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TERM STRUCTURE OF DISCOUNT RATES FOR FIRMS IN THE INSURANCE INDUSTRY

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ABSTRACT

There is strong evidence in the literature for the hypothesis that firms in the insurance industry display countercyclical risk. Interest rates and the market risk premium are known to change during the business cycle. The major implication of these results is that discount rates for risky cash flows are time varying and must obey a term structure similar to the term structure of interest rates. This paper builds on the term structure model developed by Ang and Liu (2004) and applies it to the insurance industry. We estimate discount rates for cash flows with different time horizons for the insurance industry and for different insurance sectors. We find that the term structure cost of capital takes on different shapes depending on the business cycle. It is therefore meaningful for insurers to evaluate risky projects by selecting a discount rate most appropriate for the nature and the time horizon of each project.

Keywords: Term-Structure, Discount Rate, Cost of Capital, Insurance Industry, Multi-period CAPM, Conditional CAPM

JEL Classification: G12, G22, G31

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1. INTRODUCTION

The theoretical price, or present value, of a financial asset is defined by the sequence of expected cash flows discounted at the appropriate risk adjusted discount rate. If the cash flow is certain and known in advance, such as the face value of a U.S. Treasury zero coupon bond, then the discount rate is given by the yield corresponding to the maturity date of that particular zero-coupon bond. Because Treasury securities are actively traded, and new issues enter the bond market on a regular basis, the risk-free yield curve is continuously observable for maturities up to thirty years.

Risk adjusted discount rates for uncertain cash flows are not observable; there is no active market for a single risky cash flow payable at some specific future date. As a result, one must rely on theoretical models to determine prices. These models can be broadly divided into capital market-based models and implied cost of capital models. The former are found in finance literature and include the single period Capital Asset Pricing Model or CAPM (Fama, 1977), three factor Fama-French or FF3 (Fama and French, 1992; Fama and French, 1997), and two multi-period models: Conditional CAPM or CCAPM (Jagannathan and Wang, 1996; Ferson and Harvey, 1999; Lettau and Ludvigson, 2001) and Intertemporal CAPM or ICAPM (Merton, 1973; Ang *et al.*, 2006). The CCAPM expands the standard CAPM by including a covariance term between beta and market premium (CAPM assumes this covariance to be zero), which is done by estimating each period's beta conditional on variables that predict the market return. The ICAPM is similar to the FF3 model, in that it uses market volatility instead of SMB, HML as additional factors to account for other sources of risk. The second broad category of models estimating discount rates

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– implied cost of capital (ICC) models – are found in accounting literature, which derive a firm's internal cost of capital by equating its stock price with the present values of all expected future cash flows (Gebhardt, Lee, and Swaminathan, 2001). ICC models are useful to compare values cross firms.

All of the above models yield a constant discount rate for cash flows with different maturities. But the literature also suggests that in the context of multi-period consumption-investment decisions, the discount rate cannot be constant across the spectrum of maturity dates – a flat term structure; this is because expected returns on equity are time-varying (Cochrane, 2011). A seminal work by Brennan (1997) advocates a term structure of discount rates, assuming that both the riskless rate and the market risk premium vary over time in a predictable fashion (but market beta is assumed constant). Ang and Liu (2004) generalize the work of Brennan by allowing all three parameters of the CAPM to be time varying.

The term structure cost of equity has been empirically shown to matter in asset classes such as real estate (Giglio *et al.*, 2020), and aggregate dividend strips such as those paid by the S&P 500 (van Binsbergen, Brandt, and Koijen, 2012).[1] From the perspective of shareholders, one may question the value of applying a term structure, as shareholders effectively purchase a share of the company's all dividend payments in perpetuity. In such case, the implied cost of capital models (without any term structure) may prove useful (Berry-Stölzle and Xu, 2018). From the perspective of firm managers, the common practice of discounting all expected future cash flows at a constant rate (Graham and Harvey, 2001) can be theoretically improved using a term structure of discount rate to calculate the present value of maturity-matched future cash flows. Specifically, in a similar fashion to zero coupon bonds with different maturities, we may classify equity cash flows by

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[1] However, Boguth *et al.* (2011a) argue that synthetic securities that replicate dividend strips are subject to measurement errors and these errors magnify the uncertainty in yield estimates.

maturity date and require a specific risk adjusted cost of capital corresponding to that particular maturity.

This paper uses the model and key assumptions developed by Ang and Liu (2004) to estimate a term structure of discount rates for firms in the insurance industry with a CCAPM, assuming that the riskless rate, market beta, and market risk premium are all time-varying. We also assume independence between interest rates and the state variables in the CCAPM. By doing so, we will be able to separate the cost of capital into two additive components: a riskless discount rate plus a cash flow risk premium. In addition, we model stochastic cash flows with the "Clean Surplus" accounting relationship (Feltham and Ohlson, 1995; Berry-Stölzle and Xu, 2018).

Cummins and Phillips (2005), Wen, Martin, Lai, and O'Brien (2008), and most recently Berry-Stölzle and Xu (2018), Barinov, Xu, and Pottier (2018) (henceforth, BXP, 2018) all provide cost of capital estimates for firms in the insurance industry by different sector or line of business. However, they do not estimate discount rates that vary by time horizons. BXP (2018) present strong evidence for the hypothesis that firms in the insurance industry display countercyclical risk: betas decrease when the economy expands and tend to increase during recessions when bearing risk is more costly. In addition, asset liability management and duration matching are fundamental elements of insurance business operations. Therefore, using a term structure framework built on the conditional CAPM to estimate discount rates for insurance firms would be a meaningful exercise.

Our work contributes to the insurance literature by extending the recent work by Berry-Stölzle and Xu (2018) and BXP (2018). Both papers estimate cost of capital for firms in the insurance industry using a variety of approaches, including the implied cost of capital approach and multi-period *models* such as the CCAPM and the ICAPM. However, these models are silent

about expected returns more than one period ahead. Our model fills this gap in the literature. Specifically, we show how to determine multi-period discount rates in an equilibrium setting. To estimate these discount rates, we model the dynamic behavior of insurance firms' beta and the market risk premium across the business cycle. Macroeconomic variables such as the default spread and term premium may be used to predict both beta and the risk premium. We combine these relationships with the CCAPM to derive a model for the cost of capital that varies across time *and* cash flow duration. We estimate the term structure cost of equity capital using insurance firm data ranging from the fourth quarter of 1972 thru the fourth quarter of 2018.

In theory, the term structure, graphed as a function of maturity, may display patterns similar to those observed in the term-structure of interest rates: upward-sloping, downward-sloping, hump-shaped, or U-shaped. For example, consumption-based asset pricing models (e.g., Campbell and Cochrane, 1999) predict that the equity risk premium should increase with the maturity of the cash flow. Consistent with this prediction, Ang and Liu (2004) find that the cost of capital curve is, on average, upward sloping both for value and growth stocks.

However, a consensus appears to be emerging in the finance literature that discount rates might be lower the higher the cash flow duration. In other words, the typical shape of the yield curve should be inverted. Dechow, Sloan, and Soliman (2004) provide indirect evidence for this effect by examining the relationship between cash flow duration and expected equity returns. They find that short duration assets are associated with high expected returns. Similarly, the model developed by Lettau and Wachter (2007) prescribes a risk premium of 18% for zero coupon equity with two years to maturity vs. 4% for a cash flow with 40 years to maturity. Giglio *et al.* (2020) also estimate an inverted yield curve for cash flows as far as 100 years from now. Goncalves (2019) summarizes this literature and empirically shows the terms structure of dividend discount rates is

negatively sloped especially for dividends expected far in the future. The theoretical argument for this result is that because claims to long term cash flows provide a good hedge for reinvestment risk, the risk premium required by long-term investors declines as maturity increases.

We find that for the insurance industry as a whole, the term structure of discount rates is on average hump shaped. Compared with constant discount rates estimated with CAPM, the discount rates based on our term structure model can be, on average, nearly 220 basis points lower. This difference is statistically significant. The slope of the term structure is also significantly different from zero for short and medium-term cash flows. We show that as market beta and the market risk premium display substantial time variation over the business cycle, our term structures also vary both in level and shape. This is a result of our term structure model taking into account the stochastic nature of the risk premium during the course of the business cycle, giving rise to a different discount rate for cash flows of different duration.

The rest of this paper is organized as follows: the next section shows that the Clean Surplus accounting relationship and the CCAPM may be combined to obtain the theoretical model for the cost of capital term structure. Section 3 describes the empirical methodology for estimating the model's unknown parameters. Section 4 presents the data. Section 5 presents our empirical estimates of the term structure discount rates for firms in the insurance industry and in subsectors such as property/casualty (P/C) and life/health. Section 6 concludes the paper.

2. ASSET PRICING THEORY

Consider the time-t price of a single cash flow $C_{t+\tau}$ expected at a future date $t+\tau$. The theoretical price, or present value $V(\tau)_t$ may be obtained by discounting the expected value with a corresponding risk adjusted discount rate:

$$V(\tau)_t = e^{-\tau\rho(\tau)_t}[E_t C_{t+\tau}]$$

(1)

Where $\rho(\tau)_t$ is the cost of capital as of end-of-period t for a cash flow expected τ periods from the current period. Because of the similarity between Equation (1) and the formula for the present value of a zero coupon bond, Lettau and Wachter (2007) describe the present value of a single risky cash flow as 'zero coupon equity'.

An equivalent representation of the discounting formula may be obtained as follows. Define μ_t as the continuously compounded expected return generated by holding a τ-period zero coupon equity from t to $t+1$ as: $\mu_t = E_t [ln(V(\tau-1)_{t+1} / V(\tau)_t)]$. This expression may be solved forward subject to the condition that the terminal value equal the future cash flow itself: $V(0)_{t+\tau} = C_{t+\tau}$. The solution is the well-known formula:

$$V(\tau)_t = E_t[e^{-(\mu_t+\dots+\mu_{t+\tau-1})}C_{t+\tau}]$$ (2)

To make this solution operational, we need to specify the joint evolution of cash flows and expected single period returns. We model expected returns with a conditional CAPM. Thus, the riskless rate, market beta and market risk premium are time varying. Specifically, the single period equilibrium discount rate is:

$$\mu_t = R_{f,t} + E_t[R_{m,t+1} - R_{f,t}]\beta_t$$ (3)

where $R_{f,t}$ is the risk-free rate observed at the end-of-period t, $E_t[R_{m,t+1} - R_{f,t}]$ is the market risk premium, and β_t is the current value of beta.

To model cash flow dynamics, we use the "Clean Surplus" accounting relationship. Let NI_{t+1} represent net income from period t to $t+1$, and let B_t stand for book equity value at the end of period t. Clean surplus accounting requires that the cash flow be given by accounting earnings minus the change in book value: $C_{t+1} = NI_{t+1} - [B_{t+1} - B_t]$. This condition may be restated in

terms of growth in book equity (in logs) $g_{t+1} = ln(B_{t+1} / B_t)$, and return on book equity $ROE_{t+1} = ln(1+NI_{t+1} / B_t)$: $C_{t+1} = B_t [e^{ROE_{t+1}} - e^{g_{t+1}}]$. The zero-coupon equity price depends on the joint evolution of expected single period returns, return on equity, and book equity growth:

$$V(\tau)_t = B_t E_t \{e^{-(\mu_t + \cdots + \mu_{t+\tau-1})} [e^{g_{t+1} + \cdots + g_{t+\tau-1} + ROE_{t+\tau}} - e^{g_{t+1} + \cdots + g_{t+\tau}}]\} \tag{4}$$

Modern finance literature (e.g. Ang and Liu, 2004, Cochran, 2011) argues that model parameters should be estimated jointly as in a vector autoregressive process (VAR) to capture dynamic interactions between variables. To accomplish this, let x_t be a vector of accounting book equity rate of return, growth in book value of equity, beta, and market risk premium: $x_t' = (ROE_t, g_t, \beta_t, E_t[R_{m,t+1} - R_{f,t}])$. In line with this literature, we specify a first-order VAR:

$$x_{t+1} = (I - \phi)\bar{x} + \phi x_t + \varepsilon_{t+1} \tag{5}$$

where $\bar{x}$ is the long run (unconditional mean) vector, ϕ is the companion matrix, and x_t represents observed sample values at time-t. The idiosyncratic shocks $\{\varepsilon_t\}$ form a time-series of independently distributed normal variables with zero mean and variance matrix ω.

We note that next period's cash flow may be stated as a function of the state variable as follows. Define ei' as a row vector of 0's in each cell except for 1 in the ith place. Using this vector, we may connect growth in book equity to the state vector as: $g_{t+1} = e2'x_{t+1}$. Similarly, the return on book equity may be extracted from the state vector: $ROE_{t+1} = e1'x_{t+1}$. Lastly, we note that the risk premium $E_t[R_{m,t+1} - R_{f,t}]\beta_t$ is analogous to the quadratic form $x_t'\Theta x_t$, where the 4x4 matrix Θ is defined as follows: rows one and two consist of 0s, the 3rd row is (0, 0, 0, ½), and the last row is (0, 0, ½, 0). In the next two subsections we derive the cost of capital term structure for risky equity cash flows.

2.A   Valuation with a Known Discount Rate

The price of a single cash flow may be obtained using Equation (1) assuming that the cost of capital $\rho(\tau)_t$ is a known variable; we need only the conditional expectation of $C_{t+\tau}$. This expectation depends on the evolution of return on equity and book equity growth:

$$E_t[C_{t+\tau}] = B_t E_t[e^{(g_{t+1}+\dots+g_{t+\tau-1})+ROE_{t+\tau}} - e^{g_{t+1}+\dots+g_{t+\tau}}] \tag{6}$$

By the properties of log normal variables, this expectation requires the mean and variance of the aggregate growth in book equity, and the return on equity τ periods from now. We develop a recursive algorithm to easily compute these moments (details are in Appendix A).

Define the system of vector difference equations as $\lambda'_j = \lambda'_{j-1}\phi + e2'$ starting from j=2 and working forward to j=τ. The initial condition is: $\lambda'_1 = e2'$. Thus, each vector λ_j' may be computed recursively, and the expected future growth in book equity may be computed as:

$$E_t[e^{g_{t+1}+\dots+g_{t+\tau}}] = \exp[\sum_{j=1}^{\tau}\lambda_j'(I-\phi)\bar{x} + \lambda_\tau'\phi x_t + \frac{1}{2}\sum_{j=1}^{\tau}\lambda_j'\omega\lambda_j].$$

A similar methodology applies for the conditional mean and variance of book equity return. The expected future return on book equity is $E_t[e^{g_{t+1}+\dots+g_{t+\tau-1}+ROE_{t+\tau}}] = exp[\sum_{j=1}^{\tau}\gamma_j'(I-\phi)\bar{x} + \gamma_\tau'\phi x_t + \frac{1}{2}\sum_{j=1}^{\tau}\gamma_j'\omega\gamma_j]$, where the recursive coefficients are obtained as $\gamma_j'=\gamma_{j-1}'\phi+e2'$ starting from j=2 and working forward to j=τ. The initial condition is $\gamma_1' = e1'$. Then, the present value of the expected cash flow corresponding to Equation (1) is:

$$V(\tau)_t = B_t\{e^{-\tau\rho(\tau)_t}[e^{A(\tau)^* + B(\tau)^{*'}x_t} - e^{C(\tau)^* + D(\tau)^{*'}x_t}]\} \tag{7}$$

where $A(\tau)^* = \frac{1}{2}\sum_{j=1}^{\tau}(\gamma_j'\omega\gamma_j) + \sum_{j=1}^{\tau}\gamma_j'(I-\varphi)\bar{x}$, $B(\tau)^{*'} = \gamma_\tau'\varphi$, $C(\tau)^* = \frac{1}{2}\sum_{j=1}^{\tau}(\lambda_j'\omega\lambda_j) + \sum_{j=1}^{\tau}\lambda_j'(I-\varphi)\bar{x}$, and $D(\tau)^{*'} = \lambda_\tau'\varphi$. In the next subsection we use the conditional CAPM to derive an alternative model for $V(\tau)_t$; then we set these equations equal and solve for the cost of capital.

2.B   Valuation of Expected Cash Flows with a Conditional CAPM

The price of zero-coupon equity may be obtained recursively assuming the conditional CAPM holds period by period. Let $Z(\tau)_t$ be the price of a zero coupon riskless bond with a \$1 face value expected at a future date $t+\tau$, and $y(\tau)_t$ be the corresponding yield to maturity observed as of end-of-period t. By definition, the time-t price is given by $Z(\tau)_t = e^{-\tau\, y(\tau)_t}$. We also assume independence between interest rates and the state variables in the x_t vector. By doing so, we will be able to separate the cost of capital into a riskless discount rate and a cash flow risk premium.

