent ever so slightly different - the entry being 12.51 mrad, vs the egress being 12.46 mrad [18]. At the point of injection in the transfer line, the beam size and emittance are measured, as well as the beam polarization.



(a) Technical schematic of Polarized Injection Line [17]



(b) Overhead view of Polarized Injection Line [18]

Figure 4.8: A view of the RHIC polarized injection system. We see a zoomed in technical view of the OPPIS to the booster (a), below, we see a zoomed out cartoon of the next step in the polarization injection system, including the AGS, and the feeder line to RHIC.

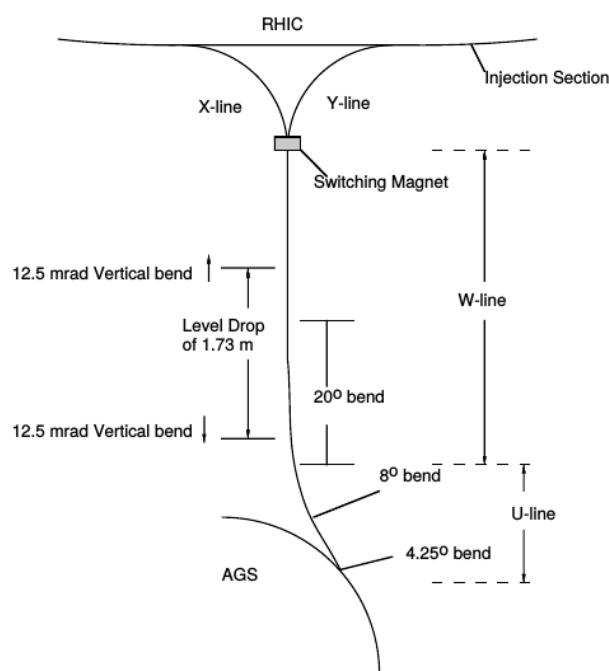


Figure 4.9: A schematic of the geometry of the AGS-to-RHIC transfer line [18].

4.3 Maintaining Beam Polarization

The creation of polarized beams is only half the battle. Depolarizing resonances in any particle beam are intrinsic in the design of any circulating beam particle accelerator - without intervention, after a few rotations, RHIC's polarized beams would be unpolarized. RHIC uses several strategies in concert to correct for the largest of these depolarizing resonances - including beam orbit corrections, the Siberian Snakes, Betatron Tune Spreading, and sextupole magnetic depolarizing resonances.

4.3.1 Siberian Snakes and Spin Rotators

The Siberian Snakes are positioned at two locations on the RHIC ring (as well as others along the injection sequence). The most stable configuration of spin injected in RHIC is such that the spin axis is perpendicular to the plane of the accelerator ring. The Siberian snake is a helical magnet which forces the spin to rotate 180 degrees every half rotation. This special configuration of snakes (see Figure 4.1) ingeniously takes advantage of the rotational precision of the spin (a depolarizing resonance) to re-polarize the beam, every half-orbit.

The spin rotators are located outside of experimental interaction regions around PHENIX and STAR. These special dipole magnets rotate the spin of the beams onto a longitudinal (parallel with beam) axis - these magnets are important for any measurement (such as this one) requiring longitudinal spin polarization. Otherwise, transverse spin effects can by studied.

4.3.2 Measuring Beam Polarization

The RHIC Collider-Accelerator Department provides several means of measuring the beam polarization over the course of the data taking period. PHENIX takes special data and studies it, to determine the real beam polarization delivered to the detector, in a yearly analysis (for years where polarized data is taken). This analysis is often called "Local Polarimetry", but is often abbreviated as LPol in PHENIX logs.

CAD will additionally measure polarization in via inelastic proton-carbon scattering in the Coulomb-Nuclear Interference (CNI) region. Relative polarization can be determined with to within 10% in only a few seconds of measurement.



Figure 4.10: As beams are longitudinally rotated into position for collision, it is crucial to keep careful track of the magnet currents rotating the beams, as well as the overall polarization pattern. Shown is a cartoon of one potential polarization pattern.

Vertical polarization is determined through the calculation of the left and right particle production, with a known analyzing power ([18], Ch 8):

$$P_B = \frac{1}{A_p} \frac{N_L - N_R}{N_L + N_R} \quad (4.1)$$

Where P_B is the beam polarization, N_L and N_R are the left-scattering produced particles, and right-scattering produced particles and A_p is the analyzing power, which can be calculated from first principals, and experimentally verified. Scattering takes place as a carbon filament is swept across the beam.

As many decisions are financially constrained, this one was too. Using a p-Carbon CNI polarimeter provides an economically viable way to measure beam polarization within the precision needed for the spin experiments.

4.3.2.1 The Spin Monitor

One of my major contributions to the PHENIX experiment was in the upkeep and development of the spin monitoring systems for the online data taking portions of the experiment.

During a RHIC run, it is crucial to keep track of the polarization patterns being collided at the PHENIX IR 4.10

4.4 The Pioneering High Energy Nuclear Interaction Experiment

4.4.1 Overview

The Pioneering High Energy Nuclear Interaction Experiment is a synthesis of many smaller detectors all of whom were commissioned for various physics goals, some of which have been repurposed from their original application once the primary physics mission of the detectors were completed.

