Linear Regression 2

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Multiple Regression

- Multiple regression is an extension of simple linear regression where the model involves more than one independent variable to predict the dependent variable. This allows for more complex relationships between the dependent variable and multiple factors.
- In essence, multiple regression is the extension of ordinary least-squares (OLS) regression because it involves more than one explanatory variable.
    - Ordinary Least Squares regression (OLS) is a common technique for estimating coefficients of linear regression equations which describe the relationship between one or more independent quantitative variables and a dependent variable (simple or multiple linear regression).

Formula and Calculation of Multiple Linear Regression

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_p x_{ip} + \epsilon$$

where, for $i = n$ observations:

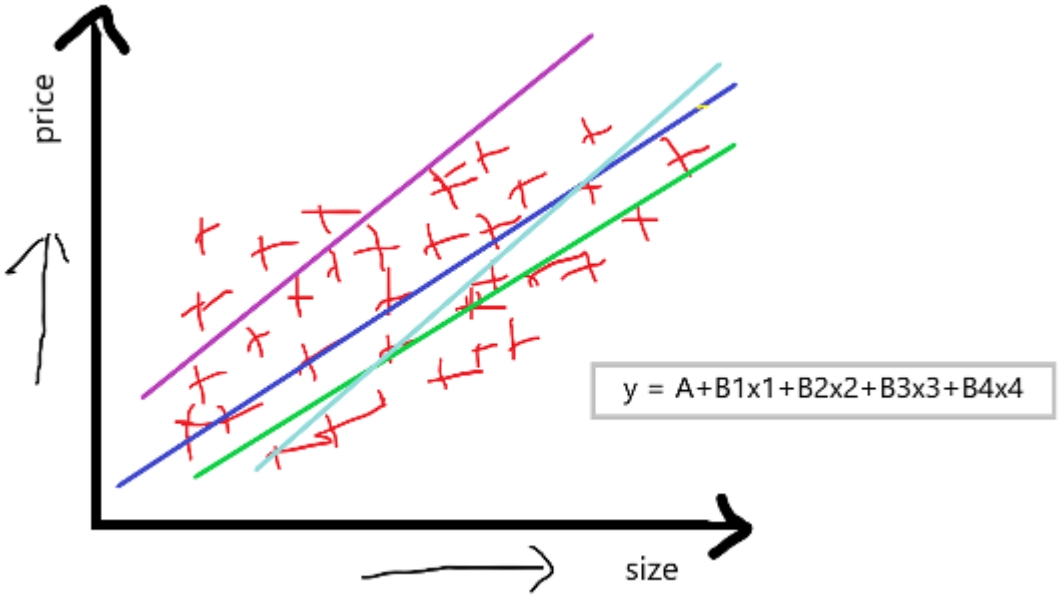
y_i = dependent variable

x_i = explanatory variables

β_0 = y-intercept (constant term)

β_p = slope coefficients for each explanatory variable

ϵ = the model's error term (also known as the residuals)



- Interpreting Coefficients
  - Significance: The coefficient's p-value indicates its statistical significance. A low p-value suggests that the variable is a significant predictor of Y.
  - Magnitude: The absolute value of the coefficient reflects the strength of the relationship between the variable and Y. A larger absolute value indicates a stronger relationship.
  - Direction: The sign of the coefficient (positive or negative) indicates the direction of the relationship. A positive coefficient means that as the independent variable increases, so does the dependent variable. A negative coefficient means that as the independent variable increases, the dependent variable decreases.
- Assumptions of Multiple Regression
  - Linearity: The relationship between Y and each Xi is linear.

- Independence: The observations are independent of each other.
- Homoscedasticity: The variance of the errors is constant across all levels of the independent variables.
- Normality: The errors are normally distributed.
- No multicollinearity: The independent variables are not perfectly correlated with each other.

Cost Function

The cost function measures how well your model fits the training data. In the case of multiple regression, it quantifies the error between the predicted values ($h_\beta(x)$) and the actual values (y) in your dataset.

The Mean Squared Error (MSE) is a commonly used cost function:

$$J(\beta) = \frac{1}{2m} \sum_{i=1}^{m} (h_\beta(x^{(i)}) - y^{(i)})^2$$

Where:

- m is the number of training examples.
- $x^{(i)}$ is the feature vector of the ith training example.
- $y^{(i)}$ is the actual output for the ith example.
- $h_{\beta(}x^{(i)})$ is the predicted output for the ith example using the current parameters β.

Gradient

The gradient of the cost function indicates the direction and magnitude of the steepest ascent (or descent) of the function. In the context of multiple regression, it tells us how to update the coefficients (β) to minimize the cost function.

The gradient of the cost function with respect to each coefficient β_j is calculated as:

$$\frac{\partial J(\beta)}{\partial \beta_j} = \frac{1}{m} \sum_{i=1}^{m} (h_\beta(x^{(i)}) - y^{(i)}) \cdot x_j^{(i)}$$
$$\text{for } j = 0, 1, \ldots, n.$$

Where:

- x_j^i is the jth feature of the ith training example
- $h_{\beta(}x^{(i)})$ is the predicted output for the ith example using the current parameters β.
- $y^{(i)}$ is the actual output for the ith example.

This expression computes the partial derivative of the cost function with respect to a specific coefficient β_j, considering the contribution of each training example.

Optimization:

With the cost function and gradient, you can employ optimization algorithms like gradient descent to iteratively update the coefficients (β) until convergence. In each iteration, the coefficients are adjusted in the direction that minimizes the cost function, ultimately leading to a set of coefficients that best fit the training data.

Exploring Data

[Download Dataset From Here]

```
# Importing all required modules
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
```

```
# reading a CSV file named "data.csv" located in the "Data" folder on the your Go
# write your data file path
df = pd.read_csv('/content/CarPred.csv')
```

Data Prepration

- In data preparation, first we will drop all categorical variables. These variables are typically not suitable for direct numerical representation and may require encoding techniques like one-hot encoding or label encoding to convert them into a format that the model can process effectively.
- Data is split into input features (X) and target variable (Y), with standardization or normalization applied to the input features to ensure uniform scaling and mitigate the influence of differences in the magnitude of features, enabling more stable and efficient model training.

```
df.make.value_counts()
```

make	count
hyundai	3375
toyota	3375
suzuki	3359
maruti	3343
honda	3314
ford	3234

dtype: int64

```
# drops the "make" and "model" columns from the DataFrame "df" along the column a
df.drop(["make", "model"], axis = 1, inplace=True)
```

```
df.head()
```

