Spacetime algebra without reference to a metric signature

Define spacetime basis vectors by $|\gamma_\mu^2| = 1$ and $\gamma_0^2 = -\gamma_i^2$. Define reciprocal basis vectors by $\gamma^\mu := \gamma_\mu^{-1}$. Define the pseudoscalar $I := \gamma_0\gamma_1\gamma_2\gamma_3$.

Define relative vectors $\sigma_i := \gamma_i\gamma^0$ so that

$$\begin{aligned}
\sigma_i^2 &= \gamma_i\gamma^0\gamma_i\gamma^0 \\
&= -(\gamma_i)^2(\gamma^0)^2 \\
&= -\gamma_i^2(\gamma_0^2)^{-1} \\
&= -\gamma_i^2(-\gamma_i^2)^{-1} = 1
\end{aligned}$$

and

$$\begin{aligned}
\sigma_1\sigma_2\sigma_3 &= \gamma_1\gamma^0\gamma_2\gamma^0\gamma_3\gamma^0 \\
&= \gamma^0\gamma_1\gamma_2\gamma_3\gamma^0\gamma^0 \\
&= \gamma_0\gamma_1\gamma_2\gamma_3\gamma_0\gamma^0 \\
&= \gamma_0\gamma_1\gamma_2\gamma_3 = I
\end{aligned}$$

Define $\sigma^i = \sigma_i^{-1}$ and find $\sigma^i = \gamma_0\gamma^i$ so that $\sigma_i\sigma^i = \gamma_i\gamma^0\gamma_0\gamma^i = 1$.

Relation to vector cross product

If $\times$ is the $\mathbb{R}^3$ cross product, then for $\vec{E} = E^i\sigma_i$ and $\vec{B} = B^i\sigma_i$ we have

$$\begin{aligned}
\left\langle \vec{E}\vec{B} \right\rangle_2 &= \sum_{i,j} E^i B^j \langle \sigma_i\sigma_j \rangle_2 \\
&= \sum_{i\neq j} E^i B^j \sigma_i\sigma_j \\
&= \sum_{i\neq j} E^i B^j \sigma^k I \varepsilon_{ijk} = \left(\vec{E}\times\vec{B}\right) I
\end{aligned}$$