COP4533 Algorithms, Abstraction, and Design
Project Milestone 1

1 Group Members

Jonathan Williams

2 Member Roles

Jonathan Williams - Work through problem sets for all milestones. Transcribe solutions for submission on a pdf, and maintain the project github page with any updates.

3 Communication Methods

Since there is only one member there will not be a main form of communication. Progress will be tracked using the project Gantt chart and the Canvas calendar.

4 Project Gantt Chart

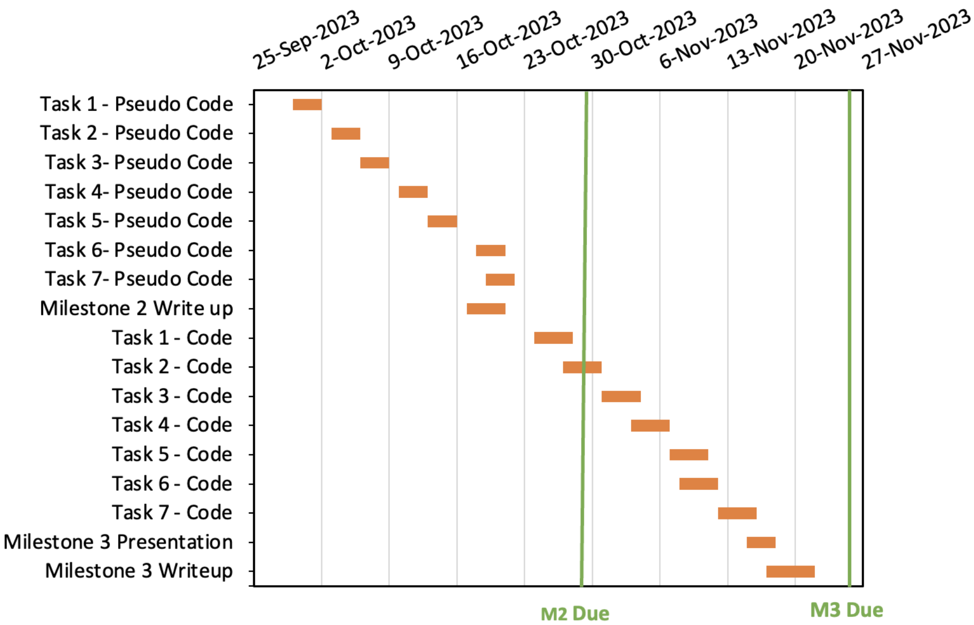

Figure 1: Proposed progress of future milestones.

5 GitHub Repository Link

https://github.com/Jon-Williams/COP4533Project

6 Problem 1

6.1 Solution

We begin with the following matrix, A, representing four stock prices over a five day period.

$$A = \begin{bmatrix} 12 & 1 & 5 & 3 & 16 \\ 4 & 4 & 13 & 4 & 9 \\ 6 & 8 & 6 & 1 & 2 \\ 14 & 3 & 4 & 8 & 10 \end{bmatrix}$$

First we calculated all possible single transactions, to see which single transaction had the maximum profit per stock.

Output \ Stock Index	1	2	3	4
1	(1, 1, 2, -11)	(2, 1, 2, 0)	(3, 1, 2, 2)	(4, 1, 2, -11)
2	(1, 1, 3, -7)	(2, 1, 3, 9)	(3, 1, 3, 0)	(4, 1, 3, -10)
3	(1, 1, 4, -9)	(2, 1, 4, 0)	(3, 1, 4, -5)	(4, 1, 4, -6)
4	(1, 1, 5, 4)	(2, 1, 5, 5)	(3, 1, 5, -4)	(4, 1, 5, -4)
5	(1, 2, 3, 4)	(2, 2, 3, 9)	(3, 2, 3, -2)	(4, 2, 3, 1)
6	(1, 2, 4, 2)	(2, 2, 4, 0)	(3, 2, 4, -7)	(4, 2, 4, 5)
7	(1, 2, 5, 15)	(2, 2, 5, 5)	(3, 2, 5, -6)	(4, 2, 5, 7)
8	(1, 3, 4, -2)	(2, 3, 4, -9)	(3, 3, 4, -5)	(4, 3, 4, 4)
9	(1, 3, 5, 11)	(2, 3, 5, -4)	(3, 3, 5, -4)	(4, 3, 5, 6)
10	(1, 4, 5, 13)	(2, 4, 5, 5)	(3, 4, 5, 1)	(4, 4, 5, 2)
Max Transaction Profit	(1, 2, 5, 15)	(2, 1, 3, 9)	(3, 1, 2, 2)	(4, 2, 5, 7)