Asset pricing theory implies that the current price of zero coupon equity obeys a recursive relation: $V(\tau)_t = E_t[e^{-\mu_t} V(\tau-1)_{t+1}]$, where the discount rate μ_t is given by Equation (3). For the general valuation model with $\tau > 1$, Ang and Liu (2004) show the solution is:

$$V(\tau)_t = Z(\tau)_t\ B_t\left[e^{A(\tau)+B(\tau)'x_t+x_t'G(\tau)x_t} - e^{C(\tau)+D(\tau)'x_t+x_t'G(\tau)x_t}\right] \tag{8}$$

where $Z(\tau)_t$ is the current riskless bond price, and the coefficients $A(\tau)$ thru $G(\tau)$ are:

$A(\tau) = A(\tau-1) + [\,e2' + B(\tau-1)'\,](I-\phi)\bar{x} + [(I-\phi)\bar{x}\,]'\,G(\tau-1)(I-\phi)\bar{x}]$

$\quad + 1/2\{e2' + B(\tau-1)' + 2[(I-\phi)\bar{x}\,]'G(\tau-1)\}V\,\{e2 + B(\tau-1) + 2G(\tau-1)'\,[(I-\phi)\bar{x}\,]\}\ - (1/2)\,\ell n|I - 2G(\tau-1)\,\omega\,|$

$B(\tau)' = [e2' + B(\tau-1)']\phi + 2[(I-\phi)\bar{x}]'G(\tau-1)\phi$
$\qquad + 2\big(e2' + B(\tau-1)' + 2[(I-\phi)\bar{x}]'G(\tau-1)\big)\,V\,G(\tau-1)'\phi$

$C(\tau) = C(\tau-1) + [\,e2' + D(\tau-1)'\,](I-\phi)\bar{x} + [(I-\phi)\bar{x}\,]'\,D(\tau-1)\ (I-\phi)\bar{x}]$

$\quad + 1/2\{e2' + D(\tau-1)' + 2[(I-\phi)\bar{x}\,]'G(\tau-1)\}V\{e2 + D(\tau-1) + 2G(\tau-1)'\,[(I-\phi)\bar{x}\,]\}\ - (1/2)\,\ell n|I - 2G(\tau-1)\,\omega\,|$

$$D(\tau)'=[e2'+D(\tau-1)']\phi+2[(I-\phi)\bar{x}]'G(\tau-1)\phi$$
$$+2\big(e2'+D(\tau-1)'+2[(I-\phi)\bar{x}]'G(\tau-1)\big)V\,G(\tau-1)'\phi$$

$$G(\tau)=-\Theta+\phi'\,G(\tau-1)\phi+2[G(\tau-1)'\phi]'\,V[G(\tau-1)'\phi]$$

and the matrix $V=[\omega^{-1}-2\,G(\tau-1)]^{-1}$. For the price of a single period ($\tau=1$) equity we have:

$$V(1)_t=Z(1)_t\,B_t\,[\,e^{A(1)+B(1)'x_t+x_t'G(1)x_t}-e^{C(1)+D(1)'x_t+x_t'G(1)x_t}\,]$$

where $Z(1)_t=exp(-R_{f,t})$ is the current price of a one-period riskless bond, and the coefficients $A(1)$

thru $G(1)$ are:
$$A(1)=e1'(I-\phi)\bar{x}+(\tfrac{1}{2})e1'\omega e1,\quad B(1)'=e1'\phi$$
$$C(1)=e2'(I-\phi)\bar{x}+(\tfrac{1}{2})e2'\omega e2,\quad D(1)'=e2'\phi,\quad G(1)=-\Theta\quad.$$

2.C   Cost of Capital Term Structure

The solution for the cost of capital implied by the single period conditional CAPM, and the VAR model for the state vector is obtained by setting Equation (7) equal to (8):

$$\rho(\tau)_t=y(\tau)_t+\frac{1}{\tau}\left\{\ell n\left[\frac{e^{A(\tau)^*+B(\tau)^{*\prime}x_t}-e^{C(\tau)^*+D(\tau)^{*\prime}x_t}}{e^{A(\tau)+B(\tau)'x_t+x_t'G(\tau)x_t}-e^{C(\tau)+D(\tau)'x_t+x_t'G(\tau)x_t}}\right]\right\}\qquad(9)$$

This equation shows the required rate to discount a single cash flow with time to maturity τ. We note that the discount rate consists of a riskless rate plus a risk premium commensurate with the cash flow risk. Both components depend on the term to maturity, and also on the degree of mean reversion in the state vector.

In the empirical section that follows, we show that both level and shape of the cost of capital term structure change as current market conditions, summarized by the riskfree yield curve $y(\tau)_t$ and the current state vector x_t, change over time. Also, the term structure, graphed as a function of τ, follows similar patterns observed in the term-structure of interest rates: rising, falling, or hump-shaped.

3. EMPIRICAL METHODS

In this section, we address three empirical issues: first, estimation of the market risk premium; second, estimation of market beta based on fundamental characteristics of each firm; and third, time-varying betas obtained from a rolling window regression. We then use a Bayesian approach to combine these two beta estimates. This method is proposed by Cosemans *et al.* (2016) and appears to have superior forecasting performance compared to a number of different approaches from the literature.

3.A ROLLING WINDOW BETA ESTIMATION

Fama and McBeth (1973) proposed a rolling window regression methodology to estimate time-varying betas that is quite popular to this day. Specifically, let $r_{i,s}$ be the excess daily rate of return for portfolio i for day s, and let $r_{m,s}$ be the corresponding excess market portfolio return. Consider the rolling window time series regression of portfolio return i on a constant and the market return:

$$r_{i,s} = \alpha_{i,t} + \beta_{i,t} r_{m,s} + \varsigma_{i,s} \tag{10}$$

The index s covers a six months period ending on the last trading day of month t and beginning 125 trading days earlier. The idiosyncratic return $\varsigma_{i,s}$ is modeled as a sequence of identically, independently distributed random variables with zero mean and variance σ_ς^2. The intercept $\alpha_{i,t}$ is the risk-adjusted return, and the slope $\beta_{i,t}$ measures the market risk for month t. Assuming

actual and idiosyncratic returns are normally distributed, the estimated rolling window (*RW*) betas

$\hat{\beta}_{i,t}^{RW}$ are also normal with mean $\beta_{i,t}$ and variance $V[\hat{\beta}_{i,t}^{RW}] = \hat{\sigma}_{\varsigma}^2 (\sum_s r_{m,s}^2)^{-1}$, where $\hat{\sigma}_{\varsigma}^2$ is the

residual variance estimator for the month *t* regression. We use daily returns, rather than weekly or

monthly returns, because higher frequency data lead to more efficient estimates (see Cosemans *et al.*, 2016). Furthermore, the estimation window of 125 trading days is chosen to capture short-run

variation in beta.

3.B CONDITIONAL BETA MODEL

We follow the empirical literature on the conditional CAPM (e.g., Cosemans *et al.*, 2016,

and BXP, 2018) and model time-varying beta as a linear function of five general business

conditions variables: Dividend Yield (*DIV*), Default spread (*DEF*), U.S. Treasury bill rate ($R_{f,t}$),

the term spread (*TERM*), and market volatility (Mkt Volatility), which is the volatility of the daily

returns of the value-weighted CRSP index over the 126 trading days prior to time t. We also add

to the regression model three firm-specific variables: firm size (*SIZE*), the book-to-market ratio

(*BM*), and the lagged beta ($\beta_{i,t-1}$). The rationale for including size and the book-to-market ratio

comes from the asset pricing literature (e.g., Gomes, Kogan, and Zhang, 2003). Boguth *et al.*

(2011b) show that there is a significant relationship between betas and market volatility. This

"volatility timing" may be a significant factor for the term structure. Thus, we include the market

volatility and lagged betas in our model. The beta model is

$\beta_{it|t-1}^* = \delta_{0,i} + \delta_1 DEF_{t-1} + \delta_2 DIV_{t-1} + \delta_3 R_{f,t-1} + \delta_4 TERM_{t-1} + \delta_5 Mkt\ Volatility_{t-1}$

$+\gamma_1 Size_{i,t-1} + \gamma_2 BM_{i,t-1} + \gamma_3 \beta_{i,t-1}$ (11)

We assume that the relationship between beta, firm size, book-to-market ratio, and lagged beta is constant across firms. The use of cross-sectional data on firm characteristics will lead to more efficient estimates of the parameter vector $\gamma' = (\gamma_1, \gamma_2, \gamma_3)$. On the other hand, the relationship between beta and the macro variables $\delta_i' = (\delta_{1,i}, \delta_{2,i}, \delta_{3,i}, \delta_{2,i}, \delta_{3,i})$. is allowed to vary across firms; thus, it has the potential to capture unobserved heterogeneity in systematic risk. The intercept term $\delta_{0,i}$ is also firm specific to capture effects of unobserved variables that may differ across firms (but constant over time).

Let $\hat{\beta}_{i,t-1}^{FC}$ be the firm characteristic (*FC*) Ordinary Least Squares estimator of beta (Equation 11). It may be shown that this estimator has mean $\beta_{it|t-1}^{*}$ and variance $V\left[\hat{\beta}_{i,t-1}^{FC}\right]$ (see Appendix B for details). Following standard arguments as in Cosemans *et al.* (2016), we treat $\hat{\beta}_{i,t-1}^{FC} \sim N(\beta_{it|t-1}^{*}, V[\hat{\beta}_{i,t-1}^{FC}])$ as a prior distribution and obtain the posterior market beta:

$$\hat{\beta}_{it|t}^{*} = w_{i,t}\hat{\beta}_{i,t-1}^{FC} + (1 - w_{i,t})\hat{\beta}_{i,t}^{RW} \tag{12}$$

where the corresponding weight is $w_{i,t} = \dfrac{V[\hat{\beta}_{i,t}^{RW}]}{V[\hat{\beta}_{i,t-1}^{FC}] + V[\hat{\beta}_{i,t}^{RW}]}$. Clearly, the posterior mean $\hat{\beta}_{it|t}^{*}$ is a weighted average of the prior beta, conditional on firm characteristics, and the rolling window beta.

3.C ECONOMETRIC MODEL FOR THE MARKET RISK PREMIUM

We use a standard model for the market risk premium (e.g., Petkova, 2006, and BXP, 2018). The model is based on four business cycle variables: Dividend Yield (*DIV*), Default spread (*DEF*), U.S. Treasury bill rate, and the term spread (*TERM*). Fama and French (1989) and Keim

and Stambaugh (1986) provide evidence that the dividend yield and the term spread help forecast future market returns and the market risk premium.

Our model for the market risk premium is a linear function of four macro variables :
$R_{m,t+1} - R_{f,t} = \lambda_0 + \lambda_1 Div_t + \lambda_2 DEF_t + \lambda_3 R_{f,t} + \lambda_4 TERM_t + e_t$. The estimated regression coefficients plus the observed values of the regressors for each month t are used to obtain the expected market risk premium for the following period:

$$E_t[R_{m,t+1} - R_{f,t}] = \lambda_0 + \lambda_1 Div_t + \lambda_2 DEF_t + \lambda_3 R_{f,t} + \lambda_4 TERM_t \tag{15}$$

4. DATA

We gather our data from several sources. Firms' financial variables are sourced from the Compustat quarterly database; stock return variables are from the Center for Research in Security Prices (CRSP); risk-free rates are obtained from the Ken French Library; bond yields are taken from FRED at the Federal Reserve Bank of St. Louis. We calculate an annualized market portfolio return R_m using CRSP value-weighted market portfolio. Annualized risk free rate $R_{f,t}$ is calculated from the one-month Treasury bill rate. Annual dividend rate DIV is the sum of the previous 12 months' dividends accruing to the CRSP value-weighted market portfolio divided by the level of the market index at the beginning of the 12-month period. DEF is defined by the spread between Moody's Baa and Aaa corporate bond yields, TERM is the difference in yields between the ten-year and one-year Treasury bond.

Our sample is all U.S. publicly traded insurance companies over the time period from 1972 Quarter 1 to 2018 Quarter 4.[2] Because we need the annualized return on equity, and the annual book equity growth rate, the time series of portfolios is from 1972 Q4 to 2018 Q4.

During this sample period, the U.S. economy experienced a wide range of market conditions starting with a long period of high and rising inflation throughout the 1970s. This period was followed by the Fed's drastic change in monetary policy to tame inflation expectations during the early 1980s. Another important event was the stock market crash of 1987. The following decade may be characterized as a period of steady growth for the economy with low inflation. This came to a halt with the dot-com bubble crash in 2000, and the September 11 attacks. Last, our sample period includes the so called "Great Recession" – the longest economic decline since the Great Depression.[3] We believe the wide range of economic cycles provides a great opportunity to study the cost of capital for the insurance industry.

Firms are identified based on the Standard Industrial Classification System (SIC) codes. We identify all insurance companies (SIC codes 6300-6399), as well as the three subgroups: P/C insurers (SIC codes 6330-6331), life insurers (SIC codes 6310-6319), and health insurers (SIC codes 6320-6329).[4] For these insurance firms, earnings are measured using income before extraordinary items (E_t) from Compustat data item #8. Book value of equity (B_t) is extracted from Compustat data item #59. We calculate the annualized return on equity (ROE_t) using these two

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[2] We focus on the U.S. publicly traded insurers because stock prices are needed to estimate the cost of capital, in line with the sample used in prior literature (Cummins and Phillips, 2005; Wen *et al.*, 2008; BXP, 2018). Publicly traded stock insurers and their subsidiaries account for over 50% of the premiums written in the US, making this a representative sample of all US insurers.

[3] The National Bureau of Economic Research (NBER) reports that the great depression lasted from September 1929 thru March 1933, while the Great Recession was from December 2007 thru June 2009.

[4] Besides these three subgroups, there are still a small group of other insurers such as surety insurers and title insurers with SIC codes 6350-6399. The 4-digit SIC codes identify the primary line of business of an insurer, although some insurers may have multiple lines of business that cross the aforementioned subgroups. This classification scheme is in line with the approach of Wen *et al.* (2008) and BXP (2018), but different from Cummins and Phillips (2005); we do not make efforts to estimate cost of capital by specific line of business.

variables: $ROE_t = \frac{E_t + E_{t-1/4} + E_{t-1/2} + E_{t-3/4}}{B_{t-1}}$. We also compute an annualized equity growth rate

variable: $g_t = B_t / B_{t-1}$. Lastly, firm β is the posterior beta from Section 3.B.

Table 1 summarizes our sample. Our sample consists of 391 insurance firms and 19,139 firm-quarter observations. We winsorize the one-percent tails of book-to-market ratio. In our sample of insurers, P/C insurers make up the largest group (7,928 firm-quarter observations), followed by life insurers (4,214 observations) and health insurers (3,651 observations).[5] Median quarterly earnings for all of these insurers is $13.30 millions and mean quarterly earnings is $100.06 millions. There are some very large insurers that skew the descriptive statistics for earnings, book value of equity, market value of equity, and book to market ratio as the average values are much larger than median values. Based on median values, a mid-sized insurer has a book value of $641.74 millions and a market value of $727.76 millions. The average book to market ratio shows that, in general, the insurance industry consists of value firms. Looking at different subsectors, life insurers tend to have larger book values than other types of insurers, while health companies have lower book to market ratios.

5. EMPIRICAL RESULTS

5.A TERM STRUCTURE OF DISCOUNT RATES FOR THE INSURANCE INDUSTRY

As a first step to understanding the term structure of discount rates, we estimate Equation (9) at the unconditional value, or long-term average, of the state vector $\bar{x}$ and the riskless rate $\bar{y}$. We display these estimates in solid lines in Figures 1A thru 1F. The first graph is for the insurance

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[5] Not reported in the tables are other insurers (3,346 firm-quarter observations).

industry as a whole, followed by three value weighted subsector portfolios of P/C, life and health insurers. Figure 1F is for the universe of stocks in Compustat excluding all insurers. Each graph includes also the unconditional term structure of interest rates for risk-free cash flows, a flat (horizontal) line for the unconditional single period CAPM, and another flat (horizontal) line for the single-period discount rate estimated from a conditional CAPM (CCAPM).

Figure 1A shows that the term structure of discount rate for the insurance industry starts at $\rho(\tau)_t$ = 9.61% for one-year cash flows – which is also the level for the single-period CCAPM – then gradually increases to 10.23% for ten-year cash flows before decreasing to 8.99% for a 30-year zero coupon equity strip. In contrast, we note that the unconditional single period CAPM is constant at 11.74% per year. The implication of these results is that using the traditional CAPM overstates by roughly 2.75% the long run cost of capital, while using the single-period CCAPM does not allow the cost of capital to evolve over the duration of the cash flows. Although the term structure in Figure 1A seems relatively flat and only deviates from the 1-year term CCAPM by 62 basis points at the maximum, this is only the case for term structure evaluated at a long-term average of the input variables. We show in the next few sections that not only the slope of this term structure curve is significantly different from zero for most short to medium term equity cash flows, but also that the term structure becomes more variable when it is evaluated at a certain point in time, such as in recession or expansion.

Looking across different insurance sectors, the term structures are all slightly hump-shaped and downward slopping in the long term (Figures 1B, 1C and 1D), consistent with the notion that longer duration cash flows require lower annual expected return (Dechow et al 2004; Giglio et al 2020; Binsbergen et al 2017; Weber 2018). In terms of the absolute level of discount rates for both short and long-term cash flows, life insurers have a higher cost of capital than P/C and health

insurers, which is consistent with the observation made in prior literature. Life insurers especially, need to maintain a competitive investment rate of return in relation to banks and other investment funds as some of their products (e.g. variable annuity) compete directly with other banking products. For comparison purposes, we estimate the term structure of the portfolio consisting all firms but the insurers in the Compustat Universe (Figure 1F).

5.B COMPARISON OF TERM STRUCTURE MODEL WITH OTHER MODELS

This section focuses on comparing discount rates estimated from various models for firms in the insurance industry. In Figures 2 to 4, we compare the discount rates estimated from our term structure model with estimates from BXP (2018) using CCAPM, ICAPM, CAPM, and FF5. Figure 2 compares the various estimates in 2014 (the last year of data available from BXP (2018)), Figure 3 compares the various estimates during the recession of 2009, while Figure 4 displays results at the peak of the business cycle in 2007. In figure 5, we compare our term structure discount rates with those estimated from the implied cost of capital model by Berry-Stözle and Xu (2018).

Comparing our term structure results with cost of capital estimates from other models requires some background explanation about theoretical asset pricing models. First, it is important to realize that the CCAPM (Jagannathan and Wang, 1996; Ferson and Harvey, 1999; Lettau and Ludvigson, 2001), and more importantly the ICAPM (Merton, 1973; Ang *et al.*, 2006), do not actually solve for the price of an asset. These models are basically statements about the relationship between the expected rate of return for a particular asset, and the covariance between the asset return and the fundamental state variables in the economy. Second, under ICAPM pricing with a stochastic investment opportunity set, the single period expected return depends on the dynamics of the opportunity set, the maturity of the cash flow and its risk. The theory is silent, however,

about which state variables are important for asset pricing, and the magnitude of the market price of risk for each state variable. Third, our starting point or year-one estimate of CCAPM does not yield the same result as the CCAPM estimate in BXP (2018). This is because we estimate our beta using a Bayesian approach combining a rolling window beta estimate and a conditional beta estimate (Cosemans *et al.*, 2016). We also use a different sample period and two additional conditioning variables–lagged beta and lagged market volatility (Boguth *et al.*, 2011b). Lastly, and most important, as we pointed out earlier, the models used in BXP (2018) are silent about expected returns more than one period ahead and, in turn, how multi-period discount rates should be determined in an equilibrium setting.

Figure 2 compares discount rates evaluated at 2014 quarter 4. In Figure 2A for the entire insurance industry, our cost of capital term structure based on CCAPM starts at 8.82% and decreases to 6.78% for a 30-year zero equity cash flow. For the subsector portfolios, P/C insurers (Figure 2B) display hump-shaped term structure, while life insurers (Figure 2C) display downward sloping term structure. The difference is striking between P/C insurers and life insurers. During normal times, life insurers term structure curve is inverted. This is possibly because participating life or annuity policyholders perceive that their dividend claims long into the future can hedge their reinvestment risk (Goncalves, 2019), thus lowering the long-term cost of capital for life insurers, in turn lowing the discount rate considered by firm managers.

Figure 3 compares discount rates evaluated in 2009 – the bottom of the great recession, while Figure 4 shows the same set of comparisons at the peak of the business cycle in 2007. Both figures show substantial variation in both the level and the shape of CCAPM-based term structure discount rate for the insurance industry. Generally speaking, during the recession, term structure slope would be positive for a few years then become inverted; during economic expansion, term

structure is sloping upward. Using the constant CCAMP discount rate to evaluate long-term investments during economic boom may not capture the risk adequately for firms in the insurance industry.