	transmission_type	seats_cop	seats_family	seats_large	fuel_cng	fuel_dies
0	0	0	0	1	0	
1	0	0	0	1	0	
2	1	0	0	0	0	
3	1	0	0	0	0	
4	1	0	0	1	0	

```
# separating the DataFrame "df" into input features X and target variable Y.
# X contains all columns except "selling_price", while Y contains only the "selli

X = df[df.columns.drop("selling_price")]
Y = df["selling_price"]
```

```
# converting the input features X and the target variable Y from pandas DataFrame
# This conversion is often done to prepare the data for machine learning algorith
X = X.to_numpy()
Y = Y.to_numpy()
```

```
# standard normalisation
# calculating the mean (u) and standard deviation (std) along each column (axis=0
# These statistics are used for standard normalization, where each feature is tra
# This step ensures that all features are on the same scale, which can help impro

u = np.mean(X, axis=0)
std = np.std(X, axis=0)


u
```

```
array([4.99250000e-01, 1.25150000e-01, 2.50350000e-01, 5.00400000e-01,
       3.08500000e-02, 6.35500000e-02, 1.23900000e-01, 2.51050000e-01,
       4.99600000e-01, 4.98350000e-01, 4.94100000e-01, 2.01051390e+03,
       1.00041147e+05, 2.99900990e+03, 2.74710600e+02])
```

```
std
```

```
array([4.99999437e-01, 3.30888920e-01, 4.33214586e-01, 4.99999840e-01,
       1.72911184e-01, 2.43949580e-01, 3.29467434e-01, 4.33617225e-01,
       4.99999840e-01, 4.99997277e-01, 4.99965189e-01, 6.35487268e+00,
       5.77092436e+04, 1.15653080e+03, 1.29838297e+02])
```

```
# performing standard normalization on the input features X by subtracting the me
# Standard normalization ensures that all features have a mean of 0 and a standar
# This preprocessing step is essential for many machine learning algorithms, espe

X = (X - u)/std


# adding a column of ones to the input features matrix X.
# The added column of ones represents the intercept term in linear regression mod
# Including this intercept term allows the model to learn an offset from the orig
# where the dependent variable is not zero when all independent variables are zer

ones = np.ones((X.shape[0],1))
X = np.hstack((ones, X))


print(X.shape, Y.shape)
```

```
(20000, 16) (20000,)
```

Implementation

Defining Hypothesis

```
# implementing the hypothesis equation for linear regression, where the predicted
# It iterates over each feature value in x, multiplies it by the corresponding pa

def hypothesis(x, theta):
    y_ = 0.0
    n = x.shape[0]
    for i in range(n):
      y_ += (theta[i]*x[i])
    return y_
```

Defining Error Function

```
# calculating the mean squared error (MSE) between the actual target values y and
# It iterates over each data point, computes the squared difference between the a
# Finally, it returns the average squared error by dividing the total squared err

def error(x, y, theta):
  e = 0.0
  m = X.shape[0]

  for i in range(m):
    y_ = hypothesis(X[i], theta)
    e += (y[i] - y_)**2

  return e/m
```

Defining Gradient

```
# computes the gradient of the cost function with respect to each parameter θj in
# It iterates over each parameter θj  and for each parameter, it iterates over ea
# The gradient is the sum of the product of the error (the difference between the
# Finally, it returns the average gradient for each parameter.

def gradient(X, y, theta):
  m, n = X.shape

  grad = np.zeros((n,))

  # for all values of j
  for j in range(n):
    # sum over all examples
    for i in range(m):
      y_ = hypothesis(X[i], theta)
      grad[j] += (y_ - y[i])*X[i][j]

  # out of loops
  return grad/m
```

Defining Gradient Descent

```
# implementing gradient descent to minimize the cost function iteratively.
# It initializes the parameter vector θ with zeros and iterates over a fixed numb
# Then, it updates each parameter θj by subtracting the product of the learning r
# The process continues until convergence or until the maximum number of epochs i
# Finally, it returns the optimized parameter vector θ and a list of errors at ea

def gradient_descent(X, y, learning_rate=0.1, max_epochs = 100):
  m, n = X.shape
  theta = np.zeros((n,))
  error_list = []

  for i in range(max_epochs):
    e = error(X, y, theta)
    error_list.append(e)

    # gradient descent
    grad = gradient(X, y, theta)
    for j in range(n):
      theta[j] = theta[j] - learning_rate*grad[j]


  return theta, error_list


# snippet measures the time taken to execute the gradient descent algorithm. It s
# Afterward, it stops the timer, calculates the duration of execution, and prints
# This is useful for assessing the algorithm's efficiency and comparing different

import time
start = time.time()
theta, error_list = gradient_descent(X, Y)
end = time.time()
print("Time taken is: ", end-start)
```

```
Time taken is:  274.7797317504883
```

```
print(theta)
```

```
[ 2.66264733e+05 -6.83299890e+02  1.46785635e+02  1.38290563e+02
  5.99317666e+01  5.00566845e+02  4.10095966e+01  6.94401366e+01
 -1.69202454e+02  5.47068986e+01  5.09551339e+02 -1.03189963e+03
  6.98370728e+00  2.51785560e+02 -7.37756347e+01  6.35011335e+04]
```

Linear Regression Model using Scikit-learn

```
# The linear_model module in Scikit-Learn provides an implementation of linear re
#  importing the LinearRegression class from the linear_model module in Scikit-Le
# Then, we prepare the input features (X) by dropping the "selling_price" column
# This sets up the data for training the linear regression model.

from sklearn.linear_model import LinearRegression
model = LinearRegression()

X = df[df.columns.drop('selling_price')]
Y = df["selling_price"]


# stadardization
# importing the StandardScaler and MinMaxScaler classes from the preprocessing mo
# Then, it initializes a StandardScaler object named sc.
# This scaler will be used to standardize the input features, ensuring that they
#  Standardization helps in scaling the features to a similar range, which can im

from sklearn.preprocessing import StandardScaler, MinMaxScaler
sc = StandardScaler()


# standardizes the input features stored in the DataFrame X.
# It first extracts the column names from X and then applies the fit_transform me
# standardization ensures that all features have a mean of 0 and a standard devia

cols = X.columns
X[cols] = sc.fit_transform(X[cols])
# x = sc.fit_transform(x) this will result into a numpy array
```

```
<ipython-input-27-f5ba76c9be82>:6: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/s
  X[cols] = sc.fit_transform(X[cols])
```

```
X.head()
```

	transmission_type	seats_cop	seats_family	seats_large	fuel_cng	fuel_dies
0	-0.998501	-0.378224	-0.577889	0.9992	-0.178415	-0.260
1	-0.998501	-0.378224	-0.577889	0.9992	-0.178415	-0.260
2	1.001501	-0.378224	-0.577889	-1.0008	-0.178415	-0.260
3	1.001501	-0.378224	-0.577889	-1.0008	-0.178415	3.838
4	1.001501	-0.378224	-0.577889	0.9992	-0.178415	3.838

```
# The linear_model module in Scikit-Learn provides an implementation of linear re
from sklearn.linear_model import LinearRegression
# Creating an instance of the linear regression model.
model = LinearRegression()
```

```
# fitting the linear regression model model to the standardized input features X
# The fit method trains the model by adjusting its parameters to minimize the dif
model.fit(X, Y)
```

```
LinearRegression()
```

```
# initializing an instance of the LinearRegression model from scikit-learn, which
LinearRegression()
```

```
LinearRegression()
```

```
# Retrieving the intercept from the linear regression model.
model.intercept_
```

```
266271.8057707528
```

```
# The coefficients of the linear regression model, indicating the weight/importan
model.coef_
```

```
array([-6.83234746e+02,  1.52442100e+02,  1.45427665e+02,  6.77434690e+01,
        5.01795178e+02,  4.26800365e+01,  7.19523318e+01, -1.66045123e+02,
        5.82983773e+01,  5.09457373e+02, -1.03191098e+03,  6.79470723e+00,
        2.51905362e+02, -7.38843334e+01,  6.35028215e+04])
```

```
# Calculating the coefficient of determination R square of the linear regression
model.score(X, Y)
```