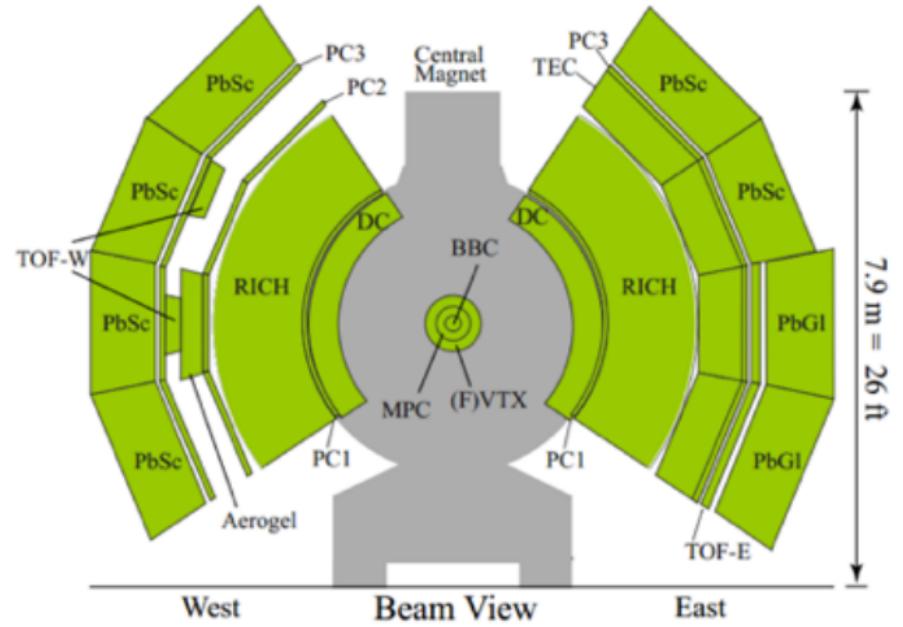
The configuration of the PHENIX spectrometer can be changed from year to year, depending on the analysis needs of the physics working groups. The configuration of the detector for the 2013 physics run is shown in Figure 4.11.

PHENIX makes use of many classic detector technologies, it contains Čerenkov light detectors, resistive plate chambers, electromagnetic calorimeters, silicon chip detectors, time of flight detectors, scintillation light detectors, cathode strip chambers, and proportional tube counters.

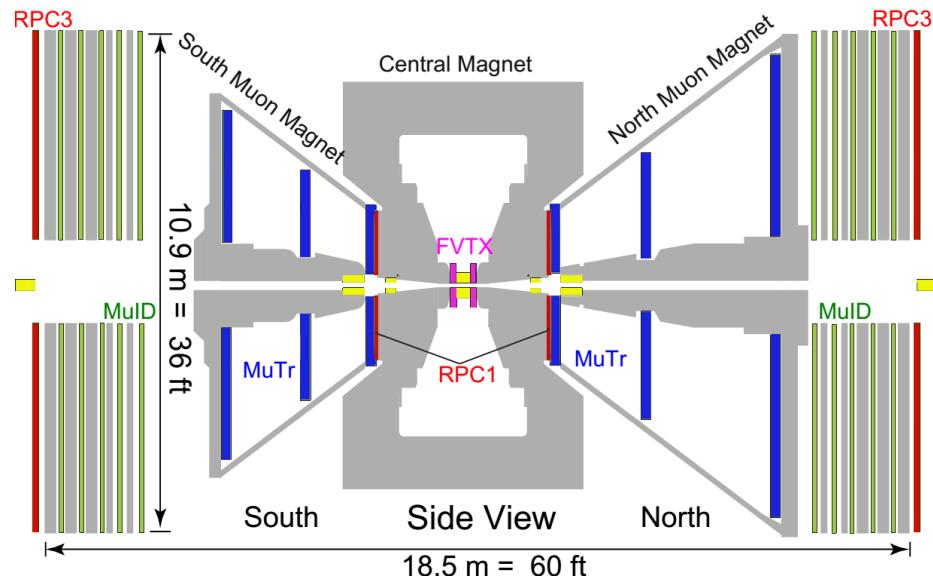
While all of these subsystems are interesting, and have produced excellent physics results, I will focus only on those pertinent to this analysis.

PHENIX is generally thought of as two 'halves', which often are used in separate analyses - the forward muon arms, and the central arms. While both halves are used for both heavy ion, and spin physics analyses, this analysis exclusively uses the forward muon arms, so the central arms will not be discussed (though, a closely related analysis, dealing with A_L for $W \rightarrow e$ exclusively uses the central arms). The different subsystems cover different rapidity ranges, so many times, complimentary results are obtained from central and forward analysis. Results from such a complementary central analysis will be presented alongside my results in later chapters.