Figure 5 compares our CCAPM-based term structure discount rate estimates with the implied cost of capital (ICC) estimates in Berry-Stözle and Xu (2018). In 2009 (during the recession), our term structure model generally suggests a higher discount rate than that suggested by the implied cost of capital model, especially for cash flows expected at 5-10 years in the future; in 2007 (during the economic expansion), our term structure is positioned much lower than the implied cost of capital estimate. These results reflect the countercyclicality in the insurance industry: betas and risk premiums are higher in bad times while lower in good times, which can be captured more easily in a CCAPM-based model than a ICC model.

5.C RISK PREMIUM FOR THE INSURANCE INDUSTRY

The empirical analysis in the last section shows that both the level and the shape of discount rate curves are highly variable over several points in our sample period. To peel off the risk-free term structure, and to show the risk premium demanded by risky cash flows with different maturities at different times across the more than 4 decades of our sample period, we consider the cost of equity risk premium at time t for a cash flow with maturity τ:

$$\text{Risk Premium}(\tau)_t = \rho(\tau)_t - y(\tau)_t$$

where $\rho(\tau)_t$ is the cost of equity capital at time t, and $y(\tau)_t$ is the riskless rate as of time t.

We plot the time series for maturity $\tau = 5$, 10, and 20 years in Figure 6 over the sample period from 1972 Q4 thru 2018 Q4 for the overall insurance industry. We observe that throughout the 1970s the risk premium follows an upward trend, but then falls dramatically by the early 1980s.

The 5-year risk premium curve lies above the 10-year and 20-year curves. This behavior correlates with the long period of high and rising unemployment and inflation experienced by the U.S. economy (the so-called stagflation). But, by the early 1980s, the Federal Reserve drastically changed monetary policy to tame inflation expectations. Figure 6 shows that during this period the risk premium for short duration cash flows ($\tau = 5$) reached a peak of 10.6% around 1977 but fell sharply to around -4.3% by 1981. This counter-intuitive result may be due to the fact that our theoretical model belongs to the class of affine asset pricing models, and we do not restrict the cost of equity capital to be greater than the riskless rate for all time periods and τ.[6] Moreover, the spread between the long- and short-term risk premiums narrowed substantially, implying that the market believed inflation expectations would continue to fall. Another interesting finding is that the stock market turmoil of the late 1980s had little impact on the insurance sector risk premium for any cash flow maturity. In fact, it appears that during the decade following the stock market crash of 1987 the level of both short- and long-term risk premia were relatively low and stable.

The last interesting episode in our sample is the "Great Recession" from December 2007 thru June 2009: the 5-year risk premium is negative again. The more interesting result, however, is the behavior of the risk premium: both level and volatility rose sharply. For example, the 5-year premium for the insurance sector reached a high of 15.29%, a level not seen since the 1970s. For comparison purpose, we added the risk premium estimated using the CCAPM from our paper and the risk premium estimated using the ICC model from Berry-Stözle and Xu (2018) for the time period between 1996 to 2012. The CCAPM model, which is essentially a 1-year term structure, displays the most volatile pattern; while the ICC model yields the least volatile estimates, which

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[6] This outcome is well known in the bond pricing literature (e.g., Dai and Singleton, 2000). Backus, Foresi, Mozumdar, and Wu (2001) dismiss this problem as follows: "we regard the possibility of negative interest rates a small price to pay for the convenience of a linear model."

is intuitive as the ICC models should capture an average cost of capital across all dividend horizons at a certain point in time.

5.D STATISTICAL TESTS

In this section we consider two important questions related to our term structure results. First, is there a statistical difference between our cost of capital estimates versus the single period unconditional CAPM? And second, is there a significant slope (either positive or negative) to the term structure? If the term structure is not significantly different form CAPM, or that the slope is not significantly different from zero, there is not much value to discounting cash flows with a maturity matched discount rate estimated from the CCAPM. In this section we investigate the statistical significance of our results; then in section 5.E we consider the economic significance.

To obtain the test statistics, we define the sequence of differences $x_t = \rho(\tau)_t - \mu_t$, where μ_t is the estimate from the unconditional CAPM computed as follows: The time-t beta estimate is based on a 60 months rolling window regression, the risk free rate is the one year rate observed during period t, and the market risk premium is the expected market risk premium from equation (15). The average difference is $\bar{x} = \overline{\rho(\tau)} - \bar{\mu}$, where $\overline{\rho(\tau)}$ is the sample average cost of capital for a specific maturity date τ, and $\bar{\mu}$ is the average rate from the single period CAPM. The t-statistic for the null hypothesis $H_0: \overline{\rho(\tau)} - \bar{\mu} = 0$ is given by $t = \frac{\bar{x}}{\sqrt{Var[\bar{x}]}}$, where $Var[\bar{x}]$ is the sample variance around the mean difference.

In Table 2 we report statistics for the null hypothesis $H_0: \overline{\rho(\tau)} - \bar{\mu} = 0$, for $\tau = 1, 5, 10, 15, 20, 25,$ and 30. The first row in each panel reports the mean cost of capital, $\overline{\rho(\tau)}$, for a specific maturity date τ (columns 2-8), followed by the unconditional CAPM mean value $\bar{\mu}$ in column 9,

followed by lower and upper values of a 95% confidence interval. The fourth row is the t-statistic for the null hypothesis, and this is followed by the corresponding p-value.

Table 2 suggests that there is overwhelming evidence against the hypothesis $H_0: \overline{\rho(\tau)} - \bar{\mu} = 0$ across the maturity spectrum. Indeed, the data point to a statistical difference between our cost of capital estimates versus the single period unconditional CAPM. Our cost of capital estimates is based on the CCAPM model, which, due to the countercyclicality of betas in the insurance industry, lead to more cyclical estimates than would be estimated by CAPM. Though this countercyclicality of cost of capital estimates (lower in good times while higher in bad times) is not shown in the average estimates in Table 2, our result is broadly consistent with BXP (2018), in that on average, CCAPM estimates are lower than CAPM estimates for insurance industry in the sample period of the last three to four decades.

For the portfolio with all firms but insurers the evidence is somewhat mixed. The confidence intervals are overlapped for all horizons but $\tau = 30$. For short and long-horizon, the corresponding t-statistics suggest that discount rates from the two models appear to be statistically different with p-values ranging from 0.00 for $\tau = 1$, 25, and 30 to 0.05 for $\tau = 10$; for medium-horizon, t-statistics are insignificant.

In Table 3 we test the second hypothesis $H_0: \overline{\rho(\tau)} = \overline{\rho(1)}$ for $\tau = 5$, 10, 15, 20, 25, and 30. The first row in each panel presents the average slope for the yield curve: $Slope = \overline{\rho(\tau)} - \overline{\rho(1)}$, for $\tau = 5$, 10, 15, 20, 25, and 30. This is followed by the corresponding t-statistic for the null hypothesis $H_0: Slope = 0$ and corresponding p-value.

For the insurance industry the t-statistics suggest that we may reject the null hypothesis at the one percent level for $\tau = 5$, 10, and 15. The slope is no longer significant except for $\tau = 30$ with a p-value of 0.08. These results are consistent with our visual observations of Figure 1A, that the

term structure of discount rate for firms in the insurance industry is hump-shaped on average and inverted in the long run. For short and medium horizons (under 20 years), we are relatively confident that our term structure slopes are positive and significantly different than zero. This pattern holds for all three insurance subsectors. It appears that investors believe that bearing risk is going to become more costly in the short and medium term, while adaptive measures can decrease that cost of risk in the long run. This is informative for managers in insurance firms, that they may select a duration-matching discount rate similar to that implied by a positive yield curve to discount future cash flows up to 15 years from the present. For life insurers, there is evidence that the term structure is eventually inverted, which gives life insurance firm managers some reason to be more opportunistic in selecting long-duration investment projects.

Our main takeaway from Table 3 is that though the constant cost of capital estimate may not be statistically different from twenty-year or twenty-five-year estimates, firms in the insurance industry can generally benefit from applying a term structure of discount rates.

5.E PRESENT VALUE ANALYSIS

Discounted cash flow analysis requires an estimate of the expected cash flow from a project, and an estimate of the risk adjusted discount rate. From a practical point of view, the current practice among large corporations is to discount all future flows with a constant rate obtained from the single period CAPM. However, our empirical analysis shows that discount rates are quite variable over time; characteristics such as location, slope and curvature evolve along with the business cycle. Moreover, we have observed large discrepancies between single-period discount rates and our term structure estimates. The purpose of this section is to quantify the impact of these differences on discounted cash flows.

We value a thirty-year ordinary annuity with an expected cash flow of $1 received at the end of each year. We first use our term structure discount rates for valuation and second using discount rates from our Conditional CAPM (CCAPM) and unconditional CAPM (CAPM). The annuity is evaluated for the entire insurance industry, three subsectors, and the portfolio of entire Compustat Universe excluding insurers. The annuity is evaluated for a starting point in 2018 (the latest year in our sample), 2014, 2009 (recession), and then again in 2007 (expansion). The valuation discrepancy is defined as:

$$\text{Valuation Discrepancy} = \frac{V_{CCAPM(CAPM)} - V_{Term\ Structure}}{V_{Term\ Structure}}$$

where $V_{CCAPM(CAPM)}$ is the annuity value when cash flows are discounted with conditional CAPM (unconditional CAPM) model. Clearly, these rates may vary by year of estimation, but they do not vary by duration of cash flows, unlike our term structure estimates.

In Table 4, our term structure cost of capital, in general, leads to much lower estimates of present values for insurers in 2018, 2009, and 2007, while higher in 2014. The level of pricing discrepancy by single-period discount rates is economically significant. This is most likely a result of significant slope to the term structure. Comparing at the point of recession (2009), the present values using our term structure are generally lower than those estimated using a constant CCAPM, and much lower than those estimated using a constant CAPM, due to a hump-shaped term structure and that medium-term discount rates are much higher. Comparing at the point of expansion (2007), the present values using our term structure estimates are still lower than their counterparts, probably due to a rising discount rate reflected in an upward sloping term structure at the point of expansion.

5. CONCLUSION AND FUTURE RESEARCH AGENDA

The literature suggests that insurance firms display countercyclical risk: betas increase during recessions and decrease during expansions. Moreover, firms in the insurance industry are sensitive to cash flow durations. All point to the necessity of discounting short-term and long-term cash flows differently and doing so for different insurance sectors. This paper provides new estimates of the cost of equity capital for the insurance industry as a whole, and several insurance subsectors such as property/casualty (P/C), life and health. We extend the previous literature by estimating a term structure discount rate for cash flows with different maturity dates, presenting them on a yield curve similar to that for riskless assets. This extension is meaningful both theoretically and empirically. Because the market risk premium, beta, riskless rate, and cash flows are all time-varying, the discount rates for equity cash flows must vary not only over time, but also across the spectrum of cash flow maturity dates.

Our empirical analysis shows that the term structure of discount rates for the insurance industry is hump-shaped, on average. Both the level and shape of the term structure also changes over the business cycle. Our statistical tests show that first, the difference between our estimates and those obtained by the traditional CAPM is statistically significant; second, the slope to our term structure is different from zero, especially for cash flow durations under 20 years and those over 30 years for life insurers. Lastly, our present value analysis reveals significant valuation discrepancies when discounting cash flows with a single period discount rate versus a term structure of discount rates.

We believe our study is but a first step in a rich research agenda. For example, it would be appropriate to develop standard errors for the cost of capital estimates using more rigorous methods, such as a Monte Carlo Markov Chain (MCMC) model (e.g., Gelfand and Smith, 1990).

A big advantage of the Bayesian approach is that the Markov chain will account for parameter uncertainty about the true market risk premium, the risk-free rate and the cash flow model.

In a second line of research, we propose that our valuation model for equity strips may be used to obtain better estimates of equity duration. Recent studies, such as Dechow, Sloan, and Soliman (2004), set the cost of capital at 12 percent for all time periods and cash flow risk. But duration measures the sensitivity of equity prices to changes in the discount rate. We may use our valuation model to study equity duration without the unreasonable assumption of a constant rate.

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Table 1: Descriptive Statistics for U.S. publicly traded insurance firms from 1972 Q4 to 2018 Q4 ($ amounts in millions). All insurers include all U.S. publicly traded insurance companies (SIC codes 6300-6399). We also report statistics on three subsector portfolios: Property/Casualty (P/C) insurers (6330-6331); life insurers (6310-6319), and accident and health insurers (6320-6329). Income before extraordinary items measures firms' quarterly earnings. Book value of equity is extracted from Compustat quarterly database and is later used to compute an annualized growth rate and annualized return on equity. Market value of equity, from the CRSP database, is obtained by multiplying each firm's shares outstanding by its share price at the end of each quarter.

Variables	N	P5	P50	P95	Mean	STD
All Insurers						
Quarterly Earnings	19,139	-15.87	13.30	467.60	100.06	618.51
Book Value of Equity	19,139	18.80	641.74	17,009.00	4,288.25	17,693.00
Market Value of Equity	19,139	19.99	727.76	19,658.00	4,475.02	14,760.00
Book to Market Ratio	19,139	0.31	0.89	2.73	1.52	6.14
P/C Insurers						
Quarterly Earnings	7,928	-15.20	10.46	278.50	53.63	159.03
Book Value of Equity	7,928	27.21	490.30	9,851.50	2,118.1	4,586.94
Market Value of Equity	7,928	22.62	596.52	12,544.00	2,572.8	5,748.13
Book to Market Ratio	7,928	0.39	0.90	2.84	1.44	5.73
Life Insurers						
Quarterly Earnings	4,214	-8.45	17.99	601.00	103.72	337.13
Book Value of Equity	4,214	26.30	812.64	23,072.00	4037.83	9435.07
Market Value of Equity	4,214	20.29	756.30	20,256.00	3553.24	8323.59
Book to Market Ratio	4,214	0.44	1.00	3.28	2.24	8.85
Health Insurers						
Quarterly Earnings	3,651	-8.76	15.41	505.00	96.10	252.54
Book Value of Equity	3,651	6.05	658.79	11,285.00	2,883.13	5,555.30
Market Value of Equity	3,651	11.19	1029.72	26,368.00	5,372.05	14,995.00
Book to Market Ratio	3,651	0.17	0.66	2.20	1.11	5.83

Table 2: Statistics for the Hypothesis: $\overline{\rho(\tau)} = \bar{\mu}$, for τ = 1, 5, 10, 15, 20, 25, and 30. $\overline{\rho(\tau)}$ is the Mean cost of capital for a specific maturity date τ (columns 2-8), and $\bar{\mu}$ is the average CAPM cost of capital (last column). The second and third rows in each panel report the lower and upper boundary of a 95% confidence interval around the mean cost of capital. The fourth row is the t-statistic for the null hypothesis H_0: $\overline{\rho(\tau)}$ - $\bar{\mu}$ = 0, and this is followed by the corresponding p-value. These results are based on the entire sample period ranging from 1972 Q4 to 2018 Q4.

Insurer type	τ=1	5	10	15	20	25	30	$\bar{\mu}$
All Insurers								
Mean	9.62	10.17	10.46	10.29	9.88	9.42	8.99	11.74
Lower Boundary	8.92	9.57	9.93	9.82	9.45	8.99	8.53	10.75
Upper Boundary	10.32	10.77	11.00	10.76	10.31	9.85	9.45	12.73
t-statistic	-9.84	-5.79	-3.78	-3.75	-4.29	-4.86	-5.33	
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
P/C Insurers								
Mean	8.75	9.44	9.85	9.78	9.45	9.04	8.64	10.76
Lower Boundary	8.11	8.88	9.33	9.33	9.03	8.61	8.18	9.86
Upper Boundary	9.39	10.01	10.36	10.23	9.87	9.47	9.10	11.65
t-statistic	-8.98	-5.37	-3.20	-3.01	-3.50	-4.08	-4.57	
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
Life Insurers								
Mean	9.95	10.79	10.89	10.70	10.21	9.67	9.17	13.75
Lower Boundary	9.22	9.83	10.36	10.25	9.80	9.25	8.71	12.47
Upper Boundary	10.68	11.75	11.42	11.14	10.62	10.09	9.63	15.02
t-statistic	-10.66	-5.83	-5.70	-5.49	-5.92	-6.43	-6.88	
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
Health Insurers								
Mean	9.07	9.63	9.84	9.74	9.45	9.10	8.75	11.59
Lower Boundary	8.39	9.01	9.26	9.22	8.98	8.64	8.28	10.57
Upper Boundary	9.75	10.26	10.41	10.25	9.92	9.56	9.23	12.62
t-statistic	-9.73	-6.35	-4.92	-4.74	-4.95	-5.25	-5.53	
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
All stocks excl Insurers								
Mean	11.35	11.99	12.13	11.69	11.01	10.28	9.58	11.56
Lower Boundary	10.45	11.17	11.40	11.08	10.49	9.81	9.12	10.64
Upper Boundary	12.25	12.81	12.86	12.30	11.52	10.74	10.04	12.49
t-statistic	-8.33	2.26	2.00	0.39	-1.58	-3.29	-4.61	
p-value	0.00	0.03	0.05	0.70	0.12	0.00	0.00	

