Simple Age-structured Operating Model

1. Stage I - Current structures of a simple operating model

1.1. Life history schedules

The length-at-age (l_a) is modeled following a von Bertalanffy growth model (E1.1) and the weight-at-age (w_a) is obtained from the weight-length relationship by the power function (E1.2). The maturity-at-age (m_a) is used to define the proportion of the mature female population at age and it follows a logistic function (E1.3). The sex ratio is assumed 1:1.

1.2. Stock-recruitment relationships (B-H based on mature female biomass)

The unfished equilibrium spawning biomass per recruit ($\emptyset_0$) is modeled as shown in E2.1 and the virgin recruitment (R_0) is 1,000,000. Expected annual recruitment of age 1 fish is computed from the Beverton-Holt stock-recruitment model (E2.2).

1.3. Initial condition

The number of fish in age group a in the initial year is modeled by considering fishing mortality in spawning biomass per recruit (E3.1-E3.5). The spawning biomass of fish in year 1 is calculated based on mature female biomass (E3.6).

1.4. Basic abundance dynamics

The abundance of fish at age 1 ($N_{1,y}$) corresponds to annual recruitment of age 1 (E2.2) with recruitment deviations (E4.1). The abundance of each subsequent age of each year is modeled assuming exponential decay (E4.2-E4.4). The spawning stock biomass of fish in each year is calculated based on mature female biomass (E4.5). The abundance and biomass of fish in each year are calculated as shown in E4.6-E4.7.

1.5. One fleet (logistic selectivity)

Time-invariant logistic selectivity function is assumed for the fishery (E5.1). The landings-at-age a during year y is determined using the Baranov catch equation (E5.2). The landings in weight is calculated as shown in E5.3.

1.6. One survey (logistic selectivity)

Time-invariant logistic selectivity function is assumed for a fishery-independent survey (E6.1). The fishery-independent survey index (I_y) is scaled to the mean I_y over time (E6.2-E6.4).

1.7. Time series of Fmult

The fully selected fishing mortality rate in each year is modeled with linear increase of f_y and its lognormal deviates (E7.1). Currently, the first year of f_y is defined as 0.01 and the last year of f_y is defined as 0.4.

1.8. Observed data

Observed landings are modeled with a lognormal observation error (E8.1). Proportion-at-age data for landings are simulated using random draws from multinomial distributions with a sample size of 200 (E8.2-E8.3).

Time series of relative abundance (in numbers) of survey is modeled with a lognormal observation error (E8.4). Proportion-at-age data for surveys are simulated using random draws from multinomial distributions with a sample size of 200 (E8.5-E8.6)

1.9. Output data

The general flow of operations in the simple age-structured operating model is presented in Figure 1. The output data include simulated data from the population, such as spawning stock biomass (mt) per year, abundance (in number) per year, biomass (mt) per year, abundance-at-age (in number) per year, landings-at-age (in number) per year, landings (×1000 in number and mt) per year.

The output data also include true reference points calculated from the simulated data, such as maximum sustainable yield (MSY), fishing rate at MSY (F_{MSY}), spawning stock biomass at MSY (SSB_{MSY}), equilibrium recruitment at MSY (R_{MSY}), biomass (both male and female) at MSY (B_{MSY}), exploitation rate at MSY (E_{MSY}), spawners per recruit at MSY (spr_{MSY}), and spawning potential ratio at MSY (SPR_{MSY}) which is virgin spawners per recruit divided by spr_{MSY}. The fishing rate, along with equilibrium landings at fishing rate and equilibrium SSB at fishing rate are also saved as output data. The Fratio (F/F_{MSY}) and SSBratio (SSB/SSB_{MSY}) are also saved as output data.

The output data from the observation model include observed landings (1000 fish or mt) per year, observed proportion-at-age for landings per year, observed survey index (scaled) per year, and observed proportion-at-age for survey per year.

Table 1. Description and current values for index variables, structure parameters, state variables, derived variables, and stochastic deviation used in the operating model. The phase of estimated means the parameter is not fixed but estimable in the estimation process.