PHENIX also utilizes a complex data acquisition system (DAQ) which streams data from each detector, assembles this data into a labeled event, compresses and stores into a proprietary storage format, the work-flow of this is summarized in Figure 4.12. A complete summary of PHENIX detector subsystems (excluding the Forward Vertex Detector, Silicon Vertex Detector, and Resistive Plate Chambers, which are new, and discussed separately)



(a) Central Arms



(b) Forward Muon Arms

Figure 4.11: Shown: The two main halves of the PHENIX Spectrometer. The central arms are shown via the beam-on view of PHENIX (left) and Forward Muon Arms are shown via the 90-degree rotated view. In both cases, the 2013 configuration is shown. The beams are brought into intersection at the geometric center of each figure (immediately between the BBCs)

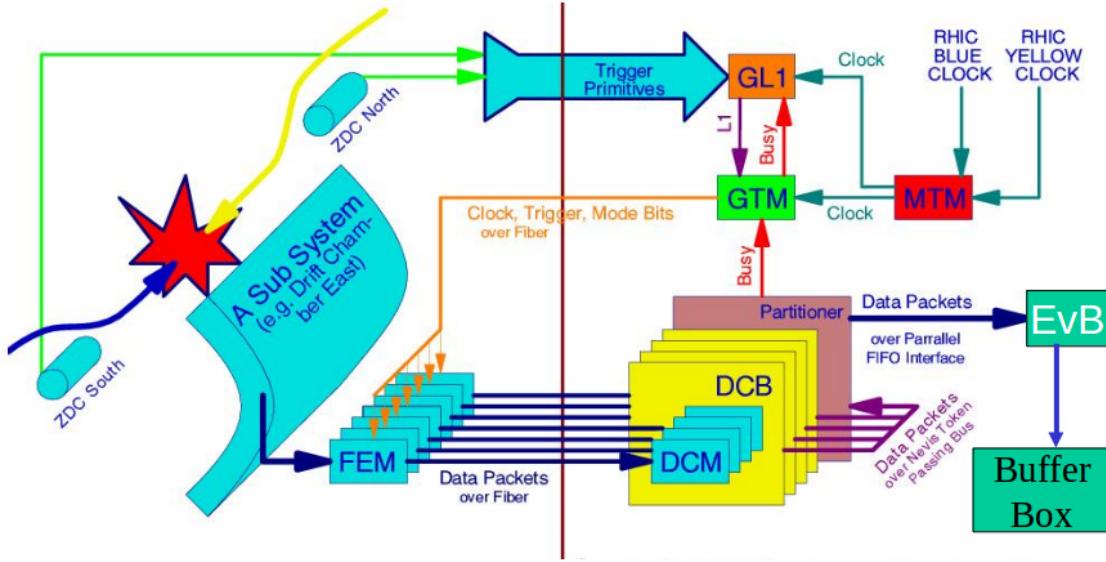


Figure 4.12: A flow chart summarizing the PHENIX DAQ [19]. From left to right, we can get a feel for the data flow at PHENIX. Shown is an event, the red splat on the far left. Particles from this event are transduced by a detector ('A Sub System'). The transduced signals are serialized into a detector specific data stream, such that the state of the detector's excitation can be recorded and reproduced later. This information is stored on the front-end-electronics modules (FEMs), and synchronized with timing information from the clock (ticks once every time there is a bunch crossing) and a Global Trigger decision, i.e. whether or not the right parts of the detector lit up to make this particular event worth keeping. After this, if the event is designed by the heuristics to be worthy of keeping, the uncompressed serialized information is sent to the DCMs, where it is assembled into a packet, and then sent to the event builder (EvB), where all packets sharing a common collision are assembled into an event. The event is compressed into a proprietary PRDF format, and sent to the Buffer Boxes, which are a cache of high density local storage, which is later sent off to cold storage on magnetic tape drives.

Element	$\Delta\eta$	$\Delta\phi$	Features
Magnets			
Central Magent	± 0.35	360°	1.15 T
Muon Magnet North	$-1.1 - -2.2$	360°	0.72 T
Muon Magnet South	$1.1 - 2.4$	360°	0.72 T
Minimum Bias			
Beam Beam Counter	$\pm(3.1 - 3.9)$	360°	Vertex Reconstruction
Zero Degree Calorimeter	$\pm 2\text{mrad}$	360°	Minimum Bias Trigger
Central Detectors			
Drift Chambers	± 0.35	$90^\circ \times 2$	Central p and m resolution
Pad Chambers	± 0.35	$90^\circ \times 2$	Pattern Recognition, Tracking
Ring Imaging Čerenkov	± 0.35	$90^\circ \times 2$	Electron ID
Time of Flight	± 0.35	45°	Hadron ID, $\sigma < 100\text{pm}$
PbSc EMCal	± 0.35	$90^\circ \ 45^\circ$	Calorimetry, photon and electron energy
PbGl EMCal	± 0.35	45°	e^\pm, μ^\pm separation at $p > 1\text{GeV}/c$, EM Shower and $p < 0.35\text{GeV}$, K^\pm, π^\pm separation up to $1\text{GeV}/c$
Muon Arms			
Muon Tracker South	-1.15 to -2.25	360°	North installed 2003
Muon Tracker North	1.15 to 2.44	360°	
Muon ID South	-1.15 to -2.25	360°	Steel absorbers, larocci tubes
Muon ID North	1.15 to 2.44	360°	""

Table 4.1: A summary of PHENIX hardware [22]. Electron/pion separation and Pion/Kaon separation requires the Time of Flight (ToF) working with PbGl and PbSc data. PbGl refers to "Lead Glass Scintillator" and PbSc refers to "Lead Scintillator". The Muon Identifier (Muon ID, MuID) can help separate muons from hadrons.

can by found in Table 4.1.

4.4.2 The Spin Program

The PHENIX spin program was planned as part of the RHIC upgrade to produce polarized proton beams. The major analysis thrust of the PHENIX spin program has been to understand the spin structure of the proton, and has historically used various flavors of particle production asymmetries (left-right and forward-backward) as an experimental probe for polarized parton distribution functions (as we saw in Chapter 3.1).