Table 3: Statistics for the Hypothesis: $\overline{\rho(\tau)} = \overline{\rho(1)}$, for τ = 5, 10, 15, 20, 25, and 30. The first row in each panel reports the average slope for the yield curve: $Slope = \overline{\rho(\tau)} - \overline{\rho(1)}$, for τ = 5, 10, 15, 20, 25, and 30. This is followed by the corresponding t-statistic for the null hypothesis H_0: $Slope = 0$ and corresponding p-value. These results are based on the entire sample period ranging from 1972 Q4 to 2018 Q4.
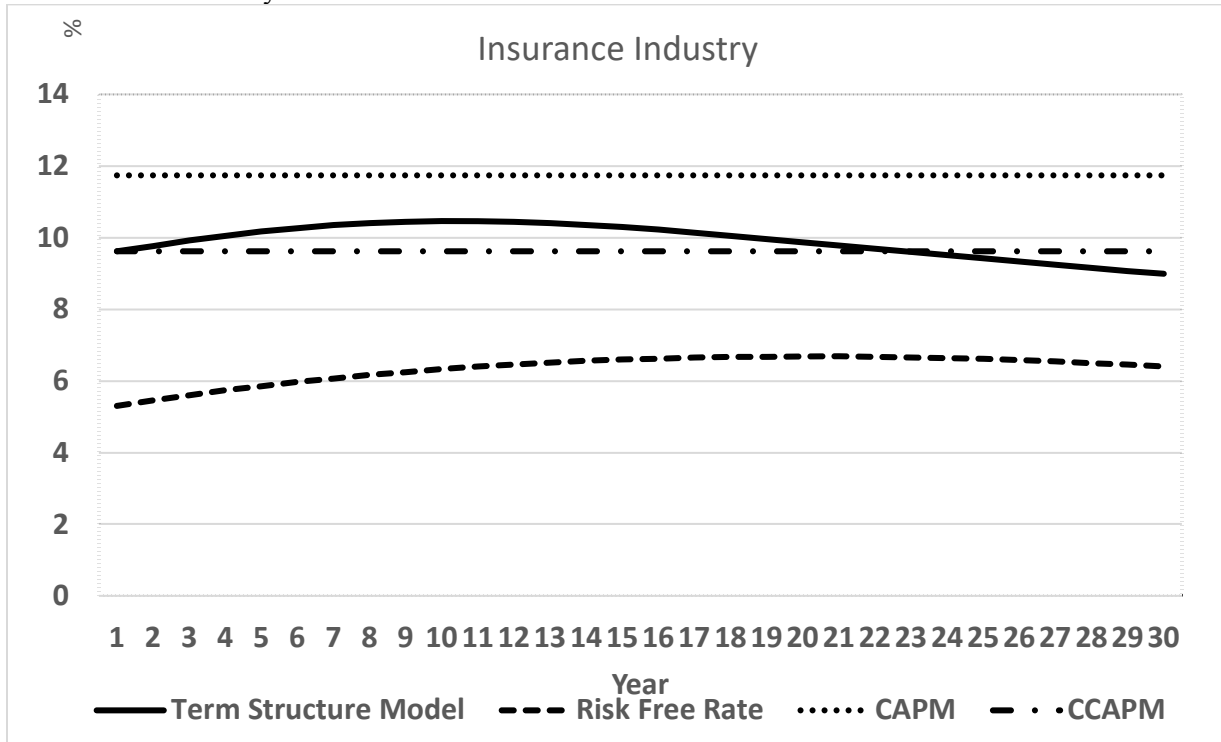
Insurer type	τ=5	10	15	20	25	30
All Insurers						
Slope	0.55	0.84	0.67	0.26	-0.20	-0.63
t-statistic	4.55	4.28	2.82	0.92	-0.62	-1.76
p-value	0.00	0.00	0.01	0.36	0.53	0.08
P/C Insurers						
Slope	0.69	1.09	1.03	0.70	0.29	-0.11
t-statistic	6.89	7.05	5.53	3.06	1.07	-0.35
p-value	0.00	0.00	0.00	0.00	0.29	0.72
Life Insurers						
Slope	0.84	0.94	0.74	0.26	-0.28	-0.78
t-statistic	2.56	4.41	2.86	0.84	-0.82	-2.05
p-value	0.01	0.00	0.00	0.40	0.41	0.04
Health Insurers						
Slope	0.57	0.77	0.67	0.38	0.04	-0.31
t-statistic	5.37	4.76	3.51	1.65	0.13	-0.97
p-value	0.00	0.00	0.00	0.10	0.90	0.33
All stocks excl Insurers						
Slope	0.64	0.79	0.34	-0.34	-1.07	-1.77
t-statistic	3.64	2.89	1.10	-1.01	-2.81	-4.21
p-value	0.00	0.00	0.27	0.32	0.01	0.00

Table 4: Valuation Discrepancies when Cash Flows are Discounted Using Single Period Discount Rates obtained from CCAPM and CAPM. This table presents the Present Value of a 30-year ordinary annuity with an expected cash flow of $1 paid at the end of each year. In column 1 cash flows are discounted with our cost of equity term structure, whereas in column 2 we use the Conditional CAPM (CCAPM) estimates. We report percentage valuation discrepancies between our term structure model and CCAPM estimates in column 3. Column 4 and 5 report unconditional CAPM estimates and corresponding percentage value discrepancies. Results are shown for the insurance industry, its three subsectors, and all firms in the Compustat universe excluding insurers, evaluated in 2018, 2014, 2009 (recession) and then again in 2007 (expansion).

2018 Q4	Term Structure	CCAPM	Valuation Discrepancies	CAPM	Valuation Discrepancies
All Insurers	14.19	18.73	24.24%	17.99	26.73%
P/C	14.72	19.12	22.98%	18.37	24.73%
Life	13.15	18.63	29.42%	17.46	32.79%
Health	14.75	18.41	19.90%	17.89	21.35%
All stocks excl Insurers	14.26	18.12	21.31%	18.02	26.36%
2014 Q4					
All Insurers	11.30	10.44	-8.23%	8.75	-22.60%
P/C	13.06	13.72	4.86%	10.41	-20.30%
Life	9.81	8.55	-14.77%	5.00	-49.03%
Health	11.25	10.36	-8.55%	8.97	-20.27%
All stocks excl Insurers	7.70	8.54	9.80%	8.24	7.02%
2009 Q2					
All Insurers	6.01	7.26	17.21%	8.14	35.57%
P/C	8.02	8.83	9.20%	11.28	40.66%
Life	4.31	5.41	20.38%	5.13	19.10%
Health	7.20	7.42	2.91%	7.01	-2.68%
All stocks excl Insurers	5.92	8.73	32.15%	8.68	46.42%
2007 Q3					
All Insurers	13.28	17.70	24.95%	17.88	34.63%
P/C	13.54	17.66	23.34%	17.88	32.04%
Life	12.75	17.83	28.46%	17.76	39.28%
Health	13.63	17.56	22.35%	17.54	28.66%
All stocks excl Insurers	13.01	17.91	27.38%	17.92	37.79%

Figure 1: Average term structure of discount rates. Panel A thru E show discount rates on the vertical axis and years to maturity on the x-axis. The solid curve indicates the term structure for risky cash flows evaluated at the long-term average value of the state vector. The constant discount rate estimated from the Conditional CAPM (CCAPM), the constant discount rate estimated from the unconditional CAPM, and the term structure for risk-free cash flows are also plotted.
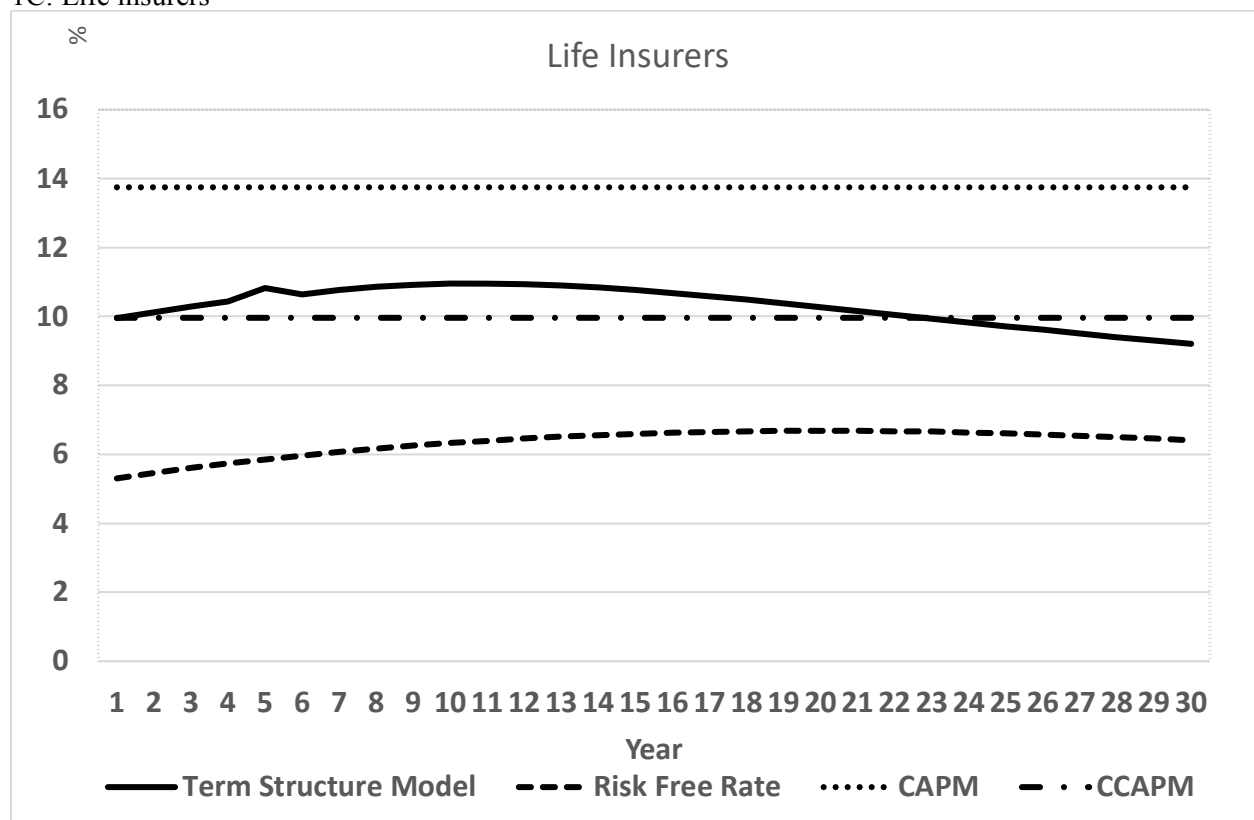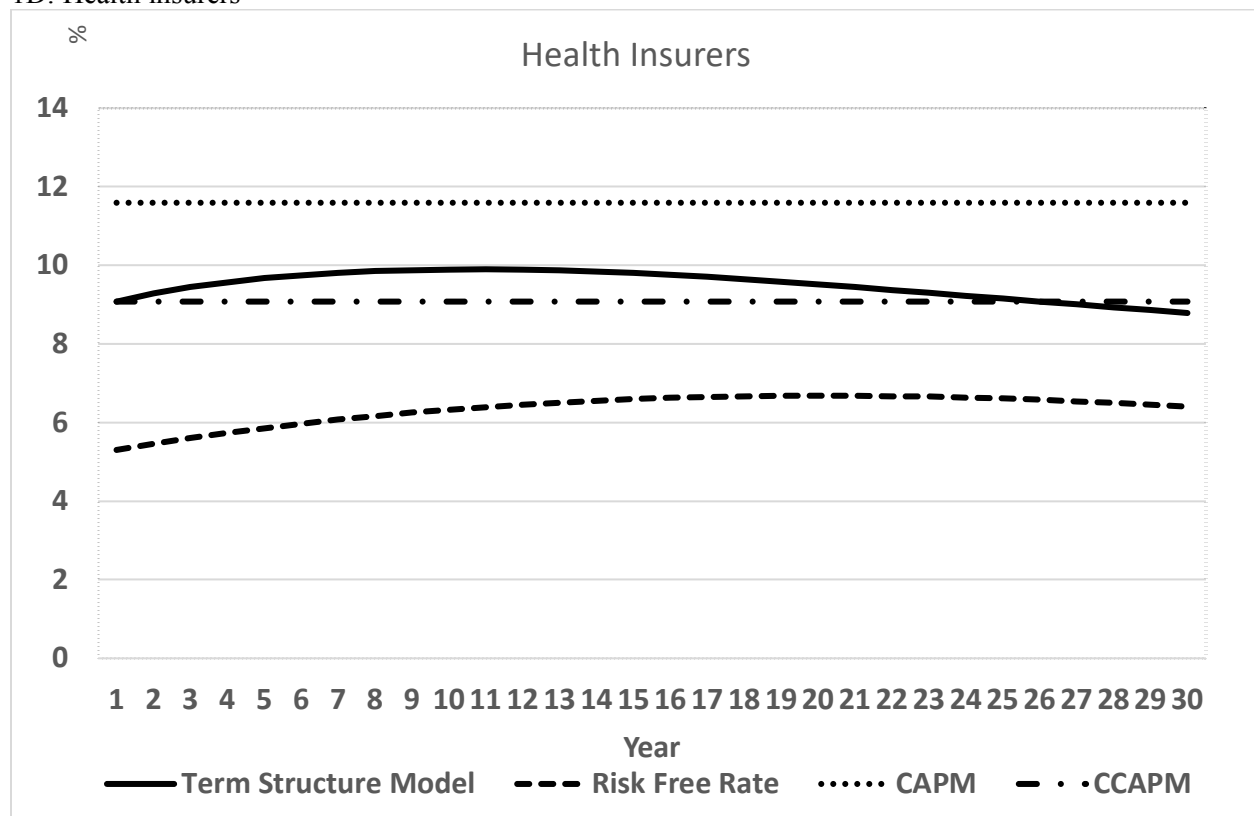
1A: Insurance Industry

1B: P/C insurers

1C: Life insurers

Life Insurers

%

Year

Term Structure Model — Risk Free Rate ····· CAPM — · ·CCAPM

1D: Health insurers

Health Insurers

%

Year

Term Structure Model — Risk Free Rate ····· CAPM — · ·CCAPM
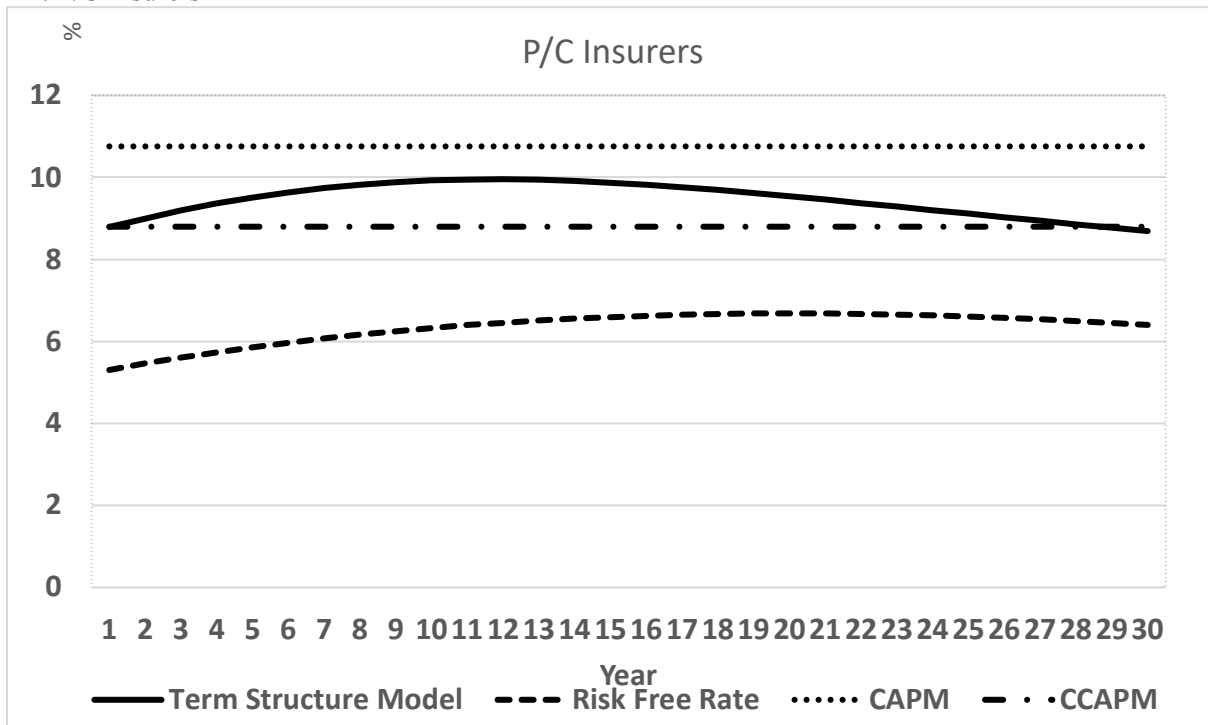
1F: All stocks in Compustat excluding insurance industry

All stocks excluding insurers
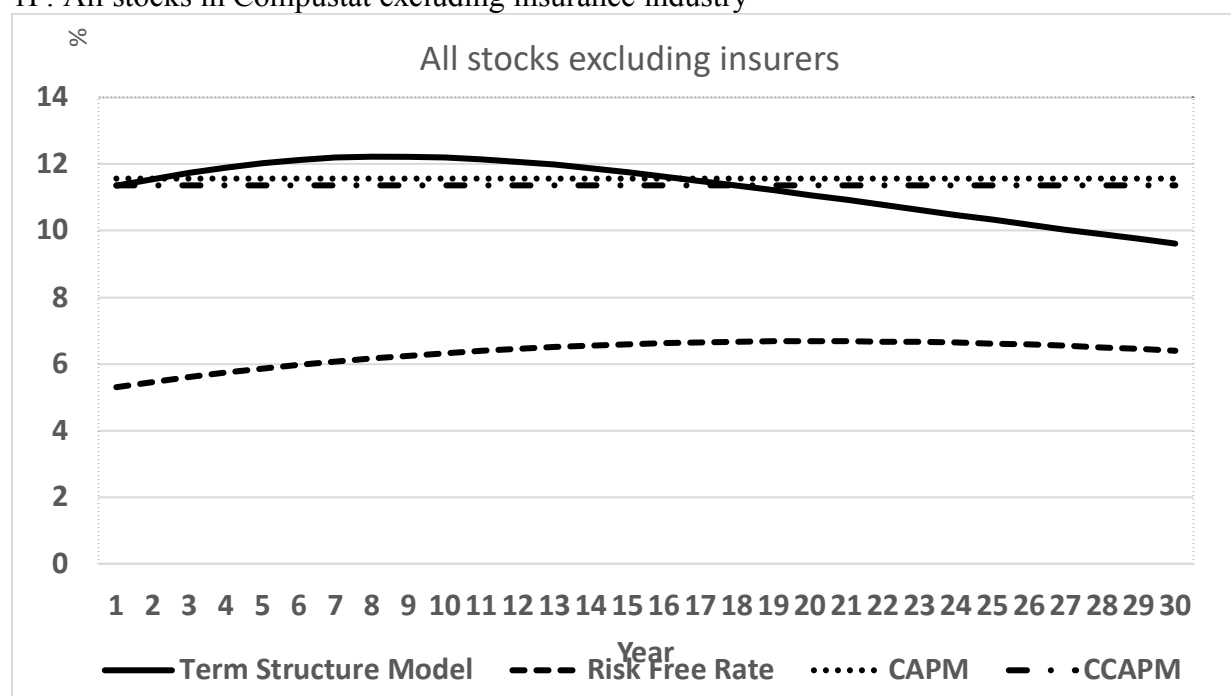
Figure 2: Term structure of discount rates vs. cost of equity capital estimates from BXP (2018) evaluated at 2014 quarter 4. The estimates from BXP (2018) are based on the unconditional CAPM (triangle), FF5 (diamond), the Conditional CAPM (CCAPM, circle), and the Intertemporal CAPM (ICAPM, hollow square). Their cost of equity capital estimates are constant (flat lines) for cash flows with varying durations, while our term structure is a curved solid line.

