Thermodynamic Systems Theory Manual

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Contents

Chapter 1

Component Effects on Fluid

This chapter will describe the mathematical theory used to model the effect each component has on a fluid. The components of interest in this chapter are those that are common in gas-flow thermodynamic systems such as jet engines and electrical power plants. All models describe in this document assume adiabatic, isentropic conditions.

1.1 Background

A full survey of thermodynamics is beyond the scope of this document; however, the following relationships form the basis for all the adiabatic-isentropic relationships described in this document.

All equations in the *Thermodynamic Systems* code assume ideal gas conditions, which are achieved by assuming one dimensional flow. An ideal gas can be approximated by Equation 1.1 where ρ is density, R is the gas constant and T is the static temperature. The constant R_o is the ideal gas constant of 8.314 J/(kg·mole) and M is the molar mass of the fluid in units of grams per mole.

$$p = \rho R T \tag{1.1}$$

where

$$R = \frac{R_o}{M} \tag{1.2}$$

In addition, ideal, non-compressible flow is assumed, which implies the rule of continuity in Equation 1.3, where $\dot{m}$ represents the mass flow rate, u represents the flow velocity, and A represents the cross-sectional area of the flow-channel. The subscripts 1 and 2 imply the up-stream and down-stream conditions respectively.

$$\dot{m}_1 = \dot{m}_2 = \rho_1 u_1 A_1 = \rho_2 u_2 A_2 \tag{1.3}$$

In an adiabatic system the stagnation conditions can be estimated from the fluid Mach number via equations 1.4 and 1.5 where M represents the Mach number, γ represents the ratio of specific heats (c_p/c_v), P_o is stagnation pressure, P is static pressure, T is static temperature and T_o represents the stagnation temperature. The terms c_p and c_v represents the specific heats at constant pressure and volume respectively.

$$\frac{P_o}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/(\gamma-1)} \tag{1.4}$$

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2 \tag{1.5}$$

where;

$$M = \frac{u}{\sqrt{\gamma R T}} \tag{1.6}$$

Assuming that the ratio of specific heats (γ) is constant across a process where work is done, then we can assume a relationship between the stagnation conditions at the entrance and exit to the work process.

$$\frac{P_{o1}}{P_{o2}} = \left(\frac{T_{o1}}{T_{o2}}\right)^{\gamma/(\gamma-1)} \tag{1.7}$$

1.2 Diffuser/Nozzle

A diffuser slows the fluid velocity by increasing the flow area and a nozzle executes the inverse. According to Equation 1.3, assuming the fluid density does not change, which is consistent with non-compressible flow, an increase in the area of a piping system will necessitate a decrease in the fluid velocity. In theory, no work is executed in a nozzle or a diffuser, which implies that the stagnation conditions should be equal at the entrance and the exit to a diffuser or nozzle. However, real systems will always incur some thermodynamic losses, which will decrease the stagnation conditions across the nozzle or diffuser, which can be approximated through isentropic efficiencies (η) . The isentropic efficiency, inlet area and exit area can be considered as attributes of a diffuser. The models described in this section were derived from *Mechanics and Thermodynamics of Propulsion* by Hill and Peterson[1] and a Masters Thesis by Webb[2]. All models calculate exit conditions with an assumption that the inlet conditions are known.

1.2.1 Fluid Stagnation Pressure and Temperature

The stagnation pressure at the diffuser exit can be determined via Eq. 1.8 where η_d is the isentropic efficiency of the diffuser. The value of η_d can be replaced with η_n for a nozzle.

$$P_{o2} = P_{o1} \left(\eta_d \left[\frac{\gamma - 1}{2}\right] M^2 + 1\right)^{\gamma/(\gamma-1)} \tag{1.8}$$

The stagnation temperature is determined via Eq. 1.7

1.2.2 Fluid Static Pressure and Temperature

The static temperature (T) is calculated via Eq. 1.9 where the derivation of the velocity is described in Section 1.2.3.

$$T = T_o - \frac{u^2}{2c_p} \tag{1.9}$$

The static pressure is determine via Eq. 1.4 where the stagnation pressure (P_o) is determined via Eq. 1.8 and the Mach number (M) is determined through the method described in Section 1.2.3

1.2.3 Fluid Velocity and Mach Number

The fluid velocity (u) leaving the diffuser or nozzle is determined via Eq. 1.3 and the Mach number (M) is calculated using Eq. 1.6. The static temperature (T) used in Eq. 1.6 is calculated from 1.9. The calculation of velocity and Mach number assume that the flow is subsonic and non-compressible.

1.2.4 Fluid Density

The fluid density is determined with a-priori knowledge of the mass flow rate ($\dot{m}$) and Eq. 1.3. The fluid velocity u required in Eq. 1.3 is determined in Section 1.2.3.

1.3 Compressor

A compressor does work upon a system and increases the stagnation conditions (i.e. T_o, P_o), and thereby the fluid density. There are isentropic losses in a compressor, which are approximated by the term η_c. The isentropic efficiency and compression ratio (i.e $P_{cr} = P_{o2}/P_{o1}$) are considered as compressor attributes. The models described in this section were derived from *Mechanics and Thermodynamics of Propulsion* by Hill and Peterson[1] and a Masters Thesis by Webb[2]. All models calculate exit conditions with an assumption that the inlet conditions are known.

1.3.1 Fluid Stagnation Pressure and Temperature

The stagnation pressure is determined with knowledge of the inlet stagnation pressure and the compression ratio (P_{cr}) and is shown in Eq. 1.10.

$$P_{o2} = P_{o1} P_{cr} \tag{1.10}$$

The stagnation temperature at the compressor exit can also be expressed in terms of the inlet stagnation temperature and the compression ration as shown in Eq. 1.11

$$T_{o2} = T_{o1} \left[1 + \frac{1}{\eta_c} \left(p_{cr}^{\frac{\gamma-1}{\gamma}} \right) - 1 \right] \tag{1.11}$$

1.3.2 Fluid Static Pressure and Temperature

1.3.3 Fluid Velocity and Mach Number

1.3.4 Fluid Density

Bibliography

[1] Hill, P. and Peterson, C., *Machanics and Thermodynamics of Propulsion*, Addison-Wesley, Reading, MA, 1992.

[2] Webb, J., A., *Radioisotope Heated Air-breathing Engine Design for Flight Applications on Titan*, A M.S. Thesis at the Idaho State University, Dept. of Nuclear Engineering, 2009.