Much of PHENIX collaboration's early published work focused on creating and studying quark gluon plasma in heavy ion collisions, but in following years spin papers came too. Major question in physics that PHENIX set out to answer with its heavy-ion program include studying confinement - i.e. why are quark color charges confined to exist in the nucleus, baryons and mesons? PHENIX sought to study this via examination of the J/Ψ and measuring screening length in heavy ion collisions. Additional research topics included the study of chiral symmetry restoration, thermal radiation of hot gasses, QCD Phase transition, Strangeness and Charm Production, Jet Quenching, and Space-time evolution [49].

The spin program came shortly after the 2001 commissioning run. The first polarized proton run was produced by RHIC for PHENIX in 2002, with 8.3 total weeks of data, run discontinuously. The primary goals of the PHENIX spin program are to study the polarization of the proton, specifically in the context of the Ellis-Jaffe sum rule, which decomposes the proton spin into various contributions from its substructure. The main structures studied by PHENIX are the gluon polarization, Δg and the anti-quark polarization, $\Delta \bar{q}$. Additionally, the 'nature of parity non-conservation itself can be directly studied' using polarized beams, and spin asymmetries in collisions. Though this measurement generally requires a means of reconstructing jets (PHENIX doesn't have this), inclusive or leading particle production can be used as a substitute with some small asymmetry remaining [50].

4.4.3 Subsystems

The major subsystems contributing to this work include the Muon Arms, the Beam Beam Counters (BBCs), and the Forward Vertex Detector, since the analysis is totally characterized with calculating the A_L for $W \rightarrow \mu$ interactions, only muon reconstruction

and identification is required. The Central arms are used in the $W \rightarrow e$ analysis.

The Beam-Beam counters (BBCs, Figure 4.13) are photomultiplier tubes with scintillating lead-glass crystals. These detectors are situated about 144 cm on either side of the nominal center of the PHENIX interaction region, and typically are used to trigger on events with minimal bias towards any particular physics goal. This is important as a means for reconstructing the relative abundance of particle production, which is as you might expect, crucial for determination of any inelastic scattering cross section. The Beam-Beam counters provide us with vertex reconstruction by way of analyzing the time delay between triggering of the North BBC and the South BBC. The delay window is then used to reconstruct the event vertex by assuming particles are travelling at near light speed, Figure 4.14.

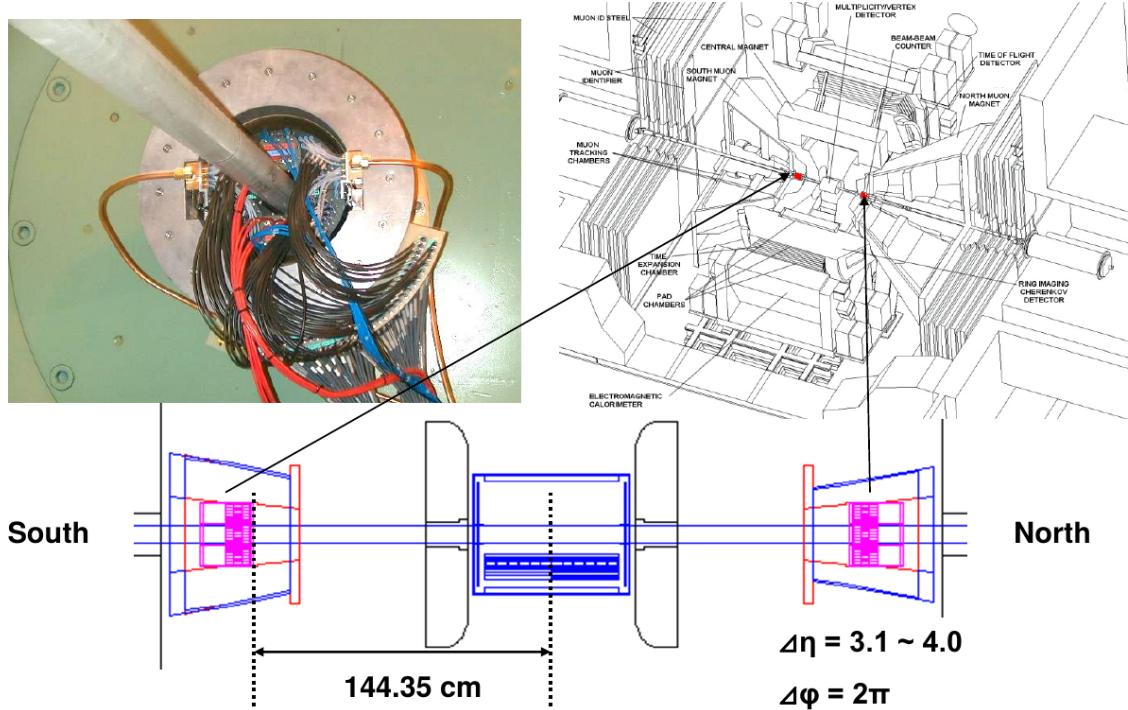


Figure 4.13: Shown: a photograph of the BBC hugging the beryllium beam pipe near the center of PHENIX (top left), a schematic showing the relative size and location of the BBCs as compared to the rest of PHENIX (top right), and a schematic of the exact proportions of the detector as viewed alongside the beam pipe (bottom), along with the rapidity and azimuthal coverage [20]