2A: Insurance Industry
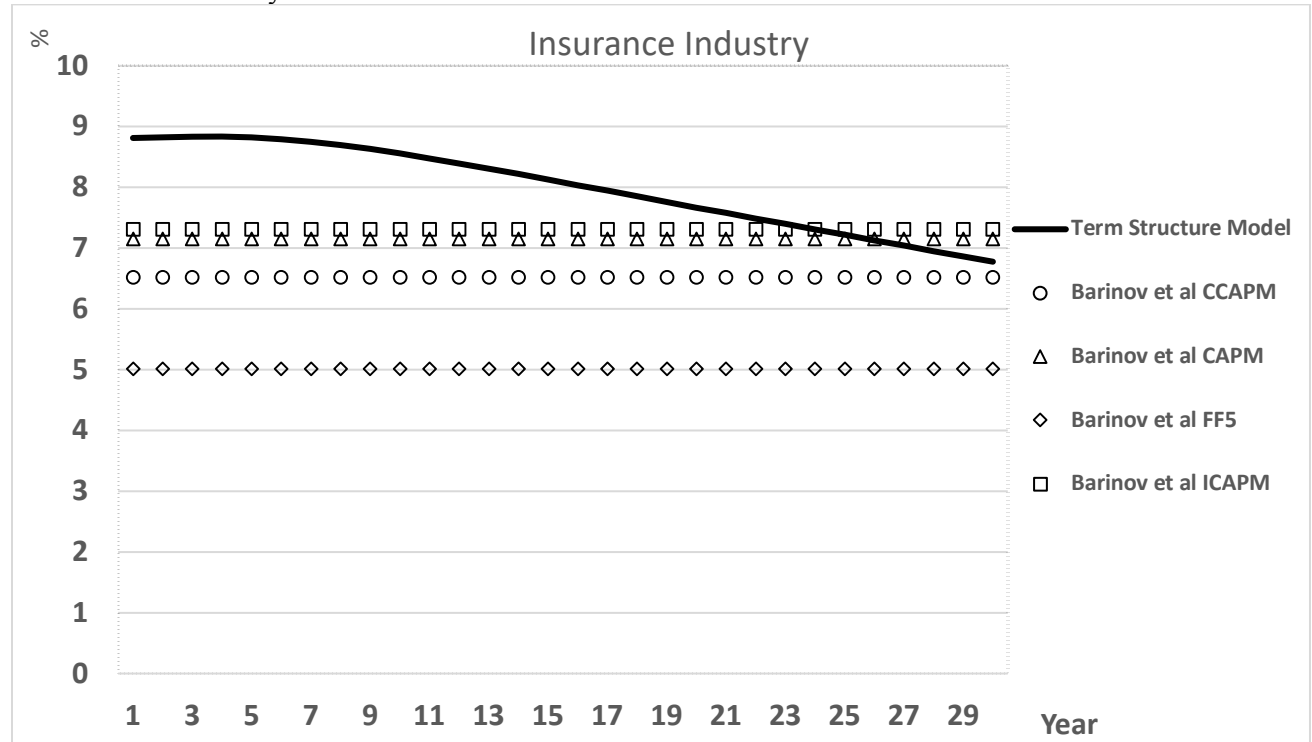
2B: P/C insurers
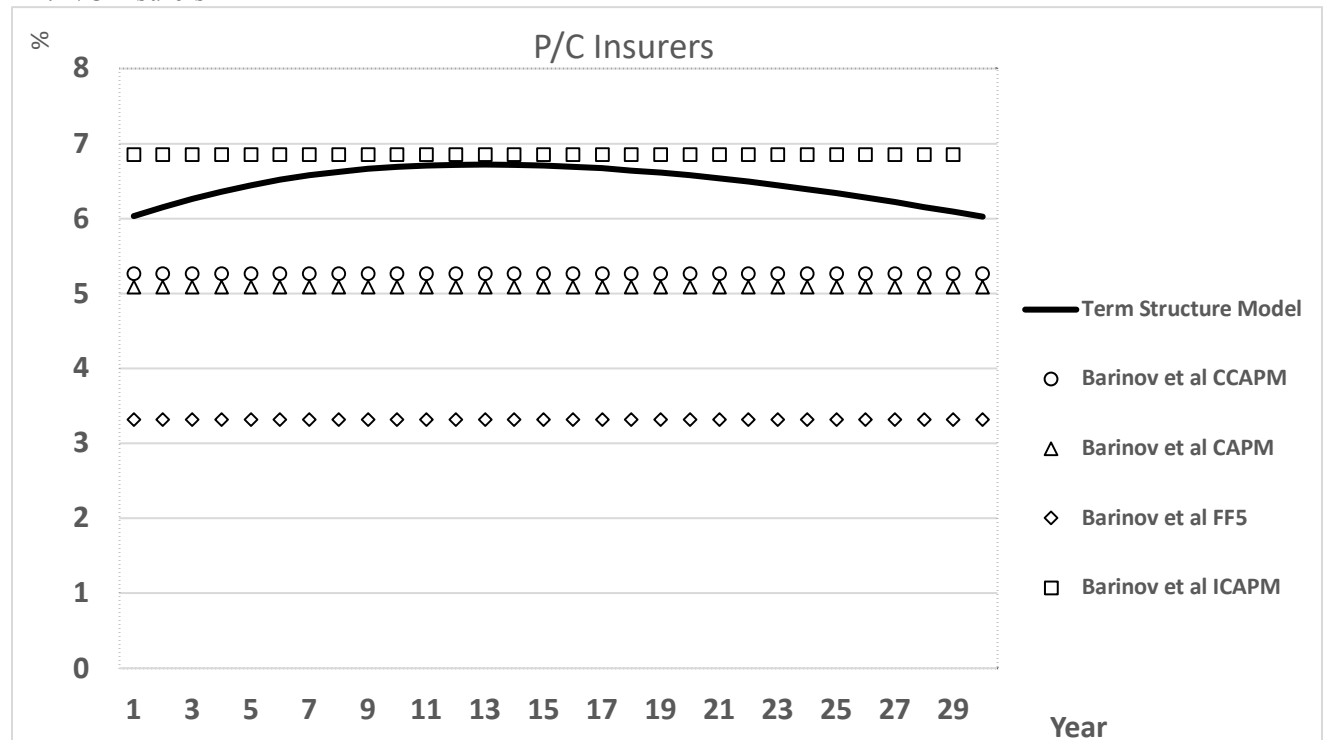
2C: Life insurers
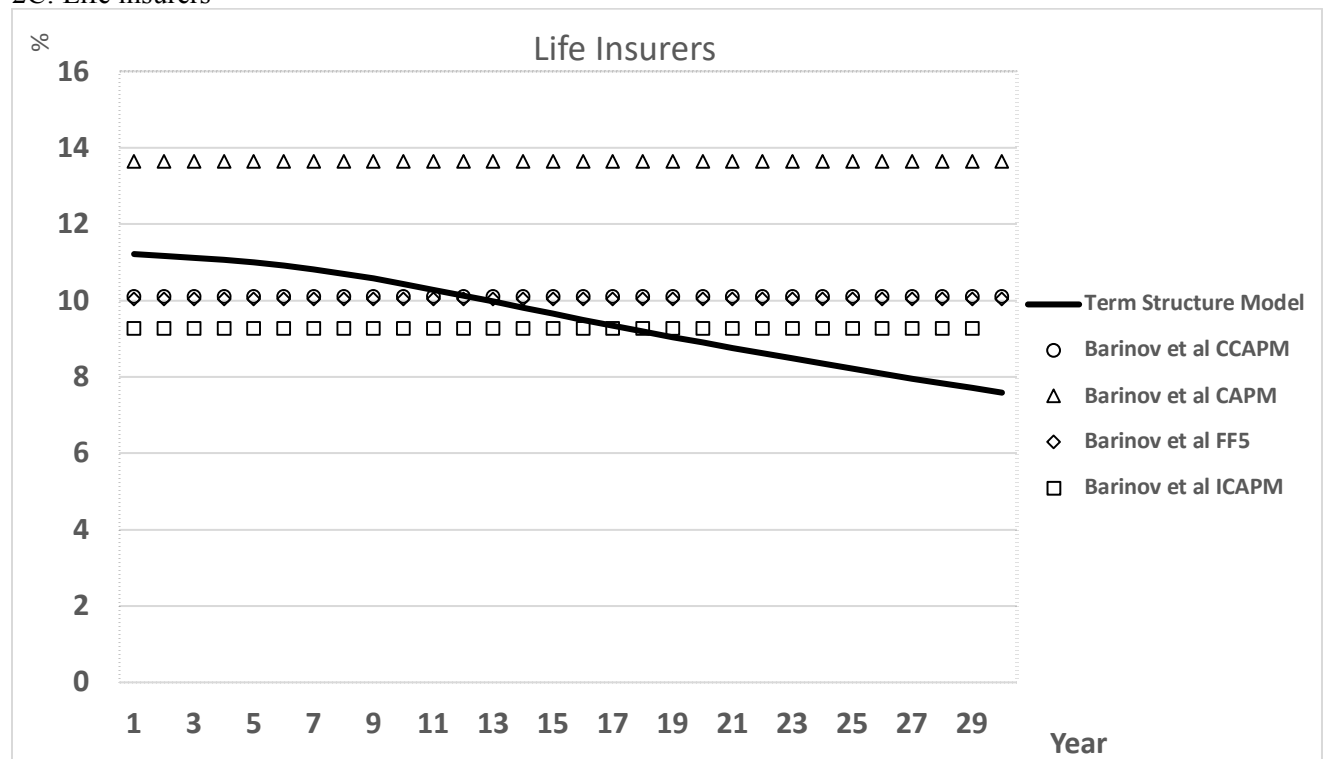
Figure 3: Term structure of discount rates vs. cost of equity capital estimates from BXP (2018) for 2009 when the U.S. economy was in a deep recession. The estimates from BXP are based on the unconditional CAPM (triangle), FF5 (diamond), the Conditional CAPM (CCAPM, circle), and the Intertemporal CAPM (ICAPM, hollow square). Their cost of equity capital estimates is constant (flat lines) for cash flows with varying durations, while our term structure is a curved solid line, evaluated in 2009.

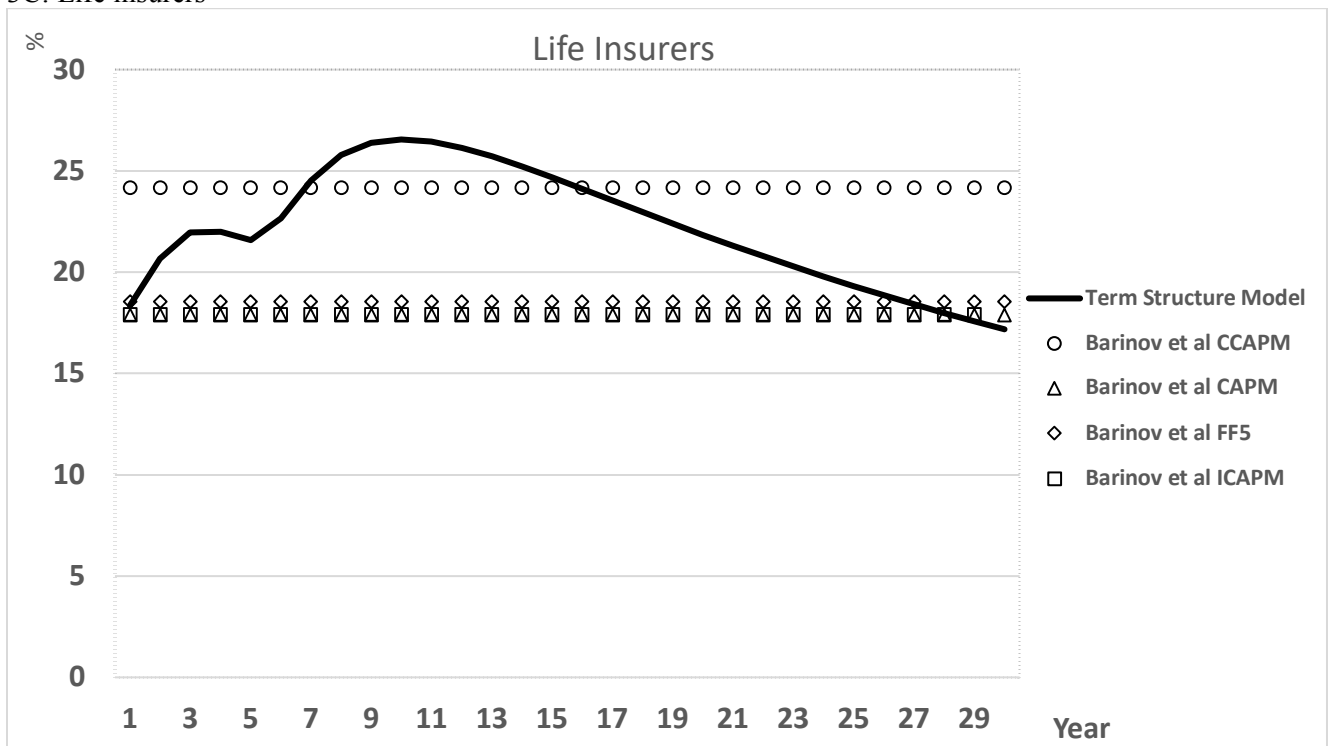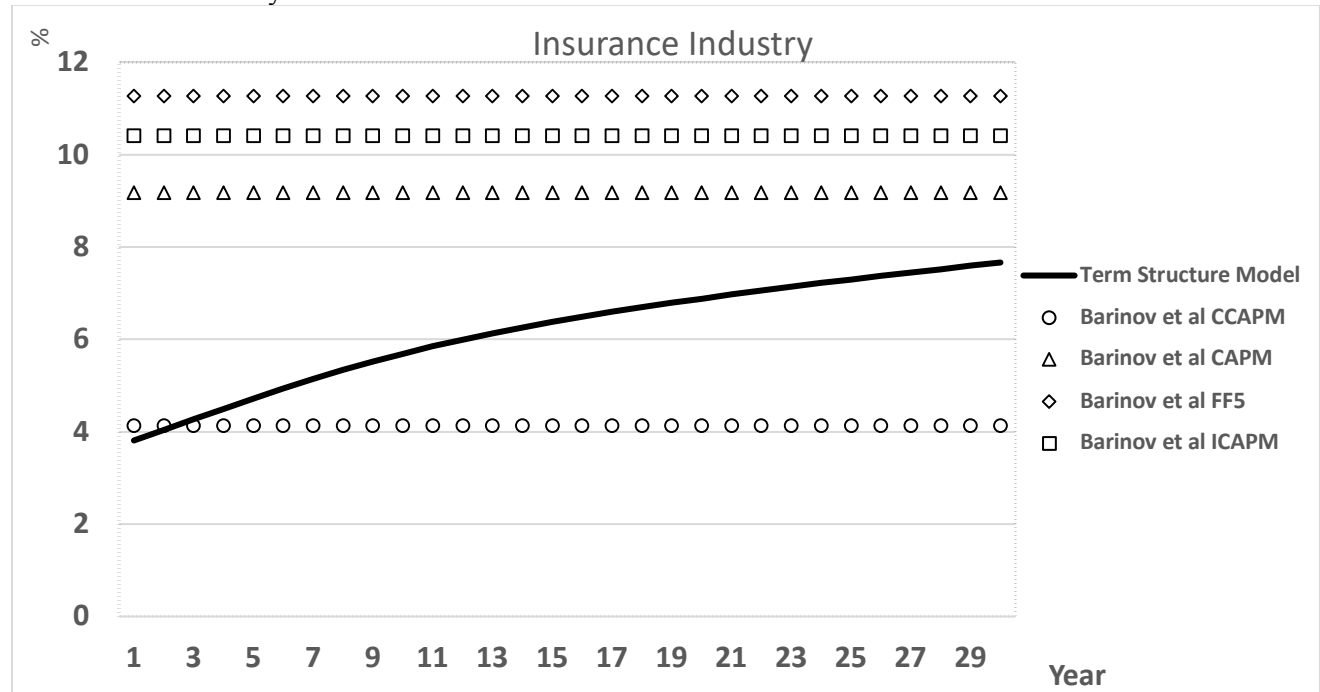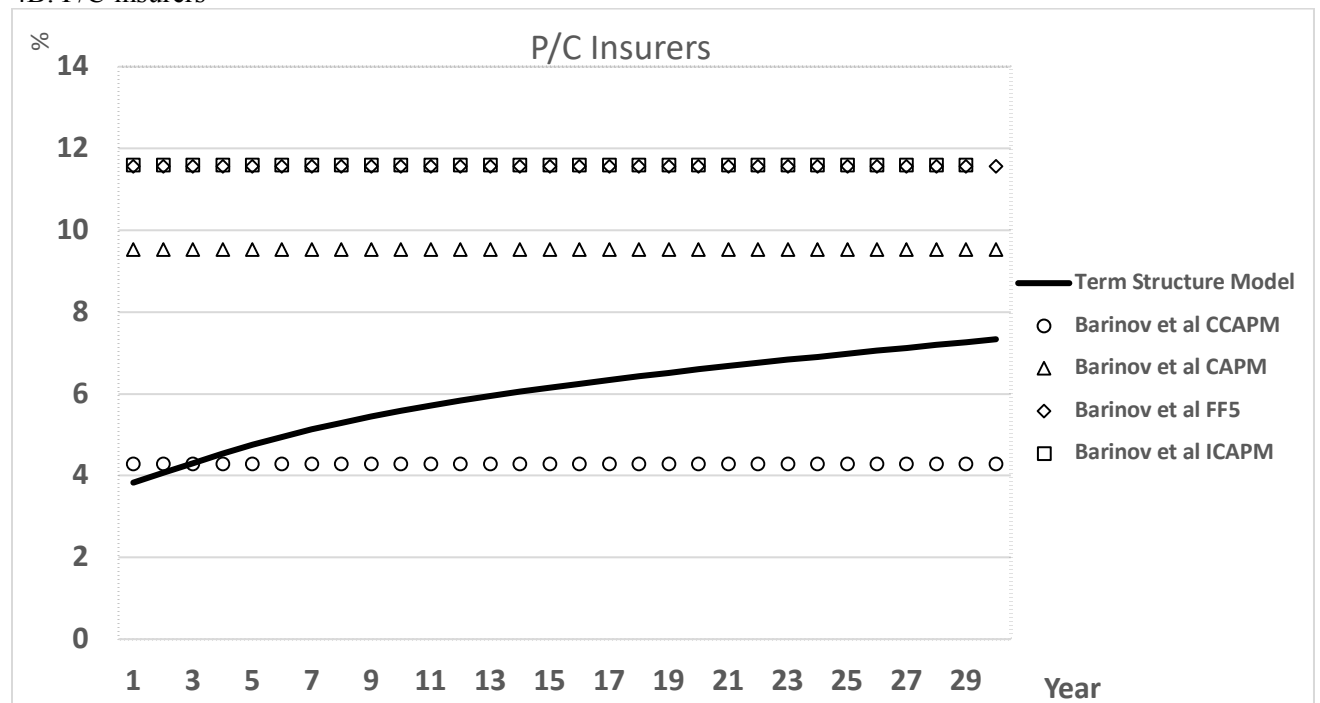3A: Insurance Industry
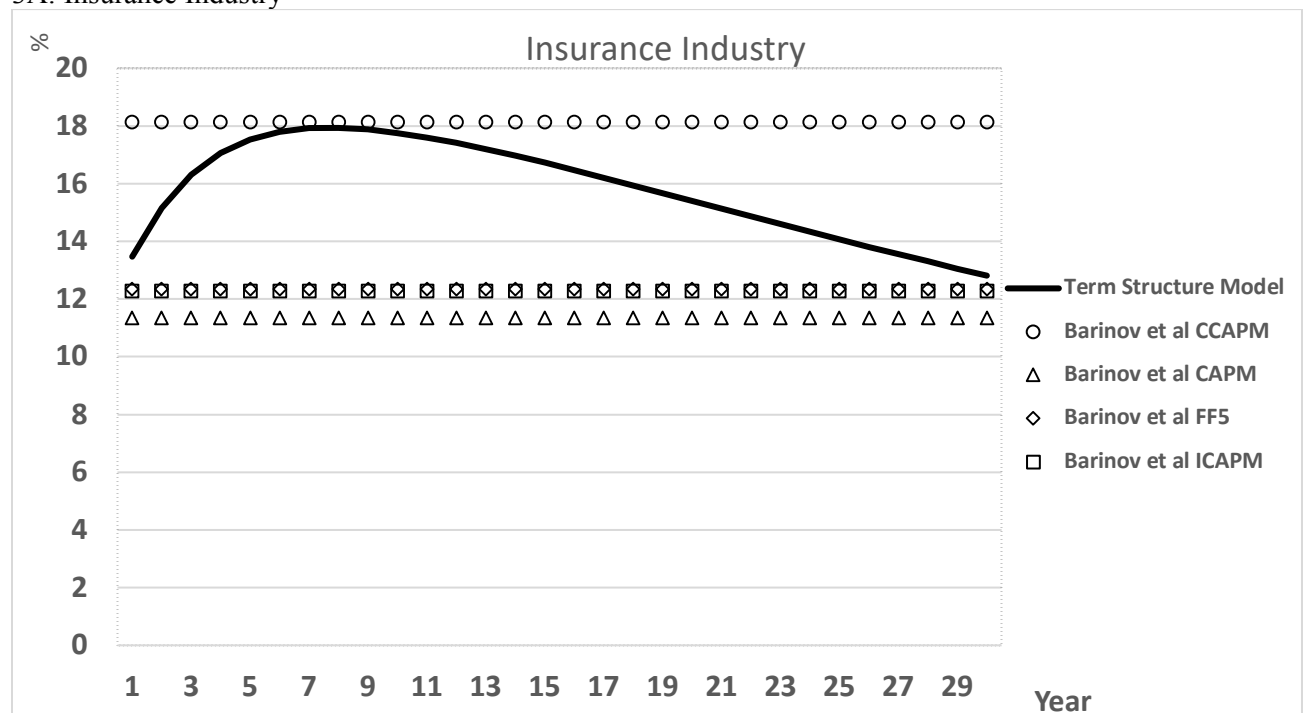
3B: P/C insurers
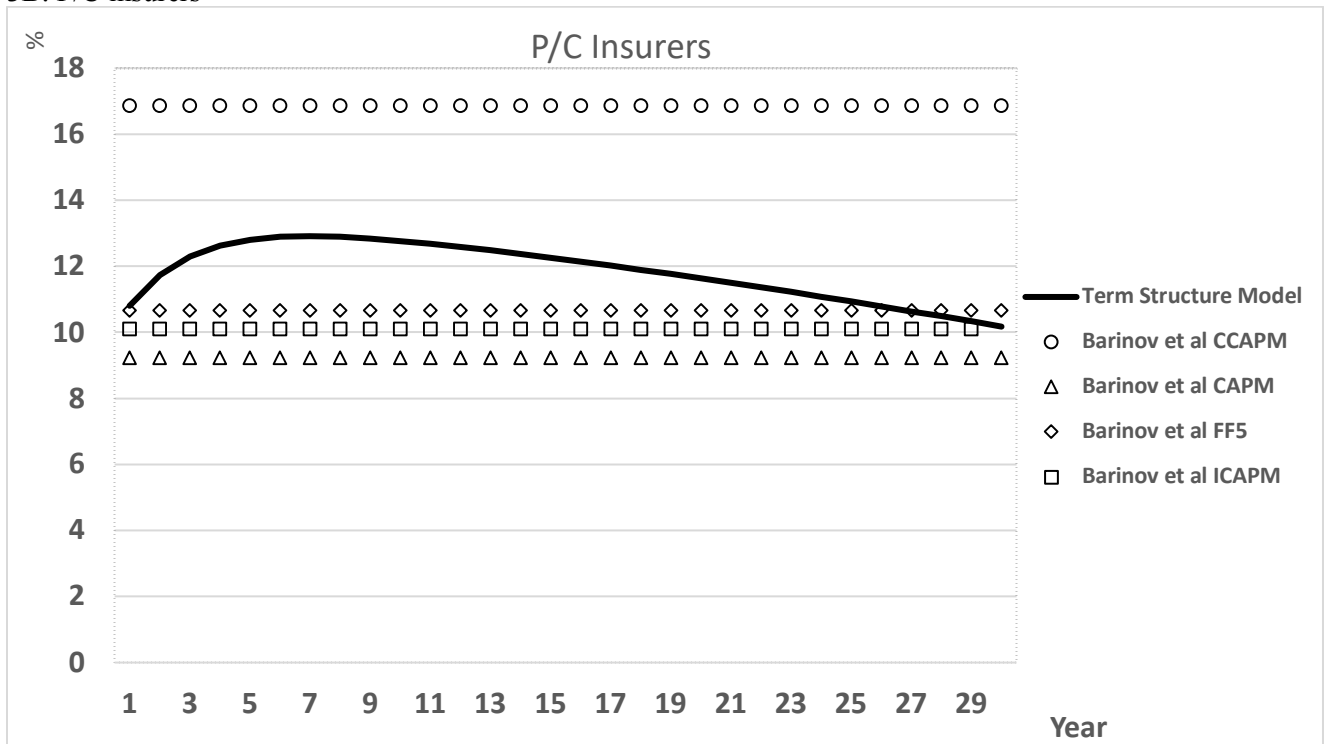
3C: Life insurers