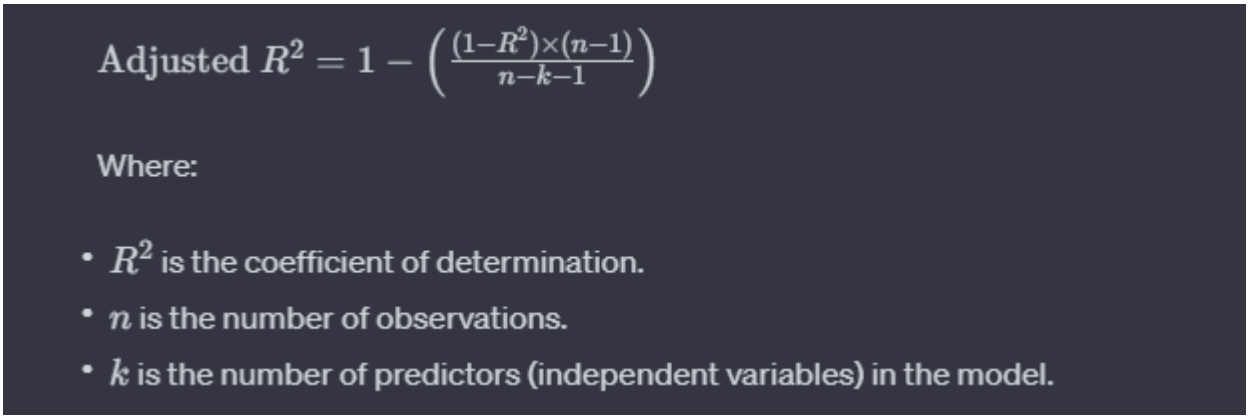
```
0.5729801349938517
```

Adjusted R_Square

Adjusted R_Square is a modified version of the coefficient of determination (R^2) that adjusts for the number of predictors in a regression model.

While (R^2) measures the proportion of variance in the dependent variable explained by the independent variables, adjusted (R^2) penalizes for the inclusion of unnecessary predictors that do not significantly improve the model's explanatory power.

The formula for adjusted (R^2):



Adjusted (R^2) increases only if the new term improves the model more than would be expected by chance. It avoids overestimating the performance of the model when additional predictors are added, as (R^2) typically increases with the addition of predictors regardless of their actual usefulness.

```
print("Adjusted R-squared: ", 1 - (1-model.score(X,Y))*(len(Y) - 1)/(len(Y) - 1 -
```

```
Adjusted R-squared:  0.5726382284813112
```

Statsmodel Implementation

```
# importing the statsmodels module, providing access to its API for statistical m
import statsmodels.api as sm


# adds a constant term to the feature matrix X, then performs ordinary least squa

X_sm = sm.add_constant(X)
# OLS -> ordinary least square
sm_model = sm.OLS(Y, X_sm).fit()


# summary of the regression model sm_model
# providing detailed information about the model's performance, coefficients, sta
# It gives insights into how well the model fits the data and the significance of

print(sm_model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:          selling_price   R-squared:                       0.57
Model:                            OLS   Adj. R-squared:                  0.57
Method:                 Least Squares   F-statistic:                     1788
Date:                Tue, 17 Sep 2024   Prob (F-statistic):               0.0
Time:                        07:04:19   Log-Likelihood:            -2.4662e+0
No. Observations:               20000   AIC:                         4.933e+0
Df Residuals:                   19984   BIC:                         4.934e+0
Df Model:                          15
Covariance Type:            nonrobust
==============================================================================
```

```
                         coef    std err          t      P>|t|      [0.025
------------------------------------------------------------------------------
const                2.663e+05    387.920    686.409      0.000    2.66e+05
transmission_type   -683.2347    388.042     -1.761      0.078   -1443.829
seats_cop            152.4421    514.449      0.296      0.767    -855.921
seats_family         145.4277    583.598      0.249      0.803    -998.472
seats_large           67.7435    615.347      0.110      0.912   -1138.388
fuel_cng             501.7952    539.423      0.930      0.352    -555.519
fuel_diesel           42.6800    655.506      0.065      0.948   -1242.165
fuel_electric         71.9523    811.278      0.089      0.929   -1518.220
fuel_lpg            -166.0451   1012.091     -0.164      0.870   -2149.828
fuel_patrol           58.2984   1134.654      0.051      0.959   -2165.718
seller_dealer        509.4574    388.108      1.313      0.189    -251.265
seller_self        -1031.9110    387.989     -2.660      0.008   -1792.402
year                   6.7947    388.006      0.018      0.986    -753.729
km_driven            251.9054    388.054      0.649      0.516    -508.712
engine               -73.8843    388.096     -0.190      0.849    -834.585
max_power           6.35e+04    387.998    163.668      0.000    6.27e+04
==============================================================================
Omnibus:                        1.574   Durbin-Watson:                   2.014
Prob(Omnibus):                  0.455   Jarque-Bera (JB):                1.577
Skew:                          -0.008   Prob(JB):                        0.455
Kurtosis:                       2.959   Cond. No.                         5.87
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correct
```

Assumptions in Data

- Input and Target Variable Linear Relationship:
  - This assumption assumes that there is a linear relationship between the input variables (also known as independent variables or features) and the target variable.
  - Linear regression models, such as ordinary least squares (OLS), assume that the relationship between the predictors and the response variable is linear. This means that a change in the predictor variable leads to a proportional change in the response variable.
  - Scatter plots or correlation coefficients can be used to visualize or quantify the linear relationship between the input variables and the target variable. If the relationship appears nonlinear, transformations or more complex modeling techniques may be necessary.
- No Multicollinearity:
  - Multicollinearity occurs when two or more predictor variables in a regression model are highly correlated with each other. This can cause problems in the estimation of the regression coefficients.
  - Multicollinearity can inflate the standard errors of the regression coefficients, making them unreliable for inference. It can also make it difficult to interpret the

individual effects of predictor variables on the target variable.
    - Variance inflation factor (VIF) and correlation matrices are common tools used to detect multicollinearity. A VIF greater than 10 or high pairwise correlations (typically above 0.7 or 0.8) between predictor variables may indicate multicollinearity.
    - To address multicollinearity, one approach is to remove one of the correlated variables from the model. Alternatively, techniques such as ridge regression or principal component analysis (PCA) can be used to mitigate its effects while retaining all variables in the model.
- Target Variable Follows Normal Distribution:
    - This assumption implies that the values of the target variable (also known as the dependent variable or response variable) are normally distributed within the population.
    - Normality is important because many statistical tests and estimators assume that the data are normally distributed. It facilitates the interpretation of statistical tests and helps ensure the validity of statistical inference.

VIF - Variance Inflation Factor

- The Variance Inflation Factor (VIF) is a statistical tool used in regression analysis to detect and measure the severity of multicollinearity.
  Multicollinearity occurs when two or more independent variables in a regression model are highly correlated with each other.
- This can lead to several problems, including:
    - Inflated variances of the regression coefficients: This makes it difficult to accurately estimate the true effect of each independent variable on the dependent variable.
    - Unreliable coefficient estimates: Even if the coefficients are statistically significant, they may not be reliable enough to draw meaningful conclusions.
    - Instability of the model: Small changes in the data can lead to large changes in the estimated coefficients.

The VIF calculates how much the variance of a regression coefficient is inflated due to multicollinearity. It is calculated for each independent variable in the model. A VIF value of 1 indicates no multicollinearity, while values greater than 1 indicate some degree of multicollinearity. The higher the VIF value, the more severe the multicollinearity.

Here are some general guidelines for interpreting VIF values:

- VIF < 5: No multicollinearity issue.
- 5 ≤ VIF < 10: Moderate multicollinearity, may not be a major problem.
- VIF ≥ 10: Severe multicollinearity, investigate further and consider remedial actions.

It is important to note that these are just guidelines, and the decision of whether or not to take action to address multicollinearity will depend on the specific context of your analysis.

Here are some of the ways to address multicollinearity:

- Remove one or more of the collinear variables: This is the most common approach, but it should only be done if the variable(s) being removed are not theoretically important to the model.
- Combine collinear variables: If two or more variables are measuring the same underlying construct, they can be combined into a single variable.
- Use ridge regression or LASSO: These are regression techniques that can help to reduce the impact of multicollinearity on the coefficient estimates.

