Algorithm 1 Reverse Mode With Hessian Accumulation

Input: *Tape*

$\bar{w} = [\bar{x_1}, \bar{x_2}, \bar{x_3}...\bar{x_{m-1}}] = 0$

$$\bar{h} = \begin{vmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_m} \\ \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_m} \\ \frac{\partial^2 f}{\partial x_3^2} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3 \partial x_3} & \cdots & \frac{\partial^2 f}{\partial x_3 \partial x_m} \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ \frac{\partial^2 f}{\partial x_m^2} & \frac{\partial^2 f}{\partial x_m \partial x_2} & \frac{\partial^2 f}{\partial x_m \partial x_3} & \cdots & \frac{\partial^2 f}{\partial x_m \partial x_m} \end{vmatrix} = 0$$

$S = \{w_m\}$

$\bar{w}[m] = 1$

for $i = m$ to 1 **do**

    $\frac{df}{dx_i} = \bar{w}[i]$

    $\bar{w}[i] = 0$

    $w_i \cup S_i$

    $S_i = \{\}$

    for $j = 1$ to i **do**

        $\bar{w}[j] += \frac{\partial f}{\partial x_i} \bar{w}[i]$

        for $k = 1$ to i **do**

            $temp = \bar{h}[i][k] \frac{\partial f}{\partial x_j} + \bar{h}[i][j] \frac{\partial f}{\partial x_k} + \bar{h}[i][i] \frac{\partial f}{\partial x_k} \frac{\partial f}{\partial x_j} + w \frac{\partial^2 f}{\partial x_j \partial x_k}$

            if $temp \neq 0$ **then**

                $\bar{h}[j][k] += temp$

                $j \in S_i$

                $k \in S_i$

            end if

        end for

    end for

end for

Output:

$\nabla f = \bar{w} = [\bar{x_1}, \bar{x_2}, \bar{x_3}...\bar{x_m}]$

$$\nabla f^2 = \bar{h} = \begin{vmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_m} \\ \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_m} \\ \frac{\partial^2 f}{\partial x_3^2} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3 \partial x_3} & \cdots & \frac{\partial^2 f}{\partial x_3 \partial x_m} \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ \frac{\partial^2 f}{\partial x_m^2} & \frac{\partial^2 f}{\partial x_m \partial x_2} & \frac{\partial^2 f}{\partial x_m \partial x_3} & \cdots & \frac{\partial^2 f}{\partial x_m \partial x_m} \end{vmatrix}$$