Figure 4: Term structure of discount rates vs. cost of equity capital estimates from BXP (2018) for 2007 at the top of the business cycle for the U.S. economy. The estimates from BXP (2018) are based on the unconditional CAPM (triangle), FF5 (diamond), the Conditional CAPM (CCAPM, circle), and the Intertemporal CAPM (ICAPM, hollow square). Their cost of equity capital estimates are constant (flat lines) for cash flows with varying durations, while our term structure is a solid curved line, evaluated in 2007.

4A: Insurance Industry

4B: P/C insurers

4C: Life insurers

Life Insurers

Legend:
- Term Structure Model
- ○ Barinov et al CCAPM
- △ Barinov et al CAPM
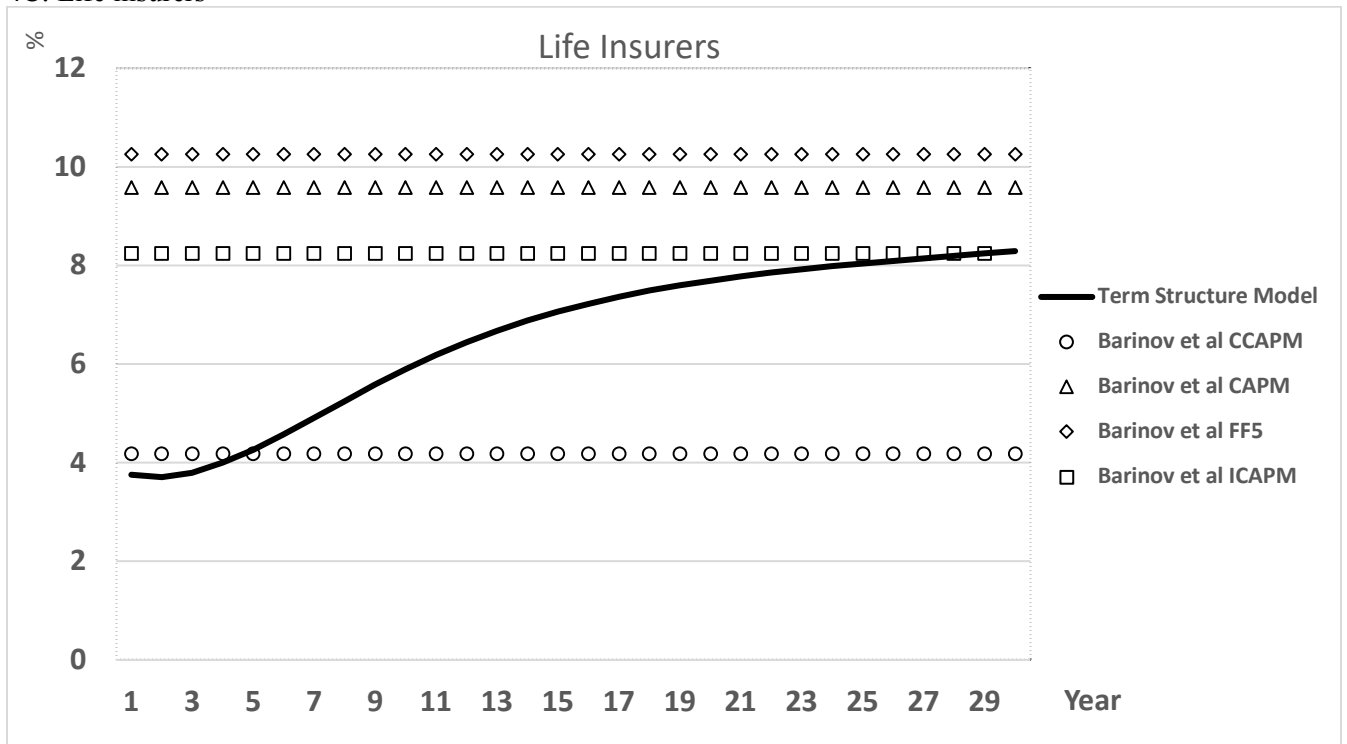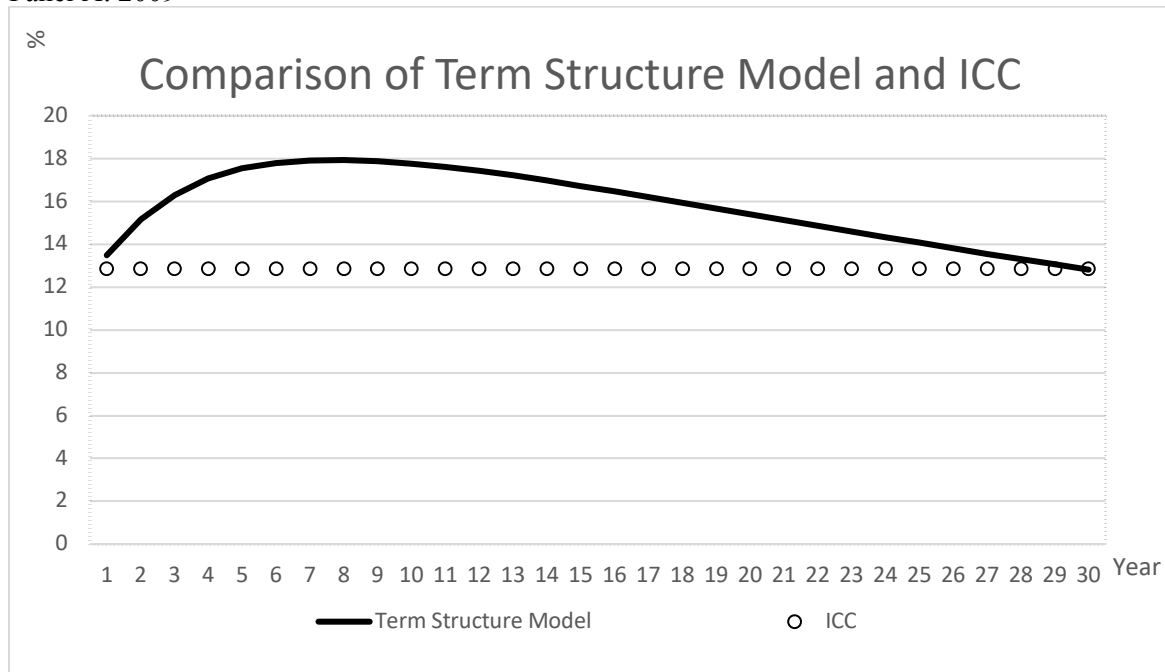- ◇ Barinov et al FF5
- □ Barinov et al ICAPM

Figure 5: Term structure of discount rates vs. the implied cost of capital (ICC) estimates from Berry-Stoelzle and Xu (2016). Comparisons are made for 2009 (when the U.S. economy was in a deep recession) and for 2007 (when the U.S. economy was at the top of the business cycle).
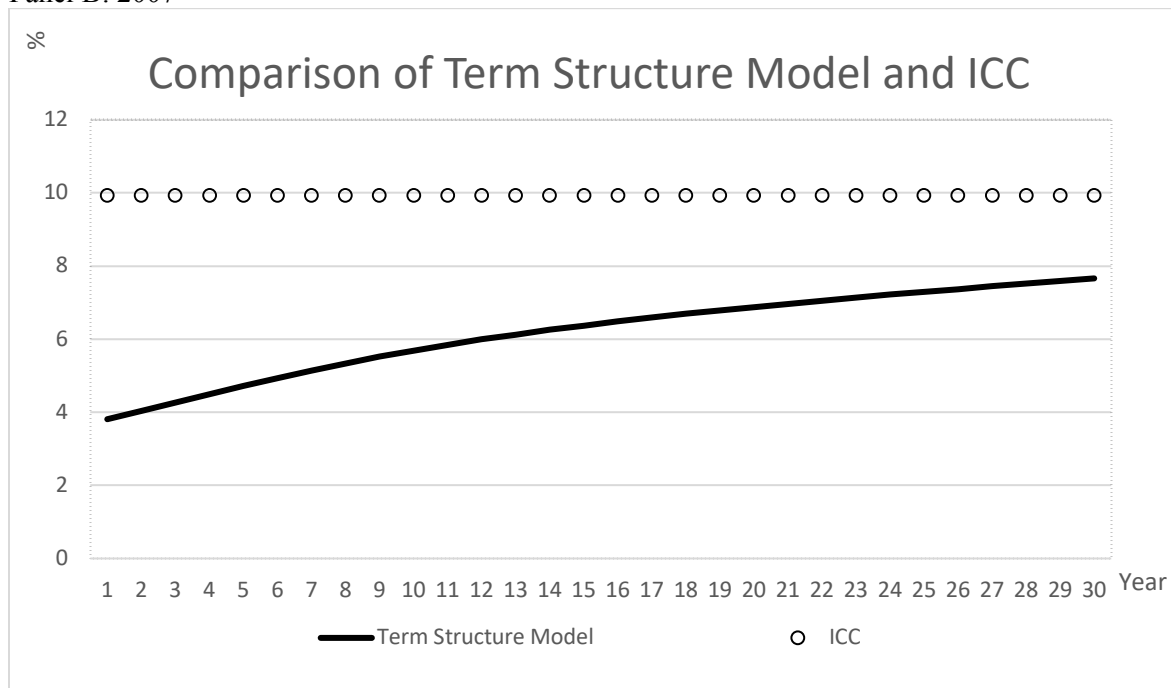
Panel A: 2009

Panel B: 2007

Figure 6: Time series of 5-, 10- and 20-year risk premium for all insurers over our sample period from 1972 Q1 to 2018 Q4. The solid line is the time series of the 5-year premium, the dash line is for 10-year, the dotted line is for the 20-year risk premium. Also included in this graph are two benchmark risk premiums: one calculated from single-period Conditional CAPM (CCAPM-Rf), and another from the implied cost of capital (ICC-Rf) in Berry-Stözle and Xu (2018).
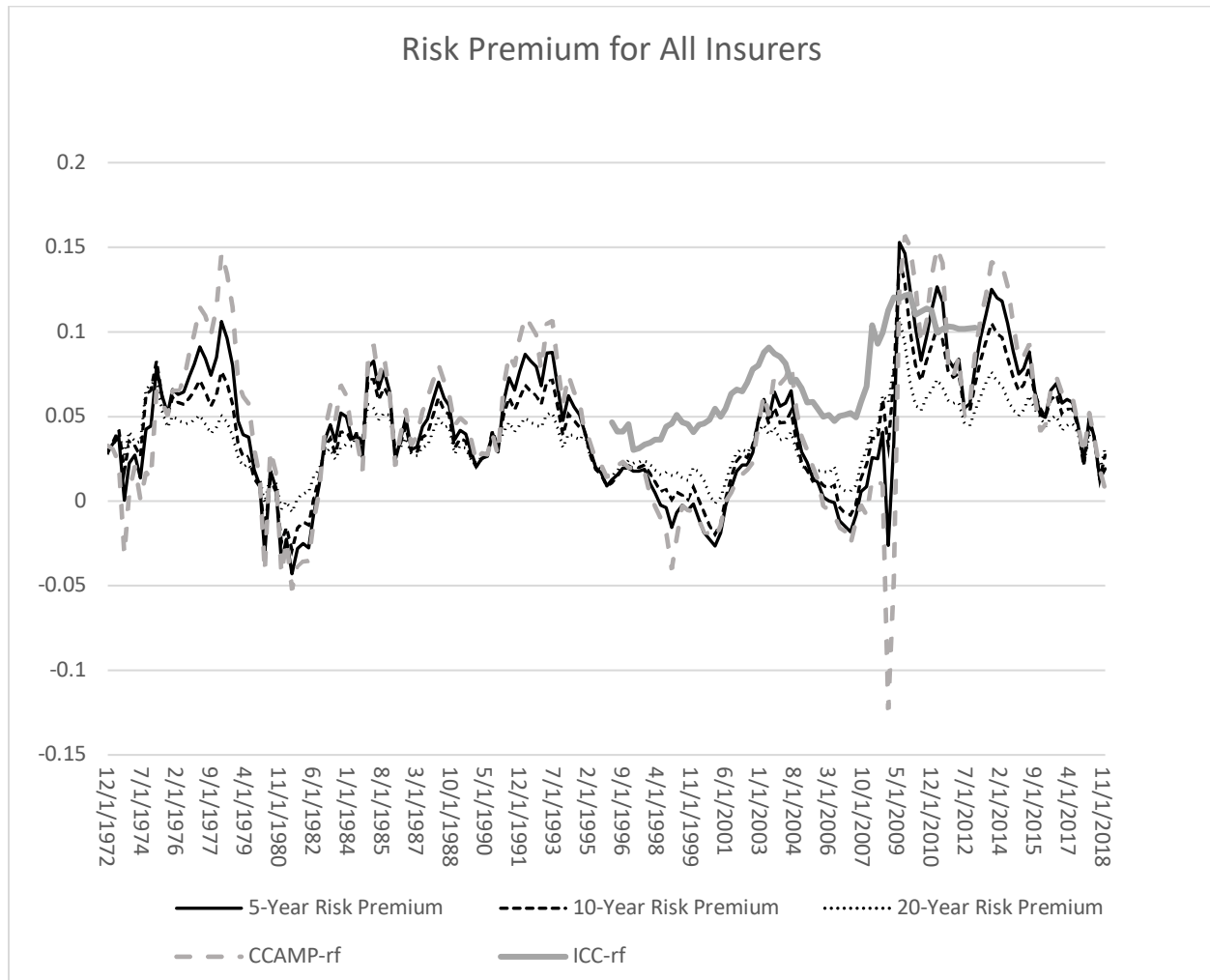
APPENDIX A

The purpose of Appendix A is to show a complete derivation of the cost of capital Term Structure model.