VIF - Variance Inflation Factor

```
# importing the function variance_inflation_factor from the statsmodels.stats.out
from statsmodels.stats.outliers_influence import variance_inflation_factor


# Compute variance inflation factor (VIF) for predictor variables
# and create a DataFrame to store the results
vif = pd.DataFrame()

# Set the features column in the DataFrame to the column names of X
vif['features'] = X.columns

# Compute the VIF for each predictor variable in X
vif['VIF'] = [variance_inflation_factor(X.values, i) for i in range(X.shape[1])]

# Round the VIF values to two decimal places
vif['VIF'] = round(vif['VIF'], 2)

# Sort the DataFrame by VIF values in descending order
vif = vif.sort_values(by='VIF', ascending=False)

# Display the DataFrame with feature names and their corresponding VIF values
vif
```

	features	VIF
8	fuel_patrol	8.56
7	fuel_lpg	6.81
6	fuel_electric	4.37
5	fuel_diesel	2.86
3	seats_large	2.52
2	seats_family	2.26
4	fuel_cng	1.93
1	seats_cop	1.76
0	transmission_type	1.00
9	seller_dealer	1.00
10	seller_self	1.00
11	year	1.00
12	km_driven	1.00
13	engine	1.00
14	max_power	1.00

Above code computes the variance inflation factor (VIF) for each predictor variable in X to check for multicollinearity.

It creates a DataFrame vif to store the results, where each row corresponds to a predictor variable and its associated VIF value.

The VIF quantifies how much the variance of an estimated regression coefficient is increased because of collinearity.

Removing all those variables which have Multicollinearity

In our dataset, although multicollinearity is not significant, we'll simulate it to demonstrate how to address it effectively.

```
# Define the columns to keep, excluding those suspected of multicollinearity
cols2 = ["max_power", "transmission_type", "year", "km_driven", "fuel_electric",
```

```
# Extract the selected columns from the feature matrix
X2 = X[cols2]

# Add a constant term for the intercept in the feature matrix
X2_sm = sm.add_constant(X2)

# Fit an Ordinary Least Squares (OLS) regression model using the selected feature
sm_model = sm.OLS(Y, X2_sm).fit()


# Print the summary statistics of the regression model
print(sm_model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:          selling_price   R-squared:                       0.57
Model:                            OLS   Adj. R-squared:                  0.57
Method:                 Least Squares   F-statistic:                     4467
Date:                Tue, 17 Sep 2024   Prob (F-statistic):               0.0
Time:                        07:12:52   Log-Likelihood:            -2.4663e+0
No. Observations:               20000   AIC:                         4.933e+0
Df Residuals:                   19993   BIC:                         4.933e+0
Df Model:                           6
Covariance Type:            nonrobust
==============================================================================
                     coef    std err          t      P>|t|      [0.025
------------------------------------------------------------------------------
const           2.663e+05    387.937    686.379      0.000    2.66e+05
max_power       6.351e+04    387.949    163.697      0.000    6.27e+04
transmission_type -678.5675    387.973     -1.749      0.080   -1439.026
year              12.8687    387.962      0.033      0.974    -747.569
km_driven        258.0840    388.017      0.665      0.506    -502.462
fuel_electric     48.2980    387.945      0.124      0.901    -712.107
seats_cop        100.0867    387.988      0.258      0.796    -660.402
==============================================================================
Omnibus:                        1.714   Durbin-Watson:                   2.01
Prob(Omnibus):                  0.424   Jarque-Bera (JB):                1.70
Skew:                          -0.007   Prob(JB):                        0.42
Kurtosis:                       2.957   Cond. No.                         1.0
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correc
```

```
X2 = X[cols2]

X2_sm = sm.add_constant(X2)

sm_model = sm.OLS(Y, X2_sm).fit()


print(sm_model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:          selling_price   R-squared:                       0.57
Model:                            OLS   Adj. R-squared:                  0.57
```

```
Method:                 Least Squares   F-statistic:                     4467
Date:                Tue, 17 Sep 2024   Prob (F-statistic):               0.00
Time:                        07:13:15   Log-Likelihood:            -2.4663e+05
No. Observations:               20000   AIC:                         4.933e+05
Df Residuals:                   19993   BIC:                         4.933e+05
Df Model:                           6
Covariance Type:            nonrobust
==========================================================================
                         coef    std err          t      P>|t|      [0.025
--------------------------------------------------------------------------
const                2.663e+05    387.937    686.379      0.000    2.66e+05
max_power            6.351e+04    387.949    163.697      0.000    6.27e+04
transmission_type    -678.5675    387.973     -1.749      0.080   -1439.026
year                   12.8687    387.962      0.033      0.974    -747.569
km_driven             258.0840    388.017      0.665      0.506    -502.462
fuel_electric          48.2980    387.945      0.124      0.901    -712.107
seats_cop             100.0867    387.988      0.258      0.796    -660.402
==========================================================================
Omnibus:                        1.714   Durbin-Watson:                   2.01
Prob(Omnibus):                  0.424   Jarque-Bera (JB):                1.70
Skew:                          -0.007   Prob(JB):                        0.42
Kurtosis:                       2.957   Cond. No.                         1.0
==========================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correct
```

```
# Calculate the Variance Inflation Factor (VIF) for each feature to check for mul
vif = pd.DataFrame()
x_t = X2
vif['features'] = x_t.columns
vif['VIF'] = [variance_inflation_factor(x_t.values, i) for i in range(x_t.shape[1
vif['VIF'] = round(vif['VIF'], 2)
vif = vif.sort_values(by='VIF', ascending=False)
vif
```

	features	VIF
0	max_power	1.0
1	transmission_type	1.0
2	year	1.0
3	km_driven	1.0
4	fuel_electric	1.0
5	seats_cop	1.0

Splitting Data for Evaluation

The process of splitting data into training and testing sets, combined with maintaining consistency using a random seed, is fundamental for developing robust and reliable machine

learning models.

It ensures that models generalize well to unseen data and enables consistent evaluation of model performance.

```
# Importing the train_test_split function from the sklearn.model_selection module
from sklearn.model_selection import train_test_split


# Splitting the data into training and testing sets with 90% for training and 10%
# while maintaining consistency using a random seed of 1
x_train, x_test, y_train, y_test = train_test_split(X2, Y, test_size = 0.1, rando


print("Training set shape X: ", x_train.shape)
print("Training set shape Y: ", y_train.shape)
print("Test set shape X: ", x_test.shape)
print("Test set shape Y: ", y_test.shape)
```

```
Training set shape X:  (18000, 6)
Training set shape Y:  (18000,)
Test set shape X:  (2000, 6)
Test set shape Y:  (2000,)
```

```
# Creating a Linear Regression model instance and fitting it to the training data
final_model = LinearRegression()
final_model.fit(x_train, y_train)
```

```
▾ LinearRegression
LinearRegression()
```

```
# Computing the R-squared score of the final model on the training data
final_model.score(x_train, y_train)
```

```
0.5721401925646443
```

```
# Getting the intercept of the final model
final_model.intercept_
```

```
266207.0902900053
```

```
# Getting the coefficients of the final model
final_model.coef_
```

```
array([ 6.35529402e+04, -6.31518829e+02,  2.44810594e+02,  1.24676617e+02,
       -1.36838783e+02, -4.51104629e+01])
```

Predictions using test data set

```
# Generating predictions on the test dataset using the final trained model
y_pred = final_model.predict(x_test)
# y_pred contains the predicted values of the target variable based on the featur
```

