

von Neumann Double Commutant Theorem

Jon Bannon

Jireh Loreaux

October 2, 2025

0.1 Introduction

Definition 1 (Blah). The commutant of a set S of operators is the set of $T \in B(\mathcal{H})$ such that $\forall s \in S, Ts = sT$.

Definition 2. For $A \subseteq B(\mathcal{H})$, the *weak-operator closure* $\overline{A}^{\text{WOT}}$ is the closure of A in the weak operator topology on $B(\mathcal{H})$.

Definition 3. A $*$ -subalgebra $A \subseteq B(\mathcal{H})$ is *self-adjoint* if $T \in A \Rightarrow T^* \in A$.

0.2 Main Theorem

Theorem 4 (von Neumann 1929). *The weak operator closure of a self-adjoint $*$ -subalgebra of $B(\mathcal{H})$ equals its double commutant.*