Projections and rejections of multivectors by vectors

Let $u \in G$ be a 1-vector. We can decompose any multivector $A \in G$ into orthogonal components $A = A^{\parallel u} + A^{\perp u}$ given by

$$A^{\parallel u} := u \wedge \left(u^{-1} \rfloor A\right)$$
$$A^{\perp u} := u \rfloor \left(u^{-1} \wedge A\right)$$

so that $A^{\parallel u}$ "contains" u and $A^{\perp u}$ is "orthogonal" to u.

Lemma.
1) $A = A^{\parallel u} + A^{\perp u}$
2) $u \wedge A^{\parallel u} = u \rfloor A^{\perp u} = 0$
3) $\left(A^{\parallel u}\right)^{\parallel u} = A^{\parallel u}$, $\left(A^{\perp u}\right)^{\perp u} = A^{\perp u}$, $\left(A^{\perp u}\right)^{\parallel u} = \left(A^{\parallel u}\right)^{\perp u} = 0$

Proof. To show $A = A^{\parallel u} + A^{\perp u}$ note that

$$A^{\perp u} = u \rfloor \left(u^{-1} \wedge A\right) = \left(u \rfloor u^{-1}\right) \wedge A - u^{-1} \wedge \left(u \rfloor A\right) = A - u \wedge \left(u^{-1} \rfloor A\right) = A - A^{\parallel u}$$

using the anti-derivation property of $u \rfloor$.

We have $u \wedge A^{\parallel u} = \cancel{u \wedge u} \wedge \left(u^{-1} \rfloor A\right) = 0$ immediately and $u \rfloor A^{\perp u} = u \rfloor \left(u \rfloor \left(u^{-1} \wedge A\right)\right) = \cancel{\left(u \wedge u\right)} \rfloor \left(u^{-1} \wedge A\right)$ by the double contraction identity.

To show that these are projections, note that

$$\begin{aligned}\left(A^{\parallel u}\right)^{\parallel u} &= u \wedge \left(u^{-1} \rfloor \left(u \wedge \left(u^{-1} \rfloor A\right)\right)\right) \\ &= u \wedge \left(u^{-1} \rfloor u\right) \wedge \left(u^{-1} \rfloor A\right) - \cancel{u \wedge u} \wedge \left(u^{-1} \rfloor \left(u^{-1} \rfloor A\right)\right) = A^{\parallel u}\end{aligned}$$

again using the anti-derivation identity. Since $\left(A^{\parallel u}\right)^{\perp u} = u \rfloor \left(\cancel{u^{-1} \wedge u} \wedge \left(u^{-1} \rfloor A\right)\right) = 0$, we have also $\left(A^{\perp u}\right)^{\perp u} = \left(A + A^{\parallel u}\right)^{\perp u} = A^{\perp u}$. ∎