Evaluating Model Performance

```
# model performance
# Importing evaluation metrics for regression models from scikit-learn
from sklearn.metrics import mean_absolute_error, mean_squared_error, mean_absolut
# These metrics will be used to assess the performance of the regression model


print("Mean absolute error: ", mean_absolute_error(y_pred, y_test))
print("Mean squared error: ", mean_squared_error(y_pred, y_test))
print("Root Mean squared error: ", np.sqrt(mean_squared_error(y_pred, y_test)))
print("Mean absolute percentage error: ", mean_absolute_percentage_error(y_pred,
```

```
Mean absolute error:  42831.8168004857
Mean squared error:  2889049562.0473557
Root Mean squared error:  53749.879646817404
Mean absolute percentage error:  0.1701119256512152
```

```
# Plotting the distribution of the target variable (y_train) using seaborn
sns.distplot(y_train)
```

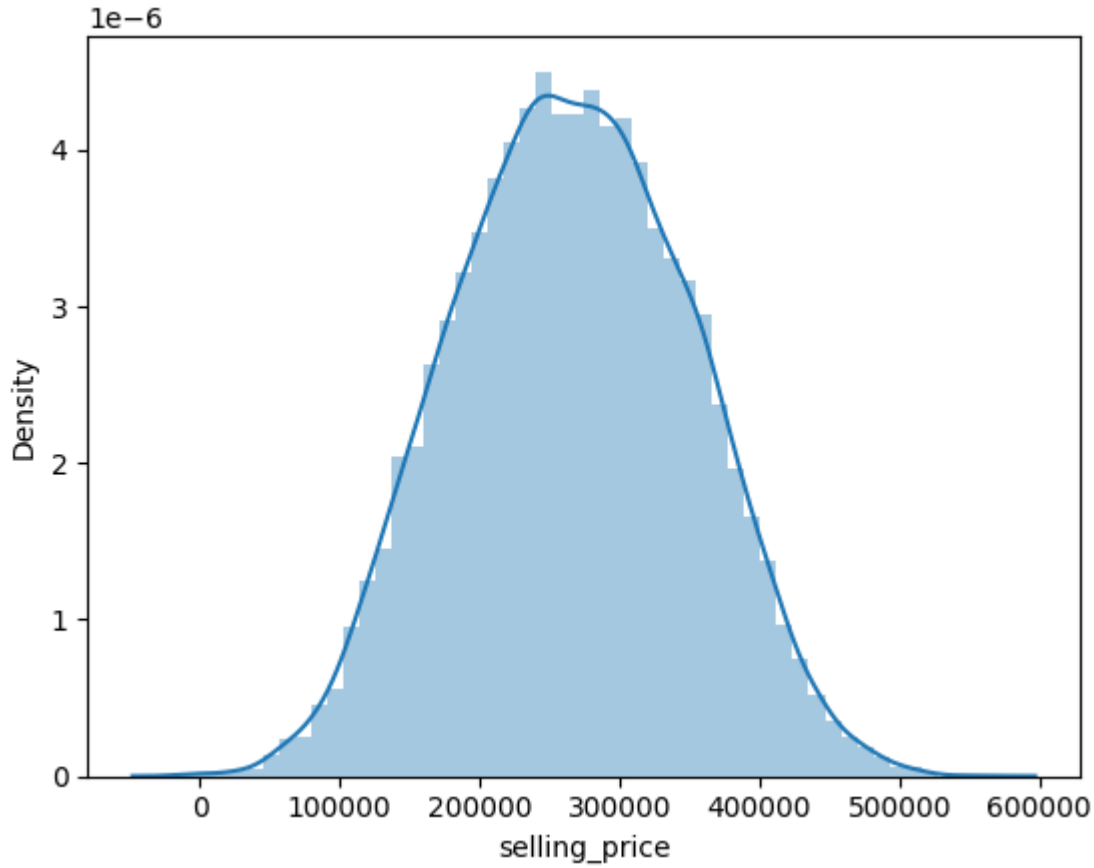
```
<ipython-input-58-f34bd58de931>:2: UserWarning:

`distplot` is a deprecated function and will be removed in seaborn v0.14.0.

Please adapt your code to use either `displot` (a figure-level function with
similar flexibility) or `histplot` (an axes-level function for histograms).

For a guide to updating your code to use the new functions, please see
https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751

  sns.distplot(y_train)
<Axes: xlabel='selling_price', ylabel='Density'>
```



- The resulting plot is a normal distribution curve, it indicates that the distribution of the target variable in the training data approximates a normal distribution.
- Combining this observation with the assumption that the target variable follows a normal distribution, we reinforce the notion that our data aligns with this assumption.
- By verifying that the target variable conforms to a normal distribution, we validate an essential assumption underlying various statistical techniques and machine learning algorithms. This validation enhances the reliability and interpretability of our analyses and predictions. Furthermore, it enables us to make informed decisions about the suitability of certain modeling approaches and the applicability of statistical tests that assume normality.

Residual Analysis

- Residual analysis is an essential step in evaluating the performance of a multiple linear regression model. It involves examining the residuals (errors) of the model to assess how well the model fits the data and whether the assumptions of linear regression hold. Residuals are the differences between the observed values and the predicted values of the dependent variable.
- What Are Residuals?
    - In a multiple linear regression model, residuals are defined as:
        - Residual = y_{actual} - $y_{predicted}$

Where:

- y_{actual} is the actual observed value of the dependent variable.
- $y_{predicted}$ is the value predicted by the regression model.

Normality of Errors

- In the context of linear regression, the errors or residuals should ideally follow a normal distribution. This means that the distribution of the discrepancies between the observed and predicted values should resemble a bell curve.
- A normal distribution of errors indicates that the model is capturing the underlying patterns in the data effectively. Deviations from normality may suggest that the model is inadequately capturing certain features of the data or that there are influential outliers skewing the distribution.

```
# errors normally distributed
# Predicting the target variable using the final linear regression model and stor
pred = final_model.predict(x_train)


# Calculating the residuals by subtracting the actual target variable values from

errors = pred - y_train # residuals


sns.distplot(errors)
```