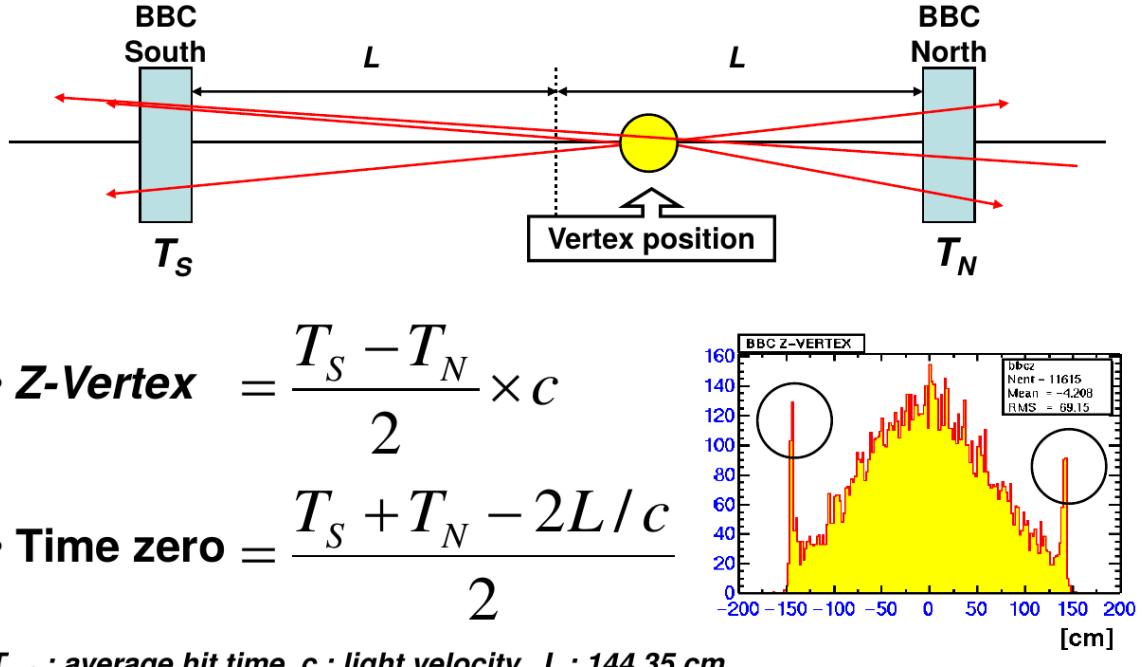


Figure 4.14: A diagram of the general strategy for reconstructing the event z-vertex for a collision. Top: a cartoon of the North and South BBCs getting some particle penetration after an event. Bottom Right: a characteristic distribution of measured z-vertices for a short run taken in 2002 [20].

The Forward Vertex Detector, Figure 4.15 is a silicon detector, which provides additional tracking to detect secondary event vertices and additional precision to the Muon Tracking system. This detector can provide an important additional layer of precision, because it can help to identify events which do not originate from the primary event vertex of a collision, so they can be rejected from a pool of candidate $W \rightarrow \mu$ events [21]. The properties of this detector are summarized in Table 4.2

The Muon Arms are composed of several subsystems, including the Muon Tracker (MuTR, cathode strip chambers), the Muon Identifier (MuID, shielding and scintillation layers), and the Resistive Plate Chambers (RPC(s), bakelite gas gaps and azimuthal oriented capacitively coupled copper readout strips). The job of the muon tracker is to identify muons with the penetration through the many layers of the MuID, and provide momentum and charge reconstruction for muon tracks. Tracks are matched to the event vertex with Kalman

Property	Value
Silicon sensor thickness (μm)	320
Strip pitch (μm)	75
Nominal operating sensor bias (V)	+70
Strips per column for small, large wedges	640, 1664
Inner radius of silicon (mm)	44.0
Strip columns per half-disk (2 per wedge)	48
Mean z-position of four stations (mm)	201.1, 261.4, 321.7, 382.0
Silicon mean z offsets from station center (mm)	5.845, 9.845

Table 4.2: A brief summary of the FVTX design parameters [21]

filter during reconstruction.

The muon arms were the subject of significant upgrades from 2011-2013. New front end electronics were added to improve triggering, and entire new detector subsystems (The RPCs) were added. The full details of these subsystems will be discussed in the forthcoming sections (Section 4.5).

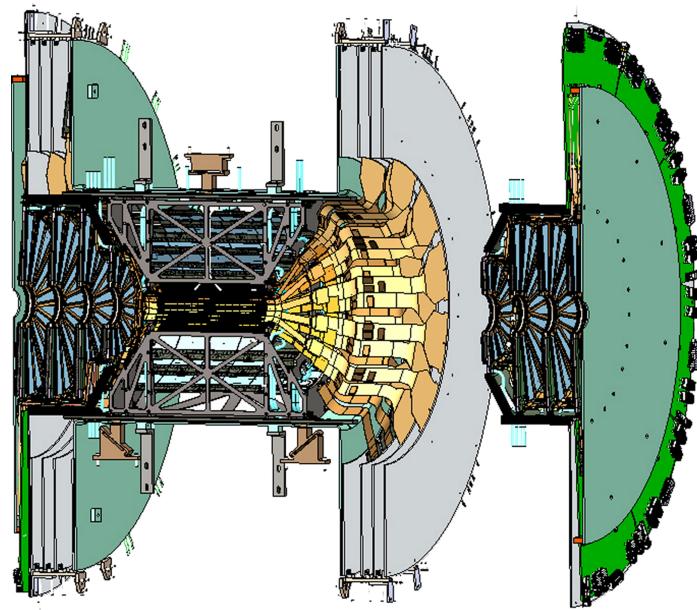


Figure 4.15: A schematic of the Forward Vertex Detector, showing the silicon chip layers (light blue wedges), and readout electronics (green). They are mounted directly onto the Silicon Vertex Detector, which was not used in this analysis. The FVTX and SVD together are used in heavy ion analyses [21].

