Thermodynamic Systems Theory Manual

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September 17, 2020

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Chapter 1

Component Effects on Fluid

This chapter will describe the mathematical theory used to model the effect each component has on a fluid. The components of interest in this chapter are those that are common in gas-flow thermodynamic systems such as jet engines and electrical power plants. All models describe in this document assume adiabatic, isentropic conditions.

1.1 Background

A full survey of thermodynamics is beyond the scope of this document; however, the following relationships form the basis for all the adiabatic-isentropic relationships described in this document.

All equations in the *Thermodynamic Systems* code assume ideal gas conditions, which are achieved by assuming one dimensional flow. An ideal gas can be approximated by Equation 1.1 where ρ is density, R is the gas constant and T is the static temperature. The constant R_o is the ideal gas constant of 8.314 J/(kg·mole) and M is the molar mass of the fluid in units of grams per mole.

$$p = \rho R T \tag{1.1}$$

where

$$R = \frac{R_o}{M} \tag{1.2}$$

In addition, ideal, non-compressible flow is assumed, which implies the rule of continuity in Equation 1.3, where $\dot{m}$ represents the mass flow rate, u represents the flow velocity, and A represents the cross-sectional area of the flow-channel. The subscripts 1 and 2 imply the up-stream and down-stream conditions respectively.

$$\dot{m}_1 = \dot{m}_2 = \rho_1 u_1 A_1 = \rho_2 u_2 A_2 \tag{1.3}$$

In an adiabatic system the stagnation conditions can be estimated from the fluid Mach number via equations 1.4 and 1.5 where M represents the Mach number, γ represents the ratio of specific heats (c_p/c_v), P_o is stagnation pressure, P is static pressure, T is static temperature and T_o represents the stagnation temperature. The terms c_p and c_v represents the specific heats at constant pressure and volume respectively.

$$\frac{P_o}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/(\gamma-1)} \tag{1.4}$$

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2 \tag{1.5}$$

where;

$$M = \frac{u}{\sqrt{\gamma R T}} \tag{1.6}$$

Assuming that the ratio of specific heats (γ) is constant across a process where work is done, then we can assume a relationship between the stagnation conditions at the entrance and exit to the work process.

$$\frac{P_{o1}}{P_{o2}} = \left(\frac{T_{o1}}{T_{o2}}\right)^{\gamma/(\gamma-1)} \tag{1.7}$$

1.2 Diffuser/Nozzle

A diffuser slows the fluid velocity by increasing the flow area and a nozzle executes the inverse. According to Equation 1.3, assuming the fluid density does not change, which is consistent with non-compressible flow, an increase in the area of a piping system will necessitate a decrease in the fluid velocity. In theory, no work is executed in a nozzle or a diffuser, which implies that the stagnation conditions should be equal at the entrance and the exit to a diffuser or nozzle. However, real systems will always incur some thermodynamic losses, which will decrease the stagnation conditions across the nozzle or diffuser, which can be approximated through isentropic efficiencies (η) . The isentropic efficiency, inlet area and exit area can be considered as attributes of a diffuser. The models described in this section were derived from *Mechanics and Thermodynamics of Propulsion* by Hill and Peterson[1] and a Masters Thesis by Webb[2]. All models calculate exit conditions with an assumption that the inlet conditions are known.

1.2.1 Fluid Stagnation Pressure and Temperature

The stagnation pressure at the diffuser exit can be determined via Eq. 1.8 where η_d is the isentropic efficiency of the diffuser. The value of η_d can be replaced with η_n for a nozzle.

$$P_{o2} = P_{o1} \left(\eta_d \left[\frac{\gamma - 1}{2}\right] M^2 + 1\right)^{\gamma/(\gamma-1)} \tag{1.8}$$

The stagnation temperature is determined via Eq. 1.7

1.2.2 Fluid Static Pressure and Temperature

The static temperature (T) is calculated via Eq. 1.9 where the derivation of the velocity is described in Section 1.2.3.

$$T = T_o - \frac{u^2}{2c_p} \tag{1.9}$$

The static pressure is determine via Eq. 1.4 where the stagnation pressure (P_o) is determined via Eq. 1.8 and the Mach number (M) is determined through the method described in Section 1.2.3

1.2.3 Fluid Velocity and Mach Number

The fluid velocity (u) leaving the diffuser or nozzle is determined via Eq. 1.3 and the Mach number (M) is calculated using Eq. 1.6. The static temperature (T) used in Eq. 1.6 is calculated from 1.9. The calculation of velocity and Mach number assume that the flow is subsonic and non-compressible.