Symbols	Description	Current Value	R output
Index variables			
y	Years	$\{1,2,\ldots,Y\}$ and $Y = 30$	sim1$yr
a	Ages	$\{1,2,\ldots A\}$ and $A = 12+$	par.sim1$ages
Structural parameters			
l_∞	Asymptotic average length (mm)	800	par.sim1$Linf
K	Growth coefficient (year^{-1})	0.18	par.sim1$K
t_0	Mean length-at-age at zero age (year)	-1.36	par.sim1$a0
θ_1	Length-weight coefficient	2.5e^{-8}	par.sim1$a.lw
θ_2	Length-weight exponent	3	par.sim1$b.lw
θ_3	Slope of maturity ogive	3	par.sim1$slope.mat
a_{50}	Age at 50% maturity	2.25	par.sim1$A50.mat
M_a	Natural mortality rate-at-age (year^{-1})[a]	0.2	par.sim1$M.age
r_a	Proportion of female-at-age	0.5	par.sim1$proportion.female
R_0	Virgin recruitment	1,000,000	par.sim1$R0
h	Steepness	0.75	par.sim1$h
x_1	Slope of selectivity for landings	1	par.sim1$slope.sel
x_2	Age-at-50% selection for landings	2	par.sim1$A50.sel
x_3	Slope of selectivity for survey	2	par.sim1$slope.sel.survey
x_4	Age-at-50% selection for survey	1.5	par.sim1$A50.sel.survey
f_y	Shape of the fully selected fishing mortality rate in year y	Linear increase with $f_1 = 0.01$ and $f_Y = 0.39$	
φ_F	Annual sample size for age composition of landings	200	par.sim1$n.L
φ_I	Annual sample size for age composition of survey	200	par.sim1$n.survey
State variables			
R_y	Annual recruitment in year y		sim1$N.age[,1]
SSB_y	Spawning biomass in year y (mt)		sim1$SSB
$N_{a,y}$	Abundance-at-age a in year y		sim1$N.age

Table 1. Continued.

Symbols	Description	Current Value	R output
A_y	Abundance in year y		sim1$abundance
B_y	Biomass in year y		sim1$biomass.mt
$L_{a,y}$	Landings-at-age a in year y in numbers		sim1$L.age
L_y	Landings in year y (mt)		sim1$L.mt
$I_{a,y}$	Survey abundance-at-age a in year y		survey.sim1.age
I_y	Survey abundance (sum across ages) in year y		survey.sim1
L'_y	Observed landings in year y (mt) with noise		dat.input$L.obs
$P_{L_{a,y}}$	Proportion-at-age a in year y for landings		
$P'_{L_{a,y}}$	Observed proportion-at-age a in year y for landings		dat.input$L.age.obs
I'_y	Observed survey abundance in year y with noise		dat.input$survey.obs
$P_{I_{a,y}}$	Proportion-at-age a in year y for survey		
$P'_{I_{a,y}}$	Observed proportion-at-age a in year y for survey		dat.input$survey.age.obs
Derived variables			
l_a	Length-at-age a (mm)		par.sim1$len
w_a	Weight-at-age a (mt)		par.sim1$W.mt
m_a	Maturity-at-age a		par.sim1$mat.age
$\emptyset_0$	Unfished spawning biomass per recruit		par.sim1$phi.0
$Z_{a,y}$	Total mortality rate-at-age a in year y		
$Fmult_y$	Fully selected fishing mortality rate in year y ($year^{-1}$)		sim1$F
S_{F_a}	Selectivity-at-age a for landings		par.sim1$selex
S_{I_a}	Selectivity-at-age a for survey		par.sim1$selex.survey
$\emptyset_F$	Spawning biomass per recruit given F		
R_{eq}	Equilibrium recruitment		

q	Catchability coefficient for survey		dat.sim1$q

Table 1. Continued.

Symbols	**Description**	**Current Value**	**R output**
Stochastic deviation			
Process error			
σ_R	Standard deviation of log recruitment	0.2	par.sim1$logR.sd
$Rdev_y$	Recruitment deviates in year y	$Rdev_y \sim N(0, {\sigma_R}^2)$	par.sim1$logR.resid
σ_f	Standard deviation of log Fmult	0.2	
$fdev_y$	Fully selected fishing mortality deviates in year y	$fdev_y \sim N(0, {\sigma_f}^2)$	
Observation error			
CV_L	CV of landings	0.005	par.sim1$cv.L
ε_{1_y}	Landings deviates in year y	$\varepsilon_{1_y} \sim N\left(0, \log\left(1 + {CV_L}^2\right)\right)$	
CV_I	CV of survey	0.1	par.sim1$cv.survey
ε_{2_y}	Survey abundance deviates in year y	$\varepsilon_{2_y} \sim N\left(0, \log\left(1 + {CV_I}^2\right)\right)$	

[a] Currently assumed constant across years.