Consider the time-t price of a single cash flow $C_{t+\tau}$ expected at a future date $t+\tau$. The theoretical price, or present value $V(\tau)_t$ may be obtained by discounting the expected value with a corresponding risk adjusted discount rate:

$$V(\tau)_t = e^{-\tau\rho(\tau)t}[E_t C_{t+\tau}] \tag{1}$$

where $\rho(\tau)_t$ is the cost of capital as of end-of-period t. Because of the similarity between Equation (1) and the formula for the present value of a zero coupon bond, Lettau and Wachter (2007) describe the present value of a single risky cash flow as 'zero coupon equity'. Unfortunately, such assets are not traded therefore one must rely on theoretical models to extract and analyze the implicit term structure cost of capital. The goal of this section is to develop such a model and provide empirical estimates of $\rho(\tau)_t$ for different sectors of the insurance industry.

An equivalent representation of the discounting formula may be obtained as follows. Define μ_t as the continuously compounded expected return generated by holding a τ-period zero coupon equity from t to $t+1$ as: $\mu_t = E_t[\ln(V(\tau-1)_{t+1}/(V(\tau)_t))]$. This expression may be solved forward subject to the condition that the terminal value equal the cash flow itself: $V(0)_{t+\tau} = C_{t+\tau}$. The solution is the well-known present value textbook formula $V(\tau)_t = E_t[e^{-(\mu_t + \ldots + \mu_{t+\tau-1})}C_{t+\tau}]$. To make this formula operational, we need specify the joint evolution of cash flows and expected single period returns – we address these two issues next.

To identify the cost of capital we need an equilibrium model for expected returns. We model expected returns with a conditional CAPM. Thus, the riskless rate, market beta and market

risk premium are time-varying. Specifically, the single period equilibrium discount rate is defined as:

$$\mu_t = R_{f,t} + E_t[R_{m,t+1} - R_{f,t}]\beta_t \tag{2}$$

where $R_{f,t}$ is the risk-free rate observed at the end-of-period t, $E_t[R_{m,t+1} - R_{f,t}]$ is the market risk premium, and β_t is current value of beta.

To model cash flows, we use the "Clean Surplus" accounting relationship. Let NI_{t+1} represent net income from period t to $t+1$, and let B_t stand for the book equity value at the end of period t. Clean surplus accounting requires that net cash flow is given by accounting earnings minus the change in book value: $CF_{t+1} = NI_{t+1} - (B_{t+1} - B_t)$. This condition may be restated in terms of log of growth in book equity $g_{t+1} = \ln(B_{t+1}/B_t)$, and return on book equity $ROE_{t+1} = \ln(1 + NI_{t+1}/B_t)$:

$$CF_{t+1} = B_t[e^{ROE_{t+1}} - e^{g_{t+1}}] \tag{3}$$

This model has been used extensively both in the accounting and finance literature (e.g., Feltham and Ohlson, 1995, and Pastor and Veronesi, 2003). Assuming that cash flows grow as prescribed by clean surplus accounting, then the current zero coupon equity price depends on the joint evolution of expected single period returns, return on equity, and book equity growth:

$$V(\tau)_t = B_t E_t\{e^{-(\mu_t + \ldots + \mu_{t+\tau-1})}[e^{g_{t+1} + \ldots + g_{t+\tau-1} + ROE_{t+\tau}} - e^{g_{t+1} + \ldots + g_{t+\tau}}]\} \tag{4}$$

In the next two subsections we derive the cost of capital term structure for risky equity cash flows. This term structure is the analog of the term structure of interest rates.

A.1   Valuation with a Known Discount Rate

In this section we assume that the cost of capital is known, therefore to obtain the current price of a single cash flow we need only the conditional expectation of $C_{t+\tau}$. This value may be obtained using well known properties of log-normal random variables; but first we need to specify the state vector x_t and specify its time series behavior of cash flows, beta and market risk premium. We then develop a recursive algorithm to compute the present value

$$V(\tau)_t = e^{-\tau\rho(\tau)_t}[E_t C_{t+\tau}].$$

Let x_t be a vector of accounting book equity rate of return, growth in book equity value, equity β_t, and market risk premium: $x_t' = (ROE_t, g_t, \beta_t, E_t(R_{m,t+1} - R_{f,t}))$. Modern finance literature (e.g. Campbell (1991), Ang and Liu (2004), Cochran (2011)) argues that model parameters should be estimated jointly as in a vector autoregressive process (VAR) to capture dynamic interactions between variables such as return on equity, growth in book equity, beta, and the market risk premium. In line with this literature, we specify a first-order VAR to model the time series dynamics of the state vector

$$x_{t+1} = (I - \phi)\bar{x} + \phi x_t + \varepsilon_{t+1} \tag{5}$$

where $\bar{x}$ is the long run (unconditional mean) vector, ϕ is the companion matrix, and x_t represents observed sample values at time-t. The idiosyncratic shocks $\{\varepsilon_t\}$ form a time-series of independently distributed normal variables with zero mean and variance matrix ω.

To describe potential sample paths of the four-equation system from time period t+1 through t+τ we use a multivariate VAR defined as

$$\begin{bmatrix} I & 0 & 0 & . & . & 0 \\ -\phi & I & 0 & . & . & 0 \\ 0 & -\phi & I & 0 & . & 0 \\ & . & . & . & . & \\ 0 & . & . & 0 & -\phi & I \end{bmatrix} \begin{bmatrix} x_{t+1} \\ x_{t+2} \\ . \\ . \\ x_{t+\tau} \end{bmatrix} = \begin{bmatrix} (I-\phi)\bar{x} \\ (I-\phi)\bar{x} \\ . \\ . \\ (I-\phi)\bar{x} \end{bmatrix} + \begin{bmatrix} \phi x_t \\ 0 \\ . \\ . \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ \varepsilon_{t+2} \\ . \\ . \\ \varepsilon_{t+\tau} \end{bmatrix}$$

Expressed in more compact form, we have: $\Phi X = (I-\phi)\bar{X} + X^* + \varepsilon$, where the row vector

$X' = [x_{t+1}, \ldots, x_{t+\tau}]$ captures the future state variables, the initial condition X^* consists of the

vector ϕx_t in the first four rows, followed by 0's in the remaining rows. The entire sequence of

random shocks $\{\varepsilon_{t+1}, ..., \varepsilon_{t+\tau}\}$ are normal variables with zero mean and covariance matrix

$\Omega = diag(\omega, \omega, \ldots, \omega)$.

By clean surplus accounting, the expected cash flow depends on the evolution of return on

equity and book equity growth:

$$E_t[CF_{t+\tau}] = B_t \, E_t[e^{(g_{t+1} + \ldots + g_{t+\tau-1}) + ROE_{t+\tau}} - e^{g_{t+1} + \ldots + g_{t+\tau}}] \tag{6}$$

We note that by the properties of log normal random variables, this expectation requires the mean

and variance of the aggregate growth in book equity $g_{t+1} + \ldots + g_{t+\tau}$, and the predicted return on

equity τ periods from now $(g_{t+1} + \ldots + g_{t+\tau-1}) + ROE_{t+\tau}$. Next, we develop a recursive

algorithm to easily compute these moments.

A convenient way to sum growth rates is obtained by first defining the row vector

$\Lambda' \equiv (e2', \ldots, e2')$ where $e2'$ is a 1x4 vector of 0s except for a 1 in the 2nd place. It follows

that the cumulative growth in book equity is a linear function of the X vector $\sum_{j=1}^{\tau} g_{t+j} = \Lambda'X$ --

provided Λ' has τ individual $e2'$ vectors. Thus, the $e2'$ vectors in Λ' extract book equity growth from period 1 to τ.

The conditional mean and variance of this sum are:

$E_t[\Lambda'X] = \Lambda'\Phi^{-1}(I-\phi)\bar{X} + \Lambda'\Phi^{-1}X*$ and $Var_t[\Lambda'X] = \Lambda'\Phi^{-1}\Omega(\Phi^{-1})'\Lambda$, respectively. We now show that these moments have a simple closed form solution that does not require the inversion of the Φ matrix. Define the vector $(\lambda_\tau', \lambda_{\tau-1}', \ldots, \lambda_1') \equiv \Lambda'\Phi^{-1}$ from which it follows that $(\lambda_\tau', \lambda_{\tau-1}', \ldots, \lambda_1')\Phi = \Lambda'$ represents a system of vector difference equations. The solution may be obtained recursively as $\lambda_j' = \lambda_{j-1}'\phi + e2'$ starting from j=1 and working forward to j=τ. The initial condition is a 1x4 vector of 0s: $\lambda_0' = 0$. Thus, each vector λ_j' may be computed recursively with a simple Excel spreadsheet, and the expected future growth in book equity may be computed as:

$$E_t[e^{g_{t+1} + \cdots + g_{t+\tau}}] = \exp[\sum_{j=1}^{\tau}\lambda_j'(I-\phi)\bar{x} + \lambda_\tau'\phi x_t + \frac{1}{2}\sum_{j=1}^{\tau}\lambda_j'\omega\lambda_j] \tag{7}$$

A similar methodology applies to the predicted equity return τ-periods from now $(g_{t+1} + \cdots + g_{t+\tau-1}) + ROE_{t+\tau}$. To develop the recursive algorithm, we define the row vector $\Gamma' \equiv (e2', \ldots, e2', e1')$ where the number of $e2'$ vectors included is $\tau-1$, and the last element of Γ' is $e1'$ which is defined as a 1x4 vector of 0s except for a 1 in the first place. Note that $e1'$ is used to extract the return on equity from the vector Γ'. Then, the conditional mean and variance of book equity return are: $E_t[\sum_{j=1}^{\tau-1}g_{t+j} + ROE_{t+\tau}] = \Gamma'\Phi^{-1}(I-\phi)\bar{X} + \Gamma'\Phi^{-1}X*$ and

$$Var_t[\sum_{j=1}^{\tau-1} g_{t+j} + ROE_{t+\tau}] = \Gamma'\Phi^{-1}\Omega(\Phi^{-1})'\Gamma \text{ , respectively. To compute these moments, define}$$

$(\gamma_\tau{}', \gamma_{\tau-1}{}', \ldots, \gamma_1{}') \equiv \Gamma'\Phi^{-1}$ from which it follows that $(\gamma_\tau{}', \gamma_{\tau-1}{}', \ldots, \gamma_1{}')\Phi = \Gamma'$. The

solution may be obtained recursively as $\gamma_j{}' = \gamma_{j-1}{}'\phi + e2'$ starting from j=2 and working forward

to j=τ. The initial condition for j = 1 is $\gamma_1{}' = e1'$. The expected future return on book equity may

be easily computed as

$$E_t[e^{g_{t+1} + \ldots + g_{t+\tau-1} + ROE_{t+\tau}}] = \exp[\sum_{j=1}^{\tau} \gamma_j{}'(I-\phi)\bar{x} + \gamma_\tau{}'\phi x_t + \frac{1}{2}\sum_{j=1}^{\tau} \gamma_j{}'\omega\gamma_j] \qquad (8)$$

Thus, the present value of the expected cash flow corresponding to Equation (1) is:

$$V(\tau)_t = B_t \{e^{-\tau\rho(\tau)_t}[e^{A(\tau)^* + B(\tau)^*'x_t} - e^{C(\tau)^* + D(\tau)^*'x_t}] \qquad (9)$$

where

$$A(\tau)^* = \frac{1}{2}\sum_{j=1}^{\tau}(\gamma_j{}'\omega\gamma_j) + \sum_{j=1}^{\tau}\gamma_j{}'(I-\phi)\bar{x}, \qquad\qquad B(\tau)^{*'} = \gamma_\tau{}'\phi$$

and

$$C(\tau)^* = \frac{1}{2}\sum_{j=1}^{\tau}(\lambda_j{}'\omega\lambda_j) + \sum_{j=1}^{\tau}\lambda_j{}'(I-\phi)\bar{x}, \qquad\qquad D(\tau)^{*'} = \lambda_\tau{}'\phi$$

A.2   Valuation of Expected Cash Flows with a Conditional CAPM

In this section, we develop a model that breaks down zero coupon equity into a zero coupon

bond plus a claim to a risky cash flow. Let $Z(\tau)_t$ be the price of a zero coupon riskless bond with

a $1 face value expected at a future date t+τ, and $y(\tau)_t$ be the corresponding yield to maturity

observed as of end-of-period t. By definition, the time-t price is given by $Z(\tau)_t = e^{-\tau \, y(\tau)_t}$. We also assume independence between interest rates and the state variables in the x_t vector. By doing so, we will be able to separate the cost of capital into a riskless discount rate and a cash flow risk premium component.

Asset pricing theory implies that the current price of zero coupon equity obeys a recursive relation:

$$V(\tau)_t = E_t[e^{-\mu_t} V(\tau-1)_{t+1}] \tag{10}$$

To value a cash flow expected one period from now $(\tau = 1)$, the solution is intuitive: price is the discounted value of the expected cash flow $V(1)_t = E_t[e^{-\mu_t} C_{t+1}]$. Assuming further that expected returns follow the conditional CAPM, and cash flows obey clean surplus accounting, we have:

$V(1)_t = B_t \, E_t[e^{-\mu_t + ROE_{t+1}} - e^{-\mu_t + g_{t+1}}]$. We find a closed form solution in terms of the state vector X in two steps. First, define Θ as a 4×4 matrix such that $x_t' \Theta x_t$ captures the risk premium $E_t[R_{m,t+1} - R_{f,t}]\beta_t$:

$$\Theta = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 \\ 0 & 0 & 1/2 & 0 \end{bmatrix} \tag{11}$$

Then, the current price is $V(1)_t = e^{-R_{f,t}} e^{-x_t' \Theta x_t} B_t \, E_t[e^{e1' x_{t+1}} - e^{e2' x_{t+1}}]$, where we use the fact that return on equity is the first element of the state vector ($ROE_{t+1} = e1' x_{t+1}$), and growth in book equity is the second element ($g_{t+1} = e2' x_{t+1}$). For the second step, we observe that by the properties of log normal variables, the expected cash flow follows immediately as

$E_t[e^{e1'x_{t+1}} - e^{e2'x_{t+1}}] = e^{e1'[(I-\phi)\bar{x}+\phi x_t]+(1/2)e1'\omega e1} - e^{e2'[(I-\phi)\bar{x}+\phi x_t]+(1/2)e2'\omega e2}$. Thus, the

solution to (10) for the price of single period $(\tau = 1)$ equity is:

$$V(1)_t = Z(1)_t \ B_t \left[e^{A(1)+B(1)'x_t+x_t'G(1)x_t} - e^{C(1)+D(1)'x_t+x_t'G(1)x_t} \right] \tag{12}$$

where $Z(1)_t = e^{-R_{f,t}}$ is the current price of a one-period riskless bond, and the coefficients $A(1)$

thru $G(1)$ are:

$$
\begin{aligned}
&A(1) = e1'(I-\phi)\bar{x} + (\tfrac{1}{2})e1'\omega e1, \quad B(1)' = e1'\phi \\
&C(1) = e2'(I-\phi)\bar{x} + (\tfrac{1}{2})e2'\omega e2, \quad D(1)' = e2'\phi, \quad G(1) = -\Theta
\end{aligned}
\tag{13}
$$

For the general valuation model with $\tau > 1$, we guess the recursive solution to Equation (10) is

as follows:

$$V(\tau)_t = Z(\tau)_t \ B_t \left[e^{A(\tau)+B(\tau)'x_t+x_t'G(\tau)x_t} - e^{C(\tau)+D(\tau)'x_t+x_t'G(\tau)x_t} \right] \tag{14}$$

where $Z(\tau)_t$ is the current riskless bond price and the coefficients $A(\tau)$ thru $G(\tau)$:

$$
\begin{aligned}
A(\tau) &= A(\tau-1) + [e2'+B(\tau-1)'](I-\phi)\bar{x} + [(I-\phi)\bar{x}]'G(\tau-1)[(I-\phi)\bar{x}] \\
&\quad + \tfrac{1}{2}\big(e2'+B(\tau-1)'+2[(I-\phi)\bar{x}]'G(\tau-1)\big)V\big(e2+B(\tau-1)+2G(\tau-1)'[(I-\phi)\bar{x}]\big) \\
&\quad - \tfrac{1}{2}\ln|I-2G(\tau-1)\omega|
\end{aligned}
$$

$$
\begin{aligned}
B(\tau)' &= [e2'+B(\tau-1)']\phi + 2[(I-\phi)\bar{x}]'G(\tau-1)\phi \\
&\quad + 2\big(e2'+B(\tau-1)'+2[(I-\phi)\bar{x}]'G(\tau-1)\big)V\,G(\tau-1)'\phi
\end{aligned}
$$

$$
\begin{aligned}
C(\tau) &= C(\tau-1) + [e2'+D(\tau-1)'](I-\phi)\bar{x} + [(I-\phi)\bar{x}]'G(\tau-1)[(I-\phi)\bar{x}] \\
&\quad + \tfrac{1}{2}\big(e2'+D(\tau-1)'+2[(I-\phi)\bar{x}]'G(\tau-1)\big)V\big(e2+D(\tau-1)+2G(\tau-1)'[(I-\phi)\bar{x}]\big) \\
&\quad - \tfrac{1}{2}\ln|I-2G(\tau-1)\omega|
\end{aligned}
$$

$$
\begin{aligned}
D(\tau)' &= [e2'+D(\tau-1)']\phi + 2[(I-\phi)\bar{x}]'G(\tau-1)\phi \\
&\quad + 2\big(e2'+D(\tau-1)'+2[(I-\phi)\bar{x}]'G(\tau-1)\big)V\,G(\tau-1)'\phi
\end{aligned}
$$

$$G(\tau) = -\Theta + \phi'G(\tau-1)\phi + 2[G(\tau-1)'\phi]'\,V\,[G(\tau-1)'\phi]$$

and the matrix $V = [\omega^{-1} - 2G(\tau-1)]^{-1}$.