4.5 The Forward Upgrade

One of the main stated physics goals of PHENIX is to constrain the sea-quark polarization of the proton spin. While this contribution is expected to be small, it is not because it is expected to be uniformly zero. Instead, the expectation is that the matter contribution to the quark sea is strongly positively polarized, while the antimatter contribution is strongly negatively polarized [14].

4.5.1 The Muon Tracker and Muon Trigger Subsystems

4.5.2 Resistive Plate Chambers

4.5.2.1 Design

4.5.2.2 Construction

4.5.2.3 Testing

4.5.2.4 Performance

4.5.3 Triggering and Data Acquisition

4.5.3.1 2013 Data Set Triggers

Chapter 5

The Data Set

5.1 Overview

Now that we have discussed the various apparatuses provided by the PHENIX experiment, we can go into more depth with the process of engineering features. For this analysis, we consider only events which are identified by the Muon Arms subsystem as being muons. The raw data provided by PHENIX is quite complex, and at the hardware level is generally not too useful for physics analysis.

In this chapter, we will discuss the process of cleaning our data set, the goal of which is to get rid of background data, while keeping any event that could possibly contribute to the $W \rightarrow \mu$ signal. This cleaning is done in three stages. The first stage concerns applying a simple basic cut to our data set to remove events which are kinematically forbidden from having W boson parent particles, this is called the "Basic Cut".

After this, we label data with W_{ness} , which is an event's likelihood for coming from a W boson decay. Although this is part of data cleaning, since W_{ness} is an important parameter in the analysis, it is discussed in Section ??.

Finally, we must estimate the overall yield of μ resulting from the various proton helicity combinations, and the signal to background ratio characterizing that yield. Again, since this is also an important part of the physics, it is discussed in Section ??.

5.2 Analysis Variables and the Basic Cut

A brief summary of the kinematic variables used later in the analysis is given in Table 5.2. In addition four sets of RPC cluster variables exist which are being used as main RPC variables. These variables contain projections from either vertex, Station 1, 3 or the MuID road to the corresponding z positions of the RPCs based on the tracks in the PHMuTracksOut node and are directly taken over from the RpcMuTracks node in the dsts:

- newsngmuons→Branch("RpcMatchVtx",0,"Rpc3dca[_RecoTracks]/F:
Rpc3time[_RecoTracks]/F:Rpc3x[_RecoTracks]/F:Rpc3y[_RecoTracks]/F:
Rpc1dca[_RecoTracks]/F:Rpc1time[_RecoTracks]/F:Rpc1x[_RecoTracks/F:
Rpc1y[_RecoTracks]/F");
- newsngmuons→Branch("RpcMatchSt1",0,"Rpc3dca[_RecoTracks]/F:
Rpc3time[_RecoTracks]/F:Rpc3x[_RecoTracks]/F:Rpc3y[_RecoTracks]/F:
Rpc1dca[_RecoTracks]/F:Rpc1time[_RecoTracks]/F:Rpc1x[_RecoTracks]/F:
Rpc1y[_RecoTracks]/F");
- newsngmuons→Branch("RpcMatchSt3",0,"Rpc3dca[_RecoTracks]/F:
Rpc3time[_RecoTracks]/F:Rpc3x[_RecoTracks]/F:Rpc3y[_RecoTracks]/F:
Rpc1dca[_RecoTracks]/F:Rpc1time[_RecoTracks]/F:Rpc1x[_RecoTracks]/F:
Rpc1y[_RecoTracks]/F");
- newsngmuons→Branch("RpcMatchMuID",0,"Rpc3dca[_RecoTracks]/F:
Rpc3time[_RecoTracks]/F:Rpc3x[_RecoTracks]/F:Rpc3y[_RecoTracks]/F:
Rpc1dca[_RecoTracks]/F:Rpc1time[_RecoTracks]/F:Rpc1x[_RecoTracks]/F:
Rpc1y[_RecoTracks]/F");

For the moment the timing and DCA distributions we use are those matching from station 1 for RPC1 and from station3 for RPC3. In addition, in order to improve the background rejection in the FVTX acceptance, for this analysis several new variables are added in relation to the FVTX-MuTr matching which were directly taken over from the corresponding methods in the PHMuTracksOut node. Those are fvtx_dr, fvtx_d ϕ and fvtx_d θ which compare the FVTX tracklets radial position, azimuthal and polar angles with

Variable	Definition
η	Pseudorapidity, used in secondary likelihood cuts
χ^2_{track}	Standard chi2 of μ track Kalman fitter
$DG0, DDG0$	Roads generated in MUID+MuTr planes. $DG0$ is distance between first gap road and track. $DDG0$ is opening angle between road and track.
DCA_r, DCA_Z	Distance of closest approach between μ track and beam axis (DCA_r). DCA_Z is the distance between the track's intersection with PHENIX's z-axis and the event vertex.
$RpcDca_{1,3}$	Distance between extrapolated track at RPC 1 or 3, and hit cluster at RPC 1 or 3.
dw_{23}	Reduced azimuthal bending angle of track. $dw_{23} = p_T \sin(\theta)(\phi_2 - \phi_3)$
$fvtx_d\theta$	
$fvtx_d\phi$	FVTX matched track matching residuals for ϕ, θ, dr .
$fvtx_dr$	

Table 5.1: Summary of engineered features from the data set used in this analysis.

those of the MuTr as an extrapolated z position between the two. Another FVTX related addition is the FVTX hit multiplicity within a cone of **INPUT RANGE HERE** around the projected track. This variable will henceforth be called FVTX_cone.