1.2.4 Fluid Density

The fluid density is determined with a-priori knowledge of the mass flow rate ($\dot{m}$) and Eq. 1.3. The fluid velocity u required in Eq. 1.3 is determined in Section 1.2.3.

1.3 Compressor

A compressor does work upon a system and increases the stagnation conditions (i.e. T_o, P_o), and thereby the fluid density. There are isentropic losses in a compressor, which are approximated by the term η_c. The isentropic efficiency and compression ratio (i.e $P_{cr} = P_{o2}/P_{o1}$) are considered as compressor attributes. The models described in this section were derived from *Mechanics and Thermodynamics of Propulsion* by Hill and Peterson[1] and a Masters Thesis by Webb[2]. All models calculate exit conditions with an assumption that the inlet conditions are known.

1.3.1 Fluid Stagnation Pressure and Temperature

The stagnation pressure is determined with knowledge of the inlet stagnation pressure and the compression ratio (P_{cr}) and is shown in Eq. 1.10.

$$P_{o2} = P_{o1} P_{cr} \tag{1.10}$$

The stagnation temperature at the compressor exit can also be expressed in terms of the inlet stagnation temperature and the compression ration as shown in Eq. 1.11

$$T_{o2} = T_{o1} \left[1 + \frac{1}{\eta_c} \left(p_{cr}^{\frac{\gamma-1}{\gamma}} \right) - 1 \right] \tag{1.11}$$

1.3.2 Fluid Static Pressure and Temperature

The fluid static pressure and temperature are determined via Equations 1.4 and 1.5 with knowledge of the inlet stagnation conditions to determine he exit stagnation conditions using Equations 1.10 and 1.11 and the Mach number which is determined from Eq. 1.12

1.3.3 Fluid Velocity and Mach Number

The compressor exit Mach number cannot be directly calculated and instead must be iteratively solved for as a root for Equation 1.12 with knowledge of the inlet stagnation temperature, outlet stagnation temperature, and inlet mach number.

$$\frac{T_{o2}}{T_{o1}} = \left[\frac{1 + \gamma M_2^2}{1 + \gamma M_1^2} \right]^2 \left(\frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_1^2} \right) \tag{1.12}$$

The fluid velocity is then determined in accordance with Equation 1.13

$$u = M\sqrt{\gamma RT} \tag{1.13}$$

1.3.4 Fluid Density

The fluid density leaving the compressor is determined via Equation 1.14

$$\rho = \frac{\dot{m}}{uA} \tag{1.14}$$

1.3.5 Compressor Work

The compressor does work on the system in order to change the stagnation conditions. The work done by the compressor on the fluid is modeled via Eq. 1.15.

$$W_c = \dot{m}c_p\left(T_{o2} - T_{o1}\right) \tag{1.15}$$

1.4 HeatAddition

Typical thermodynamic systems have a combustion chamber or a heat exchanger where energy is added to the system. As with all other thermodynamic systems a heat addition unit is accompanied by an isentropic efficiency (η_{he}) where the subscript he refers to a heat exchanger.

1.4.1 Power Addition

The required power that must be supplied to the heat exchanger or combustion chamber is determined with knowledge of the heat to be added to the fluid $\dot{Q}_{he}$ via Eq. 1.16

$$\dot{Q}_t = \frac{\dot{Q}_{he}}{\eta_{he}} \tag{1.16}$$

For a thermodynamic propulsion systems the heat addition $\dot{Q}_{he}$ is driven by the energy required to drive a nozzle exit velocity $\dot{Q}_{KE}$ and also possibly the work that must be extracted by the compressor $\dot{W}_c$. The compressor work is described in equation 1.15, the energy addition to drive a nozzle exit velocity is described in Eq. 1.17 where V_e represents the nozzle exit velocity and V_∞ represents the free field fluid velocity outside of the engine.

$$\dot{Q}_{KE} = \frac{\dot{m}}{2}\left(V_e^2 - V_\infty^2\right) \tag{1.17}$$

$$\dot{Q}_t = \dot{Q}_{KE} + \dot{W}_c \tag{1.18}$$

1.4.2 Fluid Stagnation Pressure and Temperature

The stagnation temperature at the heat addition exit T_{o2} can be determined with knowledge of the heat addition $\dot{Q}_{he}$ and the specific heat of the fluid c_p as well as the mass flow rate $\dot{m}$ as shown in Eq. 1.19

$$T_{o2} = T_{o1} + \frac{\dot{Q}_{he}}{\dot{m}c_p} \tag{1.19}$$

The stagnation temperature is determined by solving Equation 1.20 with knowledge of the inlet conditions (M_1 and P_{o1}) and the exit Mach Number (M_2).

$$P_{o2} = P_{o1} \left(\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \right) \left(\frac{1 + \frac{\gamma - 1}{2} M_2^2}{1 + \frac{\gamma - 1}{2} M_1^2} \right)^{\frac{\gamma}{\gamma - 1}} \tag{1.20}$$

1.4.3 Fluid Static Pressure and Temperature

The static temperature and pressure can be determined with Equations 1.5 and 1.4.