[b] Input CV of landings for estimation model: dat.input$cv.L. Input CV of survey for estimation model: dat.input$cv.survey.

Table 2. The operating model for generating age-structured population dynamics, indices of relative abundance, and age composition data.

1. Life history schedules	
E1.1	$l_a = l_\infty \times (1 - \exp^{(-K(a-t_0))})$
E1.2	$w_a = \theta_1 \times {l_a}^{\theta_2}/1000$
E1.3	$m_a = \frac{1}{1+exp^{(-\theta_3(a-a_{50}))}}$
2. Stock-recruitment relationships	
E2.1	$\emptyset_0 = r_1 m_1 w_1 + \sum_{a=2}^{A-1} exp^{-M_{a-1}} r_a m_a w_a + \frac{\exp^{-M_{A-1}} r_A m_A w_A}{1-exp^{-M_A}}$
E2.2	$R_{y+1} = \frac{0.8 R_0 h SSB_y}{0.2 R_0 \emptyset_0 (1-h) + SSB_y (h-0.2)}$
3. Initial condition	
E3.1	$Z_{a,1} = Fmult_1 S_{F_a} + M_a$
E3.2	$\emptyset_{F_a} = \begin{cases} 1 & a = 1 \\ \emptyset_{F_{a-1}} exp^{-Z_{a-1,1}} & 1 < a < A \\ \frac{\emptyset_{F_{A-1}} exp^{-Z_{A-1,1}}}{1-exp^{-Z_{A,1}}} & a = A \end{cases}$
E3.3	$\emptyset_F = \sum_{a=1}^{A} \emptyset_{F_a} r_a m_a w_a$
E3.4	$R_{eq} = \frac{R_0(4h\emptyset_F - (1-h)\emptyset_0)}{(5h-1)\emptyset_F}$
E3.5	$N_{a,1} = R_{eq} \emptyset_{F_a}$
E3.6	$SSB_1 = \sum_{a=1}^{A} N_{a,1} r_a m_a w_a$
4. Basic abundance dynamics	
E4.1	$N_{1,y} = R_y exp^{Rdev_y}$
E4.2	$Z_{a,y} = Fmult_y S_{F_a} + M_a$
E4.3	$N_{a+1,y+1} = N_{a,y} exp^{-Z_{a,y}} \quad where\ a < A-1$
E4.4	$N_{A,y+1} = N_{A-1,y} exp^{-Z_{A-1,y}} + N_{A,y} \exp^{-Z_{A,y}}$

Table 2. Continued.

E4.5	$SSB_y = \sum_{a=1}^{A} N_{a,y} r_a m_a w_a$
E4.6	$A_y = \sum_{a=1}^{A} N_{a,y}$
E4.7	$B_y = \sum_{a=1}^{A} N_{a,y} w_a$
5. One fleet	
E5.1	$S_{F_a} = \frac{1}{1+exp^{-x_1(a-x_2)}}$
E5.2	$L_{a,y} = \frac{Fmult_y S_{F_a}}{Z_{a,y}} N_{a,y}(1 - exp^{-Z_{a,y}})$
E5.3	$L_{W_y} = \sum_{a=1}^{A} L_{a,y} w_a$
6. One Survey	
E6.1	$S_{I_a} = \frac{1}{1+exp^{-x_3(a-x_4)}}$
E6.2	$q = \frac{1}{mean(\sum_{a=1}^{A} N_{a,y} S_{I_a})}$
E6.3	$I_{a,y} = N_{a,y} S_{I_a}$
E6.4	$I_y = q \sum_{a=1}^{A} I_{a,y}$
7. Time series of F	
E7.1	$Fmult_y = f_y exp^{fdev_y}$
8. Observed data	
E8.1	$L'_{W_y} = L_{W_y} exp^{\varepsilon_{1y}}$
E8.2	$P_{L_{a,y}} = L_{a,y} / \sum_{a=1}^{A} L_{a,y}$
E8.3	$C_{L_{a,y}} \sim Multinomial(\varphi_F, P_{L_{a,y}})$
E8.4	$I'_y = I_y exp^{\varepsilon_{2y}}$
E8.5	$P_{I_{a,y}} = I_{a,y} / \sum_{a=1}^{A} I_{a,y}$
E8.6	$C_{I_{a,y}} \sim Multinomial(\varphi_I, P_{I_{a,y}})$

Figure 1. The general flow of operations in the simple age-structured operating model.