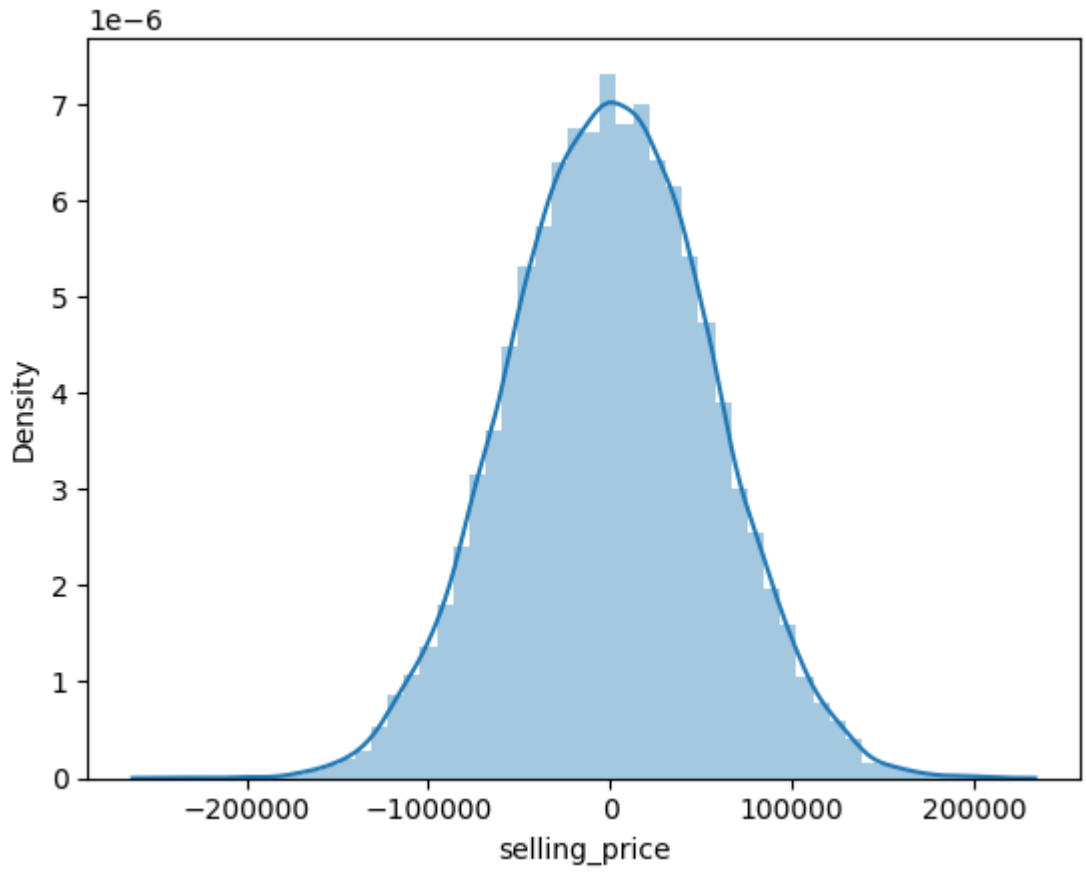
```
<ipython-input-61-4a151ca7bc24>:1: UserWarning:

`distplot` is a deprecated function and will be removed in seaborn v0.14.0.

Please adapt your code to use either `displot` (a figure-level function with
similar flexibility) or `histplot` (an axes-level function for histograms).

For a guide to updating your code to use the new functions, please see
https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751

  sns.distplot(errors)
<Axes: xlabel='selling_price', ylabel='Density'>
```



- A normally distributed error plot indicates that the linear regression model's assumptions regarding the distribution of errors are met.
- This normal distribution suggests that the model captures the underlying patterns in the data well, with errors symmetrically distributed around zero. It indicates that the model's predictions are unbiased on average and provides confidence in the model's performance.

Autocorrelation and Heteroscedasticity

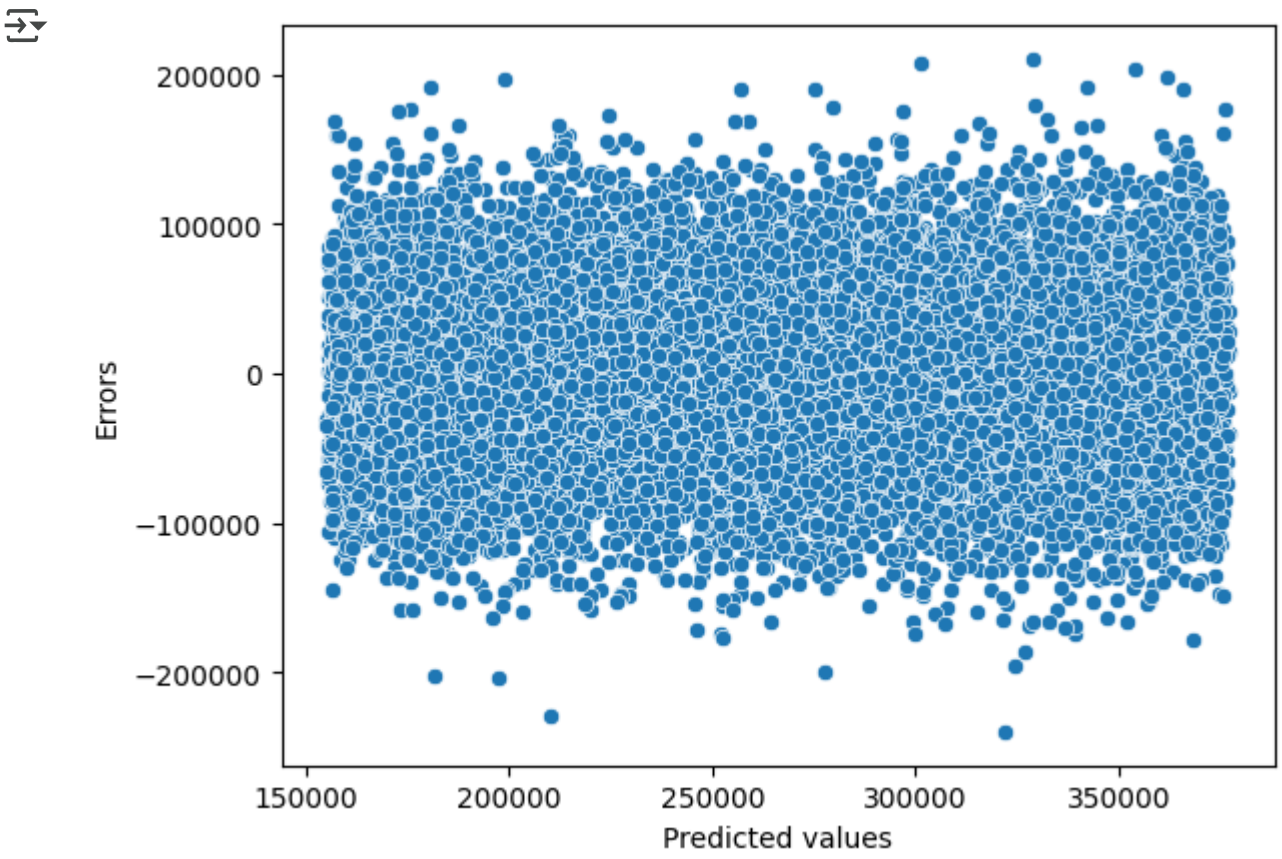
Autocorrelation: Autocorrelation refers to the correlation between the error terms in a regression model. In a time series context, it refers to the correlation between a variable's current value and its past values at different lags. In the context of linear regression, autocorrelation occurs when the error terms exhibit patterns or correlations with each other, indicating that there is still some information in the errors that the model has not captured.

Autocorrelation violates the assumption of independence of errors, which is crucial for the reliability of regression analysis.

Heteroscedasticity: Heteroscedasticity, on the other hand, refers to the situation where the variance of the errors (or residuals) in a regression model is not constant across all levels of the independent variables. In simpler terms, it means that the spread of the residuals differs for different values of the predictor variables. Heteroscedasticity violates the assumption of homoscedasticity, which states that the variance of the errors should remain constant across all levels of the independent variables. When heteroscedasticity is present, it can lead to inefficient estimates of the regression coefficients and affect the reliability of statistical inferences drawn from the model.

```
# Autocorrelation and Heteroscedasticity
# plot (predicted values vs residuals)

# Generating a scatter plot to visualize predicted values versus residuals
sns.scatterplot(x=pred, y=errors)
plt.xlabel("Predicted values")
plt.ylabel("Errors")
plt.show()
```



The plot helps us identify any patterns or trends between the predicted values and the errors, which can indicate the presence of autocorrelation or heteroskedasticity in the model's residuals.

The scatter plot appears cylindrical in shape, it suggests the presence of heteroskedasticity, where the spread of errors varies across different ranges of predicted values.

This pattern indicates that the variance of errors is not consistent, violating one of the assumptions of linear regression.

Additionally, if there are any discernible patterns or trends in the scatter plot, it could also indicate the presence of autocorrelation, suggesting that the errors are correlated with each other.

