

von Neumann Double Commutant Theorem

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[Blah] The commutant of a set S of operators is the set of $T \in B(\mathcal{H})$ such that $\forall s \in S, Ts = sT$.

For $A \subseteq B(\mathcal{H})$, the *weak-operator closure* $\overline{A}^{\text{WOT}}$ is the closure of A in the weak operator topology on $B(\mathcal{H})$.

A $*$ -subalgebra $A \subseteq B(\mathcal{H})$ is *self-adjoint* if $T \in A \Rightarrow T^* \in A$.

[von Neumann 1929] The weak operator closure of a self-adjoint $*$ -subalgebra of $B(\mathcal{H})$ equals its double commutant.