1. Embedding binary tree on 2d-mesh: Prove that a binary tree of depth k > 4 does not have a load-1, dilation-1, congestion-free embedding on a 2d-mesh. For this, show by induction that $2^{k+1}-1>2k^2+2k+1$ for k>4

Solution:

Claim: $2^{k+1}-1>2k^2+2k+1$ for k>4

Basis: n=0,1,..,5: For k = 5, $2k^2+2k+1$ =61<63 = : $2^{k+1}-1$, hence $P_0$is true.

Hypothesis: Suppose P_k: $2^{k+1}-1>2k^2+2k+1$ is true, for some K>=5.

Induction: To prove: P_{k+1}: $2^{k+2}-1>2(k+1)^2+2(k+1)+1$

L.H.S = $2(k+1)^2+2(k+1)+1$

$=(2k^2+4k+2)+2k+2+1$

$=2k^2+2k+1+(4k+1)$

$<2(2k^2+2k+1) \quad \because k>4 \rightarrow k>0, k>1 \rightarrow k(k-1)>0 \rightarrow \ 2k^2+2k+1>(4k+1)$

<$2(2^{k+1}-1)$ as P_k *is true*

$= \ 2^{k+2}-2$

$<2^{k+2}-1$ = R.H.S

Hence P_{k+1} is true .

2. Row-Column-based Matrix Multiplication on Local Memory Machines: Calculate the parallel time complexity of this algorithm for multiplying two nxn matrices using p processors con_gured in a ring C_p , showing the computation and communication phases calculations. Assume row-bands of _rst matrix and column-bands of second matrix are already distributed among processors.

Algorithm:

Two nxn matrices A and B are given, with elements aij, bij and their multiplication is calculated in C.

Row band Ri contains n/p rows, namely rows from (i-1)n/p → i(n/p)-1 , where i = 0,..p-1

Column band Cj contains n/p columns, columns from (j-1)n/p →j(n/p)-1 , where j = 0,..p-1

There are p processors, each denoted by Pi, i = 0,..p-1

Analysis:

Computational time:

n*n/p chunk of A multiplied by n*n/p chunk of B = n^3/p^2

Tcomputation = O(p*n^3/p^2) = O(n^3/p), because there are p steps of multiplications

Communication time: To send/receive message of length, m = $\frac{n^2}{p}$.

The process continues for p-1 times, hence

Tcommunication = (p-1)(t s+n^2/p*tw) = O(n^2)

Total Time = Tcomp + Tcomm = O(n^3/p) + O(n^2)