The "Basic Cut" is defined:

In this W analysis one is interested in removing most lower momentum particles which originate predominantly from background processes while keeping most of the W decay muons. With the above cuts, we aim to reduce part of the fake muons background

Variable	Lower Bound	Upper Bound
MuID lastGap	*	Gap 4
χ^2	0	20
$DG0$	0	20
$DDG0$	0	9
μ candidate	*	1

Table 5.2: The Basic Cuts used in the Run 13 analysis. lastGap refers to the last gap in the MUID which saw a μ candidate event. The fourth gap is the furthest penetration possible, therefore suggesting a high energy muon. Other parameters are described in table 5.1

assuring a good muon track reconstruction ($DG0, DDG0$ and χ^2 cuts) and selecting tracks with momentum smaller than the maximum possible physical energy. After applying these basic cuts, the background will be further reduced via a likelihood method, described in Section ??, where background and signal features will be studied in detailed.

The correlations between the several cut variables are shown in Fig. 6.2 for data and for the W-si. The only exception is the correlation between the vertex extrapolated variables DCA_z and DCA_r and the FVTX related matching variables. This is not entirely unexpected as both should be sensitive to the amount of multiple scattering in the central magnet yoke and initial shielding.

5.3 Feature Engineering

5.3.1 Discriminating Kinematic Variables

5.3.2 Simulations

Chapter 6

Spin Analysis

6.1 Classification of Signal or Background Events

After producing our data set, engineering features which help us convert our experimental data into observables, we are then tasked with the problem of separating out signal events from background events. Many processes are capable of producing muons, many of which are dominant in the W boson kinematic regime (Figure 6.1).

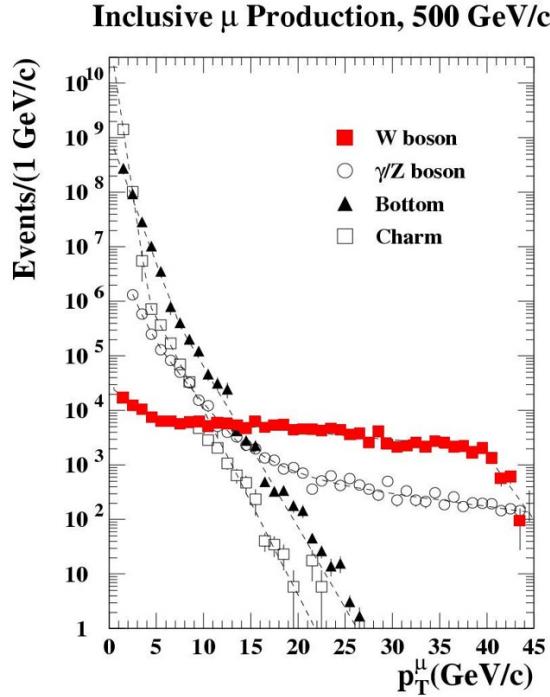


Figure 6.1: Observing the simulated production of muon as a function of p_T , we can see that in the kinematic region of W production that the dominant sources of muons come from other processes. The new PHENIX muon trigger threshold is sensitive at $10 \text{ GeV}/c$ and above.

We can divide up the total observed muon spectrum into contributions from three sources:

- Real Muon Background
 - Z, γ^*
 - $W \rightarrow \text{had}$
 - $W \rightarrow \tau\bar{\nu}$
 - onium
 - open charm
 - direct photon
- Fake Muons (Hadronic Background)

- Hadrons which are reconstructed as high p_T muons due to detector resolution.
- Signal Muons
 - Real $W \rightarrow \mu$ events.

