TA: Jonathan Woo E-mail: jonwoo@math.ucla.edu

1 Discussion Section Expectations

1. Each week, I will post discussion section notes.

2. The week of each exam, we will do a review session during the section.

3. Throughout the course, an anonymous survey is available:

   https://forms.gle/3B5DwXTPmiBUrTma8.

   This survey is meant for feedback about the discussion section only (i.e., not for lectures). Feel free to submit as many responses as you wish, including positive or negative feedback. At the end of the course, an official evaluation of instruction will be available. I encourage you to at least fill out the official survey.

4. My email is: jonwoo@math.ucla.edu. To ensure that I see your email, please title your email with "Math 31B" (e.g., Math 31B: Question about Homework).

2 Discussion Outline

(1) Review of important prerequisites.

(2) Worksheet 1.

3 Notes

3.1 Fundamental Theorem of Calculus

Definition 3.1 (Definite integral)**.** The **definite integral** of a function f over $[a, b]$ is

$$\int_a^b f(x)dx = \lim_{\|P\|\to 0} R(f, P, C)$$

where $R(f, P, C)$ is the Riemann sum of f for some partition P of $[a, b]$ and sample points C of P. If this limit exists, we say f is **integrable** over $[a, b]$.

Remark 3.1. The definite integral is just the (signed) area between the graph of a function f and the x-axis.

In practice, we often do not use the above definition. Instead we use tools to compute integrals from easier ones. Some of the tools we have are:

Proposition 3.1 (Properties of Definite Integral)**.** If f and g are integrable over $[a, b]$, then we have

(1)

$$\int_a^b f(x) + g(x)\ dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

(2)
$$\int_a^b Cf(x)dx = C\int_a^b f(x)dx$$
for any constant C.

(3)
$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

(4)
$$\int_a^a f(x)dx = 0$$

(5)
$$\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$$
for any c (assuming f is also integrable on any interval specified).

(6) If $f(x) \leq g(x)$ on $[a, b]$, then
$$\int_a^b f(x)dx \leq \int_a^b g(x)dx$$

Remark 3.2 (Integrability of functions)**.** Recall that continuous functions are integrable. In this course, we will mostly consider continuous functions.

Computing definite integrals becomes much easier when we relate integration to differentiation. To do this we need:

Definition 3.2 (Antiderivative)**.** An **antiderivative** of a function f over some open interval (a, b) is any differentiable function F such that $F'(x) = f(x)$ for all $a < x < b$.

Antiderivatives of a function differ by a constant, so we can define the general antiderivative:

Definition 3.3 (Indefinite integral)**.** Given some antiderivative F of f, we say that the **indefinite integral** of f is $F(x) + C$, write
$$\int f(x)dx = F(x) + C.$$

Proposition 3.2 (Useful integration formulas)**.** Some useful antiderivatives are:

(1) Power rule: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ when $n \neq -1$.

(2) $\int \sin(x)dx = -\cos(x) + C$

(3) $\int \cos(x)dx = \sin(x) + C$

(4) $\int \sec^2(x)dx = \tan(x) + C$

(5) $\int \csc^2(x)dx = -\cot(x) + C$

(6) $\int \sec(x)\tan(x)dx = \sec(x) + C$

(7) $\int \csc(x)\cot(x)dx = -\csc(x) + C$

(8) Linearity: $\int cf(x)dx = c\int f(x)dx$ and $\int f(x) + g(x)\ dx = \int f(x)dx + \int g(x)dx$.

For continuous functions, antiderivatives become a powerful tool for computing definite integrals:

Theorem 3.1 (Fundamental Theorem of Calculus I)**.** If f is continuous on $[a, b]$ and F is an antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Theorem 3.2 (Fundamental Theorem of Calculus II)**.** If f is continuous on an open interval I with $a \in I$, then

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

3.2 Substitution Method

Theorem 3.3 (Substitution Method)**.** If F is an derivative of f and u is differentiable, then

$$\int f(u(x))u'(x)dx = \int f(u)du$$

and

$$\int_a^b f(u(x))u'(x)dx = \int_{u(a)}^{u(b)} f(u)du.$$