The single transaction that will lead the maximum profit is (1, 2, 5, 15), buying stock 1 on day 2 and selling on day 5 with a profit of 15.

7 Problem 2

7.1 Solution

Next, we are given matrix A, with up to K transactions.

$$A = \begin{bmatrix} 25 & 30 & 15 & 40 & 50 \\ 10 & 20 & 30 & 25 & 5 \\ 30 & 45 & 35 & 10 & 15 \\ 5 & 50 & 35 & 25 & 45 \end{bmatrix}$$

The same as before, we calculated the potential profit for each of the stocks. Next, transactions that resulted in no profit were omitted, and finally the results were sorted in ascending profit.

Output \ Stock Index	1	2	3	4
1	(1, 1, 2, 5)	(2, 2, 4, 5)	(3, 1, 3, 5)	(4, 3, 5, 10)
2	(1, 2, 4, 10)	(2, 1, 2, 10)	(3, 4, 5, 5)	(4, 1, 4, 20)
3	(1, 4, 5, 10)	(2, 2, 3, 10)	(3, 1, 2, 15)	(4, 4, 5, 20)
4	(1, 1, 4, 15)	(2, 1, 4, 15)		(4, 1, 3, 30)
5	(1, 2, 5, 20)	(2, 1, 3, 20)		(4, 1, 5, 40)
6	(1, 1, 5, 25)			(4, 1, 2, 45)
7	(1, 3, 4, 25)			
8	(1, 3, 5, 35)			

For k=1, the problem simplifies to finding the transaction with the largest profit: (4, 1, 2, 45). However, for k=2, we cannot simply look at the next highest transaction, (4, 1, 5, 40), as it conflicts with our first choice. In situations where k>1, we must also consider the duration for which the stock was held. Transactions with a longer holding period could lead us to ignoring shorter transactions with a higher net profit. Therefore, when making a choice, we must carefully evaluate all other conflicting transactions at both the buy and sell stages. Since we have the freedom to trade multiple stocks in one day, we only need to examine transactions involving the same stock when addressing conflicts. One way to evaluate the trade offs could be to group complementary choices together.

Complementary choices for Stock 4 (k > 1)

$$\left| \begin{array}{c} (4,\ 1,\ 2,\ 45) \rightarrow (4,3,5,10) \\ (4,\ 1,\ 2,\ 45) \rightarrow (4,4,5,20) \end{array} \right|$$

Complementary choices for Stock 3 (k > 1)

$$\left| \begin{array}{c} (3,\ 1,\ 2,\ 15) \rightarrow (3,4,5,5) \\ (3,\ 1,\ 3,\ 5) \rightarrow (3,4,5,5) \end{array} \right|$$

Complementary choices for Stock 1 (k > 1)

$$\left| \begin{array}{c} (1,\ 1,\ 2,\ 45) \rightarrow (1,3,4,25) \\ (1,\ 1,\ 2,\ 45) \rightarrow (1,3,5,35) \\ (1,\ 1,\ 2,\ 45) \rightarrow (1,4,5,10) \end{array} \right|$$

Using the tables above we can easily determine the trade-off of a particular choice. For example, we can conclude that (1, 3, 5, 35) is the best choice after (4, 1, 2, 45) for k = 2, because (4, 1, 5, 40) can be ruled out, as it conflicts and has no benefit over (4, 1, 2, 45). If a sequence of choices within one stock has a higher net value than a current choice we know that it will be present as K increases.