1.4.4 Fluid Velocity and Mach Number

The fluid Mach number leaving the heat addition component is determine with knowledge of the inlet conditions (T_{o1} and M_1) as well as the exit Stagnation temperature (T_{o2}) and Equation 1.12. The equation is implicit and cannot be solved directly, so a root finder must be employed to solve this equation. The ThermSystems code uses the bisect method to determine the roots for M_2. The fluid velocity is determined by multiplying the Mach number by the speed of sound as determined in Eq. 1.6

1.4.5 Fluid Density

The fluid density leaving the heat addition component is determined via Eq. 1.15

1.5 Turbine

A turbine extracts energy from a fluid field, either to run an alternator that produces electricity or to supply mechanical energy to a compressor. As with all other thermodynamic systems a turbine is accompanied by an isentropic efficiency (η_t) where the subscript he refers to a heat exchanger.

1.5.1 Work Extraction

A turbine must extract a certain amount ($\dot{W}_t$) of work from the flow field to supply the mechanical energy needs of an alternator or a compressor. In order to ensure that the amount of work extracted is correct, we must divide the compressor work $\dot{W}_t$ by the turbine isentropic efficiency to understand the total work to be extracted as shown in Eq. 1.21. If the work is to be supplied to a compressor then $\dot{W}_t$ is equal to $\dot{W}_c$

$$\dot{W}_{total} = \frac{\dot{W}_t}{\eta_t} \tag{1.21}$$

1.5.2 Fluid Stagnation Temperature and Pressure

The stagnation temperature at the turbine exit can be determine via Eq. 1.22

$$T_{o2} = T_{o1} - \frac{\dot{W}_{total}}{\dot{m} c_p} \tag{1.22}$$

The stagnation temperature at the turbine exit can be detemrined via Eq. 1.23

$$P_{o2} = P_{o1}\left[1 - \frac{1}{\eta_t}\left(1 - \frac{T_{o2}}{T_{o1}}\right)\right]^{\frac{\gamma}{\gamma-1}} \tag{1.23}$$

1.5.3 Fluid StaticTemperature and Pressure

The fluid static temperature and pressure are determined from Equations 1.5 and 1.4.

1.5.4 Fluid Velocity and Mach Number

The fluid Mach number is determined from Equation 1.12

1.5.5 Fluid Density

The fluid density leaving the turbine is determined via Eq. 1.15

1.6 Propeller

A propeller extracts work from the atmosphere and transforms it into thrust. As with all other thermodynamic systems a turbine is accompanied by an isentropic efficiency (η_p) where the subscript he refers to a heat exchanger.

1.6.1 Work Extraction

The amount of work to be extracted from the fluid $\dot{W}$ is based on the amount of thrust to be produced. However, due to isentropic inefficiencies, the propeller must extract $\dot{W}_p$ Watts of power as defined by eq. 1.24.

$$\dot{W}_p = \frac{\dot{W}}{\eta_p} \tag{1.24}$$

1.6.2 Fluid Stagnation Temperature and Pressure

The stagnation temperature at the propeller exit T_{o2} is defined in accordance with Equation 1.25.

$$T_{o2} = T_{o1} + \frac{\dot{W}_p}{\dot{m}c_p} \tag{1.25}$$

The stagnation pressure is defined in Eq. 1.26

$$P_{o2} = P_{o1}\left]1 + \eta_p\left(\frac{T_{o2}}{T_{o1}} - 1\right)\right]^{\frac{\gamma}{\gamma-1}} \tag{1.26}$$

1.6.3 Fluid Static Temperature and Pressure

The static temperature and pressure are determined via Equations 1.5 and 1.4

1.6.4 Fluid Velocity and Mach Number

The fluid Mach number is determined from Equation 1.12

1.6.5 Fluid Density

The fluid density leaving the turbine is determined via Eq. 1.15

Bibliography

[1] Hill, P. and Peterson, C., *Machanics and Thermodynamics of Propulsion*, Addison-Wesley, Reading, MA, 1992.

[2] Webb, J., A., *Radioisotope Heated Air-breathing Engine Design for Flight Applications on Titan*, A M.S. Thesis at the Idaho State University, Dept. of Nuclear Engineering, 2009.