To prove that this solution is indeed correct, we note that the current price of a zero coupon equity with a cash flow expected at time τ may be written as:

$$V(\tau)_t = E_t[e^{-\mu_t} V(\tau-1)_{t+1}]$$
$$= Z(\tau)_t \, e^{x_t'[-\Theta]x_t}$$
$$\times E_t\{B_t e^{e2'x_{t+1}} [e^{A(\tau-1)+B(\tau-1)'x_{t+1}+x_{t+1}'G(\tau-1)x_{t+1}} - e^{C(\tau-1)+D(\tau-1)'x_{t+1}+x_{t+1}'G(\tau-1)x_{t+1}}]\}$$

This result follows by the assumption that Equation (14) also determines the equity price with maturity date $\tau-1$. We also use the Expectation Hypothesis to show that current price of a zero coupon bond equals next period's price discounted with a one-period riskless rate $Z(\tau)_t = E_t[e^{-R_{f,t}} Z(\tau-1)_{t+1}]$. We provide a detailed discussion of this assumption in the next section. Next, we show that the first expectation, related to the return on book equity, may be simplified as follows. Set $H' = e2' + B(\tau-1)' + 2[(I-\phi)\bar{x} + \phi x_t]'G(\tau-1)$, then

$$E_t\{e^{x_t'[-\Theta]x_t} B_t \, e^{e2'x_{t+1}} [e^{A(\tau-1)+B(\tau-1)'x_{t+1}+x_{t+1}'G(\tau-1)x_{t+1}}]\}$$
$$= B_t e^{x_t'[-\Theta]x_t + A(\tau-1)}$$
$$\times e^{[e2'+B(\tau-1)'][(I-\phi)\bar{x}+\phi x_t]+[(I-\phi)\bar{x}+\phi x_t]'G(\tau-1)[(I-\phi)\bar{x}+\phi x_t]}$$
$$\times E_t[e^{H'\varepsilon_{t+1}+\varepsilon_{t+1}'G(\tau-1)\varepsilon_{t+1}}]$$

Since ε is a multivariate normal random variable, the expected value of a linear plus a quadratic exponential form is straight forward to compute:

$$E[e^{H'\varepsilon+\varepsilon'G\varepsilon}] = \int[e^{H'\varepsilon+\varepsilon'G\varepsilon}]\frac{e^{-(\frac{1}{2})\varepsilon'\omega^{-1}\varepsilon}}{(\sqrt{2\pi})^k |\omega|^{\frac{1}{2}}} d\varepsilon = \int\frac{e^{-(\frac{1}{2})\{\varepsilon'[\omega^{-1}-2G]\varepsilon-2H'\varepsilon\}}}{(\sqrt{2\pi})^k |\omega|^{\frac{1}{2}}} d\varepsilon$$

Define the mean vector $m' = H'V$, and the matrix $V^{-1} = \omega^{-1} - 2G$; by completing the square in the exponent we obtain the following solution:[7]

---

[7] Equation (15) is analogous to Lemma I.1 in Ang and Liu (2004).

$$E\left[e^{-(H'\varepsilon+\varepsilon'\Omega\varepsilon)}\right] = \frac{e^{-(\frac{1}{2})m'V^{-1}m}|V|^{\frac{1}{2}}}{|\omega|^{\frac{1}{2}}}\left[\int \frac{e^{-(\frac{1}{2})(\varepsilon-m)'V^{-1}(\varepsilon-m)}}{\left(\sqrt{2\pi}\right)^{k}|V|^{\frac{1}{2}}}\,d\varepsilon\right]$$

$$= \frac{e^{(\frac{1}{2})H'(\omega^{-1}-2G(\tau-1)\,)^{-1}H}}{|I-2G(\tau-1)\,\omega|^{\frac{1}{2}}}. \tag{15}$$

The second equality follows from the fact that the integral inside the square brackets is the total probability of a normal variable with mean vector m and variance matrix V, hence it must add up to 1. Therefore,

$$E_t\{e^{x_t'[-\Theta]x_t}B_t\,e^{e2'x_{t+1}}[e^{A(\tau-1)+B(\tau-1)'x_{t+1}+x_{t+1}'G(\tau-1)x_{t+1}}]\}$$

$$= \exp\left\{\begin{array}{l} A(\tau-1)+[e2'+B(\tau-1)'](I-\phi)\bar{x}+[(I-\phi)\bar{x}]'G(\tau-1)[(I-\phi)\bar{x}] \\ +\frac{1}{2}(e2'+B(\tau-1)'+2[(I-\phi)\bar{x}]'G(\tau-1))V(e2+B(\tau-1)+2\,G(\tau-1)'\,[(I-\phi)\bar{x}]) \\ -\frac{1}{2}\ell\mathrm{n}|I-2G(\tau-1)\,\omega| \end{array}\right\}$$

$$\times\exp\left\{\begin{array}{l}[e2'\phi+B(\tau-1)'\phi+2[(I-\phi)\bar{x}]'G(\tau-1)\phi \\ +2(e2'+B(\tau-1)'+2[(I-\phi)\bar{x}]'G(\tau-1))VG(\tau-1)'\phi]x_t\end{array}\right\}$$

$$\times\exp\{x_t'[-\Theta+\phi'G(\tau-1)\phi+2\phi'G(\tau-1)V\,G(\tau-1)'\phi]x_t\}$$

Matching coefficients separately on the constant, the row vector multiplying x_t, and the matrix for the quadratic form with the coefficients in $e^{A(\tau)+B(\tau)'x_t+x_t'G(\tau)x_t}$ shows that Equations (14A), (14B) and (14G) hold. A similar exercise may be use to show that the coefficients in the exponential $e^{C(\tau)+D(\tau)'x_t+x_t'G(\tau)x_t}$ obey the recursions in Equations (14C) and (14D), and the proof is complete.

A.3 Cost of Capital Term Structure

The justification in Equation (14) for the discount factor $e^{-\tau y(\tau)_t}$ is two-fold: First, the Expectation Hypothesis implies that $Z(\tau)_t = E_t[e^{-R_{f,t}} Z(\tau-1)_{t+1}] = e^{-\tau y(\tau)_t}$.[8] The second reason is more pragmatic: a VAR model requires a very large number of unknown parameters that may be difficult to estimate with any precision. For example, a ten variable VAR system requires more than two hundred parameters (e.g., Ang and Liu, 2004). Assuming independence between interest rates and the state variable x_t we reduce the size of the VAR, and by default, the number of unknown parameters. As an additional advantage, we are able to separate the cost of capital into a riskless discount rate and a cash flow risk premium component.

By setting Eq. (9A) equal to (14) we find a closed form solution for the cost of capital implied by the single period conditional CAPM and the VAR model for the state variables. We repeat these equations below for convenience.

$$V(\tau)_t = B_t \{ e^{-\tau \rho(\tau)_t} [e^{A(\tau)^*+B(\tau)^{*'}x_t} - e^{C(\tau)^*+D(\tau)^{*'}x_t}]$$

(9)

$$V(\tau)_t = Z(\tau)_t B_t [e^{A(\tau)+B(\tau)'x_t+x_t'G(\tau)x_t} - e^{C(\tau)+D(\tau)'x_t+x_t'G(\tau)x_t}]$$

(14)

The solution for the cost of capital, required to discount a single cash flow with time to maturity τ, is:

$$\rho(\tau)_t = y(\tau)_t + \frac{1}{\tau} \left\{ \ell n[\frac{e^{A(\tau)^*+B(\tau)^{*'}x_t} - e^{C(\tau)^*+D(\tau)^{*'}x_t}}{e^{A(\tau)+B(\tau)'x_t+x_t'G(\tau)x_t} - e^{C(\tau)+D(\tau)'x_t+x_t'G(\tau)x_t}}] \right\}$$

This equation shows that the discount rate consists of a riskless rate plus a risk premium commensurate with the cash flow risk. Both components depend on the term to maturity.

---

[8] The literature related to the Expectation Hypothesis (EH) is vast and ever evolving; Singleton (2006) and Sangvinatsos (2010) provide reviews of the theory and empirical evidence. Sangvinatsos and Wachter (2005) show that even though the empirical evidence against the EH is overwhelming, for long term investors this evidence is irrelevant.

APPENDIX B

The purpose of Appendix B is to provide details of the Conditional Beta Model.

Modern asset pricing literature suggests that a firm's beta should be related to firm characteristics such as firm size and book-to-market ratio; in addition, there is empirical evidence to show that for insurance firms beta varies over the business cycle (e.g., Barinov, Xu, and Pottier, 2018). We follow the empirical literature on the conditional CAPM (e.g., Cosemans *et al.*, 2015, and Barinov, Xu, and Pottier, 2018) and model time-varying beta as a linear function of five general business conditions variables: Dividend Yield (*DIV*), Default spread (*DEF*), U.S. Treasury bill rate ($R_{f,t}$), the term spread (*TERM*) and), and market volatility (Mkt Volatility). We also add to the regression model two firm-specific variables: firm size (*SIZE*), the book-to-market ratio (*BM*) and the lagged beta ($\beta_{i,t-1}$). The rationale for including these three variables comes from the asset pricing literature. The equilibrium model by Gomes, Kogan, and Zhang (2003), shows that the risk of the firm's existing assets is related to the book-to-market ratio; they also show that firm size is related to the impact of growth options on systematic risk. Thus, our linear model for beta is

$$\beta^*_{it|t-1} = \delta_{0,i} + \delta_1 DEF_{t-1} + \delta_2 DIV_{t-1} + \delta_3 R_{f,t-1} + \delta_4 TERM_{t-1} + \delta_5 Mkt\ Volatility_{t-1}$$

$$+\gamma_1 Size_{i,t-1} + \gamma_2 BM_{i,t-1} + \gamma_3 \beta_{i,t-1} \tag{14}$$

Consistent with the literature (Cosemans *et al.*, 2015), we assume that the relationship between beta, firm size, book-to-market ratio and lagged beta is constant across firms. The use of cross-sectional data on firm characteristics will lead to more efficient estimates of the parameter vector $\gamma' = (\gamma_1, \gamma_2, \gamma_3)$. On the other hand, the relationship between beta and the macro variables $\delta_i' = (\delta_{0,i}, \delta_{1,i}, \delta_{2,i}, \delta_{3,i}, \delta_{4,i}, \delta_{5,i})$ is allowed to vary across firms; thus, it has the

potential to capture unobserved heterogeneity in systematic risk. The intercept term $\delta_{0,i}$ is also firm specific to capture effects of unobserved variables that may differ across firms (but constant over time).

To make this model operational, let $R_{i,t} - R_{f,t}$ be the monthly excess return for portfolio i, and define $R_{m,t} - R_{f,t}$ as the corresponding monthly market portfolio excess return. We assume there are N insurance sector portfolios numbered by $i = 1, 2, \ldots, N$. The beta characterized by macro variables and firm specific fundamentals may be estimated from the linear regression:

$$R_{i,t} - R_{f,t} = \alpha^{*}_{i,\,t} + \beta^{*}_{it|\,t-1}[R_{m,t} - R_{f,t}] + \zeta_{it}$$

(15)

The idiosyncratic returns ζ_{it} are identically distributed with zero mean and variance σ^2_ζ. We assume also independence across time and across firms.

The combination of Equations (14) and (15) leads to cross terms that should capture the cyclical nature of insurance firms' beta. The macro variables times the excess market return include $(DEF_{s-1}, DIV_{s-1}, R_{f,s-1}, TERM_{s-1}, MktVolatility_{s-1})[R_{m,s} - R_{f,s}]$; the firm specific characteristics terms are $(SIZE_{i,s-1}, BM_{i,s-1}, \beta_{i,s-1})[R_{m,s} - R_{f,s}]$. As observed by Barinov, Xu, and Pottier (2018), higher betas in recessions follow from a combination of positive loadings on $(DEF_{s-1}, DIV_{s-1}, TERM_{s-1}, MktVolatility_{s-1})$ times $[R_{m,s} - R_{f,s}]$, and negative coefficient on the riskless rate times the market excess return $R_{f,\,s-1}[R_{m,s} - R_{f,s}]$. These signs are due to relatively high values of (*DEF, DIV, TERM, MktVolatility)* and low values of R_f. Similarly, we

expect a positive loading on $SIZE_{i,\,s-1}[R_{m,t}-R_{f,s}]$ and negative on $BM_{i,\,s-1}[R_{m,t}-R_{f,s}]$ because market equity is lower in recessions.

    This model requires estimation of ten unknown parameters for each portfolio i: nine slope coefficients plus the intercept term. Estimation may be easily obtained by means of a panel time-series cross sectional regression for each set period ending in month t. To show this, define the column vector of excess returns $\Re_i$ with s^{th} element $R_{i,s}-R_{f,s}$, $(s = t\,,\,t-1,\,t-2,\,\ldots\,,\,t-T+1)$. Define $\Pi 1$ as a matrix of corresponding macro variables with s^{th} row given by $(1,\,DEF_{s-1},$ $DIV_{s-1},\,R_{f,\,s-1},\,TERM_{s-1},\,MktVolatility_{s-1})[R_{m,\,s}-R_{f,\,s}]$, and let $\Pi 2_i$ be a matrix of macro variables and specific characteristics for portfolio i with s^{th} row given by $(\,SIZE_{i,\,s-1},\,BM_{i,\,s-1},\,\beta_{i,s-1})[\,R_{m,\,t}-R_{f,\,s}]$. Then the panel regression analog of model (15) is

$$
\begin{pmatrix} \Re_1 \\ \Re_2 \\ . \\ . \\ \Re_N \end{pmatrix} = \begin{pmatrix} \alpha_1^* \\ \alpha_2^* \\ . \\ . \\ \alpha_N^* \end{pmatrix} + \begin{bmatrix} \Pi 1 & 0 & 0 & . & 0 & \Pi 2_1 \\ 0 & \Pi 1 & 0 & . & 0 & \Pi 2_2 \\ 0 & & . & . & 0 & . \\ . & . & . & . & . & . \\ 0 & & & 0 & \Pi 1 & \Pi 2_N \end{bmatrix} \begin{pmatrix} \delta_1 \\ \delta_2 \\ . \\ . \\ \delta_N \\ \gamma \end{pmatrix} + \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ . \\ . \\ \zeta_N \end{pmatrix} \tag{16}
$$

where α_i^* is a column vector of constants corresponding to the intercept in Equation (15). Let $\hat\gamma$ and $\hat\delta_i$ be the ordinary least squares (OLS) estimators of the slope coefficients in the panel cross section regression, also define the (row) data vectors for the month prior to the current month t as

$$\pi 1'_{t-1}(1, DEF_{t-1}, DIV_{t-1}, R_{f,t-1}, TERM_{t-1}, MktVolatility_{t-1})$$

$$\pi 2'_{i,t-1} = (SIZE_{i,s-1}, BM_{i,s-1}, \beta_{i,s-1})$$

Then, the firm characteristic (FC) beta model estimator is

$$\hat\beta_{i,\,t-1}^{FC} = \pi 1'_{t-1}\hat\delta_i + \pi 2'_{i,t-1}\hat\gamma \tag{17}$$

Using standard arguments as in Fama and French (1997), this OLS beta estimator has mean $\beta^*_{it|\,t-1}$. It follows immediately from Equation (17) that its variance consists of three components

$$V[\hat{\beta}^{FC}_{i,\,t-1}] = \pi 1'_{t-1} V[\hat{\delta}_i]\,\pi 1_{t-1} + \pi 2'_{i,t-1} V[\hat{\gamma}] \pi 2_{i,\,t-1} + 2\,\pi 1'_{t-1} Cov[\hat{\delta}_i,\,\hat{\gamma}\,'] \pi 2_{i,\,t-1}\ .$$ The two

individual variances $V[\hat{\delta}_i]$, $V[\hat{\gamma}]$ and the $Cov[\hat{\delta}_i,\,\hat{\gamma}\,']$ terms may be obtained from the panel

data regression error variance $\hat{\sigma}^2_{\zeta}\left[\Pi'\Pi\right]^{-1}$, where Π is the data matrix on the right hand side of

equation (16), and $\hat{\sigma}^2_{\zeta}$ is the sample variance of the error terms in (15).

We follow the Bayesian methodology proposed by Fama and French (1997), and especially

Cosemans *et al.* (2015), and treat $\hat{\beta}^{FC}_{i,\,t-1} \sim N(\beta^*_{it|\,t-1},\ V[\hat{\beta}^{FC}_{i,\,t-1}])$ as the prior distribution.

Combining this information with the rolling window estimator, we obtain posterior market beta

for portfolio i and end of month t:

$$\hat{\beta}^*_{it|\,t} = w_{i,\,t}\hat{\beta}^{FC}_{i,\,t-1} + (1-w_{i,\,t})\hat{\beta}^{RW}_{i,\,t} \tag{18}$$

the corresponding shrinkage weight is $w_{i,t}$

$$w_{i,\,t} = \frac{V[\hat{\beta}^{RW}_{i,\,t}]}{V[\hat{\beta}^{FC}_{i,\,t-1}] + V[\hat{\beta}^{RW}_{i,\,t}]} \tag{19}$$

Equation (18) and (19) are standard results in Bayesian econometrics. Clearly, the posterior mean

$\hat{\beta}^*_{it|\,t}$ is a weighted average of the prior beta, conditional on firm characteristics, and the rolling

window beta. It can be shown that the weights are the reciprocals of the respective variances. If

the variance $V[\hat{\beta}^{FC}_{i,\,t-1}]$ is relatively small, then the prior information contained in the firm

characteristics plays a dominant role in determining the posterior beta. Conversely, precise

estimates from the rolling window beta, low variance $V[\hat{\beta}_{i,\,t}^{RW}]$, lead to a larger role for time series

information.