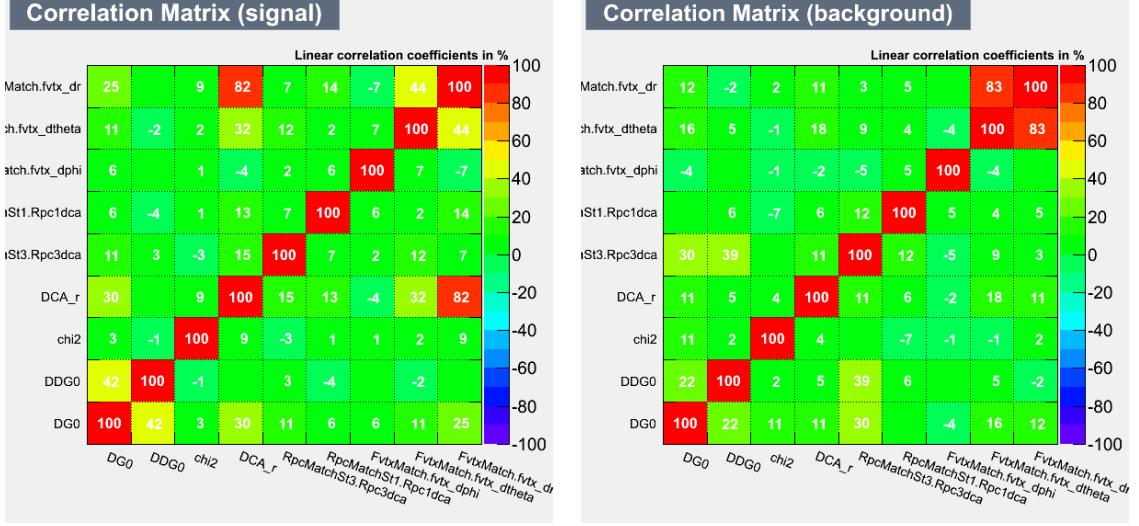
Previous analyses have attempted to separate the muon spectrum into p_T bins, to estimate the composition, however, because the $W \rightarrow \mu$ signal is so small in the forward kinematic regime, these methods are not sufficient, as there is no 'visible' cutoff in the spectrum. However, by using simulations. However, we may use other methods to split up our spectrum, with the ultimate goal of calculating A_L , and correcting for background dilution using the signal to background ratio. We must use another method to effectively describe the difference between an event which comes from a signal, vs background event.

6.1.1 Naive Bayes Classification

There are many techniques available for classifying a collection of variables (a feature set) into categories. Naive Bayes classification is an excellent candidate for classification, in cases where we have two classifications with distributions of featuresets which are uncorrelated. Naive Bayes even works when feature sets are slightly correlated. It is a robust, fast, scalable machine learning technique. Traditionally used for classification of text documents, Naive Bayes is also able to handle numeric features whose distributions are known [51].

In our analysis, we begin with a Naive Bayes classifier which is trained to classify two signal muons, vs background muons. We combine both Real Muon Background muons and Fake Muons (Hadronic Background Muons) in the label of "Background Muons" at this stage, though, later, we will separate out the muons further.

The discriminating variables described in 5 were chosen from the multitude of possible physical event parameters, because they were all maximally uncorrelated. Concretely, these correlations are presented in Figure 6.2.



(a) Correlations between kinematic variables, produced from simulated data.

(b) Correlations between kinematic variables, produced from the data, which is composed mostly of hadronic background

Figure 6.2: Low correlations between the signal variable distributions (from simulation), and the background variable distributions make this data set a good candidate for classification using Naive Bayes

Briefly, a Naive Bayes classifier may be constructed from the core of the familiar Bayes Theorem from probability and statistics.

In our case, we understand Naive Bayes as a conditional probability. Concretely, we consider a vector of features (i.e. our discriminating kinematic variables):

$$\mathbf{x} = (x_1, \dots, x_n) \quad (6.1)$$

and assume independence between each feature x_n . We then define the probability of a given classification, C_k given a set of features x_n :

$$p(C_k | x_1, \dots, x_n) \quad (6.2)$$

This conditional probability is defined in terms of Bayes Theorem:

$$p(C_k | \mathbf{x}) = \frac{p(C_k) p(\mathbf{x} | C_k)}{p(\mathbf{x})} \quad (6.3)$$

The terms here are defined as:

- $p(C_k) \rightarrow$ prior probability
- $p(\mathbf{x}|C_k) \rightarrow$ likelihood
- $p(\mathbf{x}) \rightarrow$ evidence

In principal, the final step in a classifier is to assign a class. This is done by computing the probability of a feature-set belonging to one class, or to another class, using Bayes Theorem. The class with the larger probability is then taken as the defacto classification of that particular feature set. However, we may instead observe these probabilities directly, and label data with this probability. This is what we ultimately call our " W_{ness} " parameter. This will be discussed in section 6.1.3.

6.1.2 Composition of Probability Distribution Functions

After we have engineered appropriate features to use in the analysis, we can proceed with composing probability density functions so we can proceed with the calculation of likelihoods, which will label our data set, allowing us to reduce our data set further from the basic cuts, without removing any signal events.

6.1.3 Labeling Data With Likelihood Ratio: W_{ness}

6.2 Extended Unbinned Maximum Likelihood Fits

6.2.1 Modeling The Hadronic Background

6.2.2 Modeling the Muon Background

6.2.3 Modeling the W-Signal

6.2.4 Overview

6.2.5 Fit Performance

6.2.6 S/BG and Muon Backgrounds

6.2.7 W_{ness} Dependence of S/BG

6.3 Calculation of A_L for $W \rightarrow \mu$

6.3.1 Overview

6.3.2 Asymmetry Calculation

6.3.3 Discussion of Work Done By Analysis Team

6.4 Data Validation

Mention Daniel's GPR, Ralf's PEPSI, Abraham's FVTX work, and Francesca's cross-checks.

6.4.1 Simulations and The Signal to Background Ratio**6.4.2 Gaussian Process Regression****6.4.3 Four Way Cross Validation****6.4.4 Asymmetry Consistency Check****6.4.5 Beam Polarization****6.4.6 Beam Luminosity****6.4.7 Code Cross Validation**

Chapter 7

The Vernier Analysis

7.1 Overview

7.2 Analysis Note Here

7.3 W Cross Section

Chapter 8

Discussion and Conclusion

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.1 First Thingie

.2 Second Thingie