K	Transactions	Profit
k_1	[(4, 1, 2)]	45
k_2	[(4, 1, 2) (1, 3, 5)]	80
k_3	[(4, 1, 2) (1, 3, 5) ((4, 4, 5, 20) OR (2, 1, 3, 20))]	100
k_4	[(4, 1, 2) (1, 3, 5) (4, 4, 5) (2, 1, 3)]	120
k_5	[(4, 1, 2) (1, 3, 5) (4, 4, 5) (2, 1, 3) (3, 1, 2)]	135
k_6	[(4, 1, 2) (1, 3, 5) (4, 4, 5) (2, 1, 3) (3, 1, 2) ((1, 1, 2) OR (3, 4, 5))]	140
k_7	[(4, 1, 2) (1, 3, 5) (4, 4, 5) (2, 1, 3) (3, 1, 2) (1, 1, 2) (3, 4, 5)]	145

8 Problem 3

8.1 Solution

$$A = \begin{bmatrix} 7 & 1 & 5 & 3 & 6 & 8 & 9 \\ 2 & 4 & 3 & 7 & 9 & 1 & 8 \\ 5 & 8 & 9 & 1 & 2 & 3 & 10 \\ 9 & 3 & 4 & 8 & 7 & 4 & 1 \\ 3 & 1 & 5 & 8 & 9 & 6 & 4 \end{bmatrix}$$

c = 2

We can calculate the maximum profit possible if a stock was purchased on a given day, given the constraint of not being able to sell until c + 1 days. We ignore any purchases done at $days \geq totaldays - (c+1)$

Stock Index \ Days	1	2	3	4
1	(1, 7, 2)	(2, 7, 8)	(3, 7, 4)	(4, 7, 6)
2	(1, 5, 7)	(2, 5, 5)	(3, 7, 5)	(4, 7, 1)
3	(1, 7, 5)	(2, 7, 2)	(3, 7, 1)	(4, 7, 9)
4	(1, 4, -1)	(2, 5, 4)	(3, 6, 0)	(4, 7, -7)
5	(1, 5, 6)	(2, 5, 8)	(3, 6, 1)	(4, 7, -4)
Maximum profit	7	8	5	9

Next, we can calculate the maximum profit after a stock purchase on a given day. Assuming we can sell a stock before cool down period c, having c only affect purchases.

Stock Index \ Days	1	2	3	4	5	6
1	2	8	4	6	3	1
2	7	5	6	2	-1	7
3	5	2	1	9	8	7
4	-1	5	4	-1	-3	-3
5	6	8	4	1	-3	-2
Maximum profit	7	8	6	9	8	7

Next we can think about how a sequence of transactions could be strung together given constraint c = 2. Looking at the maximum profit of a single transaction we can narrow down the maximum profit to be ≥ 9 from transaction (3, 4, 7, 9). Given constraint c, there is a very limited combinations of multiple transactions.

Stock Index \ Days	1	2	3	4	5	6
1	Buy	Sell	-	Buy	Sell	-
2	Buy	Sell	-	Buy	-	Sell
3	Buy	Sell	-	-	Buy	Sell
4	Buy	-	Sell	-	Buy	Sell
5	-	Buy	Sell	-	Buy	Sell

Finding the maximum profit of a transaction for each of the individual buy/sell timings shown above, we can start to combine transactions to achieve a value greater than 9.

Transaction Timing \ Days	1	2	3	4	5	6	Max Profit Transaction
1	Buy	Sell	-	-	-	-	(3, 1, 2, 3)
2	Buy	-	Sell	-	-	-	(5, 1, 3, 2)
3	-	Buy	Sell	-	-	-	(5, 2, 3, 4)
4	-	-	-	Buy	Sell	-	(1, 5, 6, 2)
5	-	-	-	Buy	-	Sell	(3, 5, 7, 8)
6	-	-	-	-	Buy	Sell	(2, 6, 7, 7), (3, 6, 7, 7)

Comparing the two previous tables we can determine the maximum profit from multiple transactions. The resulting profit, 11, is higher than the previously identified highest value of 9.

$$MaximumProfit = \big[(3,1,2,3)(3,5,7,8)\big]\ OR\ \big[(5,2,3,4)(2,6,7,7)\big]\ OR\ \big[(5,2,3,4)(3,6,7,7)\big]$$