Both heteroskedasticity and autocorrelation can affect the reliability and accuracy of the linear regression model's predictions.

Handling High-End Car Data

Filtering out high-end car data (selling price > 25 lakhs) from the dataset to mitigate skewness and potential outliers that may not align with the resale market being analyzed.

Filtering out high-end car data with selling prices greater than 25 lakhs is important according to the text provided because it helps address two main issues:

- Skewness Mitigation: If the distribution of selling prices in the dataset is highly right-skewed. By removing high-end car data, which likely represents outliers in terms of price, the skewness of the distribution is reduced. This is crucial because skewed data distributions can affect the performance of linear regression models and violate the assumption of normality.
- Outlier Removal: High-end cars may not align with the resale market being analyzed. Their inclusion could introduce outliers that disproportionately influence model fitting and prediction accuracy. By filtering out these high-end cars, the author aims to focus the analysis on the majority of cars within the resale market, potentially improving the model's ability to generalize and make accurate predictions for typical resale scenarios.

```
df3 = df[df["selling_price"] > 25]
```

```
df3.head()
```

	transmission_type	seats_cop	seats_family	seats_large	fuel_cng	fuel_dies
0	0	0	0	1	0	
1	0	0	0	1	0	
2	1	0	0	0	0	
3	1	0	0	0	0	
4	1	0	0	1	0	

```
df3.shape
```

```
(19993, 16)
```

```
sns.distplot(df3["selling_price"])
```

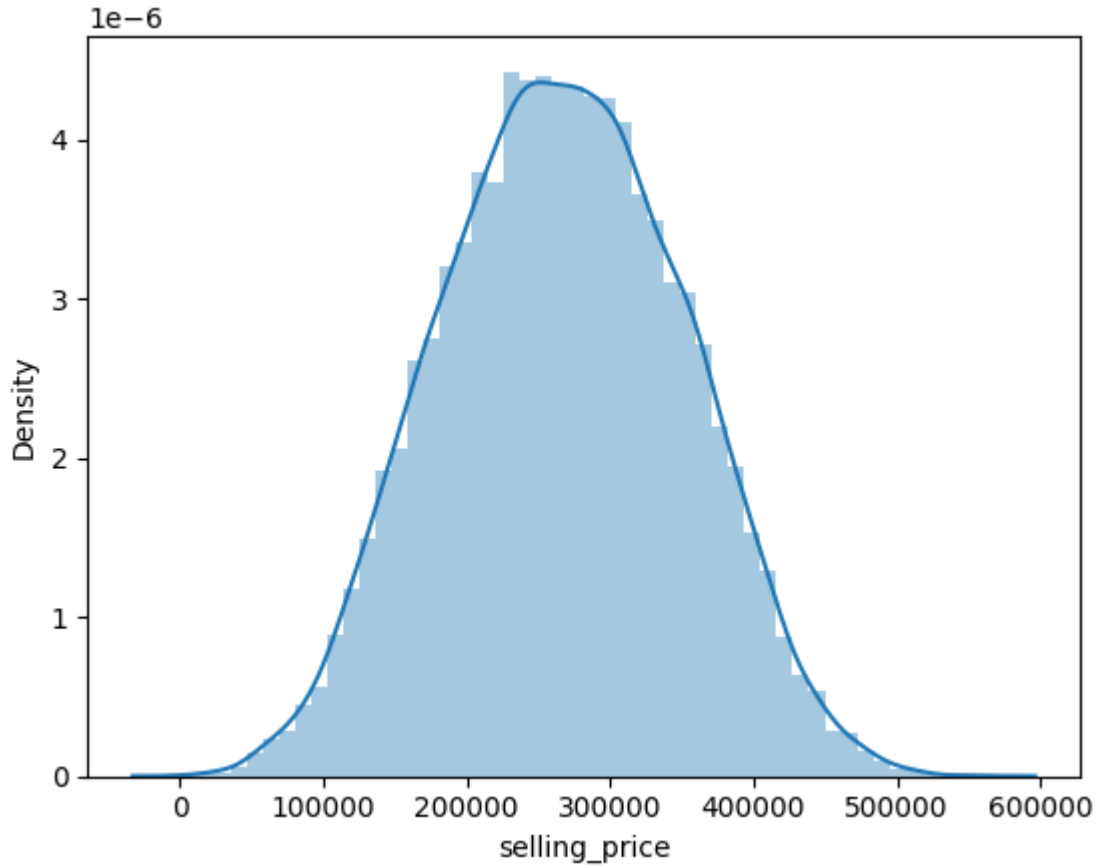
```
<ipython-input-66-79e869ad7a3a>:1: UserWarning:

`distplot` is a deprecated function and will be removed in seaborn v0.14.0.

Please adapt your code to use either `displot` (a figure-level function with
similar flexibility) or `histplot` (an axes-level function for histograms).

For a guide to updating your code to use the new functions, please see
https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751

  sns.distplot(df3["selling_price"])
<Axes: xlabel='selling_price', ylabel='Density'>
```



Again the plot of Target data is Normal Distribution.

Statsmodel Implementation(Again)

```
X = df3[cols2]
Y = df3["selling_price"]
```

```
X.shape
```

```
(19993, 6)
```

```
sc = StandardScaler()
cols = X.columns
X[cols] = sc.fit_transform(X[cols])
```

```
<ipython-input-69-53e462b909d4>:3: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/s
  X[cols] = sc.fit_transform(X[cols])
```

```
X.head()
```

	max_power	transmission_type	year	km_driven	fuel_electric	seats_cop
0	0.094124	-0.998251	-0.238299	-0.989571	2.659221	-0.378299
1	-1.438958	-0.998251	-1.182444	1.323320	-0.376050	-0.378299
2	-0.814939	1.001752	1.177919	-0.921860	-0.376050	-0.378299
3	-1.354214	1.001752	0.548489	1.083792	-0.376050	-0.378299
4	-0.606933	1.001752	-1.025087	-1.538626	-0.376050	-0.378299

```
x_train, x_test, y_train, y_test = train_test_split(X, Y, test_size=0.1, random_s
```

```
final = LinearRegression()
```

```
final.fit(x_train, y_train)
```

```
▾ LinearRegression
LinearRegression()
```

```
LinearRegression()
```

```
▾ LinearRegression
LinearRegression()
```

