

# von Neumann Double Commutant Theorem

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## 0.1 Introduction

[Blah] The commutant of a set  $S$  of operators is the set of  $T \in B(\mathcal{H})$  such that  $\forall s \in S, Ts = sT$ .

For  $A \subseteq B(\mathcal{H})$ , the *weak-operator closure*  $\overline{A}^{\text{WOT}}$  is the closure of  $A$  in the weak operator topology on  $B(\mathcal{H})$ .

A  $*$ -subalgebra  $A \subseteq B(\mathcal{H})$  is *self-adjoint* if  $T \in A \Rightarrow T^* \in A$ .

## 0.2 Main Theorem

[von Neumann 1929] The weak operator closure of a self-adjoint  $*$ -subalgebra of  $B(\mathcal{H})$  equals its double commutant.