A LinearRegression model is instantiated and trained on the training data (x_train, y_train) using the fit() method.

```
final.score(x_train, y_train)
```

```
0.5719163569005816
```

```
print("Adjusted R-squared: ", 1 - (1-final.score(x_train,y_train))*(len(y_train)
```

```
Adjusted R-squared:  0.5717497410817495
```

```
y_pred = final.predict(x_test)
```

```
print("Mean absolute error: ", mean_absolute_error(y_pred, y_test))
print("Mean squared error: ", mean_squared_error(y_pred, y_test))
print("Root Mean squared error: ", np.sqrt(mean_squared_error(y_pred, y_test)))
print("Mean absolute percentage error: ", mean_absolute_percentage_error(y_pred,
```

```
Mean absolute error:  43195.71061838411
Mean squared error:  2946232477.820871
Root Mean squared error:  54279.20852242478
Mean absolute percentage error:  0.17115604235240225
```

```
preds = final.predict(x_train)
```

```
errors = preds - y_train
```

```
# Residual analysis
sns.distplot(errors)
```

```
<ipython-input-81-c5b59cd38f4f>:2: UserWarning:

`distplot` is a deprecated function and will be removed in seaborn v0.14.0.

Please adapt your code to use either `displot` (a figure-level function with
similar flexibility) or `histplot` (an axes-level function for histograms).

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https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751

  sns.distplot(errors)
<Axes: xlabel='selling_price', ylabel='Density'>
```
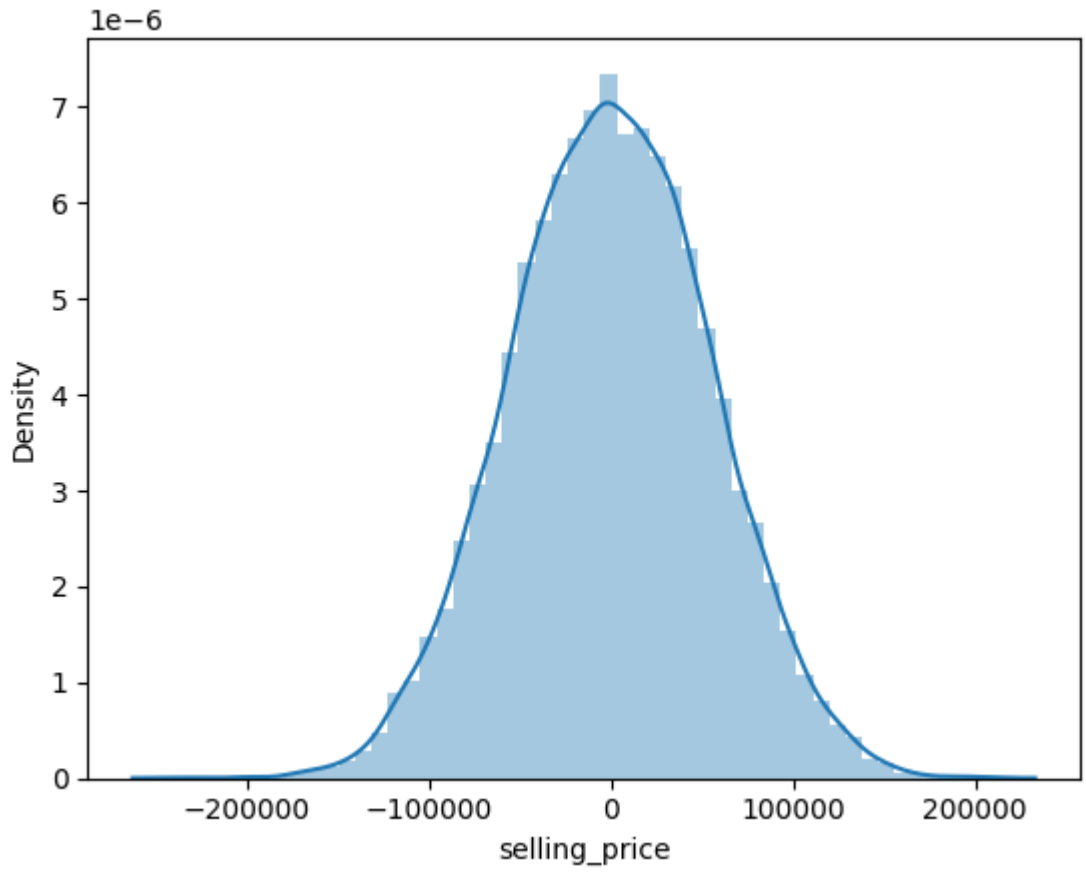
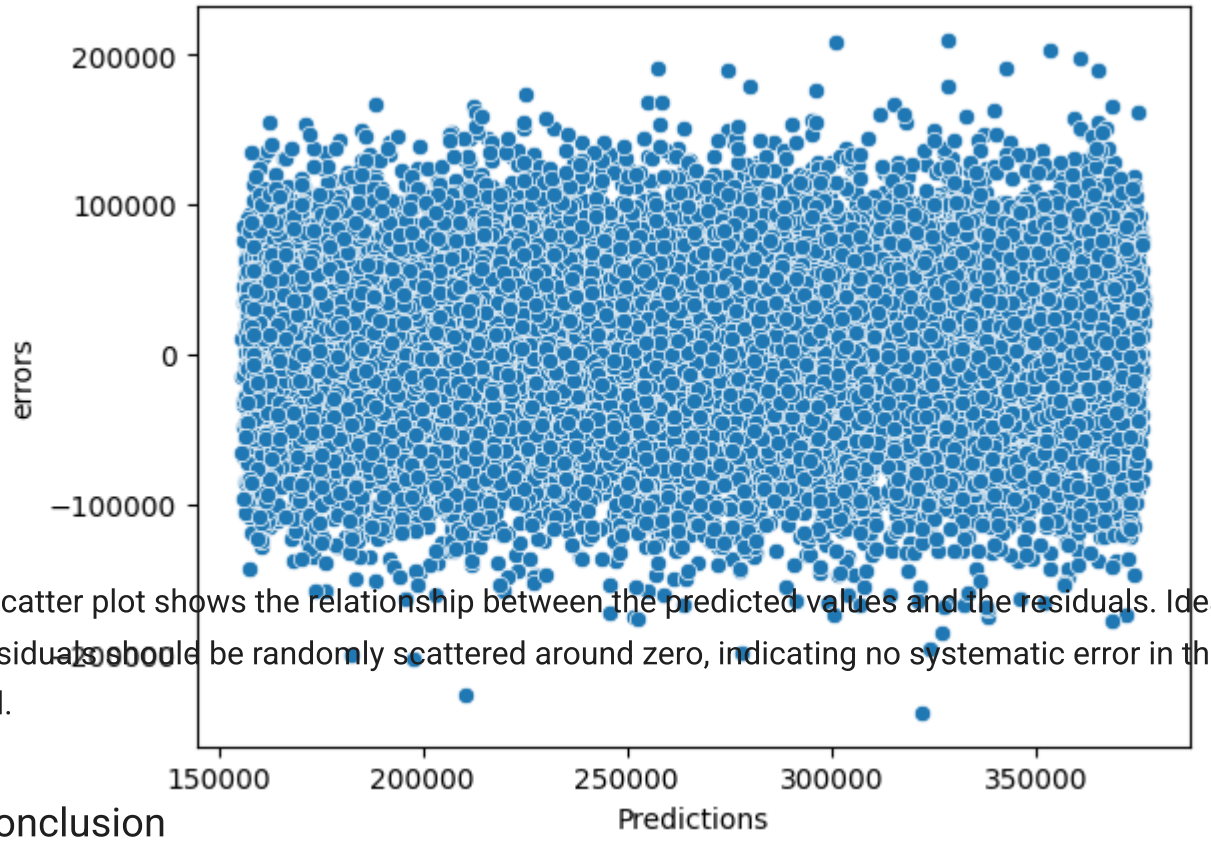



This plot visualizes the distribution of the residuals. Ideally, the residuals should follow a normal distribution, indicating that the model errors are random and not biased.

```
# Predictions vs Residuals Plot

sns.scatterplot(x=preds, y=errors)
plt.xlabel("Predictions")
plt.ylabel("errors")
```

```
Text(0, 0.5, 'errors')
```



This scatter plot shows the relationship between the predicted values and the residuals. Ideally, the residuals should be randomly scattered around zero, indicating no systematic error in the model.

Conclusion

- The code performs multiple linear regression on a dataset after standardizing the features. It evaluates the model performance using both R-squared and Adjusted R-squared, followed by calculating common error metrics like MAE, MSE, RMSE, and MAPE.
- Residual analysis is conducted to assess the quality of the model's predictions, helping identify potential issues like non-linearity or heteroscedasticity.

Log Transformation

- Log transformation is a mathematical operation applied to data to modify its scale, particularly useful when dealing with skewed distributions or heteroskedasticity.
- In a log transformation, each data point is replaced with the natural logarithm of that